Hiking through glassy phases: physics beyond aging

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Experiments performed on a wide range of glassy materials display many interesting phenomena, such as aging behavior. In recent years, a large body of experiments probed this nonequilibrium glassy dynamics through elaborate protocols, in which external parameters are shifted, or cycled in the course of the experiment. We review here these protocols, as well as experimental and numerical results. Then, we critically discuss various theoretical approaches put forward in this context. Emphasis is put more on the generality of the phenomena than on a specific system. Experiments are also suggested.

I. INTRODUCTION

In this summer school, we were given many examples of glassy systems, glassy dynamics, and glass transitions, even though a proper definition of the word 'glassy' was not really provided. However, all glassy materials share the property that their relaxation times are extremely large compared to the time scale of a typical experiment, at least in a part of their phase diagram. For practical purposes, they are thus out of equilibrium, meaning that in principle the whole sample history is relevant to a description of their physical properties. This paper is dedicated to the study of some specific histories applied to various glassy materials.

As physicists, we want to study the simplest histories that allow for an understanding of all the relevant mechanisms at work. Hopefully, an understanding of simple protocols will also allow for prediction or calculation of the behavior resulting from increasingly elaborate procedures.

Over the last decades, the experiment most often performed has been the *simple aging* experiment, see Fig. 1. The system is quenched from a non-glassy part of the phase diagram, E, into the glassy phase, A. The system relaxation time is so large that all its physical properties continue to evolve slowly with time (aging). This phenomenon has been known for a long time in the field of structural glasses [1] before its rediscovery in the field of spin glasses [2]. Interestingly, aging is observed in a still increasing number of experimental systems such as soft materials (like pastes [3], colloidal suspensions [4], or clays [5]), dipolar glasses [6], disordered ferromagnets [7] and ferroelectrics [8], granular matter [9], superconductors [10], etc. Loss of stationarity is best illustrated by the study of two-time quantities. One typically computes the correlation between times t and w, $C(t, w)$, or the response of the system at time t to a perturbation applied at w, or equivalently the time evolution of
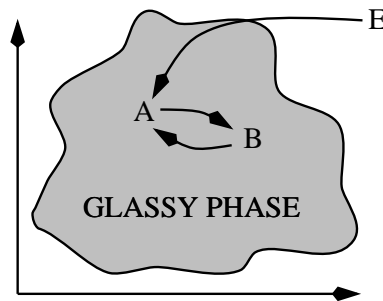

FIG. 1: Schematic view of the phase diagram of a glassy material. The relaxation time of the system in the shaded area is too large for the system to equilibrate on an experimental time scale. The arrows depict simple aging ($E \to A$), shifts ($E \to A \to B$), or cycling ($E \to A \to B \to A$) experiments. We do not label the axis, since the paper deals with an Anderson insulator, a colloidal suspension, a spin glass model, and a ferromagnet at criticality.

a susceptibility at frequency ω, $\chi(\omega, t)$. In most common experimental regimes, two-time quantities scale as $C(t, w) \sim \mathcal{F}_C(t/w)$, $\chi(\omega, t) \sim \mathcal{F}_\chi(\omega t)$, where $\mathcal{F}_*(x)$ denotes a scaling function. This scaling indicates that after a time t the only relevant time scale in the system is the time t itself.

In order to probe the dynamics of the glassy phase in greater detail, more elaborate and systematic experimental protocols have been performed [11], in which some external parameters are shifted, or cycled, during the experiment, see Fig. 1. Such experiments reveal spectacular new phenomena, such as rejuvenation and memory effects. Such new effects must be accounted for by any theory of aging, possibly allowing for discrimination between different theoretical approaches to glasses and aging phenomena. In addition, more detailed experiments may allow for discrimination between different families of glassy systems, and thus may help theoreticians refine their description of specific glassy systems. A large number of recent experimental papers are dedicated to such experiments, on a wide variety of systems, making the subject a very active one. By contrast, these protocols are barely mentioned in the classic theoretical review in the field [12] and it is one of this paper's purpose to fill this little gap.

In the first part of the paper we review briefly the basic experimental facts. Next we discuss two mean-field theoretic approaches to the problem. We then formulate the rudiments of a simple, but fairly robust, phenomenology in terms of length scales which grow with time and discuss realizations of this scenario. Last, we show that a nice account of both recent experiments performed on an Anderson insulator and on a colloidal suspension can be given in terms of such growing length scales.

II.   EXPERIMENTAL FACTS

In this section, we present the main experimental facts observed when the protocols of Fig. 1 are actually performed. This will allow us also to define precisely the vocabulary used throughout the paper. We use our own data to describe these phenomena, but emphasize that a similar phenomenology has been observed in many different systems. Such experiments were first performed on spin glasses (rejuvenation and memory effects [11, 13, 14, 15]) and polymer glasses (mechanical rejuvenation [16] and Kovacs effect [1, 17, 18]) in temperature shift or cycling experiments. Thus, we adopt 'temperature' as a control parameter, but also discuss the case of other control parameters. The degree to which different such external parameters are equivalent is a completely open question. It was recently asked in the present context [19], as a part of a more general research line [20, 21]. Here, we take a pragmatic approach and elaborate on experimental similarities. Sections II A, II B, and II C deal with effects encountered in shift experiments, while section II D deals with cycling experiments.

A.   Rejuvenation

First consider a shift experiment, see Fig. 1. The system is quenched at initial time $t = 0$ from a high temperature to a temperature T_A in the glassy phase. For $0 < t < t_A$, the temperature is kept constant. Up to this point, this is a simple aging experiment, manifested by the slow evolution of physical quantities. Such typical slow evolution is shown in the left part of Fig. 2 where a quantity analogous to a magnetic susceptibility at given frequency is computed in the numerical simulation of a microscopic spin glass model [22].

