Acta Materialia 128 (2017) 351-364

Contents lists available at ScienceDirect

Acta Materialia

journal homepage: www.elsevier.com/locate/actamat





CrossMark

Full length article Taming intermittent plasticity at small scales

Peng Zhang ^{a, 1}, Oguz Umut Salman ^{b, 1}, Jin-Yu Zhang ^{a, 1}, Gang Liu ^{a, *}, Jérôme Weiss ^{c, **}, Lev Truskinovsky ^{d, ***}, Jun Sun ^{a, ****}

^a State Key Laboratory for Mechanical Behavior of Materials, Xi'an Jiaotong University, Xi'an, 710049, China

^b CNRS, LSPM UPR3407, Université Paris 13, Sorbonne Paris Cité, 93430, Villetaneuse, France

^c IsTerre, CNRS/Université Grenoble Alpes, 38401, Grenoble, France

^d PMMH, CNRS – UMR 7636, ESPCI ParisTech, 10 Rue Vauquelin, 75005, Paris, France

ARTICLE INFO

Article history: Received 20 September 2016 Received in revised form 20 January 2017 Accepted 13 February 2017 Available online 20 February 2017

Keywords: Dislocation dynamics Size effects Plastic fluctuations Hardening

ABSTRACT

The extreme miniaturization in modern technology calls for deeper insights into the non-conventional, fluctuation dominated mechanics of materials operating at microscale. For instance, both experiments and simulations show that sub-micron face-centered-cubic (FCC) crystals exhibit high yield strength accompanied by intermittent, power law distributed strain fluctuations. At macro-scales, the same bulk materials show bounded, uncorrelated fluctuations. Both anomalous strength and intermittency appear therefore as size effects: while the former is highly desirable, the latter is detrimental because stochastic dislocation avalanches interfere with forming processes and endanger structural stability. In this paper we quantify the coexistence of correlated and uncorrelated fluctuations in compressed Al alloys micropillars, demonstrate that the partition between the two is determined by sample size, and propose quantitative strategies allowing one to temper plastic intermittency by artificially tailored disorder. Our experimental results are rationalized using a theoretical framework that quantifies the competition between external (size related) and internal (disorder related) length scales.

© 2017 Acta Materialia Inc. Published by Elsevier Ltd. All rights reserved.

1. Introduction

The classical paradigm of dislocation-mediated plasticity in crystalline solids is that of a smooth *flow* [1,2] where strain fluctuations are small and uncorrelated. This vision of *mild* plasticity was fundamentally challenged by the discovery that plastic fluctuations may be power law distributed in size and energy [3-6], with clustering in space [7] and time [8]. The fact that dislocations self-organize and plasticity proceeds through collective avalanches implies that the flow is wild in the terminology of Mandelbrot who distinguished in this way stochastic processes with infinite moments [9]. Following Ref. [10] we use this language to differentiate between Gaussian (mild) and power law distributed (wild) plastic fluctuations.

http://dx.doi.org/10.1016/j.actamat.2017.02.039

1359-6454/© 2017 Acta Materialia Inc. Published by Elsevier Ltd. All rights reserved.

The two apparently conflicting pictures of smooth and jerky plasticity have been recently reconciled as it was shown that, in bulk materials, mild and wild fluctuations can coexist, with a degree of wildness depending on crystal structure [10]. In hexagonal close-packed (HCP) crystals, long-ranged elastic interactions dominate, leading to cooperative behavior of dislocations. Instead, in face-centered cubic (FCC) crystals, short-range interactions, enhanced by the multiplicity of slip systems, quench plastic avalanches. Plastic flow then proceeds through mainly small and uncorrelated dislocation motions, confined inside the transient microstructural features (e.g. dislocation cells), which give rise to Gaussian (mild) fluctuations. Those coexist with rare power-law distributed (wild) fluctuations associated with sudden rearrangements of the dislocation substructures [10].

In view of the growing interest towards building progressively smaller technological devices, classical approaches of sizeindependent material engineering have to be reconsidered [1]. In particular, metal plasticity has to be reassessed to meet the demands posed by the manufacturing of components at the micro/ nano scales [11] and experiments with ultra-small pillars have become a standard tool in the study of the corresponding fluctuations and size effects [6,12–15]. Besides the initial observation that "smaller is stronger" [12], it has been recently argued that "smaller

^{*} Corresponding author.

^{**} Corresponding author.

^{***} Corresponding author.

^{****} Corresponding author.

E-mail addresses: lgsammer@mail.xjtu.edu.cn (G. Liu), jerome.weiss@ujfgrenoble.fr (J. Weiss), lev.truskinovsky@espci.fr (L. Truskinovsky), junsun@mail. xjtu.edu.cn (J. Sun).

These authors contributed equally to this work.

is wilder" [10], as, in contrast to the observations showing Gaussian plastic fluctuations in bulk FCC samples [10], scale-free intermittency has been confirmed at micro and nano scales for the same materials by a wealth of experiments [6,13,15–17] and simulations [11,18]. The abrupt strain jumps in quasi-statically loaded micro-/ nano-components endanger structural stability and the associated unpredictability raises serious challenges for plastic-forming processes [18]. It has been realized that tempering plastic deformation at ultra-small scales requires new approaches going beyond the phenomenological continuum theory [19].

In bulk materials, grain boundaries (GBs) hinder the propagation of dislocation avalanches, introducing grain-size related upper cut-offs on their size distribution [20]. At micro- and nano-scales, the level of poly-crystallinity cannot be controlled with the same confidence as in bulk materials [21], which limits our ability to use GB for mitigating size-induced intermittency. Considering these limitations, we focus here on a different strategy of controlling deleterious intermittency, motivated by recent simulations which showed that quenched disorder may suppress scale-free behavior in bulk materials [22]. We use the fact that the pinning strength of solutes and precipitates can be artificially tailored within metals by simple aging treatments [23].

Despite many observations that at sub-micro scales quenched disorder suppress plastic fluctuations [24–27], this effect has not been quantified so far in terms of avalanche statistics. We begin by studying the effects of miniaturization on strain fluctuations in Alalloys single crystals strengthened by different types of solutes or precipitates in the conditions when the grain size is not a relevant length scale of the problem. We experimentally *quantify* the "smaller is wilder" effect in pure crystals, tracing the evolution from mild plastic behavior at large pillar diameters *L*, to wild plasticity at small *L*. We then provide evidence that the transition between mild to wild regimes shifts towards smaller *L* with the increase of the pinning strength of quenched disorder. Translating the pinning strength into a characteristic length scale *l*, we show that the competition between *external* (due to *L*) and *internal* (due to *l*) scale effects can be quantified by a single nondimensional parameter *R* = L/l allowing one to collapse the data for materials with different degree of defectiveness on a single curve. We rationalize this collapse within a simple theoretical framework that builds an unexpected bridge between wildness and strength. Our study suggests specific semi-quantitative strategies for controlling intermittency in sub-µm plasticity.

2. Experimental procedures

2.1. Materials

The experiments were performed on four different types of Al crystals: (i) pure Al, (ii) Al-0.3 wt%Sc alloy with Sc solute clusters (referred to as Al-Sc cluster in Fig. 1a), (iii) Al-0.3 wt%Sc alloy with fine sphere-like Al₃Sc precipitates of size ~ 3-8 nm (referred to as Al-Sc precipitate in Fig. 1b), and (iv) Al-2.5 wt%Cu-0.1 wt%Sn with coarse plate-like θ' -Al₂Cu precipitates of diameter ~ 10–40 nm (referred to as Al-Cu-Sn in Fig. 1c). The pure Al, Al-0.3 wt%Sc alloy, and Al-2.5 wt%Cu-0.1 wt%Sn alloys were respectively melted and cast in a stream argon, by using 99.99 wt% pure Al, mast Al-50 wt% Cu alloy, 99.99 wt% pure Sn, and mast Al-2.0 wt% Sc alloy. The cast Al-Sc ingots were solutionized at 921 K for 3 h and then quenched in cold water. Immediately after quenching, one part of the Al-Sc ingots was aged at relatively low temperature of 523 K for 8 h to form Sc clusters. The other part of the Al-Sc ingots was aged at high temperature of 623 K for duration of 24 h, in order to precipitate Al₃Sc particles. The cast Al-Cu-Sn ingots were solutionized at 823 K for 3 h, followed by a cold water quench and subsequently aged at 473 K for 8 h to precipitate plate-like θ' -Al₂Cu particles. Minor addition of micro-alloying element Sn was used to catalyze the precipitation of θ' -Al₂Cu particles with relatively uniform size and homogeneous distribution.

2.2. Microstructure characterization

Three-dimensional atom probe tomography (3DAP) analyses were performed using an Imago Scientific Instruments 3000HR



Fig. 1. Microstructural characteristics of Al alloys. (a) Bright field TEM image (left) and 3DAP result (right) for the Al-Sc alloy. No precipitates can be observed but a large number of Sc atom clusters are detectable (referred as Al-Sc cluster alloy). The blue points represent Sc atoms. (b) Dark field TEM image (left) and HRTEM image (right) for the Al-Sc alloy dispersed with spherical Al₃Sc precipitates with average diameter ~5 nm (referred as Al-Sc precipitate alloy). (c) Dark field TEM (left) and 3DAP (right) images for the Al-Cu-Sn alloy with plate-like θ' -Al₂Cu precipitate with a diameter ~25 nm (Al-Cu-Sn). The pupe points represent Sn atoms and orange ones are & precipitates. Insets in the TEM images are the corresponding selected area diffraction patterns. The statistical results of obstacle size in Al-Sc cluster (a), Al-Sc precipitate (b), and Al-Cu-Sn alloys (c) are shown in (d), (e), (f) respectively. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)



Fig. 2. Fabrication methodology of micro-pillar. (a) Representative EBSD orientation map of Al-Cu-Sn alloy, from which the <110>-orientated grain, marked by dash lines along the grain boundaries, is locked to fabricate micro-pillar inside. **(b)** A series of micro-pillars are fabricated within the <110>-orientated grain marked in (a). **(c)** Magnified SEM image to show the morphology of micro-pillar.

local electrode atom probe (LEAP) to examine the threedimensional distribution of Sc atoms in the Al-Sc cluster alloy (Fig. 1a). The 3DAP experiments routine can be found elsewhere [28]. The reconstruction and quantitative analysis of 3DAP data were performed using the IVAS 3.4.3 software. The precipitates in aged alloys were quantitatively characterized by using transmission electron microscope (TEM) and high resolution TEM (HRTEM) (Fig. 1). TEM foils were prepared following standard electropolishing techniques for Al alloys. Quantitative measures of number density and size of the precipitates are reported as average values over more than 200 measurements. The foil thickness in the beam direction was determined through convergent beam electron diffraction pattern [29]. Volume fraction of the particles was determined by using corrected projection method [30]. Details about the microstructural measurements can be found in our previous publications [2,23].

