Soft Solids

PLATES (and shells)

I Bending & Stretching for plates

1) stretching energy
2) Bending energy
3) Mechanics of tearing

II Small slope equations for plates

1) what to expect (small slope eq. for beams)
2) plate equations
3) A geometrical coupling

III Geometry of surfaces

1) Gauss theorem
2) crumpling
3) adhesion on a sphere
4) Actuation of thin sheets

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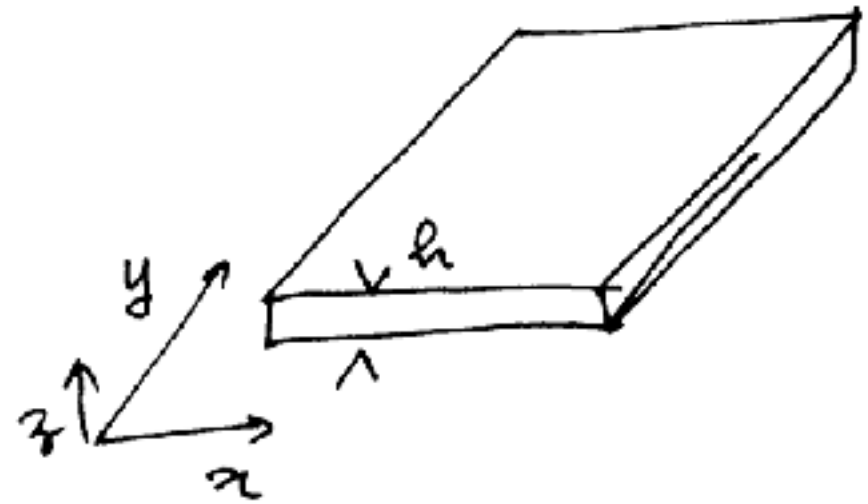
1) stretching energy



2D version of elastic equations in the plane

$\sigma_{xx}, \sigma_{yy}, \sigma_{yx}$ with $\sigma_{zz} = \sigma_{zx} = \sigma_{yz} = 0$

(forces $\perp$ to plate are zero)

Hooke law

$$\begin{cases} \sigma_{xx} = \dfrac{E}{1-\nu^2}\left(\varepsilon_{xx} + \nu\,\varepsilon_{yy}\right) \\ \sigma_{yy} = \dfrac{E}{1-\nu^2}\left(\varepsilon_{yy} + \nu\,\varepsilon_{xx}\right) \\ \sigma_{xy} = \dfrac{E}{1+\nu}\left(\varepsilon_{xy}\right) \end{cases}$$

1) stretching energy

Elastic stretching energy

$$\mathcal{U}_s = \iint \frac{h}{2}\,\sigma : \varepsilon \, dx\,dy = \frac{h}{2} \iint \left[\sigma_{xx}\varepsilon_{xx} + \sigma_{yy}\varepsilon_{yy} + 2\sigma_{xy}\varepsilon_{yx} \right] dx\,dy$$

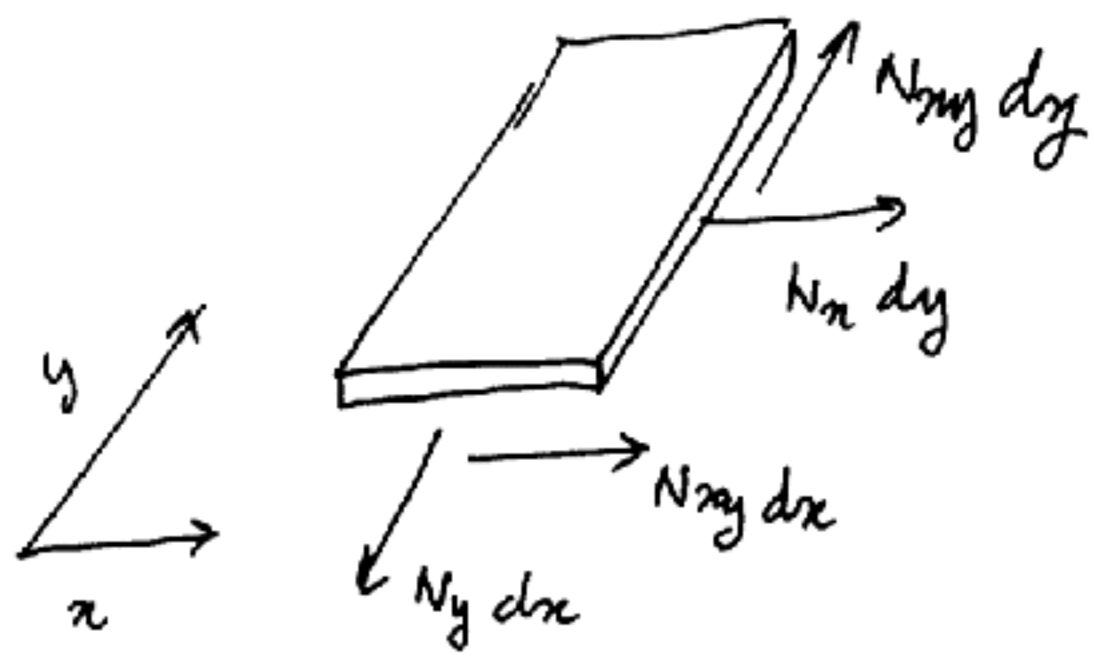
$$\boxed{\mathcal{U}_s = \iint \frac{1}{2}\,\frac{Eh}{1-\nu^2} \left[(\varepsilon_x + \varepsilon_y)^2 - 2(1-\nu)\left(\varepsilon_x \varepsilon_y - \varepsilon_{xy}^2\right) \right] dx\,dy}$$

$$\mathcal{U}_s = \iint \frac{1}{2}\,\frac{Eh}{1-\nu^2} \left[(\mathrm{Tr}\,\bar{\bar{\varepsilon}})^2 - 2(1-\nu) \det \bar{\bar{\varepsilon}} \right]$$

≥ 0 if $\nu < 1$

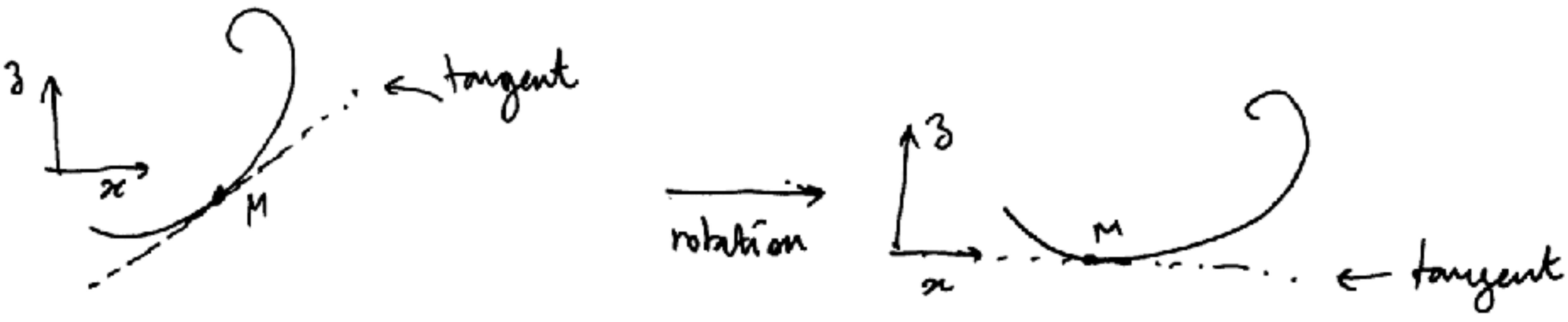
1) stretching energy

$$N_x = \int_{\text{thickness}} \sigma_{xx}\, dz \qquad N_y = \int_{\text{thickness}} \sigma_{yy}\, dz \qquad N_{xy} = \int_{\text{thickness}} \sigma_{xy}\, dz$$



2) Bending energy

RECAP : curvature for a line in 2D



$$z = z(M) + \alpha(x-x_M) + \frac{\beta}{2}(x-x_M)^2 + \cdots$$

Taylor expansion

$$K = \frac{d^2z}{dx^2}\frac{1}{\left(1+\left(\frac{dz}{dx}\right)^2\right)^{3/2}} = \frac{\beta}{(1+\alpha^2)^{3/2}}$$

$$z = z(M) + \frac{K(x-x_M)^2}{2} + \cdots$$

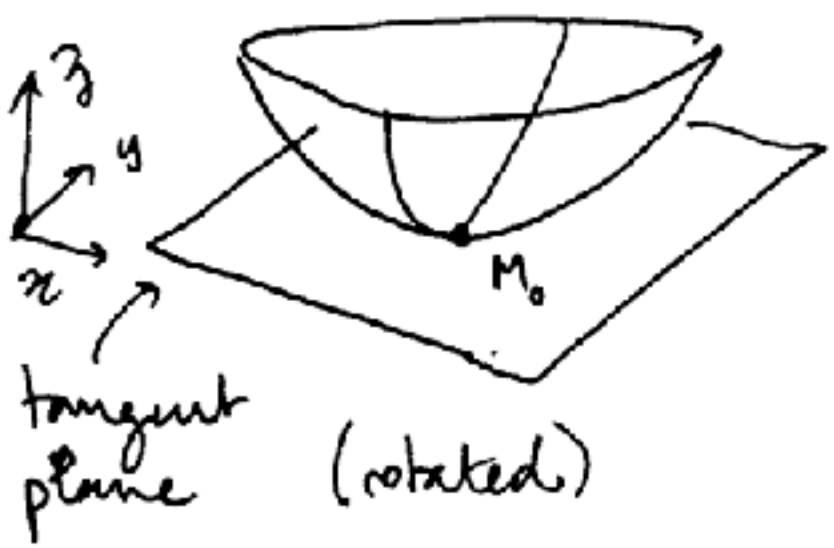
K curvature

$$K = \frac{d^2z}{dx^2}$$

what is curvature for a surface?

2) Bending energy

Curvature of surface



$$z(x,y) = z_{M_0} + \frac{1}{2}\left[a x^2 + 2bxy + c y^2\right] + \cdots$$

$$= \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} a & b \\ b & c \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

curvature tensor

rotation in plane $\quad z(x,y) = z_{M_0} + \frac{1}{2}\left[K_1 x^2 + K_2 y^2\right] + \cdots$

(K_1, K_2) principal curvature.