At $t = t_A$, the temperature is shifted to T_B. As seen in Fig. 2, aging is restarted by a negative shift $(T_A > T_B)$ in the sense that the resulting curve is similar to that obtained in a direct quench to T_B. This restart of the dynamics is called *rejuvenation effect* because the time t_A spent at T_A—the sample 'age'—seems to have no influence on the dynamics following the shift. The same effect is obtained if $T_B > T_A$. We note also that the term 'rejuvenation' was first employed to describe the effect of large stresses on the aging of polymers [1, 16].

B.   Overaging and underaging

Obviously, if $T_B = T_A$, no rejuvenation takes place, meaning that in order for rejuvenation to be observed in a shift experiment, $|T_A - T_B|$ must be 'large enough'. For small or intermediate $|T_A - T_B|$, a phenomenon recently called *overaging* is observed [19], when $T_A > T_B$. This is illustrated in Fig. 3 (left), where results obtained with a polymer are presented [23]. In this figure, two-time linear
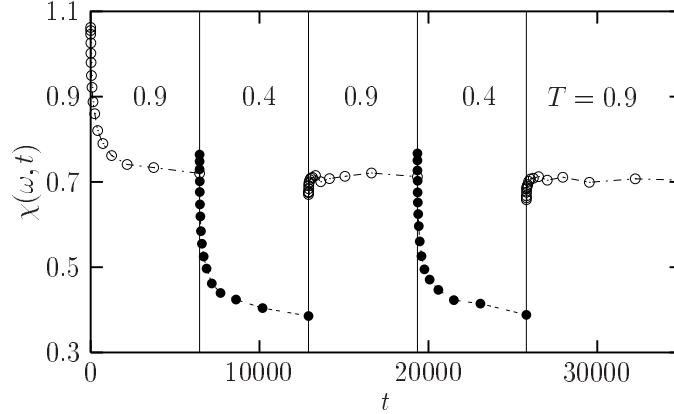
FIG. 2: A temperature cycling experiment performed numerically in a spin glass model. The system is quenched at $t = 0$ in the spin glass phase, $T_A < 1.0$. It ages at the temperature $T_A = 0.9$ as demonstrated by the time evolution of a quantity analogous to a magnetic susceptibility. The temperature is then shifted at $t_A = 6450$ to $T_B = 0.4$ where rejuvenation takes place. At $t_B = 12900$, the temperature is shifted back to $T_A = 0.9$, demonstrating, after a very short transient, the memory effect. A second cycle in then performed.
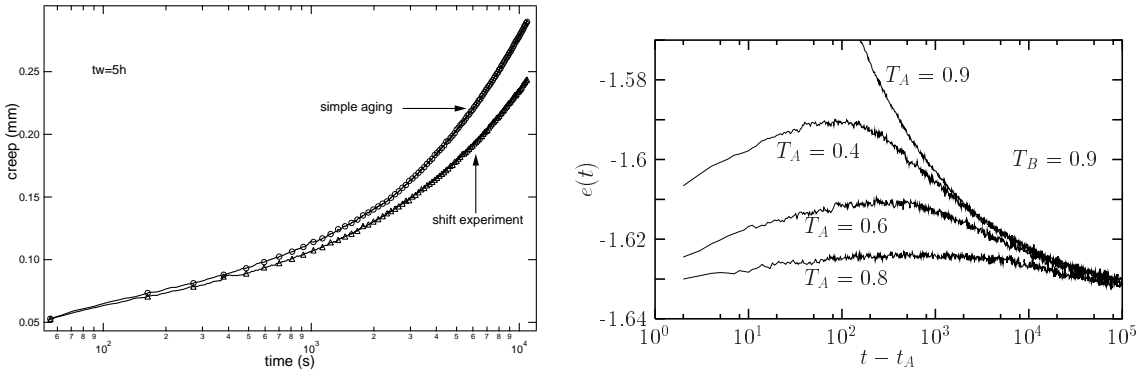

FIG. 3: Left: overaging observed in a shift experiment with $T_B < T_A$. The creep compliance of a polymer after the shift (triangles) is compared to the curve obtained in a simple aging experiment at T_B (circles) and the same aging time, $t_A = 5$h. The long-time response is clearly slower after the shift. Right: Kovacs effect observed in a shift experiment with $T_B > T_A$. The energy density of a spin glass model recorded after the shift exhibits a typical nonmonotonic behavior.

response to a stress step (creep compliance) after a shift from T_A to T_B is compared to that obtained in a simple aging experiment at T_B with the same aging time. The response after the shift is slower than that obtained in the the simple aging experiment, and the system looks 'older', or 'overaged'. Similarly, an *underaging* would be obtained if $T_A < T_B$.

These effects were also observed experimentally in a colloidal suspension submitted to a transient oscillatory shear [19], as well as in temperature shift protocols in experimental [11, 14] and numerical [22, 24] studies of spin glasses.

C. Memory effect of the first kind, or 'Kovacs effect'

A *memory effect* takes place in the shift protocol when $T_B > T_A$. It was first observed by Kovacs in polymers [17]. To distinguish it from a second memory effect (see below), this effect was called *Kovacs effect* in Ref. [25]. Kovacs measured the specific volume, $V(t)$, of the polymer during the experiment, but other physical quantities (index of refraction, energy density,...) can be investigated. The effect is particularly striking when t_A is chosen so that $V(t_A) = V_{eq}(T_B)$, which means that immediately

after the quench, the volume has already reached its equilibrium value at the new temperature. Hence, a naive expectation would be that $V(t > t_A) = const = V_{\mathrm{eq}}(T_B)$. Instead Kovacs observed a nonmonotonic variation of the volume showing that the system has some *memory* of its state at the initial temperature. This experiment is reproduced in Fig. 3 using the energy density of a microscopic spin glass model in a numerical simulation [22]. Similar results have been obtained in supercooled liquids [26], granular materials [9], foams [27] or dipolar glasses [6].