2.3. Micro-pillar fabrication and compression

Micro-pillars with the same <110> orientation were fabricated by focus iron beam (FIB) within <110>-oriented grains, which were locked by Electron back scattered diffraction (EBSD) on the electropolished surface of each material (Fig. 2). The micro-pillar diameter ranged from about 500 up to about 6000 nm, and the height-todiameter ratio of all the micro-pillars was kept between 2.5:1 and 3.5:1 (Fig. 2c). Fabrication procedures for micro-pillars have been detailed in previous publications [12,14].

The micro-compression tests were performed on a Hysitron Ti 950 with a 10 μ m side-flat quadrilateral cross-section diamond indenter. The micro-pillars were compressed under the displacement-controlled mode at a strain rate of 2 \times 10⁻⁴ s⁻¹ up to 20% strain. The cross-sectional area at half height of the pillar and the initial height were used to calculate true stresses and strains, following a well-known methodology proposed in Ref. [14].

3. Characterization of tested materials

3.1. Microstructure of aged Al alloys

Fig. 1a, b and c show representative microstructural images of Al-Sc cluster, Al-Sc precipitate, and Al-Cu-Sn alloys, respectively. In the Al-Sc cluster alloy, careful examinations found no perceptible precipitates formed (TEM image of Fig. 1a). According to previous works [31], the aging temperature should be greater than ~ 623 K in order to precipitate Al₃Sc second phase particles. In the present work, the Al-Sc cluster alloy was aged at a much lower temperature (523 K). However, abundant Sc solute clusters can be detected from

3DAP analyses, as shown in Fig. 1a. Core-Linkage (CL) algorithm [28] was used to quantify the size of Sc clusters. The statistical results given in Fig. 1d show that most of the detected Sc clusters contain less than 10 Sc atoms. As the dislocations can shear through the solute clusters, the interaction between the solute clusters and dislocations is relatively weak.

In the Al-Sc precipitate alloy that was aged at 623 K for 24 h, a large number of nano-sized Al₃Sc particles are precipitated and dispersed in the matrix, see the TEM image of Fig. 1b. The coherent Al₃Sc precipitates have an equilibrium shape of Great Rhombicuboctahedron [31], with a total of 26 facets on the {100}, {110} and {111} planes (refer to the HRTEM image of Fig. 1b). Considering these precipitates as spherical particles, their diameter distributions were quantitatively measured. The statistical results are presented in Fig. 1e, with an average diameter of about 5.0 nm. Previous work on Al₃Sc particle strengthening [32] has shown that the transition from dislocation shearing of precipitates to Orowan's looping occurs for a critical precipitate diameter of about 4.2 nm. Hence, in the present case, the interactions between dislocations and the Al₃Sc precipitates, dominated by bypassing mechanisms, are strong.

The Al-Cu-Sn alloy was aged at 473 K for 8 h, which led to the precipitation of plate-like θ' -Al₂Cu particles (see the TEM image of Fig. 1c). The micro-alloying element Sn was doped to refine the distribution of θ' precipitates by promoting θ' nucleation [30]. Representative 3DAP results shown on Fig. 1c illustrate the microalloying mechanism: fine Sn particles are firstly formed by Sn atoms segregation, and then these Sn particles provide preferential nucleation sites for the θ' precipitates. Due to the Sn micro-alloying effect, the θ' precipitates in the Al-Cu-Sn alloy have much reduced sizes compared with those in the Al-Cu counterpart [33]. In addition, the Sn-promoted θ' precipitates also have a narrower distribution in size. This optimization of the size of precipitates improves the repeatability of micro-pillar testing, and therefore is suitable for investigating the precipitate-dislocation interactions at small length scales. The statistical results in Fig. 1f show an average diameter of ~25 nm for the θ' precipitates. Since the θ' precipitates are intrinsically shear-resistant [30,33], the θ' precipitates exhibit a typical bypassing strengthening mechanism with a strong precipitate-dislocation interaction.

The measured parameters of the precipitates/solute clusters, including sizes, density, volume fraction, have been summarized in Table 1.

3.2. Evaluation of obstacle resistance to dislocation motion

We will show that obstacle resistance to dislocations motion,

Materials	Disorder Diameter d^s , d ^p _d (nm)	Disorder Thickness t^p (nm)	Number density N_v (10 ²² m ⁻³)	Volume fraction %	Disorder spacing λ_s , λ_p (nm)	Shearing or bypassing	Pinning strength (MPa)	internal scale <i>l</i> (nm)
Pure Al	_	_	_			_	4.5 (0.7)	1536.9
Al-Sc cluster	1.81 (0.04) ^a	_	131.00 (5.01)	0.41 (0.04)	20.6 (0.6)	Shearing	27.6 (2.2)	250.6 (18.5)
Al-Sc precipitate	5.08 (0.24)	_	1.86 (0.11)	0.13 (0.03)	107.0 (6.0)	Bypassing	38.3 (1.3)	180.6 (5.9)
Al-Cu-Sn	24.51 (3.02)	2.31 (0.21)	0.99 (0.05)	1.21 (0.42)	66.4 (6.1)	Bypassing	70.5 (3.5)	98.1 (4.6)

 Table 1

 Statistical results on microstructure features and evaluation of internal scale.

The values in the bracket stand for the measurement error of TEM examination (microstructural parameter) and bulk tension tests (pinning strength).

^a Individual cluster diameters were calculated as the Guinier's diameter ($d_d = l_d \sqrt{5/3}$), where l_d is the diameter of gyration²⁸.

which arises from disorders (solution atoms, clusters and precipitates), forest dislocations, and lattice friction for the studied materials, is an important controlling parameter in the following analysis. This pinning strength to dislocation motion can be evaluated from experimental measurements of the yield strength at 0.2% offset under tension in bulk polycrystalline samples. We used bulk sample testing rather than micro-pillar testing to determine pinning strengths in order to eliminate the inevitable external size effect on strength that exists in case of micro-pillars. The bulk samples were cut from as-cast ingots that were exposed only to heat treatments and underwent no warm/cold deformation. This resulted in: (a) nearly equiaxial grains, free of texture, ensuring a relatively homogeneous and isotropic deformation, and (b) a large average grain size ($L_g \sim 1$ mm, Fig. 2a), leading to a small grain boundary strengthening. For coarse-grained materials, we can evaluate the pinning strength (τ_{pin}) as follows:

$$\tau_{\rm pin} \approx \sigma_{\rm y}/M - kL_{\rm g}^{-1/2} \tag{1}$$

where σ_y is the bulk yield strength derived from the stress-strain curves under uniaxial tension (right half of Fig. 3a), *M* the Taylor factor (3.06 for FCC metals [34]), and *k* the Hall-Petch constant for Al alloy (60 MPa μ m^{1/2} [35]). The evaluated values are listed in Table 1. Note that these values are effective strengths, including the contributions from disorder strengthening, forest dislocation strengthening, and lattice friction.

To support these experimental estimates, we also calculated the pinning strength based on dislocation strengthening theory and microstructural statistics. The lattice friction stress τ_l is ~1.4 MPa [36]; the forest dislocation strengthening $\tau_{\rho} \sim \alpha G b \sqrt{\rho_f}$ is around 3.45 MPa (taking an initial forest dislocation density of ~10¹² m⁻²