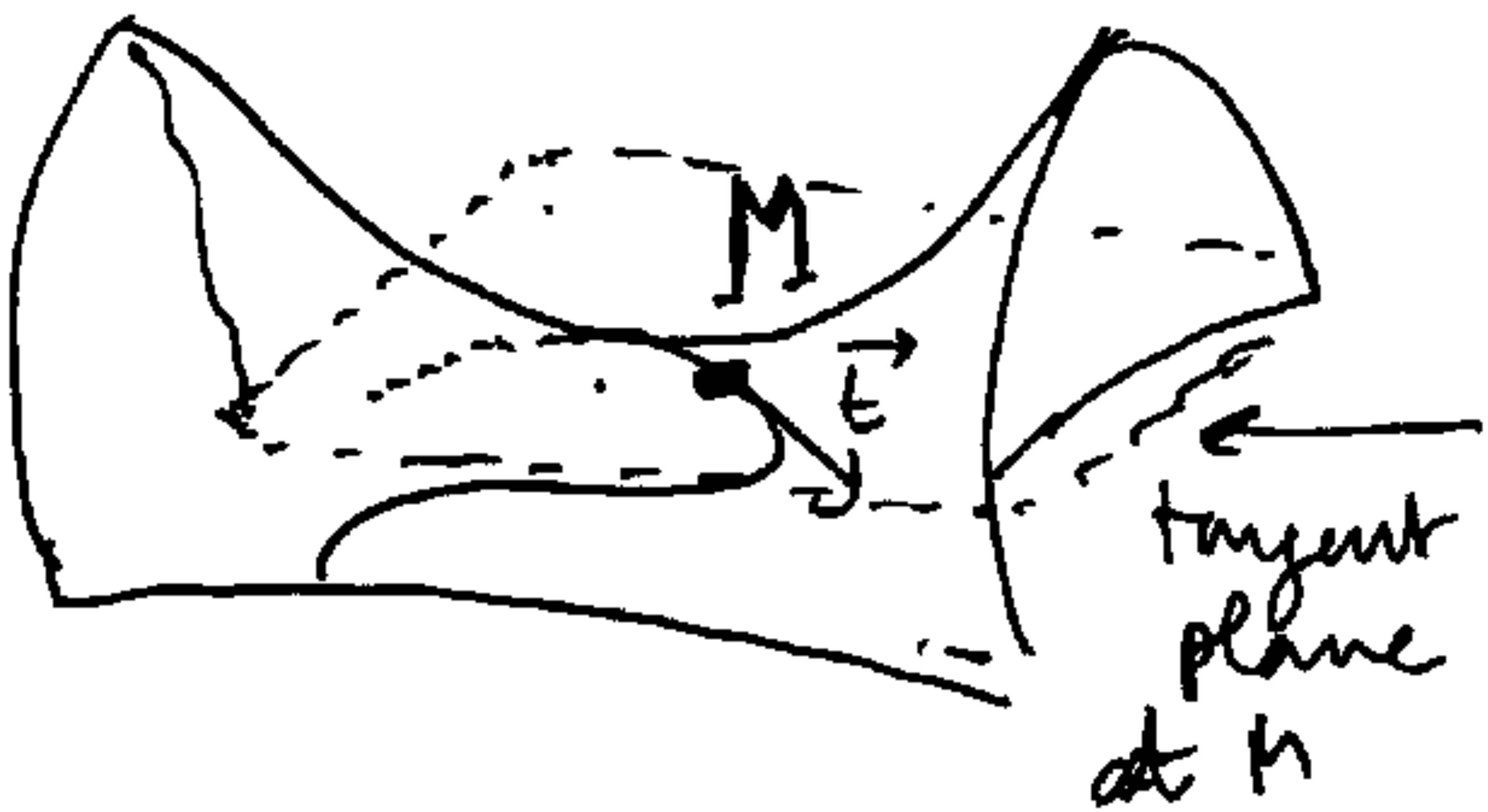
2) Bending energy

Interpretation

trace a curve on the surface

$$M = \left| \begin{array}{l} x(s) \\ y(s) \\ z[x(s), y(s)] \end{array} \right. \qquad z(x,y) = \frac{k_1}{2} x^2 + \frac{k_2}{2} y^2$$

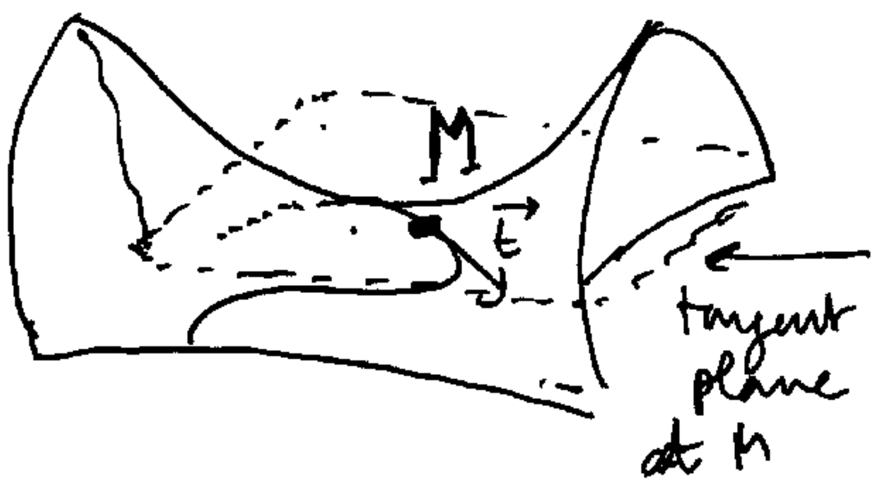
[s = curvilinear abscissa]



2) Bending energy

Interpretation

trace a curve on the surface



$$M = \begin{vmatrix} x(s) \\ y(s) \\ z[x(s), y(s)] \end{vmatrix} \qquad z(x,y) = \frac{k_1}{2} x^2 + \frac{k_2}{2} y^2$$

[s = curvilinear abscissa]

$$M_0 = M(s=0)$$

tangent : $$\vec{t} = \frac{dM}{ds} = \begin{vmatrix} dx/ds = x' \\ dy/ds = y' \\ k_1 x \frac{dx}{ds} + k_2 y \frac{dy}{ds} \end{vmatrix}$$

$$\frac{d\vec{t}}{ds} = \vec{K} = \begin{vmatrix} x'' \\ y'' \\ k_1 x'^2 + k_2 y'^2 + k_1 x x'' + k_2 y y'' \end{vmatrix}$$

Vector curvature for a curve in space

2) Bending energy

at s=0 (in M)

$$\vec{K} = \left| \begin{array}{l} x'' \\ y'' \\ k_1 \cos^2\theta + k_2 \sin^2\theta \end{array} \right.$$

x'', y'' } in-plane curvature = $\boxed{\text{"geodesic curvature"}}$

$k_1 \cos^2\theta + k_2 \sin^2\theta \rightarrow \boxed{\text{"normal" curvature}}$ only depends on orientation of tangent θ.

(k_1, k_2) . principal curvature,

. $\perp$ directions.

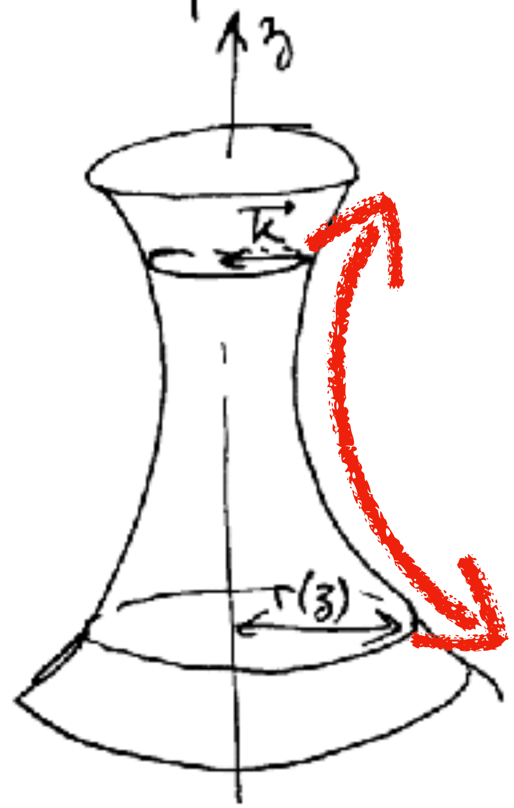
. maximal and minimal normal curvature

. $\frac{k_1+k_2}{2}$ = mean curvature; $k_1 k_2$ = Gaussian curvature

Laplace law $p = \gamma(k_1+k_2)$

2) Bending energy

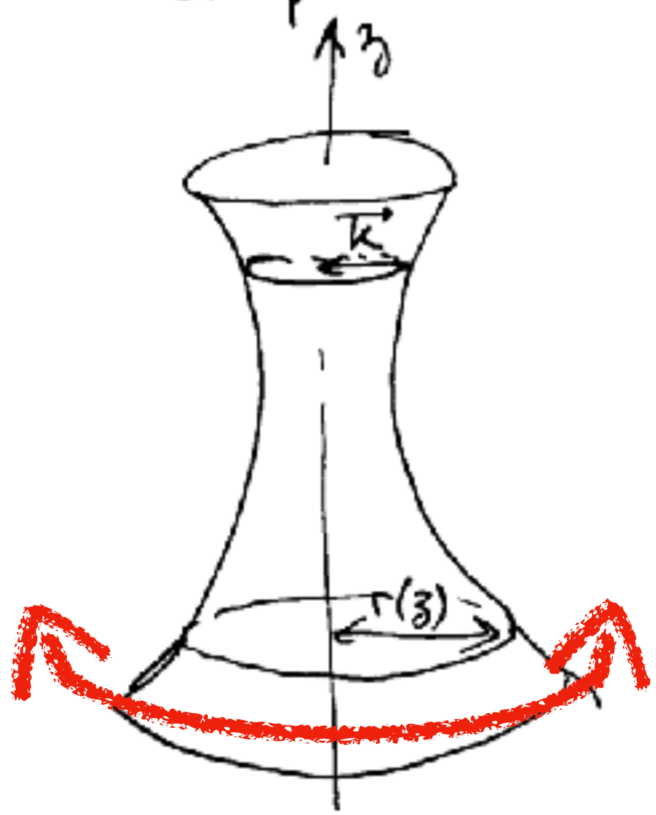
Example surface of revolution



$$k_1 = \frac{d^2 r}{dz^2}\left[1+\left(\frac{dr}{dz}\right)^2\right]^{-3/2}$$

2) Bending energy

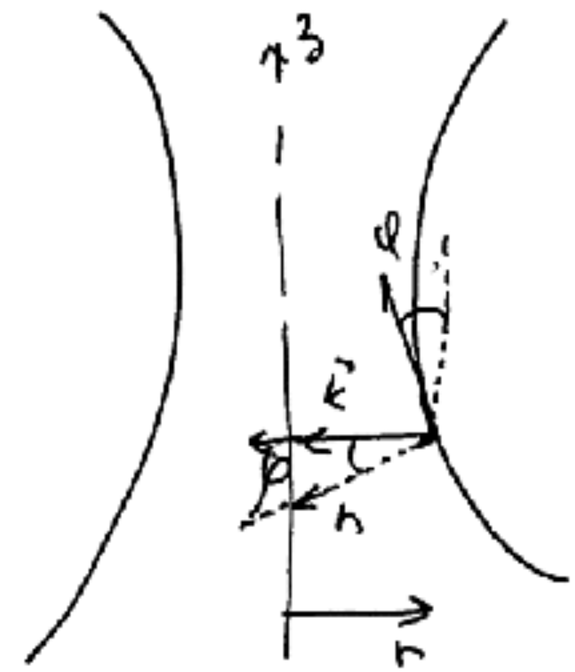
Example surface of revolution



$$K_1 = \frac{d^2r}{dz^2}\left[1+\left(\frac{dr}{dz}\right)^2\right]^{-3/2}$$

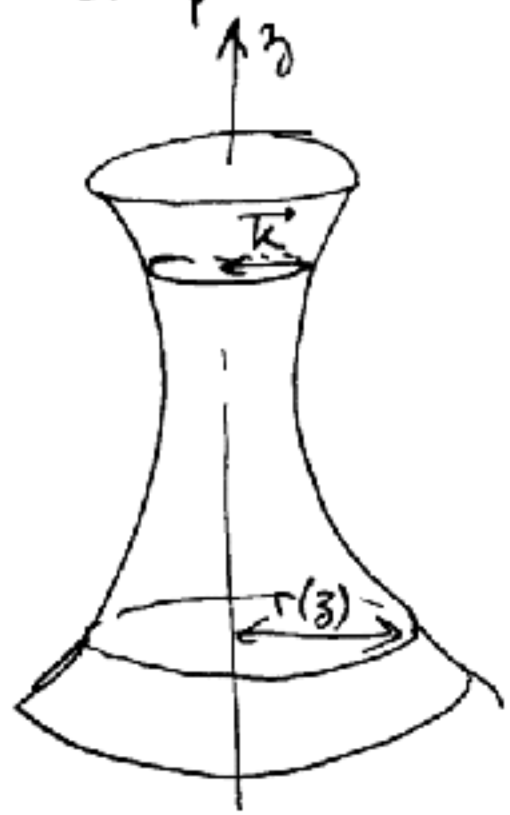
$$K_2 = \frac{1}{r}\cos\varphi$$ ← projection

$$\cos\varphi = \frac{dz}{ds} = \frac{1}{\sqrt{1+\left(\frac{dr}{dz}\right)^2}}$$



2) Bending energy

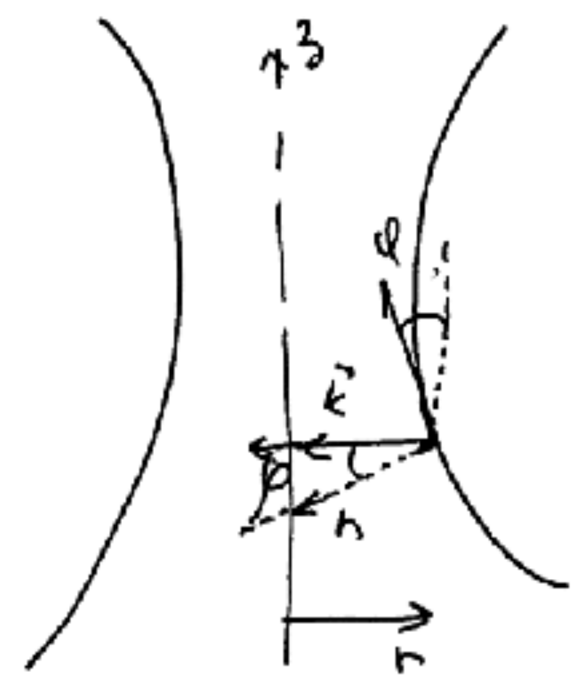
Example surface of revolution



$$K_1 = \frac{d^2r}{dz^2}\left[1+\left(\frac{dr}{dz}\right)^2\right]^{-3/2}$$

$$K_2 = \frac{1}{r}\cos\varphi \leftarrow \text{projection}$$

$$\cos\varphi = \frac{dz}{ds} = \frac{1}{\sqrt{1+\left(\frac{dr}{dz}\right)^2}}$$