D. Memory effect of the second kind

Let us describe the continuation of the experiment shown in Fig. 2. At time $t_B > t_A$, T is shifted backed to its initial value T_A. After a very short transient, $\chi(\omega, t)$ resumes its evolution as if there had been no aging at T_B. The system has a *memory* of the first stage, despite the strong rejuvenation observed in the intermediate stage of the cycle. Simultaneous observation of rejuvenation and memory is spectacularly demonstrated in the 'dip experiment' [15]. The protocol is essentially a cycling experiment in which the temperature is decreased at a fixed, finite rate (instead of at an infinite rate as in an idealized cycle). The ramp from high temperature stops at T_A, where the temperature is kept constant a time t_A after which cooling is resumed. In this context rejuvenation means that the further evolution of the system is almost the same with or without the stop at T_A. The temperature is then raised back at a constant rate. Memory means that near T_A, the system 'remembers' its stop [15, 28] and behaves differently from a system not held at T_A. However, this experiment is less simple to analyze theoretically because it mixes cooling rate effects with rejuvenation and memory. For this reason, we stick to cycles in what follows.

E. Need for a generic and robust phenomenology

This quick experimental review shows that a number of effects are both highly non-trivial as well as generically observed in a wide range of materials, suggesting that these phenomena are intrinsic to nonequilibrium glassy dynamics. This has two immediate consequences. (1) Any phenomenological theory of aging must account for these effects in addition to the results of simple aging experiments. (2) The phenomenology should be based upon general considerations, which themselves proscribe the range of experimental systems to which the theory applies. Ideally this would be the large range of quite different experimental systems in which glassy dynamics is observed.

III. TWO MEAN-FIELD THEORETICAL APPROACHES

As reviewed in Ref. [12], theoretical approaches to aging phenomena can be classified in three large families. We analyze two mean-field ones in this subsection in view of the above experimental facts.

A. Trap and multi-trap models

A first approach, made popular through the 'trap model' formulated in Ref. [29], considers the dynamics from the point of view of phase space and describes the dynamics of a point particle evolving through a given energy landscape. The reduction of a many-body system to a one-particle problem classifies this approach in the mean-field family. The 'complexity' stems from an assumed large distribution for the free energy of the metastable states, typically chosen to be exponential, $\rho(E) \sim \exp(-E/T_g)$. If one further assumes that energy barriers are related to the free energy of the states, one gets the distribution of trapping times $\rho(\tau) \sim \tau^{-(1+T/T_g)}$, the first moment of which diverges for $T < T_g$. The absence of a mean trapping time then results in the typical t/w scaling of two-time quantities. Obtaining this scaling in a one-body problem is a remarkable result. The crucial drawback of the approach is, obviously, the lack of a precise interpretation of what 'traps' are.

To account for the effects described above, the model was phenomenologically extended to multi-trap models [30], each level having its own glass temperature T_g. The phenomenology of cycles is straightforward. When the temperature is lowered, new levels of the hierarchy start to age (rejuvenation effect), while higher levels are completely frozen and unaffected by the stay at low temperature

(memory). This 'hierarchical phase space picture' has often been invoked by the Saclay group to interpret experiments on spin glasses [14]. Furthermore, numerical simulations of concrete realizations of multi-trap models confirm that this picture satisfactorily reproduces the experiments [31]. As a similar realization of this picture, the Sinai model was recently studied in this context [32].

B. Infinite-range models

A more microscopic approach can also be used to describe aging [12]. The aim is to solve exactly the dynamics starting from the Hamiltonian of a glassy system. This ambitious program has been successful for systems with infinite-range interactions. They are thus mean-field realizations of realistic systems. Simple aging experiments can be accounted for, as described in detail in Cugliandolo's lectures [33]. These solvable models have proved to be extremely rich, but by construction they are incomplete descriptions since real space is completely ignored.

Two families of models have emerged. Schematically, the first has the phenomenology of structural glasses—an example is the p-spin model—and typically exhibits t/w scalings [34]. The second family is closer to spin glasses—the prototype being the Sherrington-Kirkpatrick model. The behavior of the latter in simple aging experiments is quite involved, since it displays 'dynamic ultrametricity' [35]. Without giving the details [33], this implies a complex scaling of two-time functions, with the presence of a continuous hierarchy of diverging time scales. When submitted to shifts and cycles, this second family (but not the first one) was shown to exhibit rejuvenation and memory asymptotically [36]. It would be useful to have simulations of these models in order to go beyond the asymptotic analysis of Ref. [36] and to get some comparison to experiment. Within this approach, the glassy effects of the previous section are explained through the existence of a hierarchy of time scales.

However, as honestly noted in the conclusion of Ref. [36], a strong drawback is that the dynamic ultrametricity on which the whole interpretation relies is incompatible with experiments. Obviously this weakens the general validity of these results.

IV. SPATIAL APPROACHES

A. Domain growth

A third family of models describing slow dynamics focuses directly on spatial aspects [12]. Simple aging experiments are explained with reference to a characteristic length scale, $\ell(t)$, which grows with time. The physical content of $\ell(t)$ is that on scales smaller than ℓ the system appears equilibrated while on larger scales it does not.