Fig. 3. Representative stress-strain curves. (a) Typical stress-strain curves for pure Al and Al alloys. (Left) for 2000 nm diameter micro-pillars, we show the effect of alloying/ disorder on deformation. Stronger disorder suppresses the jerky character of deformation. (Right) for bulk samples, we show the effect of disorder on yield stress and hardening. **(b)** Typical stress-strain curves of the Al-Sc cluster micro-pillars with different sample sizes demonstrating the effect of the external scale. A segment of stress-strain curves (upper half) and the corresponding strain rate-time signal (black line, bottom half) for a 2000 nm diameter pure Al **(c)** and Al-Sc cluster **(d)** micro-pillars. The strain rate-time signal during initial elastic loading is also given for comparison (red line); the strain rate target $(2 \times 10^{-4} \text{ s}^{-1})$ has been removed, therefore the red line represents a background noise. The strain rate thresholds V_{th} are used to separate the bursts (marked by 1, 2 ...) from background noise. In the pure Al sample, plasticity is exclusively released through abrupt strain bursts. In contrast, a significant part of plastic deformation is released in a smooth manner without any detectable strain burst ("slow dissipation") for the Al-Sc cluster sample. Some plastic bursts are also visible ("jumps"). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

for the well annealed crystals), where *G* is the shear modulus of Al (24.7 GPa [33]), *b* the Burgers vector (0.286 nm) and $\alpha \approx 0.5$. Thus, the pinning strength in pure Al is ~4.85 MPa. For Al alloys, the main contribution comes from disorders. In case of non-shearable precipitates, the pinning strength is [23,30]:

$$\tau_p = \frac{Gb}{2\pi\sqrt{1-\upsilon}} \frac{1}{\lambda_s} \ln\left(\frac{\pi d_d^s}{4b}\right) \text{(for sphere - like precipitates)}$$
(2a)

$$\tau_p = \frac{Gb}{2\pi\sqrt{1-\upsilon}} \frac{1}{\lambda_p} \ln\left(\frac{0.981\sqrt{d_d^p t_d^p}}{b}\right) \text{(for plate - like precipitates)}$$
(2b)

where v is the Poisson's ratio (0.33 [33]), d_d^s and d_d^p are the diameters of sphere- and plate-like precipitates respectively, and t_d^p is the thickness of plate-like precipitates. The interparticle spacing for sphere- and plate-like precipitates, denoted λ_s and λ_p . respectively, can be obtained by using the experimentally measured precipitate parameters in Table 1 [30]:

$$\lambda_s = \frac{1.075}{\sqrt{N_v d_d^s}} - \frac{\pi d_d^s}{4} (\text{for sphere} - \text{like precipitates})$$
(3a)

$$\lambda_p = \frac{1.2669}{\sqrt{N_v d_d^p}} - \frac{\pi d_d^p}{8} - 1.061 t_d^p \text{(for plate - like precipitates)}$$
(3b)

where N_{ν} is the number density. Note that, for all alloys, the average interparticle spacing is significantly smaller than the micro-pillar diameters (from 500 nm to 3500 nm) (Table 1). This ensures that atom clusters and precipitates dispersed within the micro-pillars can effectively pin the dislocations. The microscopic estimates of pinning strength $\tau_{pin} = \tau_p + \tau_l + \tau_\rho$ for Al-Sc precipitate (38.05 MPa) and Al-Cu-Sn (70.25 MPa) are in excellent agreement with the estimates based on experimental measurements mentioned above.

4. Statistics of the fluctuations

The ordering of obstacle pinning strengths (pure Al (~4.5 MPa) < Al-Sc cluster (~28 MPa) < Al-Sc precipitate (~38 MPa) < Al-Cu-Sn(~70 MPa) dictates the hierarchy of flow stresses for both bulk materials and micro-pillars (Fig. 3a). The stress-strain curves for pure Al micro-pillars, showing almost no bulk-like strain hardening, appear jerky over the pillar diameter range analyzed in our experiments (typically shown on Fig. 3a). In contrast, Al-Cu-Sn crystals deform (flow) smoothly and strainharden within the same diameter range. The behavior of Al-Sc cluster and Al-Sc precipitate micro-pillars lay in between these two end-member scenarios, with a jerkiness clearly decreasing with increasing pillar diameter (typically shown on Fig. 3b). Fundamentally different from the highly discontinuous deformation where plasticity reveals itself through abrupt strain bursts (Fig. 3c), smoother deformation curves reveal some strain bursts scattered among extended segments of seemingly continuous deformation (Fig. 3d). Although the stress may slowly fluctuate during the smooth segments of the stress-strain curves, plastic bursts cannot be unambiguously identified in the corresponding strain rate-time signals (bottom half of Fig. 3d), implying a much lower dissipation rate compared with the episodes containing strain avalanches.

4.1. Identification of avalanches and parameter W

We define an "avalanche" as a plastic process characterized by a dissipation rate much greater than the imposed loading rate. Given the control mode provided by the loading device [37] (Hysitron Ti 950, see details in Supplement Materials (SM)), an avalanche manifests itself by both a displacement/strain burst away from the strain-rate target (Fig. 4a and Fig. S1a, b in SM) and an abrupt stress drop (Fig. 4c and Fig. S1a, c in SM). These two different manifestations can be separately used to distinguish avalanches from smooth flow through an objective determination of threshold. A representative displacement rate vs time signal, where the target displacement rate has been removed, is shown in Fig. 4a. The peaks, which are responsible for the asymmetry of the fluctuations with respect to the horizontal axis of y = 0, are signatures of strain bursts. The distribution of positive displacement rates is compared with that of the negative ones (in absolute value) in Fig. 4b in order to determine the threshold where the two distributions diverge. For the positive displacement rates, a power-law tail is obtained. Below the threshold value, the two distributions coincide and are Gaussian-like, which means that all the asymmetry marked in Fig. 4a can be explained by the power-law tail of the positive part. This threshold can then be used to distinguish avalanches from background noise and slow dissipations. Following a similar approach we determined a threshold from the asymmetry of force/ stress-rate distributions (Fig. 4c and d).

The size of a burst can be then quantified by the axial plastic displacement resulting from dislocation motion: $X = D_e - D_s + (F_s - F_e)/K_p$, where D_s , F_s , D_e and F_e are the displacement and force when a detected displacement jump or load drop starts and ends, respectively, as shown in Fig. S1, and K_p is the stiffness of the sample. The axial displacement X during an avalanche is proportional to $b(\Delta A/A)$, where ΔA is the cumulative area swept by dislocations during the avalanche, A the slip area, and *b* the Burgers vector. While the burst occurrences (D_s, F_s) extracted by measuring either displacement or force rate are always the same, the terminations (D_e, F_e) measured in these two different ways differ significantly (Fig. S1). Moreover, for a given burst size, the termination of the force drop, (D_e, F_e) , may be sensitive to the relative rate between loading and tip retracing, hence the burst duration. Consequently, burst durations and time correlations between bursts were not analyzed. However, if we neglect the influence of background noise, the evaluated burst size X is unchanged, as long as (D_s, F_s) and (D_e, F_e) correspond to two elastic states before and after the plastic event (Fig. S1). Indeed, the results obtained via displacement-rate measurements (Fig. 4a and b) and force-rate measurements (Fig. 4c and d) are in close agreement (compare Fig. 5 with Fig. S2 in SM), arguing for the robustness of our burst identification and size estimation methodologies.

Once these bursts are extracted, a statistical analysis of the distributions of burst sizes *X*, based on a maximum likelihood methodology [38], provides a lower bound X_{min} to power law scaling for the burst size distribution. Only the bursts with size $X \ge X_{min}$ are considered as wild fluctuations. The cumulative effect of these wild fluctuations (the fraction of plastic deformation accommodated through power-law distributed fluctuations normalized by the total imposed deformation) defines the *wild-ness* measure *W*. Note that this second step of selection for wild fluctuations induces only a minor correction to the value of *W* and does not change any of the general trends. In what follows we compute the value of *W* based on the signal integrated over the



Fig. 4. Displacement/force-rate signals and corresponding distributions. (a) Displacement rate-time signal (after subtraction of the target rate) of an Al-Sc cluster micro-pillar with 1000 nm diameter. The asymmetry of the signal is a signature of wild fluctuations. **(b)** The cumulative probability of displacement positive and negative rates (in absolute value). The asymmetry of the signal is reflected by the power law tail of the positive rate distribution. **(b)** The cumulative rates, respectively. In this case, the power law tail on the negative rates (stress drops) is the signature of wild fluctuations.



Fig. 5. The dependence of the fraction of plasticity released by wild fluctuations, *W*, on the nature of the disorder, micro-pillar diameter (a) and power law exponent κ -1 = *C*/*D* (b). (a) Fraction of plasticity released by wild fluctuations, *W*, as a function of sample diameter (*L*) for the pure Al and Al alloys micro-pillars. "Smaller is wilder" and "dirtier is milder" phenomena can be clearly observed. The plotted *W*(*L*) solid lines are calculated from our model, using quantitative estimates for the parameters *C* and *D* (see section 6.2.2). Error bars are derived from measurements on multiple samples. (b) Universal (material independent) relationship between *W* and the power law exponent κ - 1 = *C*/*D* (see text for details). Error bars represent the uncertainty of power law exponents fitted by the maximum likelihood analysis [38].

entire stress-strain curves. While some characteristics, such as the forest dislocation density, are expected to evolve with deformation, a possible evolution of the distribution of strain avalanches with increasing strain has been discussed recently in Refs. [4,6,39]. We do not address these points here, which are left for future work.

4.2. Effects of size and disorder on W

Our Fig. 5a summarizes the two main results of the paper. First, we experimentally *quantify* the "smaller is wilder" phenomenon, which we interpret as an external size effect. Second, we show that the crossover range between wild (large *W*) and mild (small *W*)

plasticity can be shifted (or even suppressed down to sub-um scales, see Al-Cu-Sn) by introducing high pinning-strength disorder. Such "dirtier is milder" phenomenon reveals the presence of a disorder-related characteristic scale and suggests that the corresponding internal size effect competes with the more conventional external size effect. Recall that in bulk materials, a stronger degree of wildness correlates with a smaller value of a non-universal exponent κ in the power law distribution of strain bursts $P(s) \sim s^{-\kappa}$ [10]. The maximum likelihood analysis [38] of plastic avalanches in micro-pillars confirms this trend down to sub-µm scales (Fig. 6) and reveals a universal (material independent) relation between W and κ (Fig. 5b). This crossover behavior illustrates the gradual transition from power-law (wild) to Gaussian (mild) fluctuation regimes, and provides a unifying perspective on plastic fluctuations at both micro- and macro-scales [10]. The limiting behaviors are observed in pure Al (\leq 3500 nm), showing almost purely power-law distributed bursts with exponent κ approaching (for the smallest sizes) the previously reported value [6,18,40,41] close to the one predicted by the mean field theory, $\kappa = 1.5$ [3], and in Al-Sc precipitate (\geq 2000 nm) and Al-Cu-Sn alloy, where the empirical distributions of the (few) detected strain bursts are almost Gaussian. In this last case, the exponent κ cannot be estimated from the maximum likelihood methodology as the result of insufficient statistics in the tail of the distributions.

