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Duality, inverse problems and nonlinear problems in solid mechanics

Periodic debonding of an adhesive film

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Abstract

The debonding of a thin film, glued to a rigid substratum by a soft adhesive, is known to proceed through the development of a regular array of micro-cavities. We propose a simple model of this phenomenon in the framework of the classical linear fracture mechanics. ***To cite this article: K.C. Le, L. Truskinovsky, C. R. Mecanique 336 (2008).***

Résumé

Délaminage périodique d'un film adhésif. On sait que le délaminage d'un film mince collé à un substrat rigide au moyen d'un adhésif souple se produit par formation d'un arrangement régulier de micro-cavités. Nous proposons un modèle simple de ce phénomène dans le cadre de la mécanique linéaire classique de la rupture. ***Pour citer cet article : K.C. Le, L. Truskinovsky, C. R. Mecanique 336 (2008).***

Keywords: Rupture; Adhesive film; Debonding

Mots-clés : Rupture ; Film adhésif ; Délaminage

1. Introduction

Experiments show that when an adhesive film is pulled out from a substratum by tensile forces, multiple cavities or micro-cracks form before the terminal full separation of the two adhered surfaces [1]. The collective appearance of cavities during the rupture of adhesive bonds is also observed in molecular dynamics simulations [2]. The aim of the present paper is to reproduce such a cooperative failure mechanism within a simple phenomenological 1-D model. To maximally simplify the elastic part of the problem we replace the film by an inextensible spring and model the adhesive interaction within the classical Griffith framework.

Our final goal is to understand the kinetic mechanism for the nucleation of micro-cavities leading to the subsequent formation of a macro-crack. The implicit assumption is that under tensile loading the fully bonded configuration corresponds to the local minimum of the energy and is therefore metastable. When the homogeneous loading is supplemented by superimposed fluctuations, the bonded configuration eventually gives rise to the fully detached con-

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figuration, representing the global minimum of the energy. The classical nucleation theory [3] allows one to compute, at a given tensile stress, the probability of debonding if the optimal path connecting the undamaged configuration with the fully damaged one is known. To reconstruct the mechanism of debonding and find the realistic nucleation path one must first select the minimal set of the relevant non-equilibrium parameters and then minimize out all remaining mechanical variables. What the nucleation theory expects from the mechanical model is the knowledge of the structure of the corresponding non-equilibrium energy landscape.

In the classical Griffith fracture mechanics such landscape is usually presented in terms of the variables characterizing the geometry of a single crack, most often simply in terms of its length. In this article, in order to show the cooperative nature of the debonding process we find it necessary to deal with two non-equilibrium parameters: one related to the geometry of the micro-configuration and another one to the cumulative description of the elastic field.

More specifically, we acknowledge the intrinsic translational invariance of the problem and search for the periodic equilibrium configurations with the fixed average separation and the given period. We find two distinct families of solutions which we refer to as the Griffith family and the Koiter family by alluding to the related results in a somewhat different setting [4,5]. In the configurations of the Griffith family the separation is positive everywhere except at the end points of the period. For Koiter configurations the crack opening is positive only *strictly* inside each period which implies that the cracks may be partially closed. Each Griffith solution can be extended by zero beyond one period and may be interpreted as an equilibrium configuration in the infinite domain representing an isolated crack. Therefore, the Griffith family is entirely classical, corresponding to the conventional interpretation of the crack nucleation process. On the contrary, the Koiter solutions are essentially periodic and can not be reduced to isolated cracks. Since they represent configurations where the mutual shielding of the micro-cracks is non-negligible, these solutions describe cooperative behavior during the nucleation process.

Due to the presence of the two families of the equilibrium configurations, the ensuing partially relaxed energy landscape is not single-valued. We show that the energy of the Koiter configurations, whenever they exist, is always smaller that the energy of the Griffith configurations corresponding to the same value of the fixed non-equilibrium parameters. Such multi-valuedness of the energy landscape gives rise to two different nucleation strategies aimed at the overcoming of the same energy barrier. Thus, in addition to the classical Griffith nucleation path, there is a nonclassical Koiter–Griffith nucleation path along which the energy which is strictly lower or equal to the energy associated with the Griffith path. The nonclassical path, which appears to be optimal, necessarily contains the Koiter configurations and this implies that the cooperation of the micro-cavities during the nucleation process is essential.