NB: what is the surface with zero mean curvature (soap film)

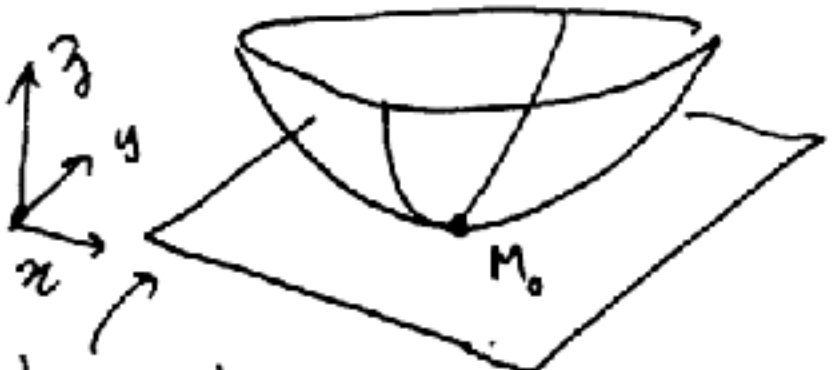
$$K_1 = K_2 \iff \frac{r''}{(1+r'^2)^{3/2}} = \frac{1}{r\,(1+r'^2)^{1/2}}$$

$$\iff r''r = 1+r'^2$$

$$r = A\,\mathrm{ch}\left(\frac{z-z_0}{A}\right)$$ catenoid!

2) Bending energy

Curvature of surface



$$z(x,y) = z_{M_0} + \frac{1}{2}\left[a x^2 + 2bxy + c y^2 \right] + \cdots$$

$$= \begin{bmatrix} x \\ y \end{bmatrix} \boxed{\begin{bmatrix} a & b \\ b & c \end{bmatrix}} \begin{bmatrix} x \\ y \end{bmatrix}$$

curvature tensor

(isotropic)

$$\mathcal{U}_{Bending} = F\left(\underset{k_1,\, k_2}{\text{invariant curvature}} \right)$$

2) Bending energy

$$\mathcal{U}_{Bending} = A(\kappa_1+\kappa_2)^2 + B\,\kappa_1\kappa_2$$

$$= A\,(\mathrm{Tr}\,C)^2 + B\,\det C$$

$$A, B \sim E h^3$$

$$\boxed{D = \frac{E h^3}{12(1-\nu^2)} \qquad \mathcal{U}_{Bending} = \frac{1}{2} D\left[(\mathrm{Tr}\,C)^2 - 2(1-\nu)\det C \right]}$$

or

$$\mathcal{U}_{Bending} = \frac{1}{2} D\left(\kappa_1^2 + \kappa_2^2 + 2\nu\,\kappa_1\kappa_2 \right)$$

$$\mathcal{U}_{\text{stretching}} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ (\mathrm{Tr}\,\varepsilon)^2 - 2(1-\nu)\det\varepsilon \right\} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ \varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 \right\}$$

principal strain

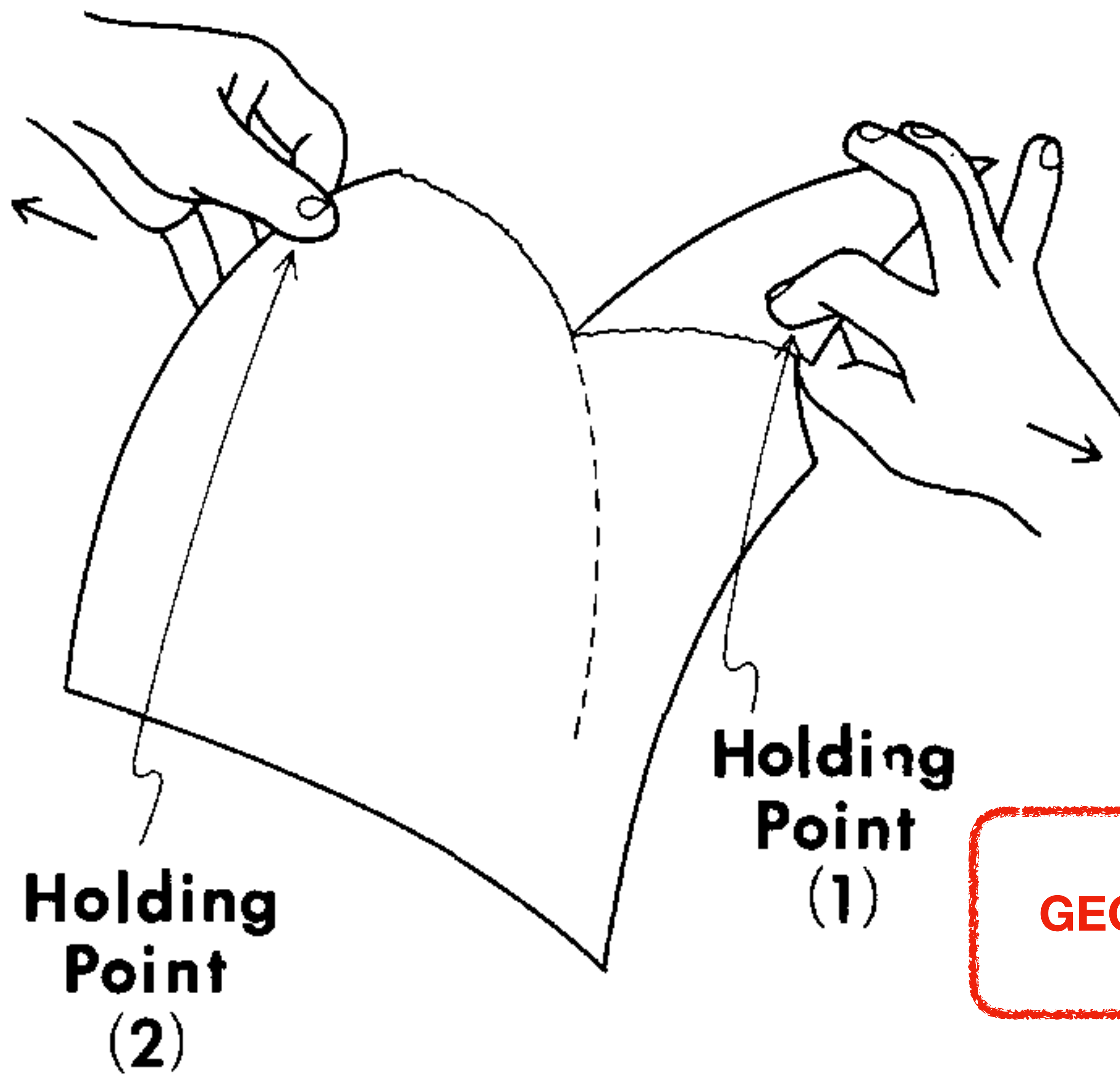
$$\mathcal{U}_{\text{bending}} = \frac{1}{2}\frac{Eh^3}{(1-\nu^2)12}\left\{ (\mathrm{Tr}\,C)^2 - 2(1-\nu)\det C \right\} = \frac{1}{2} D \left\{ k_1^2 + 2\nu k_1 k_2 + k_2^2 \right\}$$

principal curvature.

Bending + stretching energy

$\longrightarrow$ Equilibrium equations.

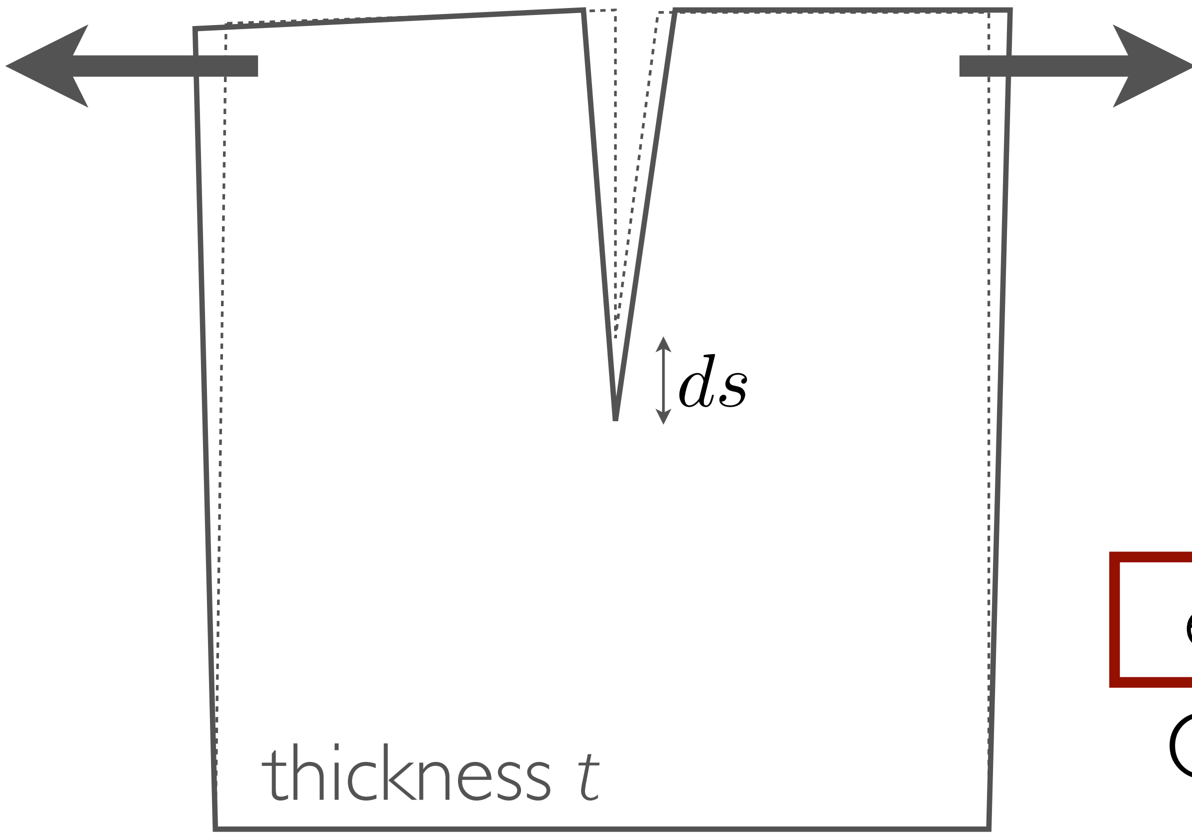
3) Mechanics of tearing



GEOMETRY

O'Keefe Am. J. Phys. 62 (4), April 1994

Griffith's criterion



propagation

$$dW = dE_{el} + G_c t\, ds$$

energy cost for crack propagation

G_c : energy / unit surface
(~ surface tension)

propagation

$$\boxed{\frac{dW - dE_{el}}{t\, ds} = G} = \boxed{G_c}$$

Energy Release Rate

Fracture energy

principal strain

$$U_{stretching} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ (\mathrm{Tr}\,\varepsilon)^2 - 2(1-\nu)\det\varepsilon \right\} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ \varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 \right\}$$

$$U_{bending} = \frac{1}{2}\frac{Eh^3}{(1-\nu^2)12}\left\{ (\mathrm{Tr}\,C)^2 - 2(1-\nu)\det C \right\} = \frac{1}{2}D\left\{ k_1^2 + 2\nu k_1 k_2 + k_2^2 \right\}$$

principal curvature.

values for a sheet of paper

$$U_{stretch} \sim EhL^2 \sim 10^5 J$$

$$U_{fracture} \sim G_c hL \sim 10^{-1} J$$

$$U_{bend} \sim Eh^3 \sim 10^{-3} J$$

«inextensible fabric» model

(0th order approximation)

The sheet is

- inextensible
- infinitely bendable

NO ELASTIC ENERGY

2D analog of a string

«inextensible fabric» model

(0th order approximation)