Note that this transition from power-law to Gaussian distributions should not be confused with an upper cut-off of a power law scaling. Indeed, we could expect a "trivial" finite-size effect to impact our displacement burst distributions from the upper side [4,18]. However, in this case, this cut-off should be more pronounced upon decreasing the sample size, in opposition with what is shown on Fig. 6. In addition, the study of the likelihoods of power-law distribution with upper cut-off, $P(s) \sim s^{-\kappa} exp\left(-\frac{s}{s_{max}}\right)$,

have shown that the possible upper cut-offs amplitude s_{max} were hardly detectable. This is likely due to the limited statistics of our datasets, but also reinforces the reliability of our estimation of the exponent κ .

4.3. Nondimensional parameter R

The "dirtier is milder" phenomenon quantified above implies the existence of a disorder-related characteristic scale that competes with the external size effect. To interpret our experiments, we first of all define this internal length scale as $l = Gb/\tau_{pin}$, where τ_{pin} is the pinning strength of all kinds of obstacles (see section 3.2). This parameter describes the length scale at which the dislocationdislocation elastic interaction stress (scaling as Gb/l) becomes equal to the dislocation-obstacle interaction stress τ_{pin} [42]. In this sense, l controls the transition from "endogeneous" (dislocation-dislocation) to "exogeneous" (dislocation-obstacle) interaction. More precisely, the implied crossover between the two fluctuation regimes should be governed by a non-dimensional parameter R = L/l. The scaling collapses of our data shown in Fig. 7a and c suggests that indeed W = W(R) and $\kappa = \kappa(R)$. In particular, this result implies that the decrease of L can be compensated by the proportional increase of τ_{pin} . From this interpretation, we can construct a new regime map (Fig. 8) to display the coupled effect of external size and disorder on intermittency for different types of defectiveness. In this regime map, the fluctuations are characterized by two important parameters, κ and W, that are linked by the universal relation shown in Fig. 5b.

It should be mentioned that we do not have grains in our samples, so we essentially assume that the corresponding characteristic scale L_g is larger than any of the above scales. Another



Fig. 6. Distributions of detected displacement burst sizes in pure Al (a), Al-Sc cluster (b), Al-Sc precipitate(c), and Al-Cu-Sn micro-pillars (d). In (b) and (c), the burst sizes follow a Gaussian distribution at small sizes, and a power law distribution at large sizes. The power law exponent is a function of external size and disorder. A crossover from a power law to a Gaussian distribution can be observed as we increase the diameter and/or the pinning strength of the disorder.



Fig. 7. The fraction of plasticity released by wild fluctuations, *W*, the normalized strain hardening rate by the bulk counterpart, $\Theta_{pillar}/\Theta_{bulk}$, and the power law exponent, $\kappa - 1 = C/D$, as a function of dimensionless ratio, *R*. (a) All data of Fig. 4a collapse into a master curve. The solid curves in (a) are calculated from the model, using the linear relationship between C/D and *R* in (c), and the value of parameter K = 0.25. (b) The hardening transition occurs around $R \approx 5$, regardless of special materials and in excellent agreement with the wild-to-mild transition in (a). (c) A linear relationship between *R* and $\kappa - 1$ or *C/D* is observed, with a saturation of *C/D* value for very small *R*.

important scale is, of course, $l_m = \rho_m^{-1/2}$, where ρ_m is the representative mobile dislocation density solving our stochastic differential equation in the model section below. We implicitly assume that in our wild regime $l_m \sim L \leq l \leq L_g$ (dislocations do not see the defects, while interacting with the surface: external size is in



Fig. 8. Tuning m ap showing the W and κ contours as a function of external size L/b and pinning strength τ_{pin}/G . Corresponding to the pinning strength, a defectiveness spectrum is developed by unifying the unshearable particles (UP, like the Al₂Cu and Al₃Sc precipitates in present work), shearable particles (SP, like fine and coherent Al₃Sc precipitates), forest dislocations (FD, like in the present pure Al), and solute clusters (SC, like the Sc solute cluster in present work; N_{ν} is the cluster number density). The numbers inside the brackets represent W and κ respectively. The color bar below illustrate the range of W and κ at $L/b = 2 \times 10^4$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

control) and in our mild regime $l_m \sim l \leq L \leq L_g$ (dislocation interaction with defects is essential while the surface plays a secondary role: internal size is in control). The crossover from mild to wild regime takes place when internal and external scales become of the same order: $L \sim l$.

For pure metals with negligible lattice friction, the tunable obstacles are represented by immobile forest dislocations of density ρ_f , hence in this case $\tau_{pin} \sim Gb\sqrt{\rho_f}$ and $R \sim L/l_f = L\sqrt{\rho_f}$, where $l_f = 1/\sqrt{\rho_f}$ is proportional to the effective mean free path of mobile dislocations [43,44]. Therefore, on the one hand, our parameter $R \sim L\sqrt{\rho_f}$ is fully compatible with scaling properties of dislocation systems in pure metals deduced from similitude principles [45]. On the other hand, the same parameter L/l_f has also been shown to control the nature of hardening at small scales for pure metals [40,46](see detailed discussions in the next section), implying a possible connection between the average hardening behavior and statistical fluctuations. In the following, we will precise the nature of this correlation and reveal that the single parameter *R* can unify both the hardening and wildness map, for both pure metals and alloys.

Note that the dependence of the forest dislocation density ρ_f on the sample size *L* [16,47,48] was not taken into account in our calculation of the pinning strength giving the value of *R*. It is known, for instance, that in pure Al micro-pillars the total dislocation density ρ_t increases when *L* is decreasing [16]. Assuming as an extreme case that $\rho_f = \rho_t (6.2 \times 10^{12} \text{ m}^{-2} \text{ for 860 nm sized micro-pillar, and <math>2.5 \times 10^{12} \text{ m}^{-2}$ for 6300 nm sized micro-pillar after 4% strain in Ref. [16]), we only obtain the shift in *l* from 1350 to 910 nm while shrinking *L* ~7 times. We checked (not shown) that such

possible effect does not change the overall picture shown in Fig. 7. For alloys, where the exogeneous disorder dominates the internal length scale, such possible dependence of the forest dislocation density on sample size would have a negligible effect on *R*.

5. Correlations between the average behaviors and the statistics of fluctuations

5.1. The link with forest hardening

In order to establish a possible link between the strain hardening behavior and the statistics of fluctuations, the dependence of the strain hardening rate (SHR) on internal/external length scales should be firstly addressed. As micro-pillars generally display jerky deformation patterns and serrated strain-stress curves, the evaluation of SHR from traditional methods used in bulk sample testing is difficult. Therefore, following previous works [49–51], we estimated the SHR by using the formula $\Theta_{pillar} = \frac{\sigma_{SL} - \sigma_{2L}}{5K - 2K}S^2$, where σ_{5K} and σ_{2K} are the stresses corresponding to the 5% and 2% strains in the strain-stress curves, respectively, and *S* is the Schmid factor (0.408 for <110> orientation). Strain-stress curves of bulk polycrystalline samples under tension were used to estimate bulk SHR for comparison with micro-pillars. The resolved stress $\tau = \sigma/M$ and resolved strain $\gamma = eM$ at slip system, where *M* is the Taylor factor, are used in the formula $\Theta_{bulk} = \frac{\tau_{T5} - \tau_{T2}}{\gamma_5 - \tau_2}$, where $\gamma_5 = \frac{5K}{5}$ and $\gamma_2 = \frac{2K}{5}$.

The dependence of SHR on the pillar diameter *L* is shown on Fig. 9 for the different alloys. For pure Al, the SHR value of the pillar of diameter L = 6000 nm is close to the bulk value, whereas values obtained for smaller sizes are extremely scattered, another manifestation of the jerkiness of plastic flow [52]. The Al-Sc cluster micro-pillars with L < 2000 nm and the Al-Sc precipitate micro-pillars with L < 1000 nm show scattered SHR values as well, with a mean value greater than that of their bulk counterpart. This suggests a breakdown of size-independent forest hardening mechanism at small samples, which is replaced by weak mutual dislocation reactions and dislocation storage [46,53], giving rise to the sources-dominating plasticity [46,49,54]. To sustain the flow in the case where weak dislocation sources are exhausted, much higher stress is required for the activation of the stronger, truncated



Fig. 9. Strain hardening rate for micro-pillars. The small samples with weak obstacles display more likely source-exhaustion hardening, characterized by *L*-dependent SHRs in high value and large scattering. Introducing obstacles suppresses source-exhaustion hardening, and extends forest hardening to small diameter. The dotted lines represent the SHR of bulk samples for comparison. The critical diameter for hardening transition is in good agreement with the wild-to-mild transition diameter (as shown in Fig. 4a).

ones [52,54,55]. Moreover, the stochastic distribution of initial dislocation sources in the small volume leads to a large scatter in the SHR at small *L*, whereas scattering decreases when SHR values reach bulk values towards large *L*. Transition from forest hardening to source-exhaustion hardening upon decreasing the size *L* has been recently studied in experiments by mean-field modeling [40], and in DDD simulations [46]. Our results shown on Fig. 9 are fully consistent with these studies.

However, we go further and show that strong disorder can also shift this transition towards smaller L, as it does for the wild-tomild transition (see Fig. 5a). Indeed, we see in Fig. 9 that the hardening transition threshold which is somewhat larger than 3500 nm for pure Al, decreases from about 2000 nm for Al-Sc cluster micro-pillars to about 1000 nm for the Al-Sc precipitate samples, and become less than 500 nm for Al-Cu-Sn. The sequence observed for the mildness transition is recovered here and, although a precise estimation of the hardening transition threshold is difficult, the values given above are in good agreement with the length scales at which most of the wild fluctuations become suppressed (see Fig. 5a). In other words, a material starts to strainharden when it becomes mild. This correspondence is further corroborated by the scaling collapse of Fig. 7a and b, showing that our non-dimensional parameter R indeed controls both the wildto-mild transition and the transition between the two hardening mechanisms.