A pure Ising ferromagnet quenched from its paramagnetic to its ferromagnetic phase provides a simple example of coarsening dynamics. In the low temperature phase, the ferromagnet has two equilibrium states, with magnetizations $+$ and $-$. As time passes, spatial domains of these equilibrium phases develop and coarsen, with a typical associated length scale $\ell(t)$. This domain growth is driven by surface tension of domain walls. The dynamic scaling hypothesis is that the only length scale involved in this process is the domain size $\ell(t)$ itself. In a pure ferromagnet, the growth law is temperature independent [37], $\ell(t) \sim t^{1/2}$, and this leads to t/w scaling of two-time correlators.

This phenomenology does not provide for the rejuvenation and memory effects discussed above and indeed such effects are not observed in ferromagnets. For instance, consider rejuvenation. Thermal reequilibration within the domains is quasi-instantaneous since thermal fluctuations in the ferromagnetic phase are short-range, of typical size $\xi_{\mathrm{eq}}(T) \ll \ell(t)$. Also, since lowering temperature during domain growth leaves the growth rate unchanged, after a temperature cycle $T_A \to T_B \to T_A$, there cannot be regions initially equilibrated at T_A that remain unchanged during the cycle.

However, coarsening in a disordered system cannot be analogous to that in a pure system [38]. Coarsened domains will no longer be simple convex objects with energy decreasing monotonically with curvature. This can be seen readily by considering a ferromagnet with some added impurities. Walls separating $+$ and $-$ domains will tend to avoid unusually strong bonds and will tend to be pinned at unusually weak bonds. In such a disordered system, domain growth will be slower than it is in a pure ferromagnet. In the case of high disorder, domain growth is likely activated, leading to a characteristic domain size $\ell(t) \sim (\ln t)^p$ [39]. Secondly, in important contrast to pure systems, renormalization of a disordered system's Hamiltonian yields statistically similar Hamiltonians. In

particular this means that we should not expect that the equilibrium states in the low temperature phase of a disordered system at temperatures T_A and T_B are related in any simple way. A simple example is given by the effective interaction between two spins on the opposite vertices of a square. Just as the sign of the effective interaction between these two spins can change with a perturbation in bond strength, so too can it change with a perturbation in temperature [40].

B.   A minimal phenomenology

These differences stated, one can ask whether there is a phenomenological coarsening theory for aging in glassy materials analogous to that in systems like pure ferromagnets. We formulate such a 'minimal' phenomenology in terms of a growing length scale. It must contain the following features.

(i) Existence of a growing coherence length ℓ. This coherence length has the same physical content as described above: objects smaller than ℓ are quasi-equilibrated while larger objects still retain their nonequilibrium initial conditions. A more precise definition of the 'objects' is not necessary at this point. For clarity we will draw compact domains in our cartoons. Furthermore, the coherence length must grow slowly with time so that equilibrium domains of size ℓ remain fixed on time scales associated with objects smaller than ℓ. This first assumption is very natural for some systems, e.g., disordered ferromagnets. Yet for other systems it is very non-trivial. For instance, no typical length scale has been invoked yet to explain the aging of supercooled liquids or colloidal suspensions.

(ii) Sensitivity of equilibrium on all length scales. Obviously, the specific equilibrium state of the system depends upon the particular values of the control parameters, but sensitivity is required in order to provide for the rejuvenation effects. Of course, even a simple two-level system has to readapt its Boltzmann weights upon a temperature change [41]. There remains to be understood what 'levels' are in a realistic system, and how those evolve with temperature. In fact as discussed above, sensitivity of the equilibrium state to control parameter values is a natural consequence of a renormalization procedure in a disordered system. Note that there are nevertheless questions outstanding. In particular, which if any glassy systems be described in terms of a glassy phase fixed point? And even if some can be, is the sort of 'chaos' that one gets automatically in fact responsible for the observed rejuvenation and memory phenomena in glassy materials? We will return to these points shortly.

(iii) Separation of length scales. The length at which a system is equilibrated after a given wait time depends strongly on external parameters. Memory effects are a consequence of such dependence. This is very natural for the thermally activated domain growth expected in disordered systems [42]:

$$t(\ell, T, \cdots) \sim t_0 \exp \left(\frac{E(\ell, T, \cdots)}{k_B T} \right), \tag{1}$$

where t is the time needed to equilibrate the length scale ℓ, t_0 a microscopic time scale, and $E(\ell, T, \cdots)$ an activation energy that in principle can depend on the coherence length itself and on temperature. The dots stand for possible control parameters in addition to T. If $E(\ell) \propto \ell/T$, the growth law is logarithmic and 'super-Arrhenius': $\ell \propto T^2 \ln(t/t_0)$. If instead barriers grow logarithmically, $E = k_B T_0 \ln \ell$, one gets a T-dependent power law growth, $\ell \sim t^{T/T_0}$. In either case, thermal activation (Eq. (1)) implies that the range of length scales 'active' in a given time window depends strongly on T and, more generally, on other external parameters in systems with glassy dynamics [41, 42].

C.   Back to experiments

Consider the cycling experiment $E \to A \to B \to A$ of Fig. 1. Again we write the control parameter as a temperature T. The three above assumptions lead to the cartoon of Fig. 4 which we now explain.

For times $0 < t < t_A$, this is a simple aging experiment at T_A. Feature (i) entails that the system equilibrates up to a growing coherence length, $\ell(t, T_A)$, leading to a $\ell(t, T_A)/\ell(w, T_A)$ scaling of two-time functions. This is the usual domain growth picture of aging. Small length scales, $\ell_s < \ell(t, T_A)$, with characteristic times $t_s \ll t$ have reached equilibrium at T_A, while big ones, $\ell_b > \ell(t, T_A)$ with characteristic times $t_b \gg t$ still retain their initial nonequilibrium state at T_E, see Fig. 4.