The sheet is

- inextensible
- infinitely bendable

NO ELASTIC ENERGY

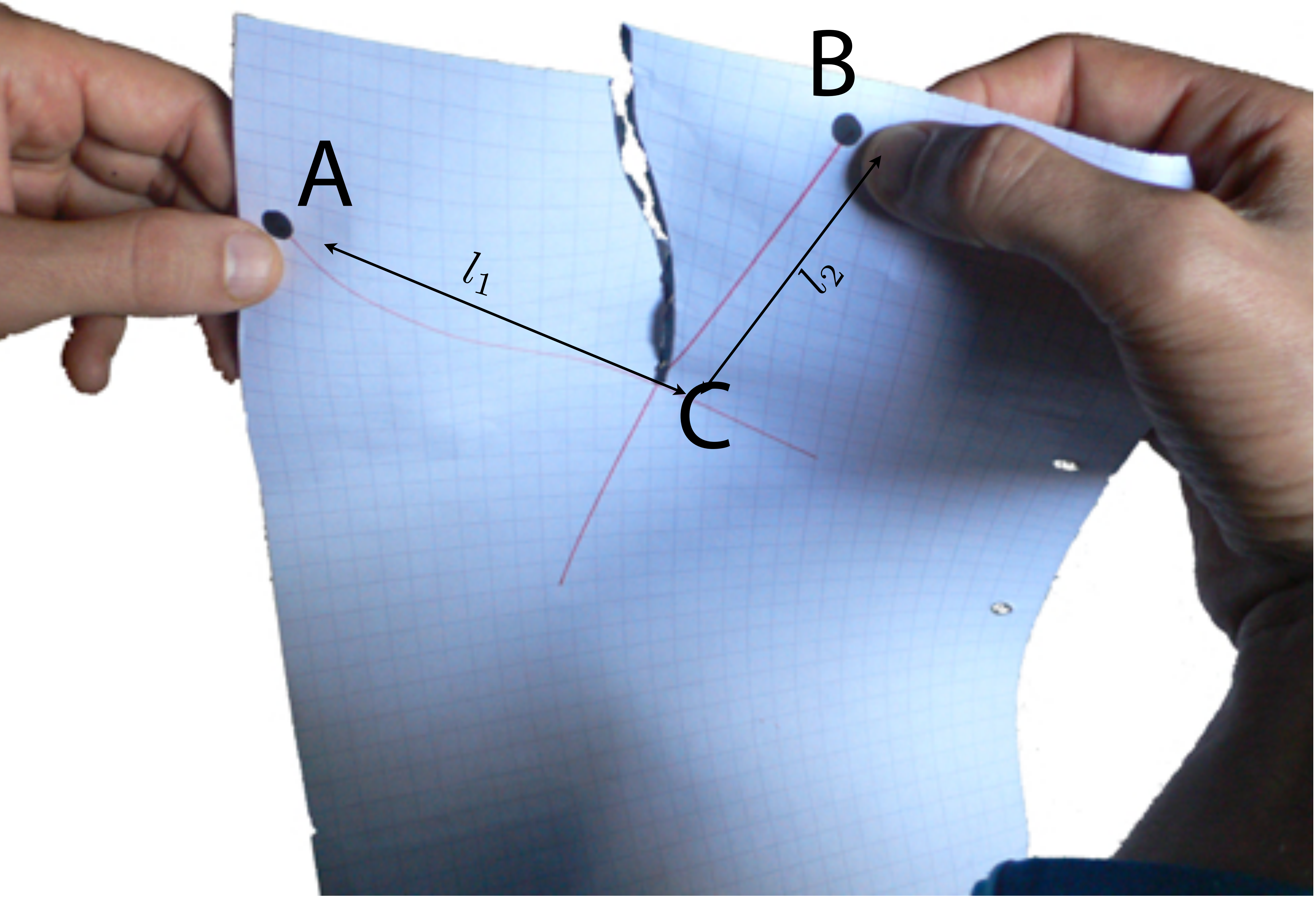
propagation

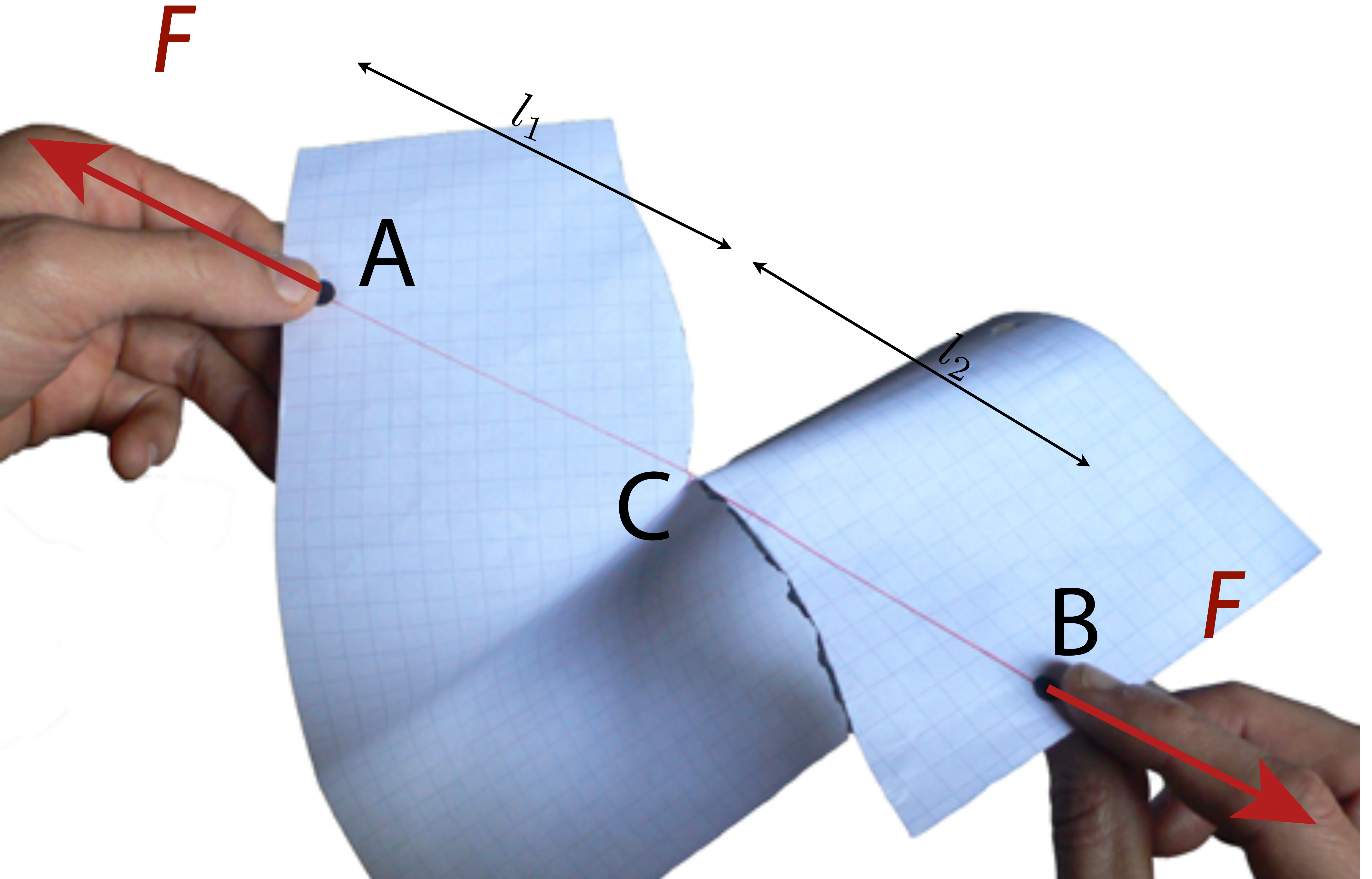
$$\boxed{\frac{dW - \cancel{dE_{el}}}{tds} = G} = \boxed{G_c}$$

