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MODEL OF A WEAKLY NON-LOCAL RELAXING COMPRESSIBLE MEDIUM*

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A model of a weakly non-local relaxing medium with viscous dispersion is considered. The relaxation kinetics are described by a Ginzburg-Landau /1/ equation which has been generalized to the case of a compressible medium. The special features of the propagation of planar acoustic waves in the medium are studied. The latter medium has an internal time scale which arises from the description of the relaxation kinetics and a spatial scale which characterizes the degree of the non-localness of the medium. General methods for constructing models of equilibrium non-local media have been developed in /2-5/. The generalization of these methods to the case of a relaxing medium enables one to describe the structure of a non-equilibrium phase discontinuity and to calculate the dissipation on the conversion front /6/.

1. Let us assume that the internal energy u of a unit mass is a function of the system of parameters

$$s, \rho g^{ij}, \xi_\alpha, \xi_\alpha^{\cdot}, \nabla_i\xi_\alpha, \nabla_j\nabla_i\xi_\alpha, \ldots \tag{1.1}$$

where s is the entropy per unit mass of the medium, ρ is the density, g^{ij} are the contravariant components of the metric tensor in the Euclidean Eulerian system of coordinates x^i $(i = 1, 2, 3)$, ξ_α $(\alpha = 1, \ldots, n)$ are additional scalar parameters (internal degrees of freedom), the total derivative with respect to time is denoted by a dot and ∇_i is a covariant derivative in the coordinate system x_i. The thermal influx equation can be written in the form /3, 6, 7/

$$du = \rho^{-1}(-pg^{ij} + \tau^{ij} + \sigma^{ij})\nabla_j v_i \, dt - \rho^{-1}\nabla_k(q^k + Q^k)\, dt \tag{1.2}$$

where p is the pressure, τ^{ij} are the components of the viscous stress tensor, v_i are the components of the velocity vector of the medium, q^k are the components of the thermal flux vector, Q^k are the components of the vector describing the flux of non-thermal forms of energy, σ^{ij} are the components of the reactive stress tensor and Q^k and σ^{ij} are functions

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of the system of parameters (1.1). The entropy balance equation has the form

$$ds = \rho^{-1}\nabla_k J^k dt + d_i s \tag{1.3}$$

where J^k are the components of the entropy flux vector which we shall assume to be equal to q^k/T and $d_i s$ is the entropy production due to irreversible processes occurring in the medium. By making use of the fact that $d_i s$ is non-negative, we find /6/

$$T = \frac{\partial u}{\partial s}, \quad p = \rho^2 \frac{\partial u}{\partial \rho}, \quad \frac{\partial u}{\partial \xi_\alpha^{\cdot}} = \frac{\partial u}{\partial \nabla_j \nabla_i \xi_\alpha} = \frac{\partial u}{\partial \nabla_k \nabla_j \nabla_i \xi_\alpha} = \ldots = 0 \tag{1.4}$$
$$Q^k = -\rho \frac{\partial u}{\partial \nabla_k \xi_\alpha} \xi_\alpha^{\cdot} + \Psi^k (s, \rho, g^{ij}, \xi_\alpha, \nabla_i \xi_\alpha, \nabla_j \nabla_i \xi_\alpha \ldots)$$
$$\nabla_k \Psi^k = 0, \quad \sigma^{ij} = -\rho \frac{\partial u}{\partial \nabla \xi_\alpha} \nabla^i \xi_\alpha$$