Then the temperature is shifted to $T_B < T_A$ at time t_A with $|T_A - T_B|$ 'large'. Feature (ii) entails that all length scales, $\ell \lesssim \ell(t_A, T_A)$, must adapt to the new temperature T_B. This occurs through the
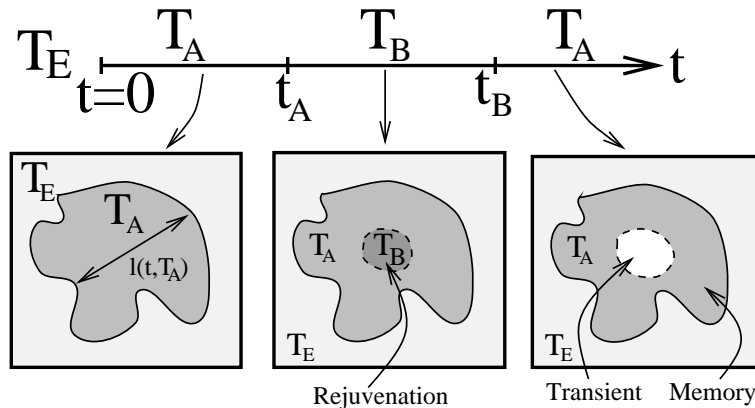
FIG. 4: Cartoon suggested by the considerations of section IV B for a temperature cycling experiment. The related effects are discussed in section IV C.

growth of a new coherence length, $\ell(t - t_A, T_B)$, as shown by a dashed line in Fig. 4. Aging is thus restarted from scratch and *rejuvenation due to the reequilibration of small length scales* is observed.

At time t_B, the system is shifted back to A. Due to (iii), $T_B < T_A$ implies a much slower growth of ℓ during the intermediate stage. Hence, for $t_B - t_A \approx t_A$, as is often the case in experiments, one has $\ell(t_B - t_A, T_B) < \ell(t_A, T_A)$ and the behavior for $t > t_B$ is due to three types of length scales. Small length scales, $\ell < \ell(t_B - t_A, T_B)$, have to reequilibrate at T_A. Intermediate lengths, $\ell(t_B - t_A, T_B) < \ell < \ell(t_A, T_A)$, are already equilibrated at T_A. *It is in these length scales that the memory resides.* Large lengths, $\ell(t_A, T_A) < \ell$, still have to equilibrate at T_A. Hence, after a short transient of duration τ given by $\ell(\tau, T_A) \sim \ell(t_B - t_A, T_B) < \ell(t_A, T_A)$, aging proceeds as the continuation of the first stage. Note that under the assumption of activated dynamics, simple inequality of length scales corresponds to a large degree of inequality in terms of time scales.

If one chooses instead $T_B > T_A$, the essential situation should be unchanged, provided that the condition $\ell(t_B - t_A, T_B) < \ell(t_A, T_A)$ is fulfilled, see Fig. 4. This property is called 'symmetrical effect' in the field of spin glasses [11]. Note, however, that this condition on the lengths is extremely difficult to satisfy due to feature (iii). Therefore, in most experiments with $T_B > T_A$ the equilibrated length grown at T_B is larger than that grown at T_A. All memory is subsequently erased, resulting in an apparent asymmetry between positive and negative cycles.

Let us finally turn to Kovacs' experiments. Two types of length scales must be distinguished in order to understand this effect. Small scales, $\ell < \ell(t_A, T_A)$, are equilibrated at T_A but must adapt to a new, *higher*, temperature T_B. Large lengths, $\ell(t_A, T_A) < \ell$, in contrast, are still in their nonequilibrium high temperature initial state and must adapt to a *smaller* temperature T_B. If the energy density $e(t)$ is computed, as in Fig. 3 (right), small length scales contribute to increase $e(t)$, while large length scales contribute to decrease $e(t)$. This accounts in simple terms for the Kovacs effect.

Schematic discussions based on (i), (ii), and (iii) like that above allow for qualitative prediction of the outcome of given experimental protocols. However, while (i), (ii) and (iii) are natural features for a phenomenological theory of coarsening in a glassy system, in the above they are put in by hand. Consequently we now discuss models that explicitly realize these properties.

D. Droplets and chaos in spin glasses

The scaling, or droplet, model of spin glasses [43, 44] possesses these three features and provides quantitative, experimentally testable predictions. These predictions are clearest in the case of Ising spin glasses, which we discuss here. At a given temperature in the spin glass phase, the assumptions made by the scaling picture resemble those made in the case of pure ferromagnetic systems, but differ due to the disorder in a spin glass.

The initial assumption of the droplet model is that Ising spin glasses have two equilibrium states, related by spin flip symmetry. Spin glass dynamics are then described in terms of 'droplets', low lying excitations about these states [44]. Because of disorder, the boundaries of low lying excitations

wander in order to take advantage of bonds not satisfied in the ground state and to avoid satisfied bonds that are particularly strong. Thus droplets are expected to be non-convex with fractal surface of dimension $d_f > d - 1$, where d is the space dimensionality. The droplet model assumes that the energy of larger droplets is, on average, larger than that of smaller ones and hence that the spin glass phase is not destroyed by arbitrarily large, arbitrarily low energy, excitations. In particular, it makes the scaling ansatz that the average free energies of these low lying excitations scales with their size, $F_\ell \sim \Upsilon(T)\ell^\theta$, $\theta > 0$. As a result of disorder, $\theta < (d-1)/2$, and the distribution of F_ℓ is expected to be broad, with weight down to zero for all ℓ. Further the droplet model assumes that dynamics is activated and makes the second scaling assumption that activation barriers scale with droplet size, $B_\ell \sim \Delta(T)\ell^\psi$ where the distribution of B_ℓ also has weight down to zero. Stiffness modulii $\Upsilon(T)$ and $\Delta(T)$ go to zero at the critical temperature T_c as $(1 - T/T_c)^{-\nu\psi}$, where ν is the standard exponent.