The results presented in Fig. 7 suggest that these two transitions are the manifestations of the same underlying phenomenon. In case of pure single crystal, Alcalá et al. [40] proposed that the nondimensional ratio L/l_{eff} controls the transition from forest hardening to source-exhaustion hardening, where l_{eff} is an "effective" mean free path for dislocations. These authors estimated l_{eff} from an expression accounting for different types of dislocation interactions. In their case, given that one can neglect the effect of lattice friction on τ_{pin} , the mean free path for mobile dislocations l_{eff} , which scales as $1/\sqrt{\rho_f}$ [43,44], can be identified with our internal length scale *l* (see section 4.3). Consequently, for pure metals, the ratio L/l_{eff} , which controls the nature of hardening [40], can be identified with our parameter R = L/l. This parameter, in turn, accounts for the wild-mild transition (Fig. 7a), arguing in favor of a non-incidental overlap of the wild-to-mild transition and the transition between two different hardening mechanisms.

Our Fig. 7 a and b extends this correlation beyond pure metals, and suggests that alloying increases dislocation storage capacity, favoring the formation of dislocation entanglements and enhancing forest hardening. Instead, it disfavors dislocation avalanches and wild plastic fluctuations. In summary, according to our unified scenario, external and internal size effects control, through the non-dimensional ratio R = L/l, the nature of both the *average* behavior (hardening) and the statistics of *fluctuations*.

5.2. The link with deformation morphology

The SEM images of Fig. 10 show the deformation morphologies of micro-pillars after uniaxial compression tests. Small micro-pillars (diameter of 1000 nm) are compared with larger ones (diameter of 3500 nm), to demonstrate the effect of external sample size. Pure Al, Al-Sc cluster, Al-Sc precipitate, and Al-Cu-Sn micro-pillars are compared to illustrate the influence of disorder characteristics. The results show that: (i) in the 1000 nm micro-pillars, plastic deformation is predominantly accommodated by localized deformation on one or two slip plane(s), except for Al-Cu-Sn in Fig. 10g, indicating that plastic deformation in these small-sized micro-pillars exhibiting a large wildness (see Fig. 5a) is strongly anisotropic. This anisotropy of plastic flow for a



Fig. 10. Effects of external size and internal disorder on deformation morphology. SEM images showing the deformation morphology of pure Al ((a) and (b)), Al-Sc cluster ((c) and (d)), Al-Sc precipitate ((e) and (f)), and Al-Cu-Sn ((g) and (h)) micro-pillars with diameter of 1000 nm ((a), (c), (e), and (g)) and of 3500 nm ((b), (d), (f), and (h)).

symmetrical multiple slip system is actually a breakdown of Schmid's law [56]. (ii) On the contrary, in 3500 nm-sized Al-Cu-Sn micro-pillars, more uniform plastic deformation is found, with barrel-like deformation morphology. The large size allows significant dislocation motion before annihilation at free surface [55,57] and so dislocation sources to be activated within the bulk on multiple slip planes [49]. 3D Dislocation networks are formed, leading to the uniform plastic deformation associated with mild fluctuations and radial dilation under compression (see Fig. 10h). (iii) In the pure Al micro-pillar with a large diameter of 3500 nm (Fig. 10b), plastic deformation is still concentrated on a single slip plane, as, in the absence of disorder, dislocations can easily escape to be annihilated at free surfaces.

The isotropy of deformation can be interpreted as a qualitative measurement of the frequency of dislocation intersections and entanglements. The correlation between isotropy and mildness hints towards their common origin that one can associate with the hindering of avalanches and the prevalence of short-range interactions.

5.3. Brief summary

The presented correlations reveal complex dependence of dislocation dynamics on sample size, disorder and crystal symmetry. Wild plasticity is rooted in anisotropy [10]: in small-sized pure crystals, symmetry is compromised by surface effects and they usually exhibit a single slip flow (Fig. 10) even when multiple slip planes are active in bulk samples [49]. Induced anisotropy can be also related to the smaller role played by short-range interactions among dislocations due to the practical absence of locks, junctions, etc. In particular, the probability of mutual dislocation reactions, resulting in entanglements and immobilization, diminishes with the sample size [14], and the size-induced high yield stress can compromise their stability [53]. This weakens the role of the self-induced dislocation microstructure, which suppresses intermittency in high symmetry bulk materials [10].

The intriguing link between the mild-to-wild transition and the transition from the regime of forest hardening to the regime of source-exhaustion hardening (Fig. 7a and b) indicates that a material becomes mild when forest hardening is favored. Our analysis of disordered samples supports these findings, suggesting that the high-strength defects can mimic forest dislocations by quenching

avalanches [22] while simultaneously strengthening the material (Fig. 3a). The disorder impedes dislocations from reaching free surface sinks and facilitates entanglements, making plasticity more isotropic (i.e. multi-slip, Fig. 10) and promoting forest hardening (Fig. 9). The data collapse suggests that both hardening-related and fluctuations-related transitions are controlled by the competition between the external scale *L* and the internal scale *l* which can be quantified in terms of a single nondimensional parameter R = L/l (Fig. 7).

The identified correlations involving strain hardening mechanisms, deformation morphology and fluctuation behavior suggest new intricate links between these seemingly unrelated phenomena.

6. The model

Our experimental results find a simple interpretation in the framework of a mean field model for the density of mobile dislocations ρ_m , first introduced in Ref. [10]. We assume that the dislocation dynamics under *constant* stress can be described by the stochastic equation:

$$\frac{d\rho_m}{d\gamma} = A - C\rho_m + \sqrt{2D} \rho_m \xi(\gamma) \tag{4}$$

Here the time-like parameter γ is the average shear strain, A > 0is the net nucleation rate (which takes into account a negative contribution from surface annihilation), C > 0 is the rate of mutual annihilation/immobilization of the dislocation pairs. While this deterministic part of the model is rather conventional, the stochastic term in Eq. (4) requires some explanation. We assumed that long range stochastic interactions can be implemented in the form of a multiplicative mechanical noise described by the last term in the right hand side of (4). We denoted by $\xi(\gamma)$ the standard white noise with zero average and delta type correlations $\langle \xi(\gamma) \rangle = 0$, $\langle \xi(\gamma_1)\xi(\gamma_2) \rangle = \delta(\gamma_1 - \gamma_2)$. This noise with intensity D describes fluctuations experienced by a representative volume due to interactions with the rest of the system. Similar approaches, relying on the idea of 'mechanical temperature' as a description of the fluctuating local stress, have been used in the modeling of athermal amorphous plasticity [58] and in the representations of crystal plasticity as a noise induced transition [59]. Various ways of representing correlations in the mechanical action of the noise in such models are expected to capture the persistent nonlocal rearrangements and avalanche-like processes triggered by local instabilities [60]. In crystal plasticity the stochastic rheological relations like Eq. (4) that can be viewed as a spatially resolved mesoscopic closure of continuum models of plasticity, still await to be validated by the rigorous upscaling procedures based on microscopic models [60,61].

6.1. Universal relation $W(\kappa)$

To link our model with mechanical measurements, we can use Orowan's relation $d\gamma = \rho_m bv dt$. Since we deal with a fixed external stress condition in our model, the average dislocation velocity *v*, set by the applied stress [62], can be taken as a constant. Thus, the distribution of strain fluctuations observed in our experiments can be associated the fluctuations of mobile dislocation density ρ_m .

The stationary probability distribution for ρ_m predicted by Eq.

(4) has the form
$$P(\rho_m) = \frac{\left(\frac{A}{D}\right)^{C/D} e^{-\frac{A}{D\rho_m}} \rho_m^{-\left(1+\frac{C}{D}\right)}}{\Gamma\left(\frac{C}{D},0\right)}$$
, where

 $\Gamma(z,x) = \int_{x}^{\infty} t^{z-1}e^{-t}dt$. We can then write $\kappa - 1 = C/D$ and define

the degree of wildness as $W = \int_{\rho_{min}}^{\infty} P(\rho_m) d\rho_m$ where ρ_{min} is the threshold distinguishing between wild and mild fluctuations. It is now natural to specify ρ_{min} by the condition $\exp(-\frac{A}{D\rho_{min}}) = K$ where K = 1 corresponds to a pure power law. At fixed K we obtain an important relation between the degree of wildness W and the power law exponent of the long tail describing intermittent fluctuations of the dislocation density

$$W = 1 - \frac{\Gamma(\frac{C}{D}, \log(\frac{1}{K}))}{\Gamma(\frac{C}{D}, 0)}$$
(5)

The fact that the ratio A/D drops from Eq. (5) is in agreement with the experimental scaling collapse, indicating that the degree of wildness W depends only on the exponent κ (see Fig. 5b). Quite remarkably, the predicted curves W(C/D) have the same sigmoidal shape as experimental data, and by choosing the value K = 0.25, we achieve an almost perfect fit; the K dependence in the range of interest is relatively weak (Fig. 5b).

6.2. The external size and disorder effect in the model

So far, an analytical study of Eq. (4) allowed to give specific predictions in terms of scaling exponent κ and degree of wildness W. In this section, we will relate our model parameters to L and l in order to rationalize the main results from our experiments, i.e., the sample size and disorder effect on W and κ .