2. The model

Consider an infinite pre-stretched string adhered to a rigid foundation. The string is uniformly loaded by the appropriately normalized external tensile tractions σ (see Fig. 1). Denoting the normalized transverse displacement of the string by $w(x)$, we express the dimensionless energy of the string in the form

$$I\big[w(x)\big] = \int \left[\frac{1}{2}w'^2 + \gamma(w) - \sigma w\right] \mathrm{d}x \tag{1}$$

where the admissible field $w(x)$ is assumed to satisfy the impenetrability constraint $w(x) \geqslant 0$. The potential of adhesive forces is taken in the classical Griffith form:

$$\gamma(w) = \begin{cases} 0 & \text{for } w = 0 \\ 1 & \text{for } w > 0 \end{cases} \tag{2}$$

We assume that the partially equilibrated configurations of interest are periodic with the period $2c$;

$$w(-c) = w(c) = 0 \tag{3}$$

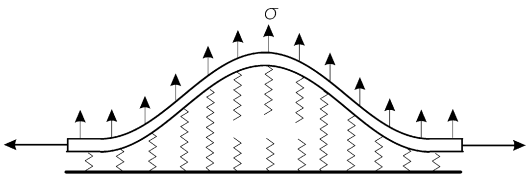

Fig. 1. The pull-out of a string from a rigid foundation.

To expose the complexity of the relevant energy landscape we impose an additional constraint on the equilibrium configuration by fixing the mean separation h

$$\frac{1}{2c}\int_{-c}^{c} w(x)\,\mathrm{d}x = h \tag{4}$$

The constraint (4) can be formally eliminated if we introduce the Lagrange multiplier λ and, using periodicity, search the extremals of

$$\tilde{I}\big[w(x)\big] = \int_{-c}^{c}\left[\frac{1}{2}w'^2 + \gamma(w) - \tau w\right]\mathrm{d}x$$

where $\tau = \sigma + \lambda/2c$. We denote by

$$E(c,h) = \tilde{I}\big[w(x)\big] = \frac{1}{2c}\int_{-c}^{c}\left[\frac{1}{2}w'^2 + \gamma(w) - \sigma w\right]\mathrm{d}x$$

the local minima of the energy (1) per period at the given c and h. Our goal is to reconstruct the non-equilibrium energy landscape $E(c,h)$ at the given σ.

The solution of the above variational problem is elementary. There are two types of equilibrium configurations which we refer to as Griffith or Koiter families. Griffith configurations are smooth inside the periodicity domain and the solution of the Euler–Lagrange equation can be written explicitly

$$w(x) = \frac{\tau}{2}(c^2 - x^2) \tag{5}$$

The Lagrange multiplier equals $\lambda = 2c(3h/c^2 - \sigma)$, and the energy per period can be written in the form

$$E_G(c,h) = \frac{3h^2}{c} + 2c - 2c\sigma h \tag{6}$$

For the Koiter configurations we assume that $w(x) = 0$ for $c \geqslant |x| \geqslant a$ and $w(x) > 0$ for $|x| < a$, where $a < c$ is an unknown internal length. Inside the interval $|x| < a$ we obtain

$$w(x) = \frac{\tau}{2}(a^2 - x^2)$$

The Lagrange multiplier is now $\lambda = 2c(3ch/a^3 - \sigma)$, which gives the following expression for the energy per period

$$E_K(c,h,a) = \frac{3c^2h^2}{a^3} + 2a - 2c\sigma h$$

We next minimize $E_K(c,h,a)$ with respect to a to obtain $a = \sqrt{3ch}/\sqrt[4]{2}$, provided $h \leqslant \sqrt{2}c/3$. The final expression for the energy of the Koiter configurations takes the form

$$E_K(c,h) = \frac{8\sqrt{3ch}}{3\sqrt[4]{2}} - 2c\sigma h \tag{7}$$

3. The energy landscape

To understand the ensuing structure of the energy surface $E(c,h)$ we first consider its cross-sections by the planes $c = \text{const}$. The general picture is presented in Fig. 2 for $c < c_n$ (a) and $c > c_n$ (b) where $c_n = \sqrt{2}/\sigma$. At each value of c we obtain two energy curves corresponding to two families of the equilibrium solutions. We observe that the Griffith energy curve always has one minimum at $h_G(c) = \sigma c^2/3$ (point G in Fig. 2(a), (b)), while the Koiter energy curve does not have extrema at $c < c_n$. For $c > c_n$ it has one maximum at $h_K(c) = 4/(3\sqrt{2}c\sigma^2)$ (point K in Fig. 2(b)). Notice also that the Griffith family is defined for all values h while the Koiter family is defined only for $h \leqslant \sqrt{2}c/3$. At $h = \sqrt{2}c/3$ the Koiter energy surface smoothly merges with the Griffith energy surface (point S in Fig. 2).