Feature (i) given above follows immediately from the assumptions of the scaling picture. Aging proceeds by equilibration of droplets in increasingly larger equilibrium domains of size $\ell(t, T) \sim \left(\frac{T}{\Delta(T)}\ln t\right)^{1/\psi}$. Feature (iii) also follows immediately both from activated dynamics and from the temperature dependent stiffness coefficients. Thus the droplet picture predicts simple aging behavior as well as memory effects, at least under certain experimental conditions.

The dynamics provided by the droplet model of spin glasses differs importantly from the dynamics of a pure ferromagnet in the role of temperature change [44]. A spin glass at $0 < T < T_c$ is indeed described in terms of a renormalized Hamiltonian with a renormalized ground state. However, as mentioned above, this renormalized finite temperature Hamiltonian is only statistically similar to the zero temperature Hamiltonian. Hence the equilibrium states and droplet excitations thereof at various temperatures are not related to one another in any simple way. In fact, a straightforward statistical argument [44, 45] provides an overlap length associated with a small temperature change from T to $T + \Delta T$, given by $\ell_o(T, \Delta T) \propto |\Delta T|^{-\zeta}$ where $\zeta = \frac{2}{d_f - 2\theta}$. The equilibrium state at T is unchanged on scales $\ell \ll \ell_o(T, \Delta T)$ but altered significantly on scales $\ell \gg \ell_o(T, \Delta T)$. This 'temperature chaos' realizes feature (ii) and predicts rejuvenation when $\ell_o(T, \Delta T)$ is sufficiently small compared to $\ell(t, T)$ [46].

Remark that the droplet picture makes certain reasonable, but as yet unverified, assumptions. It assumes that there are equilibrium states with reference to which low lying excitations can be defined, even as equilibrium domains are growing. Such assumptions are still a matter for debate [47]. Furthermore, the properties of these excitations are given by reasonable but assumed scaling laws. For work examining the validity these assumptions see Refs. [48].

Even if the assumptions made in the droplet model are valid, it is not clear whether this model is sufficient to explain observed rejuvenation and memory effects. In order for it to do so, the overlap length $\ell_o(T)$ must be small enough to affect observations corresponding to experimental length scales. We note that the characterization of $\ell_o(T)$ in spin glasses is still a very active topic at the theoretical level [49]. A cautious conclusion is that, if ℓ_o exists, it certainly is too large to be observed in numerical simulations of systems like spin glasses. However, because F_ℓ and B_ℓ are both broadly distributed, the droplet model anticipates that in rare regions, reorganization with change of temperature will take place even on unusually small length scales and experimentally viable time scales. If such anomalous reorganization is common enough, rejuvenation and memory may still be accounted for by the droplet model even if the characteristic overlap length $\ell_o(\Delta T)$ itself is not readily obtainable in experimental wait times [50]. The distribution of length scales about the overlap length $\ell_o(\Delta T)$ certainly requires further theoretical investigation.

On an experimental level, rejuvenation effects do not constitute a proof of temperature chaos. They are simply consistent with its existence. In fact, in the simulational study of Ref. [22] it is argued that a rejuvenation effect is observed in a regime in which no chaotic effect of the kind described above can be detected [51]. Hence, it is worth considering other mechanisms that might provide for rejuvenation and memory effects.

E. Surfing on a critical line

Another mechanism has recently been invoked to account for rejuvenation effects, while retaining the simplicity of the domain growth approach [25]. We showed in section IV A that the absence of rejuvenation in standard coarsening comes from the short-range character of thermal fluctuations. Since the inequality $\xi_{\mathrm{eq}}(T) \ll \ell(t)$ prevents rejuvenation, the idea of Ref. [25] is simply to consider
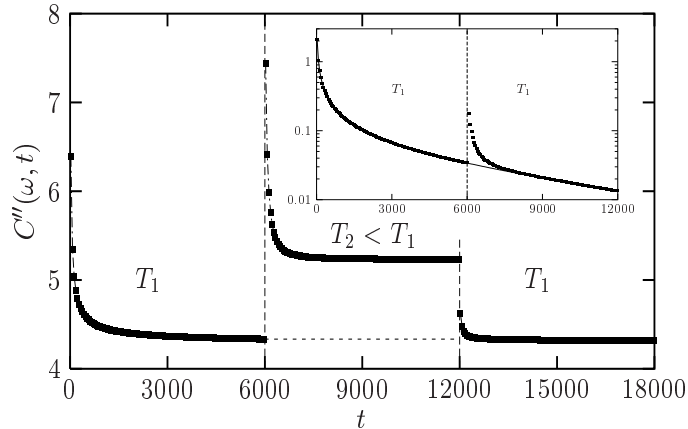
FIG. 5: Fourier transform of the spin autocorrelation function of the 2D XY model computed analytically in a temperature cycling experiment.

opposite situations where $\ell(t) \ll \xi_{eq}(T)$. Away from critical points, $\xi_{eq}(T)$ is usually small. The suggestion is hence to 'surf on a critical line' [52] where $\xi_{eq}(T) = \infty$.

A simple aging experiments corresponds then to a quench to a critical point. In this case, the coherence length is equal to the dynamic correlation length $\ell(t) = \xi(t) \sim t^{1/z_c}$, where z_c is the dynamic critical exponent. Aging is thus nothing but the successive equilibration of the critical fluctuations. Similarities between this situation and aging in glassy materials were noted [53].