Hence, the assumptions which have been made eliminate spatial derivatives higher than the first order from the list of arguments of the specific internal energy while only retaining them in the expression for the function Ψ^k which, by virtue of the penultimate relationship of (1.4) only appears in the boundary conditions. It can be made to disappear by a corresponding redefinition of the vectors q^k and J^k. As a consequence of the invariance of the internal energy with respect to solid rotations of the coordinate system x^i, the reactive stress tensor, σ^{ij}, is symmetric. The inclusion of the derivatives $\nabla_i \xi_\alpha^{\cdot}$ among the arguments of Q^k leads to a model of a moment medium with an asymmetric stress tensor. In this case,

$$\sigma^{ij} = -\rho \frac{\partial u}{\partial \nabla_j \xi_\alpha} \nabla^i \xi_\alpha - \rho \frac{\partial u}{\partial \nabla_j \nabla_k \xi_\alpha} \nabla_k \nabla^i \xi_\alpha + \nabla_k \left(\rho \frac{\partial u}{\partial \nabla_k \nabla_j \xi_\alpha}\right) \nabla^i \xi_\alpha$$

2. We will now consider the treatment of the dissipative effects. From equations (1.2)-(1.4), we have

$$\rho \frac{d_i s}{dt} = \frac{\tau^{ij}}{T} \nabla_j v_i - \frac{q^k}{T^2} \nabla_k T - \frac{\rho \xi_\alpha^{\cdot}}{T} \frac{\delta_\rho u}{\delta \xi_\alpha} \tag{2.1}$$
$$\left(\frac{\delta_\rho u}{\delta \xi_\alpha} = \frac{\partial u}{\partial \xi_\alpha} - \frac{1}{\rho} \nabla_k \left(\rho \frac{\partial u}{\partial \nabla_k \xi_\alpha}\right)\right)$$

(the expression for the variational derivative is shown in brackets /6/). We note that the internal energy $u(s, \rho, g^{ij}, \xi_\alpha, \nabla_i \xi_\alpha)$ on the right-hand side of Eq.(2.1) may be replaced by another thermodynamic potential such as $g(p, T, g^{ij}, \xi_\alpha, \nabla_i \xi_\alpha) = u - Ts + p/\rho$.

By using the phenomenological approach to the thermodynamics of irreversible processes /8/, we establish kinetic relationships which determine τ^{ij}, q^k, $\xi_\alpha^{\cdot}$. Let us now consider expression (2.1) for the rate of entropy production as a bilinear function of the generalized fluxes τ^{ij}/T, q^i/T^2, $\rho \xi_\alpha^{\cdot}/T$ and the corresponding generalized forces $\nabla_j v_i$, $-\nabla_i T$, $\delta_\rho u/\delta \xi_\alpha$. Then,

$$\tau^{ij}/T = L^{ijkl} \nabla_l v_k - L^{ijk} \nabla_k T - L_\alpha^{ij} \delta_\rho u/\delta \xi_\alpha \tag{2.2}$$
$$q^i/T^2 = L_1^{ijk} \nabla_j v_k - L^{ij} \nabla_j T - L_\alpha^{i} \delta_\rho u/\delta \xi_\alpha$$
$$\rho \xi_\alpha^{\cdot}/T = L_{1\alpha}^{ij} \nabla_i v_i - L_{1\alpha}^{i} \nabla_i T - L_{\alpha\beta} \delta_\rho u/\delta \xi_\beta$$

When account is taken of the fact that τ^{ij}/T and $\nabla_j v_i$ are an odd flux and force and that q^i/T^2 and $\rho \xi_\alpha^{\cdot}/T$ and $-\delta_\rho u/\delta \xi_\alpha$ are even fluxes and forces, the Onsager relations have the form

$$L^{ijkl} = L^{klij}, \quad L^{ij} = L^{ji}, \quad L_{\alpha\beta} = L_{\beta\alpha} \tag{2.3}$$
$$L_1^{ijk} = -L^{ijk}, \quad L_{1\alpha}^{ij} = -L_\alpha^{ij}, \quad L_{1\alpha}^{i} = L_\alpha^{i}$$

Then tensors L^{ijkl}, L^{ijk} and L_α^{ij} must be symmetrical with respect to ij (by virtue of the symmetry of τ^{ij}).