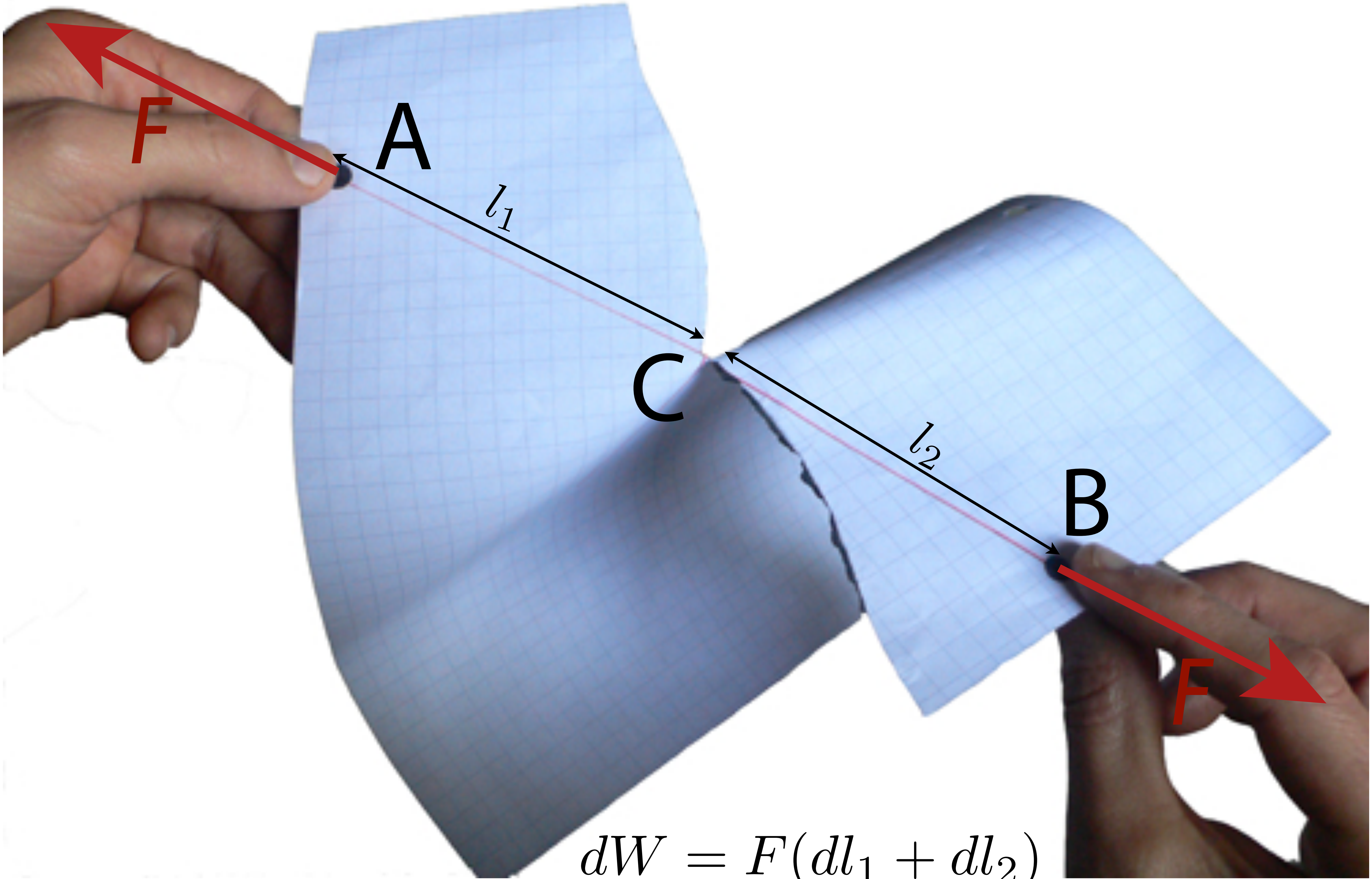
Energy Release Rate

Fracture energy

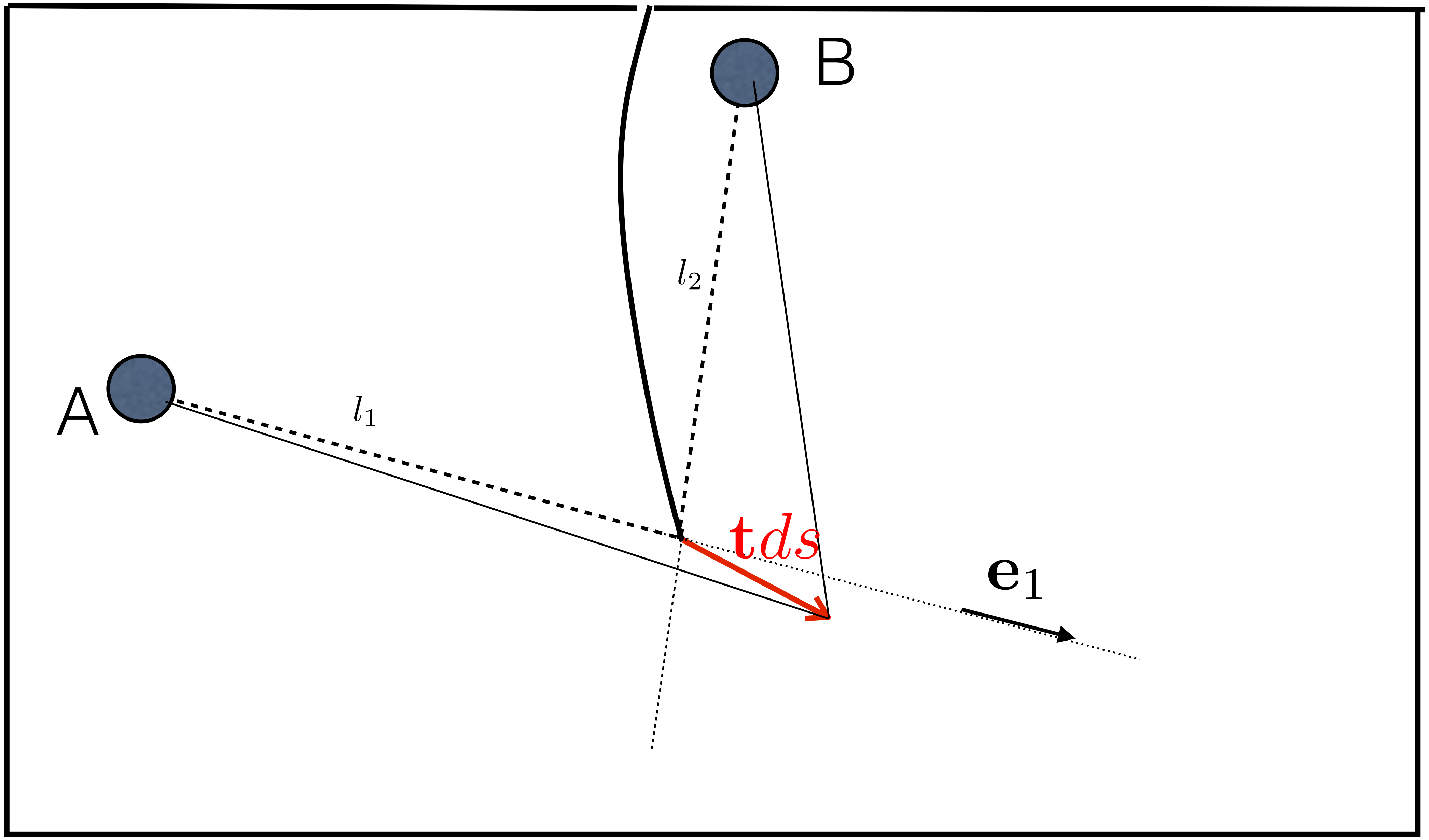
2D analog of a string


A
B
C
l_1
l_2


F
l_1
A
l_2
C
B
F

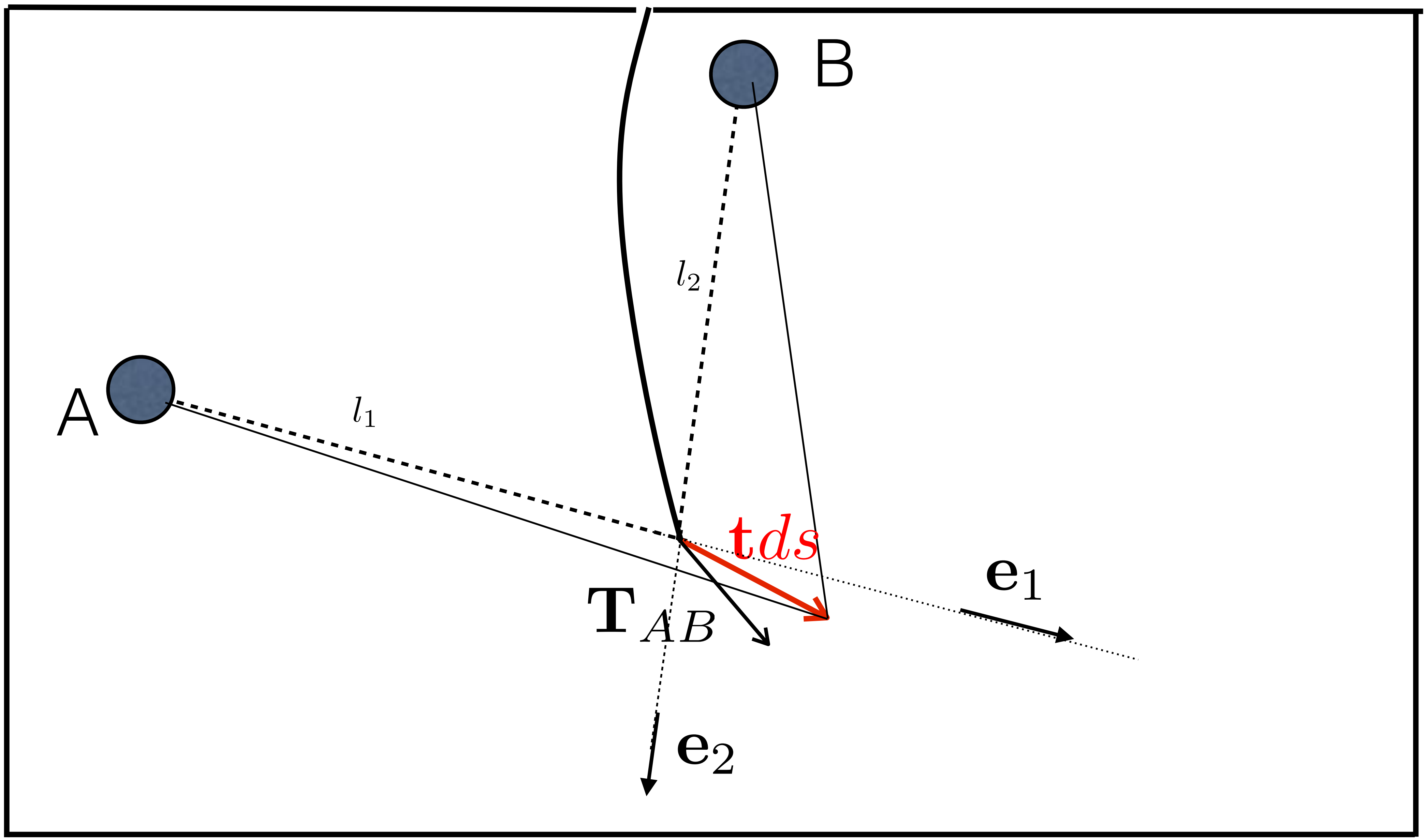


$$dW = F(dl_1 + dl_2)$$



$$dW = F(dl_1 + dl_2)$$

$$dl_1 = \mathbf{t}ds.\mathbf{e}_1$$

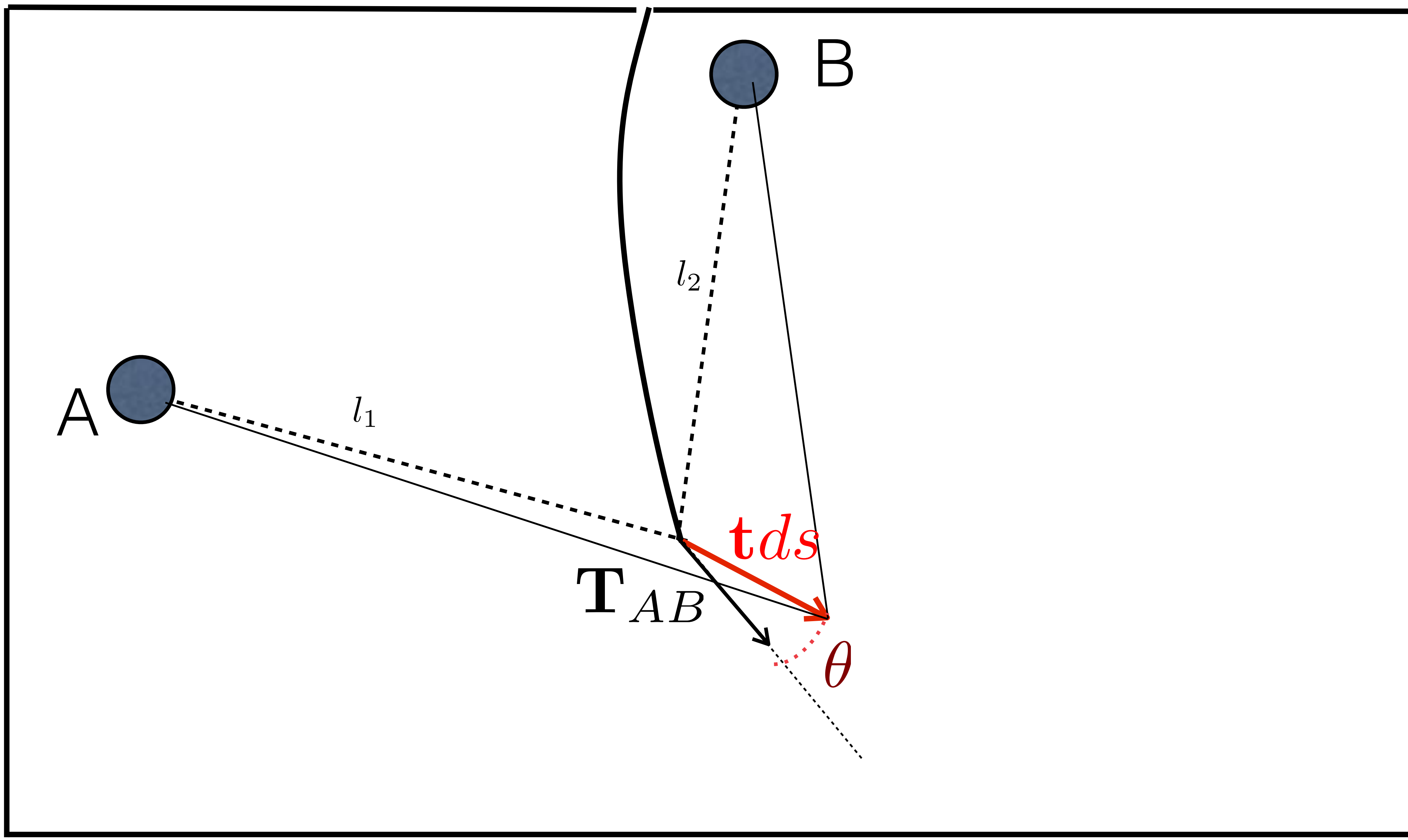


$$dW = F(dl_1 + dl_2) = F\mathbf{T}_{AB}.\mathbf{t}ds$$

bisector

$$\mathbf{e}_1 + \mathbf{e}_2$$

$$dl_1 = \mathbf{t}ds.\mathbf{e}_1$$



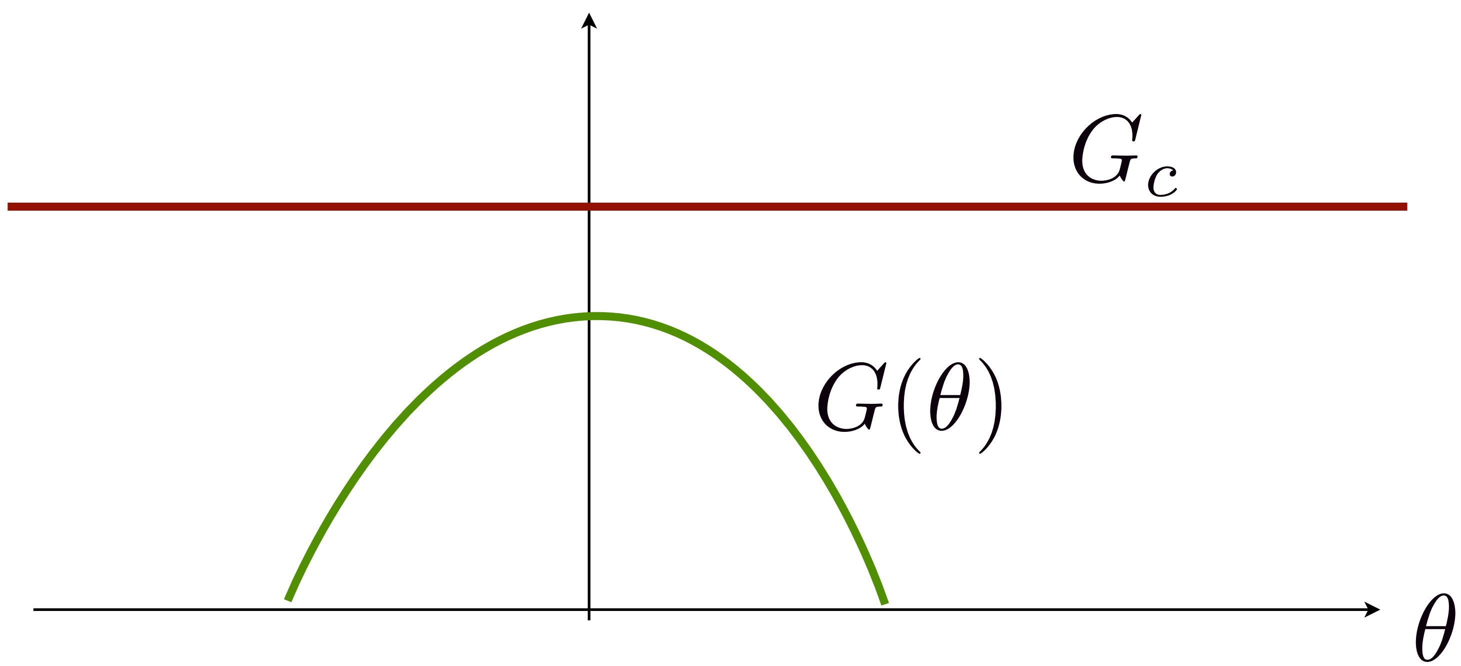
$$G = \frac{F}{t}\mathbf{T}_{AB}.\mathbf{t}$$