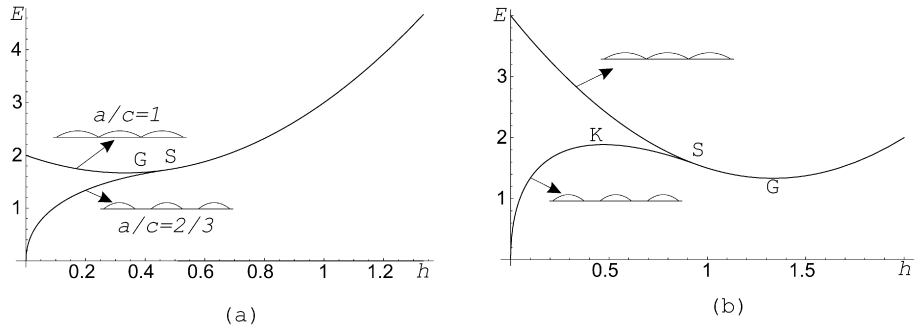

Fig. 2. The section of the energy landscape at fixed half period c and varying mean separation h; (a) $c = 1$, (b) $c = 2$. The load is $\sigma = 1$.

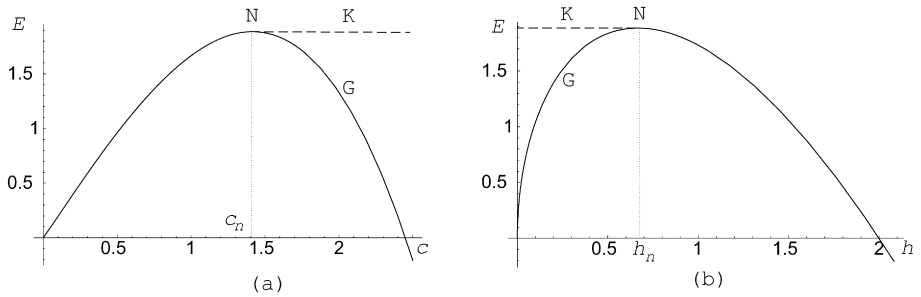

Fig. 3. Stationary values of $E(c, h)$ with respect to h at $\sigma = 1$: (a) versus c, (b) versus h.

According to the classical approach, only the stationary values of the function $E(c, h)$ with respect to h are relevant. The resulting 'relaxed' energy curve $E(c, h(c))$, presented in Fig. 3(a), (b), again contains two branches representing the Griffith configurations (solid curve) and the Koiter configurations (dashed curve). However, contrary to what is expected in the classical theory, for Koiter configurations the equilibrium value of the variable h corresponds to the maximum, rather than the minimum of the energy. One can also see that in addition to the one saddle point configuration of the classical theory (point N), the model predicts a continuous family of saddles (points K), all corresponding to the same value of the energy per period as the configuration N.

In classical nucleation theory the most probable path connecting two local minima of the energy must pass through the least energy barrier in the (c, h)-plane. We recall, however, that the energy surface is double-valued and therefore the representations of the nucleation mechanism is not unique. The classical picture can be recovered if the Koiter solutions are neglected. This gives (see Fig. 4(a)) the conventional nucleation path going through a single saddle point N at $h_n = 2/(3\sigma)$, $c_n = \sqrt{2}/\sigma$. The corresponding energy barrier is equal to $\Delta E_G = 4\sqrt{2}/(3\sigma)$. Such classical energy landscape, however, does not represent the minimum of the energy at each value of the parameters h and c. Instead, the energy minimizing landscape, presented in Fig. 4(b) can be viewed as a union of two surfaces, one corresponding to the Koiter configurations at $h \leqslant \sqrt{2}c/3$ and one, to the Griffith configurations at $h > \sqrt{2}c/3$. On the lowest energy landscape the traditional saddle point of the Griffith model (point N in Fig. 3(a)) is replaced by the ridge NL specified by the equation $h = c_n h_n/c$ (see Fig. 5).

To overcome the energy barrier the system may follow different paths depending on the dynamical model. Thus, the potentially relevant path, shown in Fig. 5(b) by the dotted line, minimizes the energy with respect to c for $c < c_n$ and with respect to h for $c > c_n$; the path minimizing energy with respect to h all the time makes sense only for the Griffith surface but not for the minimal energy surface. Alternatively, the system may choose the vicinity of the dashed path in Fig. 5 corresponding to the boundary between the Griffith and Koiter states. Finally, the nucleation mechanism may not be related at all to the classical saddle point N, and the optimal path may instead pass through one of the degenerate saddle points located along the ridge NKL. The non-classical nature of such saddle points poses an obvious problem for the application of the standard formulae of the nucleation theory. This leaves open the question of the characteristic time scale for debonding and calls for the careful reconsideration of the corresponding stochastic problem.