Let us now consider the case when $\alpha = 1$. As the arguments of the phenomenological coefficients

$$L^{ijkl}, \quad L^{ijk}, \quad L^{ij}, \quad L_\alpha^{ij}, \quad L_\alpha^{i}, \quad L_{\alpha\beta} \tag{2.4}$$

which figure in (2.2), we select the following quantities

$$s, \quad \rho, \quad \xi, \quad |\nabla \xi|, \quad \nu^i, \quad g^{ij}$$
$$|\nabla \xi| = (\nabla_k \xi \nabla^k \xi)^{1/2}, \quad \nu^i = \nabla^i \xi / |\nabla \xi|$$

The dependence of the coefficients (2.4) on the tensor arguments v^i, g^{ij} can be explicitly indicated /2/. Then,

$$\tau^{ij} = (\zeta e_k{}^k + \mu_3 e_{kl} v^k v^l - \gamma_1 v^k \nabla_k T - \zeta_1 \delta_\rho u/\delta\xi) + 2\mu (e^{ij} - {}^1/_3 e_k{}^k g^{ij}) + \tag{2.5}$$
$$2\mu_1 (e^{jk} v^i v_k + e^{ik} v^j v_k) + (\mu_3 e_k{}^k + \mu_2 e_{kl} v^k v^l - \gamma_2 v^k \nabla_k T -$$
$$\zeta_2 \delta_\rho u/\delta\xi)\, v^i v^j - \gamma_3 (v^i \nabla^j T + v^j \nabla^i T)$$
$$q^i = -\chi \nabla^i T - (T\gamma_3 e_k{}^k + T\gamma_2 e_{kl} v^k v^l + \chi_1 v^k \nabla_k T +$$
$$T\gamma_4 \delta_\rho u/\delta\xi)\, v^i - T(\gamma_1 v^k \nabla^i v_k + \gamma_3 v^k \nabla_k v^i)$$
$$\rho\xi^{\cdot} = -\zeta_1 e_k{}^k - \zeta_2 e_{kl} v^k v^l - \gamma_4 v^k \nabla_k T - \rho\Gamma \delta_\rho u/\delta\xi$$
$$(e_{ij} = {}^1/_2 (\nabla_i v_j + \nabla_j v_i))$$

Generally speaking, there are 14 kinetic parameters, which depend on s, ρ, ξ, $|\nabla\xi|$, in the model under consideration. In order to satisfy the condition $d_i s \geqslant 0$, it is necessary and sufficient to impose the following constraints on them.

$$\mu, \mu_1, \mu_2, \zeta, \chi, \chi_1, \Gamma \geqslant 0; \quad \zeta\mu_2 \geqslant \mu_3^2, \quad \rho\chi_1\Gamma \geqslant \gamma_4^2$$

We note that no constraints whatsoever are imposed on the quantities $\zeta_1, \zeta_2, \gamma_1, \gamma_2, \gamma_3$ since the corresponding terms in (2.5) do not make any contribution to the entropy production.

For simplicity, we shall subsequently neglect the dependence of the fluxes τ^{ij}/T and $\rho\xi^{\cdot}/T$ on $\nabla_i T$ and also assume that $q^k = q^k(s, \rho, \xi, |\nabla\xi|, g^{ij}, \nabla_i T)$. In this case the model is characterized by nine kinetic parameters, the number of which is reduced to six if the medium can be assumed to be incompressible $(e_k{}^k = 0)$.

3. Let us now consider the state of isothermal equilibrium. The distributions of ρ and ξ_α are described by the system of equations

$$\frac{\partial f}{\partial \xi_\alpha} - \frac{1}{\rho}\nabla\left(\rho \frac{\partial f}{\partial \nabla_j \xi_\alpha}\right) = 0, \quad \nabla_j p^{ij} + \rho F^i = 0 \tag{3.1}$$
$$f(\rho, T_0, g^{ij}, \xi_\alpha, \nabla_i \xi_\alpha) = u - T_0 s$$
$$p^{ij} = -\rho^2 \frac{\partial f}{\partial \rho} g^{ij} - \rho \frac{\partial f}{\partial \nabla_j \xi_\alpha} \nabla^i \xi_\alpha,$$