Which direction ?

independent of previous path
independent of material property

Which direction ?

pulling with a given force F

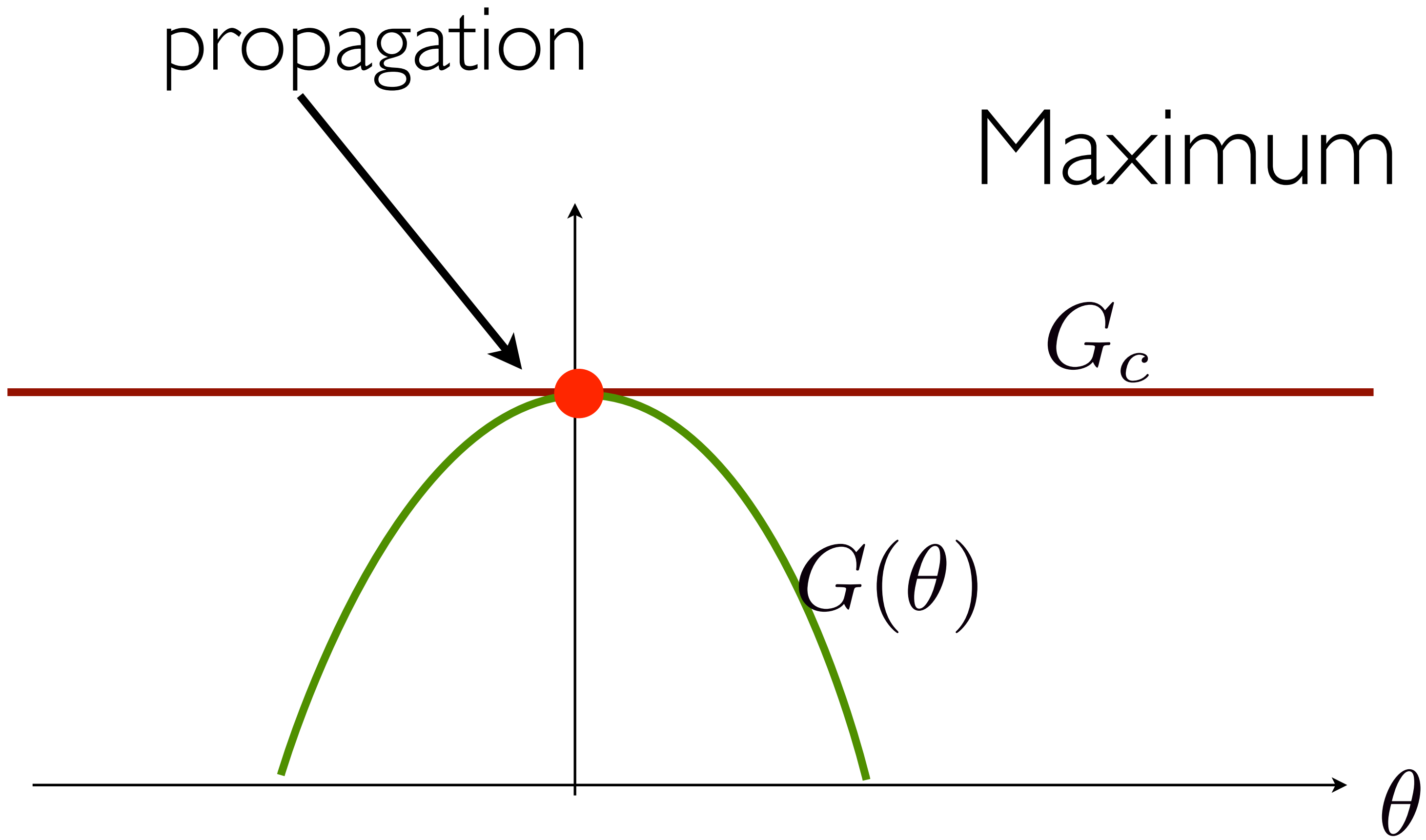


$$\boxed{G(\theta) = \frac{F}{t}\cos(\theta)} < \boxed{G_c}$$

Energy Release Rate

Fracture energy

Which direction ?