When the temperature is shifted, all the critical fluctuations have to adapt to a new critical point with a new set of critical exponents. This is the origin of the rejuvenation effect in this context. It is now obvious that all the discussion of section IV C applies, with the dictionary "objects" = "critical fluctuations" and "coherence length" = "dynamic correlation length". The mechanism for rejuvenation is subtlely different from temperature chaos, even taken close to the critical point [54]. Here, all length scales are always reshuffled without any 'overlap length' [22, 25].

The XY model is a microscopic realization of this picture. It undergoes, in two dimensions, a Kosterlitz-Thouless transition from a standard paramagnetic state to a non-magnetized state with power-law correlations. It presents then a 'line of critical points', with a continuous variation of the critical exponents with temperature. As a consequence, the 2D XY model exhibits aging, rejuvenation, memory and Kovacs effects [25]. The example of a temperature cycle, where rejuvenation and memory are present, is given in Fig. 5. Note that property (ii) is not naturally present in the model, since $\ell(t) = \sqrt{\rho(T)t}$ where $\rho(T)$ is the (renormalized) spin wave stiffness. To build Fig. 5, $\rho(T)$ was arbitrarily, but not unreasonably, chosen to ensure the separation of length scales, $\rho(T_A) = 20\rho(T_B)$.

This analogy raises the question: are glassy systems critical? A possible answer is yes they are, in which case they should have in common a line of critical points below the glass transition, in analogy with the 2D XY model. A microscopic mechanism giving rise to such a behavior in, say, spin glasses, is lacking. Another is no; but they do experimentally behave as if there is a critical line. The second, more pragmatic solution requires that $\ell(t)$ is never decoupled from $\xi_{eq}(T)$, even when the latter is finite. One might argue this if the excitations on length scales exceeding $\xi_{eq}(T)$ are dominated by activation, and hence give exponentially increasing times for $\ell(t) \geq \xi_{eq}$. These conditions may make it virtually impossible to enter the regime $\ell(t) > \xi_{eq}(T)$. Slow evolution in the crossover region could result in effective critical behavior with continuous evolution of the exponents. This is at least fully consistent with results in spin glasses [22].

V. TWO EXPERIMENTS

A. Anderson insulator

We mention the Anderson insulator, or 'electron glass', in this paper since its experimental properties were discussed at length in this school by Ovadyahu [55, 56]. Hence, we do not present the system

and directly discuss the results. Also, we only focus here on one of the many measurements performed on this system, and refer to Refs. [56] for more details. Our aims are (1) to rephrase the "aging experiment" described by Ovadyahu in the present context; (2) to show that the phenomenology of section IV B might be useful to interpret the data.

The electron glass is first prepared in its 'glassy phase'. This corresponds to a given low temperature and a gate voltage $V = V_A$, used as the 'control parameter' [55]. This corresponds then to a 'simple aging experiment'. Aging manifests itself through a logarithmic time evolution of the conductance G, used as a probe of the dynamics. Presumably, two-time functions would display in this regime interesting scaling behaviors, but no such measurements have been performed yet.

After a very large time, typically days, G is almost constant. The gate voltage is then shifted to V_B during a time t_w. Experiments show that G restarts to evolve in a logarithmic way. This corresponds to a 'rejuvenation effect', since the days already spent in the glassy phase are apparently forgotten. In a third stage of the experiment, V is shifted back to its initial value V_A. The experiments show that, after a time of order t_w, G reaches its initial value again. This is analogous to what we called the 'transient', followed by the 'memory effect of the second kind'. Furthermore, experiments have shown that during the third stage of the experiment ('transient' and 'memory') the conductance satisfies the scaling law $G = \mathcal{G}(t/t_w)$ where t is the time counted from the beginning of the third stage [55, 56]. This behavior was called 'aging' in Ref. [56], because of the t/t_w scaling, although the experiment is in fact a complete cycle in terms of V.

Can one go beyond words and analogies? If the similarity between these experiments and standard temperature cycling experiments is assumed, then the cartoon of Fig. 4 can be used. The t/t_w scaling of the conductance tells that the duration of the 'transient' has the same magnitude as the duration of the second stage. This means that there is almost no separation of time scales in the electron glass for this range of gate voltages, as noted in Ref. [56]. Building further on analogies, one can reproduce this experiment using the 2D XY model. Stay first a very long (infinite) time at temperature T_A, then do a shift of duration t_w at T_B and come back at T_A at time $t = 0$. In this case, one can compute the time dependence of the spin autocorrelation function between times t and 0, noted $C_{t_w}(t, 0)$, as a function of the duration of the shift t_w. One finds, dropping irrelevant factors and using the notations of section IV E:

$$
C_{t_w}(t, 0) = C_{\text{eq}}(t) \left(\frac{1}{\ell_{T_A}(t)} \right)^{\frac{\eta_B - \eta_A}{2}} \left(\frac{\left(2\ell_{T_B}^2(t_w) + \ell_{T_A}^2(t) \right)^2}{4\ell_{T_B}^2(t_w)\left(\ell_{T_A}^2(t) + \ell_{T_B}^2(t_w) \right)} \right)^{\frac{\eta_B - \eta_A}{4}}, \tag{2}
$$

where $C_{\text{eq}}(t)$ is the equilibrium correlation function obtained when $t_w = 0$ (no shift at all) and η_A and η_B are the usual critical exponents at temperatures T_A and T_B. The second and third term of the right hand side respectively account for the 'transient' and the 'memory'. For $\ell_{T_A}(t) \ll \ell_{T_B}(t_w)$, the transient term is dominant and aging is observed through a power law decay. When $\ell_{T_A}(t) \gg \ell_{T_B}(t_w)$, both terms combine to restore equilibrium, accounting for the memory. Interestingly, the crossover time scale t_c is given by

$$
\ell_{T_A}(t_c) \sim \ell_{T_B}(t_w). \tag{3}
$$

No separation of length scales amounts thus to $t_c \sim t_w$ as is observed in the electron glass. Deviations from t/t_w, as observed at higher gate voltages V [55], could be tentatively related via Eq. (1) to a decrease of an activation energy at high V. This effect could be more systematically investigated using Eqs. (1) and (3).