where f is the Helmholtz free energy of unit mass of the medium. It is obvious that the stress tensor is not spherical and the system of Eqs.(3.1) is therefore redefined. As a consequence of the special structure of the stress tensor of Eq.(3.1), they have a first integral

$$g - \Phi = \text{const} \tag{3.2}$$

where $g(p, T_0, g^{ij}, \xi_\alpha, \nabla_i \xi_\alpha) = f + p/\rho$ is the specific Gibbs energy and Φ is the potential due to external mass forces.

The Bernoulli integrals for stationary flows and the Cauchy-Lagrange integrals for non-stationary potential flows may be established on the basis of formula (3.2) in connection with isothermal flows of an ideal equilibrium medium in the field of external potential forces. The fact that the integrals exist is independent of the maximum order of the derivatives of the parameters ξ_α which are the arguments of the free energy function (cf. /9/) and, at the same time, in the case $f = f(\rho, T_0, g^{ij}, \xi_\alpha, \nabla_i \xi_\alpha, \nabla_j \nabla_i \xi_\alpha, \ldots)$, the second equation of (3.1) remains true and the stress tensor acquires the form

$$p^{ij} = -\rho^2 \frac{\partial f}{\partial \rho} g^{ij} - \rho\left(\frac{\partial f}{\partial \nabla_j \xi_\alpha} - \frac{1}{\rho}\nabla_k\left(\rho \frac{\partial f}{\partial \nabla_k \nabla_j \xi_\alpha}\right) +\right.$$
$$\left.\frac{1}{\rho}\nabla_k \nabla_l\left(\rho \frac{\partial f}{\partial \nabla_k \nabla_l \nabla_j \xi_\alpha}\right) - \ldots\right)\nabla^i \xi_\alpha - \rho\left(\frac{\partial f}{\partial \nabla_k \nabla_j \xi_\alpha} - \frac{1}{\rho}\nabla_l\left(\rho \frac{\partial f}{\partial \nabla_l \nabla_k \nabla_j \xi_\alpha}\right) + \ldots\right)\times$$
$$\nabla_k \nabla^i \xi_\alpha - \rho\left(\frac{\partial f}{\partial \nabla_l \nabla_k \nabla_j \xi_\alpha} - \ldots\right)\nabla_l \nabla_k \nabla^i \xi_\alpha$$

while the equilibrium condition (the first relation of (3.1)) is transformed into

$$\frac{\partial f}{\partial \xi_\alpha} - \frac{1}{\rho}\nabla_j\left(\rho \frac{\partial f}{\partial \nabla_j \xi_\alpha}\right) + \frac{1}{\rho}\nabla_k \nabla_j\left(\rho \frac{\partial f}{\partial \nabla_k \nabla_j \xi_\alpha}\right) - \ldots = 0$$

4. The features of the propagation of acoustic waves in a local relaxing medium are well-known /10/. The appearance in the theory of a time scale associated with the relaxation kinetics ensures the dispersion of the waves and their anomalous attenuation at a characteristic frequency. We will now show what changes occur when the non-local character is taken into account. We begin with the case of a non-viscous, non-thermally conducting medium and

consider the simplest expression for the internal energy, which depends on $\nabla\xi$ /6, 7/

$$u = u_0(s, \rho, \xi) + {}^1/_2\varepsilon(\nabla\xi)^2 \tag{4.1}$$

where $\varepsilon > 0$ is the parameter describing the non-local nature of the medium. We assume that the initial equilibrium state is homogeneous: $s \equiv s_0$, $\rho \equiv \rho_0$, $\xi \equiv \xi^*(s_0, \rho_0)$. The function $\xi^*(s, \rho)$ describes the dependence of the equilibrium values of the parameter ξ on the specific entropy and density. We require that

$$(\partial^2 u/\partial\xi^2)_0 > 0, \quad (2\rho\partial u/\partial\rho + \rho^2\partial^2 u/\partial\rho^2)_0 > 0$$