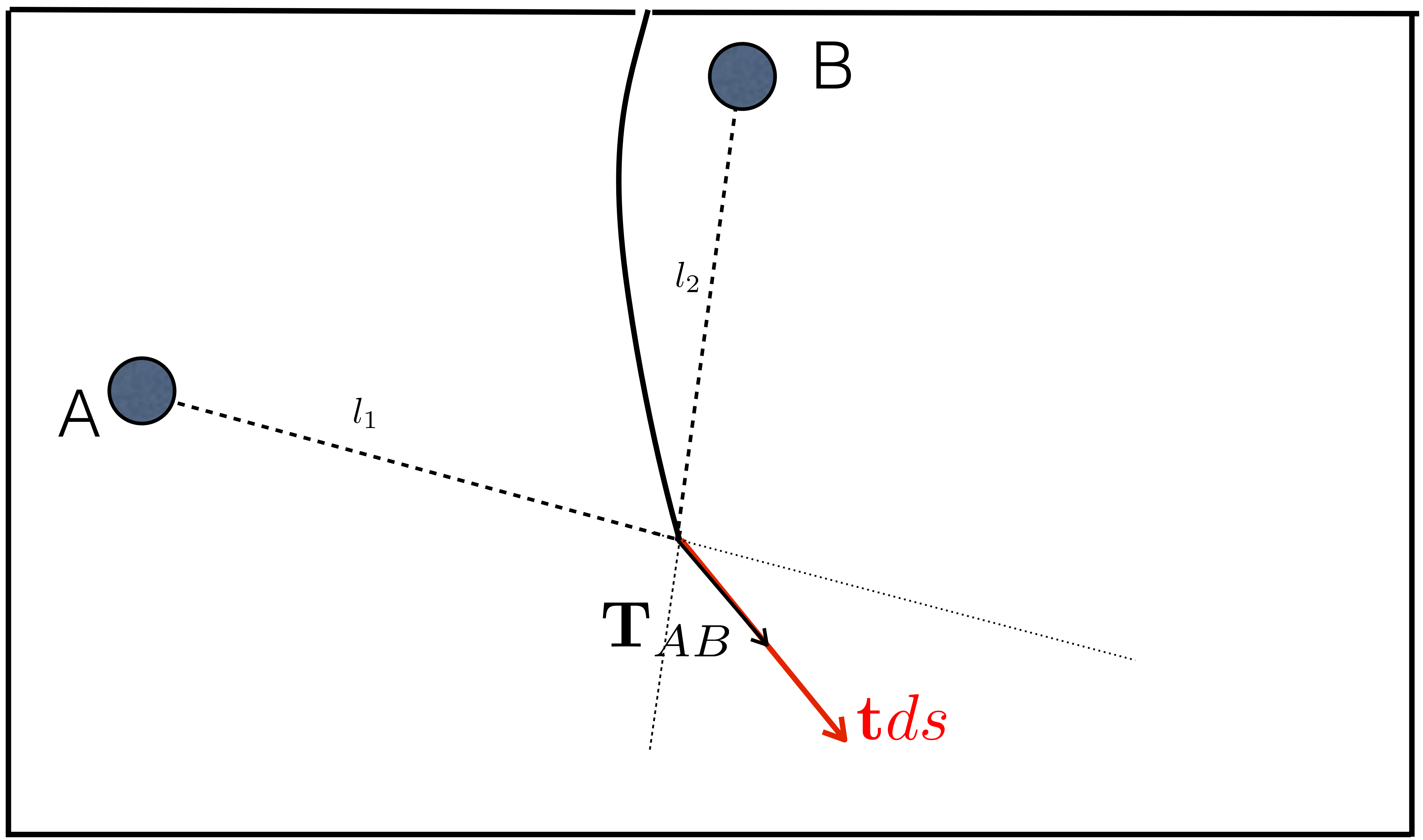


Maximum of $\boxed{\text{Energy Release Rate}}$

$$\boxed{G(\theta) = \frac{F}{t}\cos(\theta)} = \boxed{G_c}$$

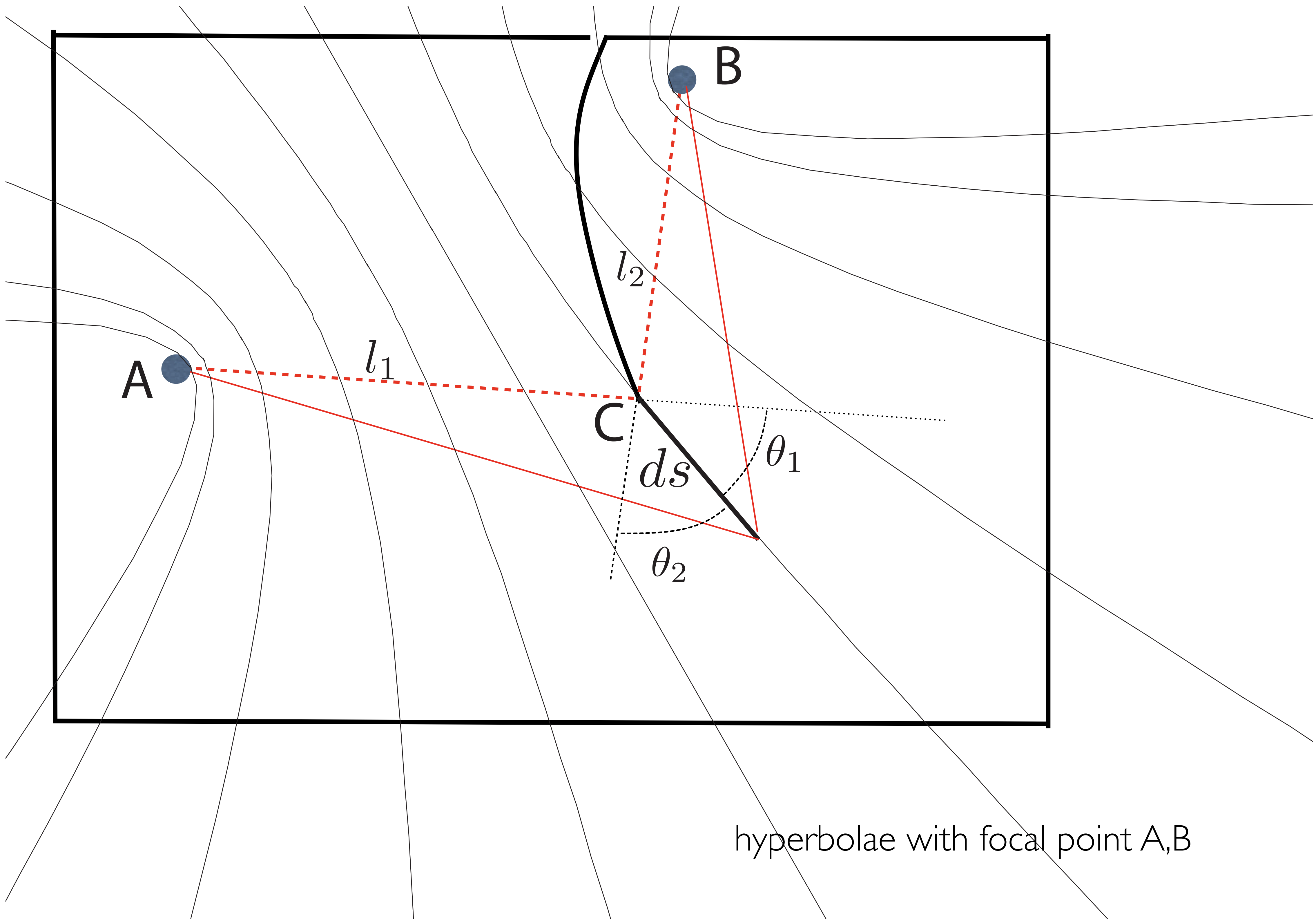
Energy Release Rate

Fracture energy

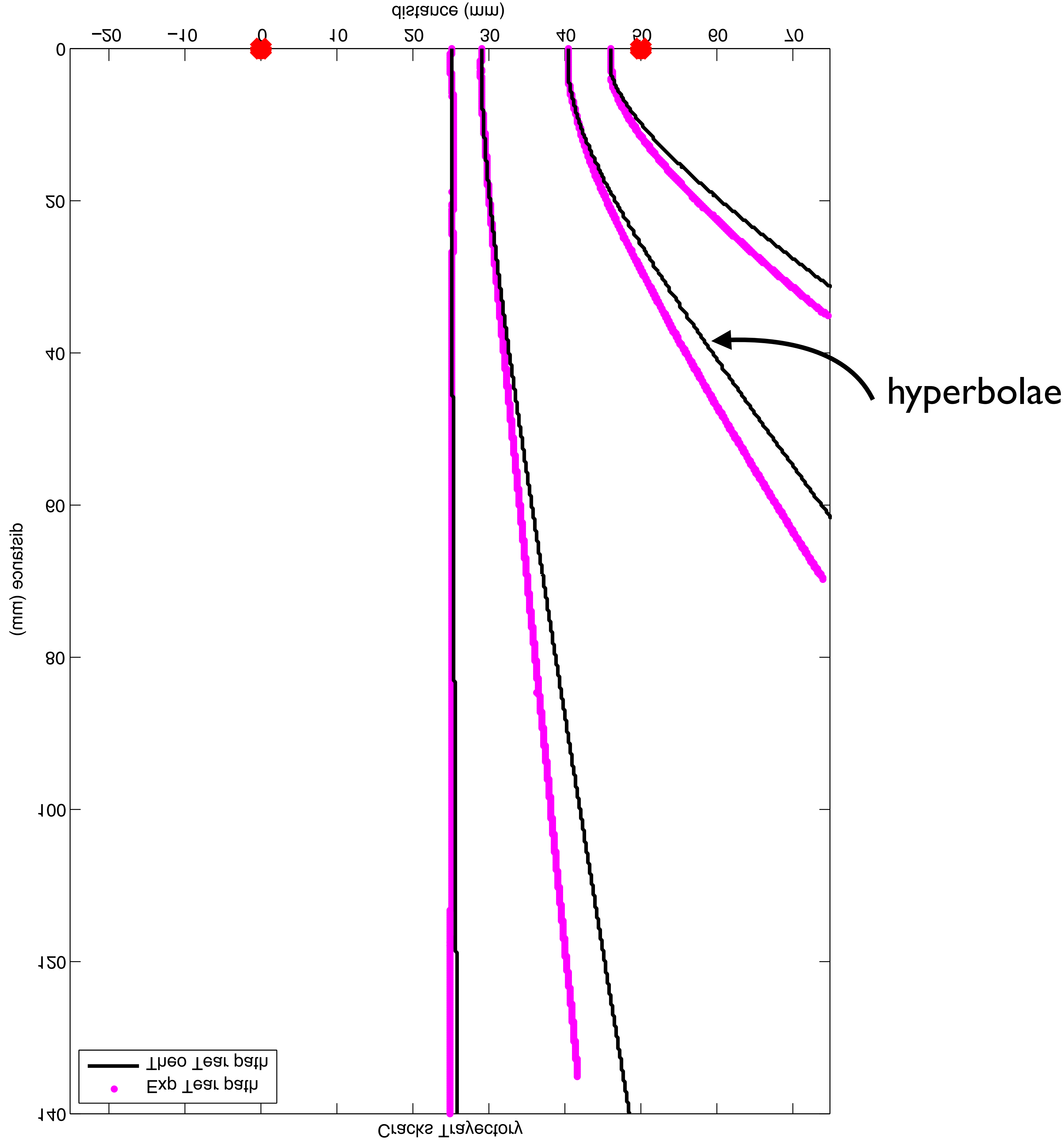


Maximum Energy Release rate : bisector

$$\theta_1 = \theta_2$$



hyperbolae with focal point A,B



J-F Fuentealba

A. Ibarra, J. F. Fuentealba, B. Roman, and F. Melo

Phys. Rev. E 100, 023002 (**2019**)

Predicting tearing paths in thin sheets

principal strain

$$\mathcal{U}_{stretching} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ (\mathrm{Tr}\,\varepsilon)^2 - 2(1-\nu)\det\varepsilon \right\} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ \varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 \right\}$$

$$\mathcal{U}_{bending} = \frac{1}{2}\frac{Eh^3}{(1-\nu^2)12}\left\{ (\mathrm{Tr}\,C)^2 - 2(1-\nu)\det C \right\} = \frac{1}{2} D \left\{ k_1^2 + 2\nu k_1 k_2 + k_2^2 \right\}$$

principal curvature.

Bending + stretching energy

$\longrightarrow$ Equilibrium equations.

<u>Hyp</u> :
- no demonstration
- <u>small slope</u> $\left|\frac{\partial w}{\partial x}\right| \ll 1$; $\left|\frac{\partial w}{\partial y}\right| \ll 1$
- no boundary conditions

<u>NB</u> : already assumed $\varepsilon \ll 1$; $hk \ll 1$

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1) what to expect (small slope eq. for beams)

rods
$$\begin{cases} \dfrac{dM}{ds} + \underline{t} \wedge \underline{R} = 0 \\ \dfrac{d\underline{R}}{ds} + p\underline{n} + f\underline{t} = 0 \end{cases}$$

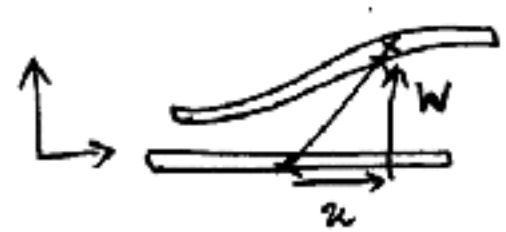
$p\underline{n}$: "pressure" normal forces/length

$f\underline{t}$: tangential forces/length

$$\underline{R} = \underbrace{N\underline{t}}_{\text{"tension"}} + \underbrace{T\underline{n}}_{\text{shearing force}} \; ; \quad M = EIK \; ; \quad N = EA\varepsilon$$

$$\frac{d^2M}{ds^2} + \underline{t} \wedge \left(-p\underline{n} + \cancel{f\underline{t}}\right) + K\underline{n} \wedge \underline{R} = 0$$

$$\frac{d^2M}{ds^2} - p - NK = 0$$



small slope : $s \sim x$; $K \sim \dfrac{d^2w}{dx^2}$

$$\begin{cases} EI\dfrac{d^4w}{dx^4} - N\dfrac{d^2w}{dx^2} = p \\ N = EA\varepsilon \quad \text{and } \varepsilon \text{ depends on geometry} \end{cases}$$

ε depends on geometry of out-of-plane displacements.

1) what to expect (small slope eq. for beams)

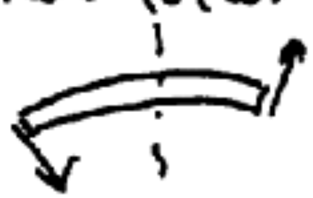
$$\begin{cases} EI \dfrac{d^4 w}{dx^4} - N \dfrac{d^2 w}{dx^2} = p \\[2ex] N = EA\,\varepsilon \end{cases}$$

and ε depends on geometry $(w\ldots)$

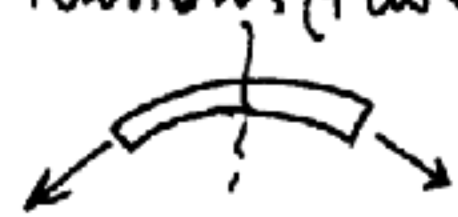
2) plate equations

$$D \Delta^2 w - \left[N_x \frac{\partial^2 w}{\partial x^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right] = p$$

Balance of shear forces

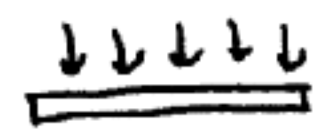


contribution of tensions (+ curvature)



("Laplace Force")

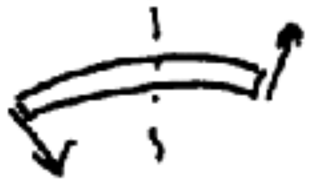
external pressure



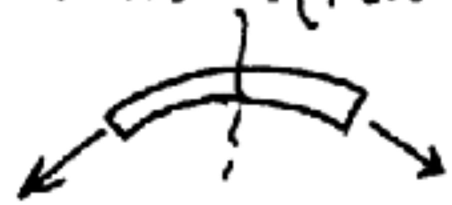
2) plate equations

$$D\Delta^2 w - \left[N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right] = p$$

Balance of shear forces

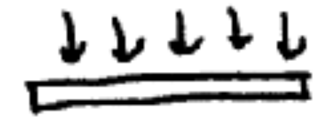


contribution of tensions (+ curvature)



("Laplace Force")

external pressure



dynamics

$$-D\Delta^2 w + \left[N_x \frac{\partial^2 w}{\partial x^2} + 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} + N_y \frac{\partial^2 w}{\partial y^2} \right] + p = \rho_s \frac{\partial^2 w}{\partial t^2}$$

mass/unit surface

3) A geometrical coupling

$$D\Delta^2 w - \left(N\frac{\partial^2 w}{\partial x^2}\dots\right) + p = 0$$

How to compute N ?

a) Equilibrium

$$\begin{cases} \dfrac{\partial N_x}{\partial x} + \dfrac{\partial N_{xy}}{\partial y} = 0 \\[2ex] \dfrac{\partial N_{xy}}{\partial x} + \dfrac{\partial N_y}{\partial y} = 0 \end{cases}$$

b) constitutive relations

$$\begin{cases} N_x = \dfrac{E}{1-\nu^2}\left(\varepsilon_x + \nu\,\varepsilon_y\right) \\[2ex] N_y = \dfrac{E}{1-\nu^2}\left(\varepsilon_y + \nu\,\varepsilon_x\right) \\[2ex] N_{xy} = \dfrac{E}{1+\nu}\,\varepsilon_{xy} \end{cases}$$

$$\begin{cases} \varepsilon_x = \dfrac{\partial u}{\partial x} \\[2ex] \varepsilon_y = \dfrac{\partial v}{\partial y} \\[2ex] \varepsilon_{xy} = \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) \end{cases} \quad +$$

Extra terms **(non-linear)**
in-plane extension
due to out-of-plane
displacement.

3) A geometrical coupling

NB :

$$D\Delta^2 w - \left(N \frac{\partial^2 w}{\partial x^2} \dots\right) + p = 0$$

strong coupling.

N depends on $\varepsilon \longrightarrow$ on w

GEOMETRY

3) A geometrical coupling

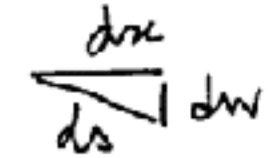
NB : $D\Delta^2 w^* - \left(N \frac{\partial^2 w}{\partial x^2} \cdots\right) + p = 0$

N depends on $\mathcal{E} \longrightarrow$ on w

strong coupling.

when is this geometrical coupling important?

$$\mathcal{E} \sim \left(\frac{W}{L}\right)^2$$



$$ds^2 = dw^2 + dx^2$$

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dw}{dx}\right)^2} \sim 1 + \frac{1}{2}\left(\frac{dw}{dx}\right)^2$$

$$\mathcal{E} = \frac{ds - dx}{dx} \sim \frac{1}{2}\left(\frac{dw}{dx}\right)^2$$

$$Eh^3 \frac{W}{L^4} \ll Eh\,\mathcal{E}\,\frac{W}{L^2}$$

coupling strong $\quad Eh^3 \frac{W}{L^4} \ll Eh \frac{W^3}{L^2} \iff W^2 \gg h^2$

- $W \ll h \rightarrow$ Simpler, decoupled plate equations
- $W \gg h \rightarrow$ strongly non-linear PDE (**Föppl–von Kármán equations**,)

[Coupled through geometry]

special geometrical coupling in surfaces...