6.2.1. The interpretation of the model parameters

The *model* has two characteristic densities $\rho_C = A/C$ and $\rho_D = A/D$, which gives rise to two characteristic lengths $l_C = \frac{1}{\sqrt{P_C}} = \sqrt{C/A}$ and $l_D = \frac{1}{\sqrt{P_D}} = \sqrt{D/A}$. The ratio of these two length scales, $r = \frac{l_C}{l_D} = \sqrt{C/D}$, is the main dimensionless parameter of the model. Our *analytical* study shows that the fraction of strain released in wild fluctuations *W* depends only on *r*, in other words that W = W(C/D). On the other hand, our *experimental* study shows that W = W(R) where the parameter R = L/l is also dimensionless. By comparing the functions W(r) and W(R) we find that $R \sim r^2$ which means that $\kappa - 1 = \frac{C}{D} \sim \frac{L}{l} = R$. This linear relation has been confirmed experimentally, as shown in Fig. 7c.

Our parameter *C* characterizes the rate of *mutual* annihilation/ immobilization, which Gilman calls 'stalemating' [63]. It is fundamentally different from surface annihilation of individual disloca-

tions, which negatively contributes to our parameter $A\left(\frac{d\rho_m}{d\gamma} = -\frac{1}{bL}\right)$

[64]. In fact, as we have shown, *A* does not affect either the degree of wildness *W* or the avalanche distribution. In small samples, the immobile configurations, such as dislocation dipoles and locks, are hard to form (or easy to break) under the external size induced high stresses, meaning that an extrinsic size effect applies on parameter *C*. Available 3-D discrete dislocation simulations (Fig. 8 in Ref. [53]) suggests an approximately linear relation between the "dislocation reaction (number) per volume" and the external size *L*. Combined with a similar dislocation density during straining for all *L* (Fig. 5 in Ref. [53]), this implies that the "dislocation reaction, we will justify this scaling based on a detailed microscopic approach and estimate the value of the proportionality coefficient.

The connection between *D* and classical (non stochastic) models of dislocation dynamics is, by nature, less straightforward. In view of the interpretation of the parameter *C*, i.e., $C \sim L/b$, we can conclude that in the range where $C/D \sim L/l$ (Fig. 7c), the mechanical temperature of the system [65] $D \sim l/b = G/\tau_{pin}$ diminishes with increasing pinning strength of obstacles and increases with stiffness responsible for the long range interactions.

The proposed interpretation of the model parameters is consistent with the effect of crystal structure on wildness in bulk materials [10]. For instance, in HCP materials like ice with essentially single slip plasticity, forest hardening is absent. Therefore C accounts for mutual annihilation only and remain small even at macro scale. Combined with a very small lattice friction, this also implies a small pinning strength and thus D is large independently of L. Hence, bulk ice crystals remain wild, and most probably outside the diagram shown in Fig. 7c as the correlation $C/D \sim L/l$ should be valid only over a limited range of L/l. At sufficiently small L/l (large degree of wildness), our exponents for the aggregate distribution (integrated over a range of stress values) appear to be saturated around C/D = 0.5 (Fig. 7c). This value is close to the prediction of the mean field theory [3,6] but may also be an indicator of a smaller stress-integrated exponent obtained in recent DDD modeling [4]. At large values of L/l (small degree of wildness), C should saturate towards a bulk value C_{bulk}; the associated departure from the scaling $C/D \sim L/l$ is hardly detectable in our experimental data, as detected avalanches become too rare to allow an estimation of. *k*.

6.2.2. The physical expression of the model parameters, and the link with the size effect on strength

To justify the scaling $C \sim L/b$ and estimate the value of C, we start by writing the expressions for the rates of mutual annihilation, dipole formation, or lock formation proposed by Roters et al. [66]:

$$\dot{\rho}_{m}^{-}(annihil) = 2\chi_{annihil}\frac{\dot{\gamma}}{b}\frac{1}{n}\rho_{m} \tag{6a}$$

$$\dot{\rho}_{m}^{-}(dipol) = 2\left(\chi_{dipol} - \chi_{annihil}\right)\frac{\dot{\gamma}}{b}\frac{1}{n}\rho_{m}$$
(6b)

$$\dot{\rho}_{m}^{-}(lock) = 4\chi_{lock}\frac{\dot{\gamma}}{b}\frac{n-1}{n}\rho_{m} \tag{6c}$$

Here $\dot{\gamma}$ is the shear strain rate, $\chi_{annihil}$ is a critical distance below which two dislocations with antiparallel burgers vectors can annihilate, and χ_{dipol} and χ_{lock} are critical distances for the spontaneous formation of dipoles and locks/junctions, respectively.

Annihilation can take place with the help of cross-slip and/or dislocation climb. As our tests were performed at room temperature, cross-slip is likely the dominant controlling mechanism in Eq. (6a). *n* is the number of active slip systems, under the assumption of an equal density of moving dislocations on each of these systems. The combination of terms due to annihilation Eq. (6a) and dipole formation Eq. (6b) yields $\dot{\rho}_m^-(annihil + dipol) = 2\chi_{dipol_D} \frac{\dot{\gamma}}{h} \frac{1}{n} \rho_m$. As $\dot{\rho}_m^- = \frac{d\rho_m}{dv}$, we get:

$$\frac{d\rho_m^-}{d\gamma}(annihil + dipol) = \frac{2\chi_{dipol}}{b}\frac{1}{n}\rho_m = C_{annihil+dipol}\rho_m$$
(7a)

$$\frac{d\rho_m^-}{d\gamma}(lock) = \frac{4\chi_{lock}}{b}\frac{n-1}{n}\rho_m = C_{lock}\ \rho_m \tag{7b}$$

By accounting for the elastic interactions between dislocations, or by invoking line tension calculations, the critical distances χ_{dipol} and χ_{lock} can be translated into critical effective shear stresses acting on dislocations, below which dipoles or locks are stable [67]:

$$\tau_{eff}(dipol) = \frac{Gb}{8\pi(1-\nu)\chi_{dipol}}$$
(8a)

$$\tau_{eff}(lock) = \frac{Gb}{2\pi\chi_{lock}}$$
(8b)

Combining Eqs. (7) and (8), we obtain for the mutual annihilation/immobilization rate *C*:

$$C = \frac{G}{\pi} \left(2 - \frac{2}{n} + \frac{1}{4(1-v)n} \right) \frac{1}{\tau_{eff}}$$
(9)

We write the effective shear stress allowing to unlock an immobile configuration as $\tau_{eff} = \tau_{yield} - \tau_{pin}$, where τ_{yield} is the yield strength of micro-pillars estimated at 0.2% of plastic strain in our compression experiments. We note that when *n* is large, the contribution of lock/junction formation in Eq. (9), $\left(2 - \frac{2}{n}\right)$, strongly dominates the contribution due to annihilation and dipole formation, $\frac{1}{4(1-\nu)n}$.

At this stage, a connection between the size effect on wildness, and the well-documented size effect on strength can be made. Although various power law exponents have been proposed to describe the scaling relation between τ_{yield} and *L* for different materials [55,68], our experimental data justify a materialindependent linear relation $\tau_{yield} - \tau_{pin} \sim Gb/L$ (Fig. 11a), which is consistent with the re-evaluation of a large set of published experimental data [69]. Actually, the data collapse shown in Fig. 11a implicitly supports the source truncation mechanism [49,70]. In this case, the effective stress $\tau_{yield} - \tau_{pin}$ required to activate a dislocation source of length λ_s goes as $1/\lambda_s$ [70], whereas recent DDD simulations [71,72] argue for a dislocation source length scaling as external size, $\lambda_s \sim L$. The combination of these relations yields the observed size effect in Fig. 11a. The scattering of source lengths contributes to the scatter of measured effective stress [70], but doesn't change the overall scaling trend. Combining Eq. (9) and our observed correlation between $\tau_{yield} - \tau_{pin}$ and 1/L (Fig. 11a), we verify the scaling $C \sim L/b$.

From the knowledge of the proportionality coefficient between $\tau_{vield} - \tau_{pin}$ and 1/L, equal to 40.8 Pa m, we obtain the proportionality coefficient in the relation $C \sim L/b$ once *n* is known. Our micro-pillars were compressed along the <110> direction, so there are two slip planes and four equal slip systems ((111) < 10-1>, (111))<01-1>, (11-1) <101> and (11-1) <011>) with the same Schmidt factor. For pure Al. Al-Sc cluster and Al-Sc precipitates, concentrated slip bands are generally observed (Fig. 10), suggesting $n \approx 2$. Instead, the much more homogeneous transversal deformation of Al-Cu-Sn samples suggests that n = 4 is more reasonable in this case. The proportionality coefficient in the relation $C \sim L/b$ changes slightly from 0.065 for n = 2 to 0.087 for n = 4. Following these numbers, we obtain for C the values ranging from ~100 for pure Al 500 nm-micropillars, to ~1000 for Al-Cu-Sn 3500 nm-micropillars. The observed correlation $\kappa - 1 = C/D \sim L/l$, with a dimensionless proportionality coefficient equal to 0.41, yields $D \sim l/b$ with a proportionality coefficient equal to 0.15 for n = 2. Our estimate shows that D ranges from ~100 for Al-Sc precipitate to ~800 for pure Al. As reliable estimates of C/D cannot be obtained for Al-Cu-Sn micro-pillars (see section 4.2), the extrapolation of the above reasoning to this material would be speculative. Therefore, the corresponding *D*-values are not presented.