The system of equations which describes the motion of a non-viscous, non-thermally conducting relaxing medium with an energy (4.1) has the form

$$\partial\rho/\partial t + \nabla_i(\rho v^i) = 0, \quad \rho dv^i/dt = -\nabla^i p - \nabla_j(\varepsilon\rho\nabla^i\xi\nabla^j\xi)$$
$$d\xi/dt = -\Gamma(\partial u/\partial\xi - \varepsilon(\Delta\xi + (\nabla\xi\rho^{-1}\nabla\rho)))$$
$$ds/dt = \Gamma(\partial u/\partial\xi - \rho^{-1}\nabla_j(\varepsilon\rho\nabla^j\xi))^2, \quad p = \rho^2\partial u/\partial\rho$$

By linearizing this system with respect to the initial state of equilibrium, we arrive at equations which describe (to a first approximation) an isentropic perturbation of the parameters in the acoustic wave. As conditions for the existence of solutions proportional to $\exp i(\mathbf{kr} - \omega t)$, we get the dispersion relationship

$$A\sigma^4 + (1 - B - i\eta - A\eta^2)\sigma^2 - \eta^2(1 - i\eta) = 0 \tag{4.2}$$
$$\sigma = c_\infty\tau|\mathbf{k}|, \quad \eta = \omega\tau, \quad A = l_\varepsilon^2/l^2, \quad B = 1 - c_*^2/c_\infty^2$$
$$\tau = 1/(\Gamma(\partial^2 u/\partial\xi^2)_0), \quad l = c_\infty\tau, \quad l_\varepsilon = (\varepsilon/(\partial^2 u/\partial\xi^2)_0)^{1/2}$$
$$c_\infty = (\partial p/\partial\rho)_0^{1/2}, \quad c_* = (\partial p/\partial\rho + (\partial p/\partial\xi)\,\partial\xi^*/\partial\rho)_0^{1/2}.$$

where σ and η are the dimensionless wavenumber and the cyclic frequency, A and B are the main dimensionless criteria of the problem, τ and l are the characteristic (kinetic) time and spatial scales, l_ε is the spatial scale associated with the non-local nature of the medium and c_∞ and c_* are the quenched and equilibrium velocities of sound. The dimensionless parameters A and B characterize the measure of the non-local nature of the medium and the difference between the quenched and equilibrium velocities of sound.

The solution of Eq.(4.2) has two branches

$$\sigma_\pm = (2A)^{-1}\{A\eta^2 + i\eta - (1 - B) \pm [(A\eta^2 + i\eta - (1 - B))^2 + 4A\eta^2(1 - i\eta)]^{1/2}\} \tag{4.3}$$

one of which (that with the plus sign) corresponds to the usual acoustic wave in an ideal relaxing compressible fluid while the other (that with the minus sign) is spherical in the case of the non-local medium being considered. Both wave modes correspond to longitudinal vibrations. The classical theory /11/ is obtained in the limit as $A \to 0$.

For the acoustic branch in the low-frequency domain $(\eta \ll 1)$, we have

$$\sigma_+ = \frac{1}{(1-B)^{1/2}}\eta + \frac{iB}{2(1-B)^{3/2}}\eta^2 + \frac{1}{8}\left[\frac{B^2 - 4B(1+A)}{(1-B)^{5/2}}\right]\eta^3 - \frac{i}{16}\left[\frac{B^3 - 4B^2(1-A) + 8B(1+2A)}{(1-B)^{7/2}}\right]\eta^4 + \ldots \tag{4.4}$$