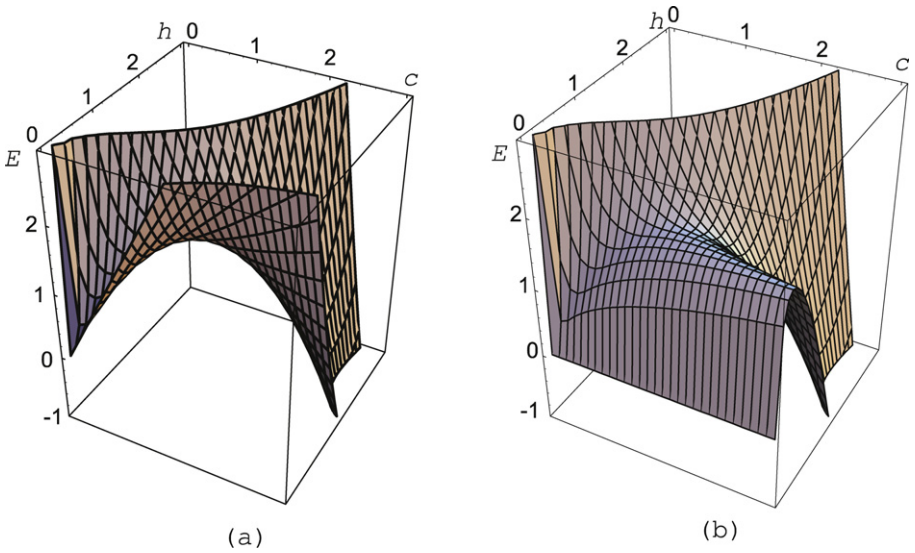

Fig. 4. Energy versus half-period c and mean separation h at $\sigma = 1$: (a) Griffith landscape, (b) the lowest energy landscape.

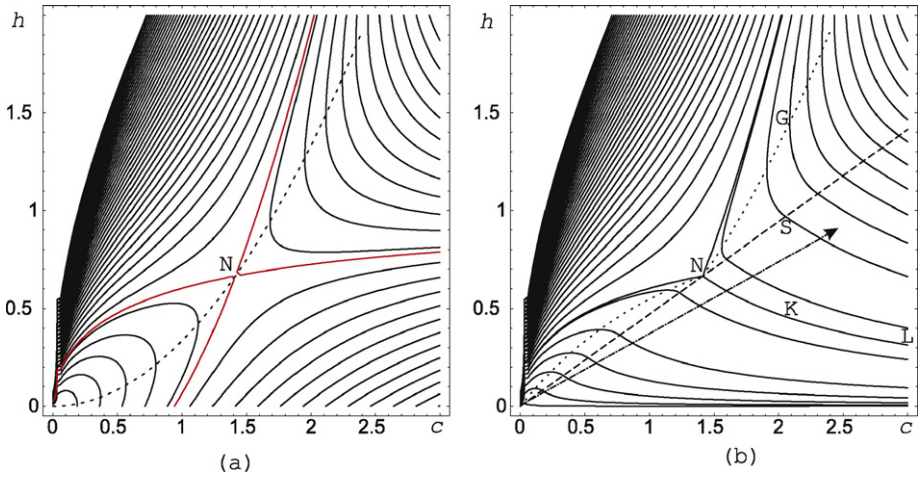

Fig. 5. Energy levels in the (c, h)-plane for $\sigma = 1$: (a) Griffith landscape, (b) the lowest energy landscape. The dash line marks the boundary between Griffith and Koiter states.

4. Conclusions

We have shown existence of the two types of paths connecting the perfectly bonded configuration with the completely de-bonded configuration, both going through the saddle points of the same height on the energy surface. One is the classical Griffith path, shown in all textbooks. According to our theory, this path is not always optimal. As we have shown, there exists an alternative Koiter–Griffith path along which the energy is almost always lower than along the Griffith path. Contrary to the Griffith path, the Koiter–Griffith path implies essential interaction of the micro-cracks during the initial stage of the nucleation process. At the later stages of the nucleation process the Koiter configurations give rise through void coalescence to Griffith configurations; the latter can be viewed as isolated cracks continuing to grow independently of each other. Yet, for sufficiently small micro-cavities the cooperative behavior appear to be unavoidable, which may be the case not only in the process of crack nucleation but also in the process of crack tip advancement, not considered here in detail.

While our findings are formulated in the simplest mechanical framework, the extensive additional computations, which will be reported elsewhere, confirm that the main results of the paper concerning the relevance of the periodic

configurations in the nucleation process remain basically unchanged if our schematic Frenkel–Kontorova type model is upgraded to the full scale 2D elasticity (Peierls–Nabarro model) and the Griffith surface energy is replaced by the more realistic Barenblatt type cohesive interactions.

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