The obtained numerical values of the parameters *C* and *D* bring the model in good agreement with experimental trends, and rationalize the competition between the effects of external size and disorder (Fig. 5a). However, the applicability of our model to the case of the mild Al-Cu-Sn alloy cannot be verified due to the lack of reliable *D*-values. In addition, the *C*/*D* value might saturate towards



Fig. 11. The correlations between the power law exponent, $\kappa - 1 = C/D$, and size dependent effective shear stress, τ_{eff} . (a) The effective shear stress defined as $\tau_{eff} = \tau_{yield} - \tau_{pin}$, where τ_{yield} is the yield strength of micro-pillars estimated as the stress at 0.2% of plastic strain, as a function of external size *L*. The relation $\tau_{eff} \sim 1/L$ can be identified. (b) The power law exponent, $\kappa - 1$, as a function of $\tau_{pin}/(\tau_{yield} - \tau_{pin})$.

the mean-field value of 0.5 for very small *R* (Fig. 7c). In this case, the above calculations would slightly underestimate *C*/*D* in this range. We also note that the crystal orientation effect on wildness can be reflected in the parameter *n*. In case of single-slip orientation where $n \approx 1$, the formation of locks/junctions is almost negligible, and the calculated coefficient of $C \sim L/b$ is only 0.020, much smaller than in our multi-slip orientation case (0.065–0.087). This move the wild-to-mild transition to larger sizes, in agreement with microcompression experiments of Ni crystals with <–269> orientation showing power-law distributed plastic bursts [13].

Finally, and most importantly, the observed correlation between the power law exponent $\kappa - 1 = C/D$ and the ratio τ_{eff}/τ_{pin} , shown on Fig. 11b, confirms the established relation $C \sim \frac{L}{b} \sim \frac{G}{\tau_{yield} - \tau_{pin}}$ and $D \sim \frac{l}{b} = \frac{G}{\tau_{pin}}$, and builds a conceptual bridge between the size effects on strength and wildness. This scaling $\frac{\tau_{eff}}{\tau_{pin}} \sim \frac{D}{C} \sim \frac{1}{R}$, which shows that our parameter R simultaneously controls the size effects on wildness (Fig. 7a), hardening mechanism (Fig. 7b) and strength (Fig. 11b), is fully compatible with the approach of Zaiser and Sanfeld [45], who argued from similitude principles that the increase of flow stress due to external size (i.e., $\frac{ au_{eff}}{ au_{pin}}$ in our case) should, for pure metals, only depend on the product $L_{\sqrt{\rho_f}}$ (i.e., our *R* for pure metal). Our results demonstrate that this can be extended to alloys, if one appropriately adjusts the definition of the dimensionless parameter $R = \frac{L}{T}$. This pivotal role of R has been independently emphasized in the study of the effect of irradiation defects on the size dependence of strength in Copper [73].

7. Concluding comments

We performed compression tests on micro-pillars of pure Aluminum and Al-alloys single crystals strengthened by different types of solutes or precipitates, with diameters ranging from 500 to 6000 nm, and quantified the statistical nature of plastic fluctuations occurring during deformation. From the obtained experimental data, we could make the following general observations:

- (1) Diminishing the external length scale (miniaturization) intensifies fluctuations and contributes to criticality. This is the "smaller is wilder" effect, that we quantified through the degree of intermittency W (the fraction of plastic deformation occurring through power law distributed avalanches) as well as through the time averaged scaling exponent of the avalanche size distributions.
- (2) Introducing quenched disorder shifts the transition from "wild" to "mild" plasticity towards smaller external length scales. This illustrates the "dirtier is milder" effect, and opens the possibility to mitigate plastic instabilities at small length scales.
- (3) The inter-relation of size and disorder effects reveals itself through a universal (material-independent) mapping between the power law exponent of avalanche size distribution and the parameter *W*.
- (4) By translating the pinning strength of obstacles into a characteristic length scale *l*, we showed that a single nondimensional parameter R = L/l controls both the transition from source-exhaustion hardening to forest hardening, and the transition from wild to mild fluctuations. It also determines the scaling exponent of the tail of the avalanche size distribution.
- (5) While jerky deformation and high flow stresses have been simultaneously observed at micro- and nano-scales by many authors, there has been so far no established link between

them, either theoretically or experimentally. We now quantify this link through our dimensionless parameter *R* which appears to be behind both phenomena.

(6) A mean field model for the stochastic evolution of mobile dislocation density is proposed, which successfully recovers the universal relationship between the scaling exponent and wildness. It also rationalizes the competition between external (size related) and internal (disorder related) length scales on plastic fluctuations as well as the link between the size effects on strength and wildness.

In summary, our results provide a new unifying perspective on micro-plasticity linking together a broad range of relevant phenomena including wildness, avalanche size distribution, hardening mechanisms, strain heterogeneity, and high flow stresses.

Acknowledgements

Financial support for this work was provided by the Natural Science Foundation of China (Grant Nos. 51621063, 51625103, 51571157, and 51201123), the 111 Project of China (Grant No. B06025) and the French ANR-2008 grant EVOCRIT. J.W. and L. T. acknowledge the hospitality of the Aspen Center for Physics, supported by the NSF Grant No. PHY-1066293. This work is also supported by the International Joint Laboratory for Micro/Nano Manufacturing and Measurement Technologies.

Appendix A. Supplementary data

Supplementary data related to this article can be found at http://dx.doi.org/10.1016/j.actamat.2017.02.039.

References

- M. Zaiser, Scale invariance in plastic flow of crystalline solids, Adv. Phys. 55 (2006) 185–245.
- [2] G. Liu, G.J. Zhang, F. Jiang, X.D. Ding, Y.J. Sun, J. Sun, E. Ma, Nanostructured high-strength molybdenum alloys with unprecedented tensile ductility, Nat. Mater. 12 (2013) 344–350.
- [3] K.A. Dahmen, Y. Ben-Zion, J.T. Uhl, Micromechanical model for deformation in solids with universal predictions for stress-strain curves and slip avalanches, Phys. Rev. Lett. 102 (2009).
- [4] P.D. Ispánovity, L. Laurson, M. Zaiser, I. Groma, S. Zapperi, M.J. Alava, Avalanches in 2D dislocation systems: plastic yielding is not depinning, Phys. Rev. Lett. 112 (2014) 235501.
- [5] J. Weiss, T. Richeton, F. Louchet, F. Chmelik, P. Dobron, D. Entemeyer, M. Lebyodkin, T. Lebedkina, C. Fressengeas, R.J. McDonald, Evidence for universal intermittent crystal plasticity from acoustic emission and highresolution extensometry experiments, Phys. Rev. B 76 (2007).
- [6] N. Friedman, A.T. Jennings, G. Tsekenis, J.-Y. Kim, M. Tao, J.T. Uhl, J.R. Greer, K.A. Dahmen, Statistics of dislocation slip avalanches in nanosized single crystals show tuned critical behavior predicted by a simple mean field model, Phys. Rev. Lett. 109 (2012).
- [7] J. Weiss, D. Marsan, Three-dimensional mapping of dislocation avalanches: clustering and space/time coupling, Science 299 (2003) 89–92.
- [8] J. Weiss, M.C. Miguel, Dislocation avalanche correlations, Mat. Sci. Eng. A 387 (2004) 292–296.
- [9] B.B. Mandelbrot, Fractals and Scaling in Finance, vol. 391, Spring, 1997, p. 758.
 [10] J. Weiss, W. Ben Rhouma, T. Richeton, S. Dechanel, F. Louchet, L. Truskinovsky,
- From mild to wild fluctuations in crystal plasticity, Phys. Rev. Lett. 114 (2015).
 M. Zaiser, Statistical aspects of microplasticity: experiments, discrete dislocation simulations and stochastic continuum models, J. Mech. Beh. Mater. 22 (2013) 89–100
- M.D. Uchic, D.M. Dimiduk, J.N. Florando, W.D. Nix, Sample dimensions influence strength and crystal plasticity, Science 305 (2004) 986–989.
- [13] D.M. Dimiduk, C. Woodward, R. LeSar, M.D. Uchic, Scale-free intermittent flow in crystal plasticity, Science 312 (2006) 1188–1190.
- [14] J.R. Greer, W.C. Oliver, W.D. Nix, Size dependence of mechanical properties of gold at the micron scale in the absence of strain gradients, Acta Mater. 53 (2005) 1821–1830.
- [15] S. Brinckmann, J.-Y. Kim, J.R. Greer, Fundamental differences in mechanical behavior between two types of crystals at the nanoscale, Phys. Rev. Lett. 100 (2008).
- [16] K.S. Ng, A.H.W. Ngan, Stochastic nature of plasticity of aluminum micro-

pillars, Acta Mater. 56 (2008) 1712–1720.