The first two terms of this asymptotic form are identical with those obtained in /11/ while, apart from terms of the order of η^2, the phase velocity of the acoustic wave $(\omega/\mathrm{Re}\,|\mathbf{k}|)$ is equal to c_*, the dimensionless absorption coefficient $(\mathrm{Im}\,\sigma)$ is of the order of η^2 and the perturbations ρ', ξ' and $\mathbf{v}'$ in the wave are connected by the relationships

$$B^{1/2}\rho'/\rho_0 + \xi' = 0, \quad |\mathbf{v}'| = c_*\rho'/\rho_0$$

In the high-frequency case $(\eta \gg 1)$, we have

$$\sigma_+ = \eta + \frac{B}{2}\frac{1}{A\eta} + \frac{iB}{2}\left(\frac{1}{A\eta}\right)^2 \tag{4.5}$$

The phase velocity of the acoustic wave tends to c_∞ (as in the classical case when

$A = 0$), the dimensionless absorption coefficient decays in proportion to η^{-2} (it is constant in a classical sense) (Fig.1) while the perturbations of the parameters in the wave, to a first approximation with respect to η^{-1} satisfy the relations

$$\xi' = 0, \quad |\mathbf{v}'| = c_\infty \rho'/\rho_0$$

The dependence of the dimensionless absorption coefficient at the wavelength ($\gamma = \operatorname{Im}\sigma/\operatorname{Re}\sigma$) of the acoustic branch σ_+ on the cyclic frequency η is shown in Fig.2. The characteristic frequency of maximum absorption of sound is practically independent of A. The values $A = 0$ and 10^{-2} (when $B = 0.5$) correspond to curves *1* and *2* in the figures.

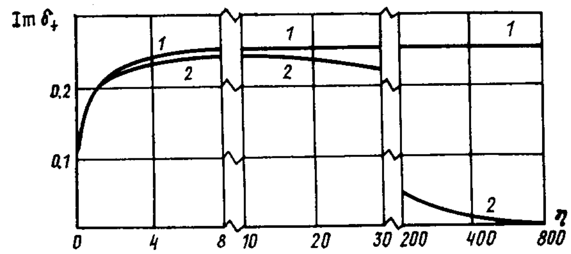

Fig.1

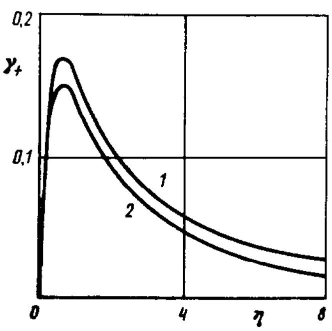

Fig.2

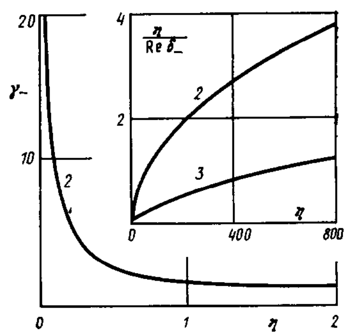

Fig.3

The asymptotic form (4.5) is uniformly suitable in the domain $\eta \gg A^{-1/4}$. An intermediate asymptotic form, which is valid when $1 \ll \eta \ll A^{-1/4}$ $(A \ll 1)$ can be obtained by passing to the limit $A \to 0$, $\eta \to +\infty$, $A^{1/4}\eta = \text{const} \stackrel{\text{def}}{=} \alpha$ in formula (4.3). In this case

$$\sigma_+ = \eta + {}^1/_2 iB + {}^1/_8 [B^2 + 4B(1-B) + \alpha]\eta^{-1} - {}^1/_{16}\, i\,[B^3 + 4B^2(1-B+\alpha) + 8B((1-B+\alpha)^2 - \alpha B)]\eta^{-2} + \ldots \tag{4.6}$$

As $A \to 0$, the left-hand boundary of the domain of applicability of relationship (4.5) goes to infinity while formulae (4.4) and (4.6) are transformed into the classical asymptotic dispersion relationships /11/.