B. Colloidal suspension

Overaging was introduced in section II. It is reproduced in Fig. 6 showing a multiple light scattering experiment on a colloidal suspension [19]. Two interesting points in this experiment are (1) the control parameter is an external oscillatory shear strain, as opposed to temperature in 'standard' shift experiments; (2) no obvious coherence length is known to grow during the aging of colloids. Therefore, such experiments could possibly be used to characterize length scales in colloidal glasses.

In this context, 'simple aging experiments' consist in a 'quench' from a shear strain of very large amplitude (typically more than 20 %) mimicking a 'high temperature'. Standard t/w scalings have
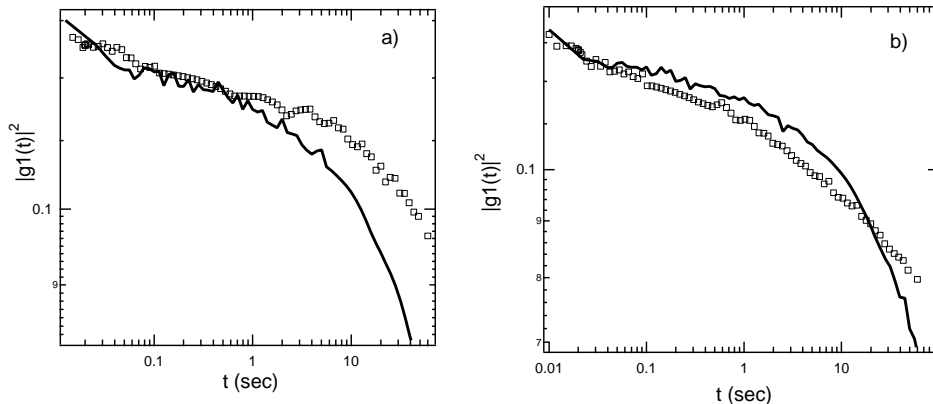
FIG. 6: Overaging in a colloidal suspension. The data show two-time intensity correlation functions obtained with a multiple scattering technique. Full lines refer to a simple aging experiment, squares to a 'strain-shift' experiment with an intermediate strain of 5.9%. (a) $w = 163$s. Overaging is observed as in Fig. 3. (b) $w = 100$s. The strain difference is strong enough to partially rejuvenate the system (square curve below the full line at short times while overaging is still observed at long times. This is the 'intermediate' shift described in the text where both rejuvenation and overaging are mixed.

been observed in many similar soft glassy materials [3, 4, 5, 19]. It is thus natural to start and investigate more complex protocols, in the spirit of shifts and cycles [19, 57, 58]. A first possibility is to quench first towards a small, but finite, shear strain during a time t_w, after which the shear is completely stopped. One can then compare the results with a temperature shift experiment, $T_E \to T_A \to T_B < T_A$, which are more commonly performed.

In spin glasses, in agreement with the phenomenology developed above, the following results are known. We already discussed the case of large $\Delta T = T_A - T_B$: rejuvenation is observed [13]. For very small ΔT, the relaxation of the system after the shift has the same shape as in a direct quench to T_B, but with an 'effective age' $t_w^{\text{eff}} > t_w$ [14]. Indeed, a small ΔT means that the objects growing at T_A will be almost unchanged at T_B (no rejuvenation). However, due the property (iii), the time spent at higher temperature is more efficient in growing the coherence length. Hence, the relaxation is slower after the shift, the system looking 'older' or 'overaged'. This is observed in the colloidal suspension in a 'strain-shift' experiment, see Fig. 6(a). More quantitatively, the effective age t_w^{eff} can be estimated in terms of the coherence length as [22, 24, 41]

$$\ell(t_w^{\text{eff}}, T_B) \sim \ell(t_w, T_A). \tag{4}$$

Repeating this protocol for various (t_w, T_A, T_B) gives thus quantitative informations on the growth law of the coherence length [22, 41, 59]. Furthermore, Eq. (1) shows that this protocol allows a quantitative study of the energy barriers encountered by the system during its nonequilibrium dynamics [41].

For intermediate ΔT, of course, the result will be a combination of rejuvenation and overaging. This has been observed in spin glasses [14, 22, 24] and in polymers [23]. That this is the case in the colloidal suspension with oscillatory strain as a control parameter is demonstrated in Fig. 6(b).

Again, systematic studies using Eqs. (1) and (4) could possibly lead to a quantitative characterization of a coherence length in this and other systems.

VI. CONCLUSION

This paper is the result of a discussion group organized during this summer school. It has become a kind of a review of what glassy dynamics looks like, as encountered in experiments performed on different materials using diverse control parameters. Although first observed in spin and structural glasses, it now is clear that these spectacular effects are quite generic. This points towards the necessity of having a simple interpretation of these phenomena and we have used the concept of a coherence length to develop one. Note that all the theoretical interpretations discussed in this paper

rely on a some sort of hierarchical picture: hierarchy of traps levels, of time scales in infinite-range glass models, of length scales in spatial approaches. We have focused on the latter because we think it gives greater physical insight. We note, however, that definition, characterization and measurement of a coherence length is still largely an open problem for most of the systems discussed here! In this respect, we mentioned several times that the experimental protocols discussed in this paper and their interpretation, for instance Eq. (3) and Eq. (4), constitute a promising starting point for progress on the crucial problem of length scales in glassy materials.

Acknowledgments

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