- [17] K.S. Ng, A.H.W. Ngan, Deformation of micron-sized aluminium bi-crystal pillars, Philos. Mag. 89 (2009) 3013–3026.
- [18] F.F. Csikor, C. Motz, D. Weygand, M. Zaiser, S. Zapperi, Dislocation avalanches, strain bursts, and the problem of plastic forming at the micrometer scale, Science 318 (2007) 251–254.
- [19] A.S. Argon, Strain avalanches in plasticity, Philos. Mag. 93 (2013) 3795-3808.
- [20] T. Richeton, J. Weiss, F. Louchet, Breakdown of avalanche critical behaviour in polycrystalline plasticity, Nat. Mater. 4 (2005) 465–469.
- [21] R.Z. Valiev, R.K. Islamgaliev, I.V. Alexandrov, Bulk nanostructured materials from severe plastic deformation, Prog. Mater. Sci. 45 (2000) 103–189.
- [22] M. Ovaska, L. Laurson, M.J. Alava, Quenched pinning and collective dislocation dynamics, Sci. Rep. 5 (2015).
- [23] G. Liu, G.-J. Zhang, R.-H. Wang, W. Hu, J. Sun, K.-H. Chen, Heat treatmentmodulated coupling effect of multi-scale second-phase particles on the ductile fracture of aged aluminum alloys, Acta Mater. 55 (2007) 273–284.
- [24] R. Gu, A. Ngan, Size effect on the deformation behavior of duralumin micropillars, Scr. Mater. 68 (2013) 861–864.
- [25] B. Girault, A.S. Schneider, C.P. Frick, E. Arzt, Strength effects in micropillars of a dispersion strengthened superalloy, Adv. Eng. Mater. 12 (2010) 385–388.
 [26] H. Ghassemi-Armaki, R. Maaß, S.P. Bhat, S. Sriram, J.R. Greer, K.S. Kumar,
- [26] H. Ghassemi-Armaki, R. Maaß, S.P. Bhat, S. Sriram, J.R. Greer, K.S. Kumar, Deformation response of ferrite and martensite in a dual-phase steel, Acta Mater. 62 (2014) 197–211.
- [27] W. Chen, J. Zhang, S. Cao, Y. Pan, M. Huang, Q. Hu, Q. Sun, L. Xiao, J. Sun, Strong deformation anisotropies of ω-precipitates and strengthening mechanisms in Ti-10V-2Fe-3Al alloy micropillars: precipitates shearing vs precipitates disordering, Acta Mater. 117 (2016) 68–80.
- [28] L.T. Stephenson, M.P. Moody, P.V. Liddicoat, S.P. Ringer, New techniques for the analysis of fine-scaled clustering phenomena within atom probe tomography (APT) data, Microsc. Microanal. 13 (2007) 448–463.
- [29] P.M. Kelly, A. Jostsons, R.G. Blake, J.G. Napier, Determination of foil thickness by scanning-transmission electron-microscopy, Phys. Status. Solidi. A 31 (1975) 771–780.
- [30] J.F. Nie, B.C. Muddle, Strengthening of an Al-Cu-Sn alloy by deformationresistant precipitate plates, Acta Mater. 56 (2008) 3490–3501.
- [31] E.A. Marquis, Microstructural Evolution and Strengthening Mechanisms in Alsc and Al-mg-sc Alloys, Ph. D thesis, Northwestern University, 2003.
- [32] D.N. Seidman, E.A. Marquis, D.C. Dunand, Precipitation strengthening at ambient and elevated temperatures of heat-treatable Al(Sc) alloys, Acta Mater. 50 (2002) 4021–4035.
- [33] B.A. Chen, G. Liu, R.H. Wang, J.Y. Zhang, L. Jiang, J.J. Song, J. Sun, Effect of interfacial solute segregation on ductile fracture of Al-Cu-Sc alloys, Acta Mater. 61 (2013) 1676–1690.
- [34] H. Justinger, G. Hirt, Estimation of grain size and grain orientation influence in microforming processes by Taylor factor considerations, J. Mater. Process. Technol. 209 (2009) 2111–2121.
- [35] S. Thangaraju, M. Heilmaier, B.S. Murty, S.S. Vadlamani, On the estimation of true hall-petch constants and their role on the superposition law exponent in Al alloys, Adv. Eng. Mater. 14 (2012) 892–897.
- [36] T.S. Noggle, J.S. Koehler, Electron microscopy of aluminum crystals deformed at various temperatures, J. Appl. Phys. 28 (1957) 53–62.
- [37] R. Maas, P.M. Derlet, J.R. Greer, Small-scale plasticity: insights into dislocation avalanche velocities, Scr. Mater. 69 (2013) 586–589.
- [38] A. Clauset, C.R. Shalizi, M.E.J. Newman, Power-law distributions in empirical data, Siam. Rev. 51 (2009) 661–703.
- [39] R. Maas, M. Wraith, J.T. Uhl, J.R. Greer, K.A. Dahmen, Slip statistics of dislocation avalanches under different loading modes, Phys. Rev. E 91 (2015) 42403.
- [40] J. Alcala, J. Ocenasek, K. Nowag, D. Esque-de los Ojos, R. Ghisleni, J. Michler, Strain hardening and dislocation avalanches in micrometer-sized dimensions, Acta Mater. 91 (2015) 255–266.
- [41] O.U. Salman, L. Truskinovsky, Minimal integer automaton behind crystal plasticity, Phys. Rev. Lett. 106 (2011).
- [42] J. Weiss, F. Louchet, Seismology of plastic deformation, Scr. Mater. 54 (2006) 747–751.
- [43] B. Devincre, T. Hoc, L. Kubin, Dislocation mean free paths and strain hardening of crystals, Sci. (New York, N.Y) 320 (2008) 1745–1748.
- [44] L. Kubin, B. Devincre, T. Hoc, Modeling dislocation storage rates and mean free paths in face-centered cubic crystals, Acta Mater. 56 (2008) 6040–6049.

- [45] M. Zaiser, S. Sandfeld, Scaling properties of dislocation simulations in the similitude regime, Model. Simul. Mater. S. C. 22 (2014) 065012.
- [46] J.A. El-Awady, Unravelling the physics of size-dependent dislocation-mediated plasticity, Nat. Commun. 6 (2015).
- [47] R. Maaß, M.D. Uchic, In-situ characterization of the dislocation-structure evolution in Ni micro-pillars, Acta Mater. 60 (2012) 1027–1037.
- [48] D.M. Norfleet, D.M. Dimiduk, S.J. Polasik, M.D. Uchic, M.J. Mills, Dislocation structures and their relationship to strength in deformed nickel microcrystals, Acta Mater. 56 (2008) 2988–3001.
- [49] D. Kiener, A.M. Minor, Source-controlled yield and hardening of Cu(100) studied by in situ transmission electron microscopy, Acta Mater. 59 (2011) 1328–1337.
- [50] C.P. Frick, B.G. Clark, S. Orso, A.S. Schneider, E. Arzt, Size effect on strength and strain hardening of small-scale [1 1 1] nickel compression pillars, Mater. Sci. Eng. A 489 (2008) 319–329.
- [51] C.A. Volkert, E.T. Lilleodden, Size effects in the deformation of sub-micron Au columns, Philos. Mag. 86 (2006) 5567–5579.
- [52] A.S. Schneider, C.P. Frick, B.G. Clark, P.A. Gruber, E. Arzt, Influence of orientation on the size effect in bcc pillars with different critical temperatures, Mater. Sci. Eng. A 528 (2011) 1540–1547.
- [53] C. Motz, D. Weygand, J. Senger, P. Gumbsch, Initial dislocation structures in 3-D discrete dislocation dynamics and their influence on microscale plasticity, Acta Mater. 57 (2009) 1744–1754.
- [54] D. Kiener, A. Minor, Source truncation and exhaustion: insights from quantitative in situ TEM tensile testing, Nano Lett. 11 (2011) 3816–3820.
- [55] J.R. Greer, J.T.M. De Hosson, Plasticity in small-sized metallic systems: intrinsic versus extrinsic size effect, Prog. Mater. Sci. 56 (2011) 654–724.
- [56] K.S. Ng, A.H.W. Ngan, Breakdown of Schmid's law in micropillars, Scr. Mater. 59 (2008) 796–799.
- [57] A.M. Hussein, S.I. Rao, M.D. Uchic, D.M. Dimiduk, J.A. El-Awady, Microstructurally based cross-slip mechanisms and their effects on dislocation microstructure evolution in fcc crystals, Acta Mater. 85 (2015) 180–190.
- [58] A. Nicolas, K. Martens, J.-L. Barrat, Rheology of athermal amorphous solids: revisiting simplified scenarios and the concept of mechanical noise temperature, EPL Europhys. Lett. 107 (2014) 44003.
- [59] M. Zaiser, P. Hähner, The flow stress of fractal dislocation arrangements, Mater. Sci. Eng. A 270 (1999) 299–307.
- [60] J. Lin, M. Wyart, Mean-field description of plastic flow in amorphous solids, Phys. Rev. X 6 (2016) 011005.
- [61] I. Groma, M. Zaiser, P.D. Ispánovity, Dislocation patterning in a twodimensional continuum theory of dislocations, Phys. Rev. B 93 (2016) 214110.
- [62] J.W. Christian, The stress dependence of dislocation velocity, and its relation to the strain rate sensitivity, Acta Metall. 12 (1964) 99–102.
- [63] P.P. Gillis, J.J. Gilman, Dynamical dislocation theory of crystal plasticity. II. Easy glide and strain hardening, J. Appl. Phys. 36 (1965) 3380–3386.
- [64] W.D. Nix, S.-W. Lee, Micro-pillar plasticity controlled by dislocation nucleation at surfaces, Philos. Mag. 91 (2011) 1084–1096.
- [65] L.F. Cugliandolo, The effective temperature, J. Phys. A-math. Theor. 44 (2011).
 [66] F. Roters, D. Raabe, G. Gottstein, Work hardening in heterogeneous alloys—a microstructural approach based on three internal state variables, Acta Mater.
- 48 (2000) 4181–4189.
 [67] R.J. Amodeo, N.M. Ghoniem, Dislocation dynamics. I. A proposed methodology
- for deformation micromechanics, Phys. Rev. B 41 (1990) 6958.
- [68] O. Kraft, P.A. Gruber, R. Mönig, D. Weygand, Plasticity in confined dimensions, Ann. Rev. Mater. Res. 40 (2010) 293–317.
- [69] D. Dunstan, A. Bushby, The scaling exponent in the size effect of small scale plastic deformation, Int. J. Plast. 40 (2013) 152–162.
- [70] T.A. Parthasarathy, S.I. Rao, D.M. Dimiduk, M.D. Uchic, D.R. Trinkle, Contribution to size effect of yield strength from the stochastics of dislocation source lengths in finite samples, Scr. Mater 56 (2007) 313–316.
- [71] Y.N. Cui, P. Lin, Z.L. Liu, Z. Zhuang, Theoretical and numerical investigations of single arm dislocation source controlled plastic flow in FCC micropillars, Int. J. Plast. 55 (2014) 279–292.
- [72] I. Ryu, W. Cai, W.D. Nix, H. Gao, Stochastic behaviors in plastic deformation of face-centered cubic micropillars governed by surface nucleation and truncated source operation, Acta Mater 95 (2015) 176–183.
- [73] D. Kiener, P. Hosemann, S.A. Maloy, A.M. Minor, In situ nanocompression testing of irradiated copper, Nat. Mater 10 (2011) 608–613.

364