We will now analyse the second branch of relationship (4.3) which describes the rapidly decaying sequence parameter waves (SPW). In the low-frequency case $(\eta \ll 1)$, we have

$$\sigma_- = i\left(\frac{1-B}{A}\right)^{1/2} + \frac{\eta}{2(1-B)^{1/2}A^{1/2}} + \ldots \tag{4.7}$$

To a first approximation with respect to η, the phase velocity and the dimensionless SPW absorption coefficient are equal to $2A^{1/2}c_*$ and $(1-B)^{1/2}/A^{1/2}$, respectively.

In the high frequency case $(\eta \gg 1)$

$$\sigma_- = (\eta/(2A))^{1/2} + i\,(\eta/(2A))^{1/2} + \ldots \tag{4.8}$$

The phase velocity and the absorption coefficient increase in proportion to $\eta^{1/2}$.

The perturbations in ρ', ξ' and $\mathbf{v}'$ in the wave are connected by the relationships

$$\rho'/\rho_0 + B^{1/2}\xi' = 0, \quad |\mathbf{v}'| = 2A^{1/2}c_*\rho'/\rho_0$$

in the case of low frequencies and

$$B^{1/2}\rho'/\rho_0 + \xi' = 0, \quad |\mathbf{v}'| = (2A\eta)^{1/2} c_\infty \rho'/\rho_e$$

in the high-frequency case.

Both in the acoustic wave and in the SPW, the perturbation of the pressure is determined by the formula $p' = c_\infty\rho' + \rho_0 c_\infty (\partial^2 u/\partial\xi^2)_0 B^{1/2}\xi'$.

The results of the numerical calculation of the dependence of the phase velocity and the absorption coefficient at the wavelength on the cyclic frequency are shown in Fig.3 for the σ_- branch of the dispersion relationship (4.3). When $B = 0.5$, the values $A = 10^{-2}$ and 10^{-3} correspond to curves 2 and 3. In the limit as $B \to 0$, the kinetic equation is separated from the remaining equations and, in this case, the perturbations ξ' and ρ', p', $\mathbf{v}'$ propagate independently.

The contribution to the dispersion and the absorption of the longitudinal acoustic waves due to the viscosity of the medium is characterized by the Reynolds number

$$\mathrm{Re} = \rho_0 c_\infty l/(4/3\mu_s + \mu_V), \quad \mu_s = \mu + \mu_1, \quad \mu_V = \zeta + 8/3\mu_1 + \mu_2 + 2\mu_3$$

The phase velocity of the low-frequency vibrations $(\eta \ll 1)$ does not change compared with the case of a non-viscous medium $(\mathrm{Re} \to +\infty)$ and the dimensionless absorption coefficient acquires the form $\eta^2 (1 - B)^{-3/2} (B + 1/\mathrm{Re})/2$.

In the low-frequency region, $(\eta \gg \mathrm{Re},\ \mathrm{Re} \gg 1)$ $\sigma_+ = (\mathrm{Re}\eta/2)^{1/2} + i(\mathrm{Re}\eta/2)^{1/2} + \ldots$

The phase velocity and the absorption coefficient increase in proportion to $\eta^{1/2}$.

If $1 \ll \eta \ll \mathrm{Re}\ (\mathrm{Re} \gg 1)$, the viscosity of the medium can be neglected in the first-order of approximation with respect to η^{-1}. At the same time, the asymptotic form (4.5) is valid in the case when $1 \ll 1/A^{1/2} \ll \eta \ll \mathrm{Re}$. If, however, $1 \ll \eta \ll 1/A^{1/2} \ll \mathrm{Re}$, then relationship (4.6) is satisfied.

In the low-frequency case, the phase velocity of the SPW has the form $2A^{1/2}(1 + B/\mathrm{Re})\, c_*$ and, to a first approximation with respect to η, the absorption coefficient does not change compared with the case of a non-viscous medium. In the high-frequency region $(\eta \gg 1)$, the effect of viscosity on the SPW does not show up in the first approximation with respect to $\eta^{-1/2}$ and the asymptotic form (1.8) therefore remains valid.

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