I Bending & Stretching for plates

1) stretching energy
2) Bending energy
3) Mechanics of tearing

II Small slope equations for plates

1) what to expect (small slope eq. for beams)
2) plate equations
3) A geometrical coupling

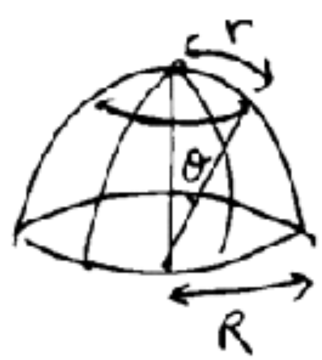
III Geometry of surfaces

1) Gauss theorem
2) crumpling
3) adhesion on a sphere
4) Actuation of thin sheets

1) Gauss *theorema egregium*

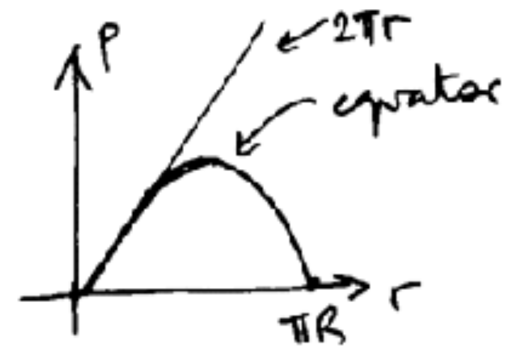
"If you know the distances between every points along a surface, then you know its Gaussian curvature"

Example sphere radius R



$\theta = r/R$

perimeter $p(r) = 2\pi R \sin(r/R)$



General formula: $p(r) = 2\pi r\left[1 - \frac{r^2}{6} K_G\right]$ on any surface.

Consequence: if K_G changes, there is a change in distances ⇒ stretching.

A strong geometrical link between 3D shape and in-plane stress.

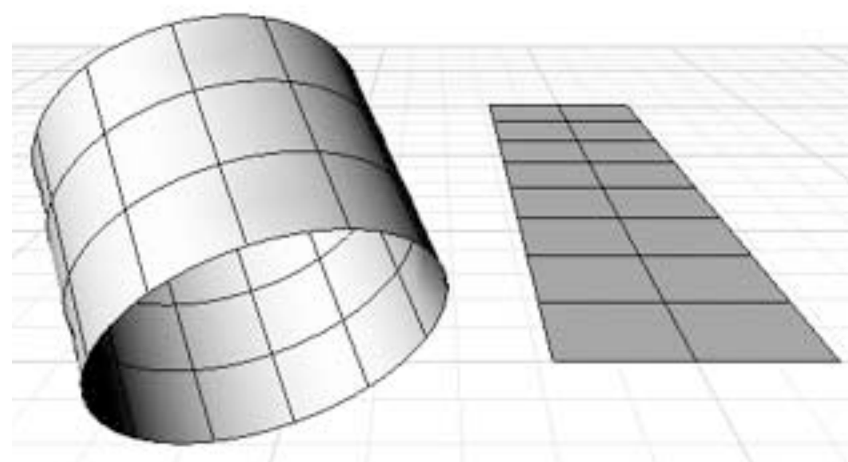
courbure de Gauss

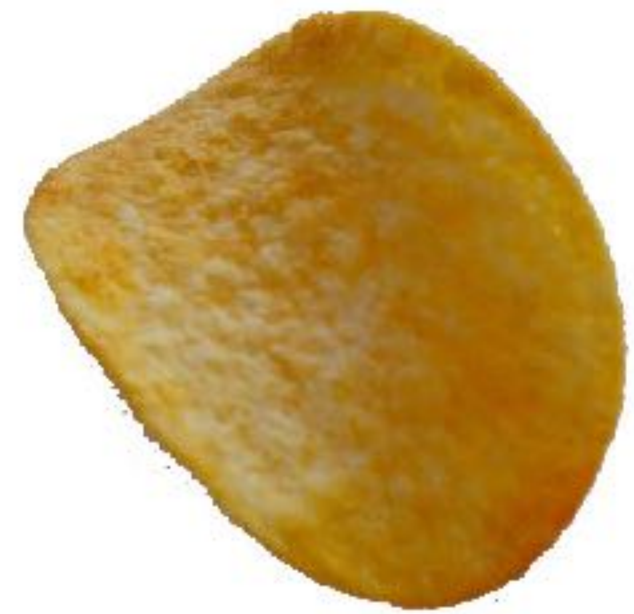
$G > 0$

$G = 0$

$G < 0$

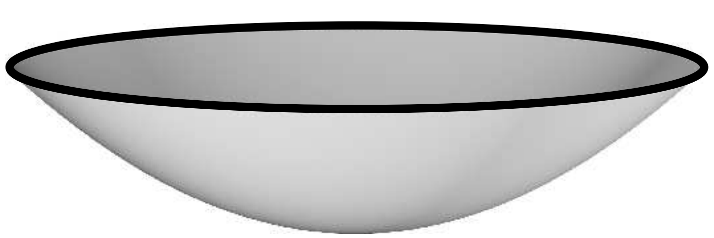
surfaces développables

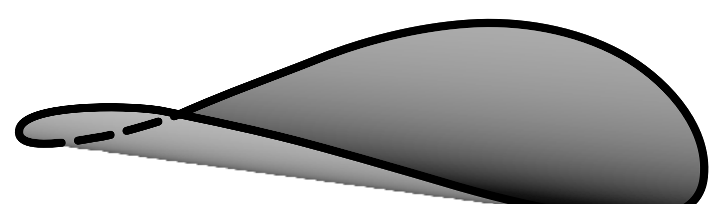


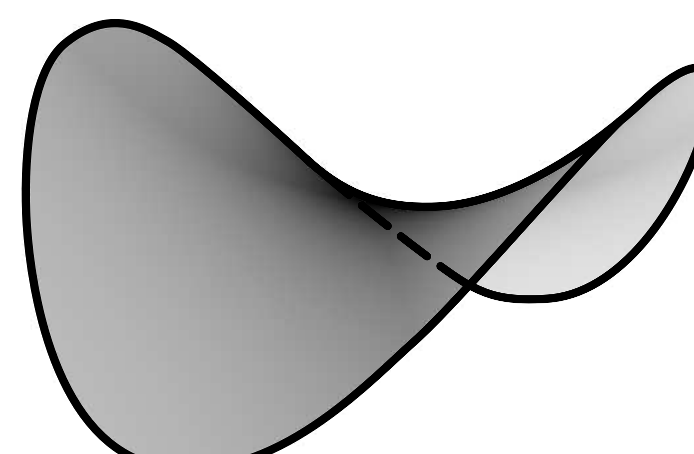


périmètres « courts »

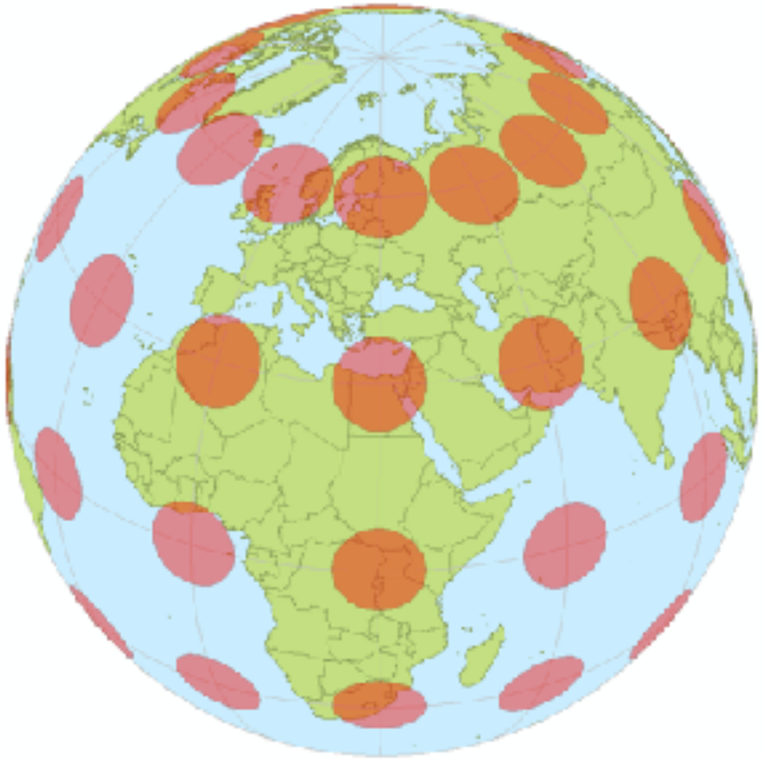
périmètres « longs »





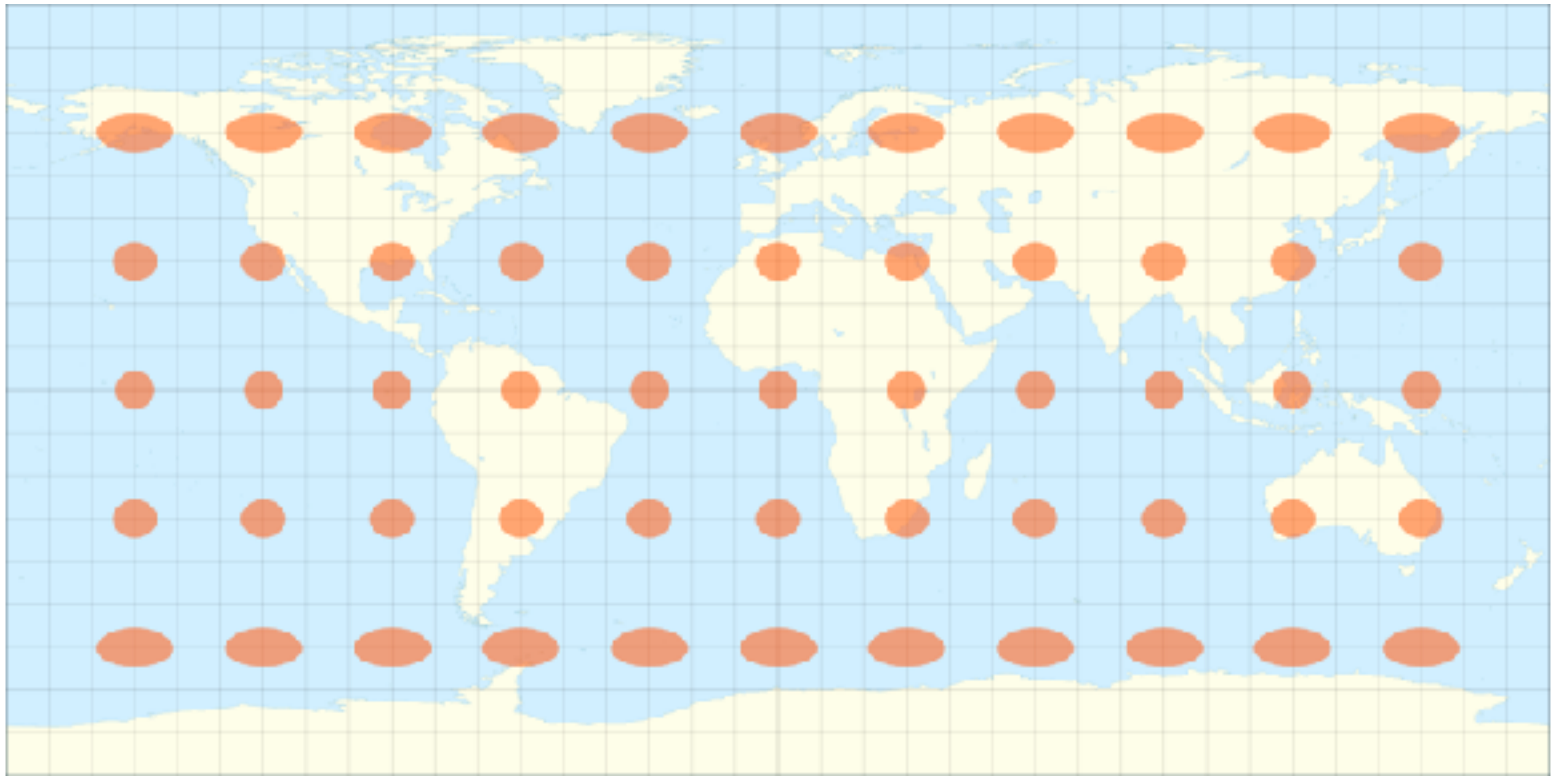


Mapping a sphere…



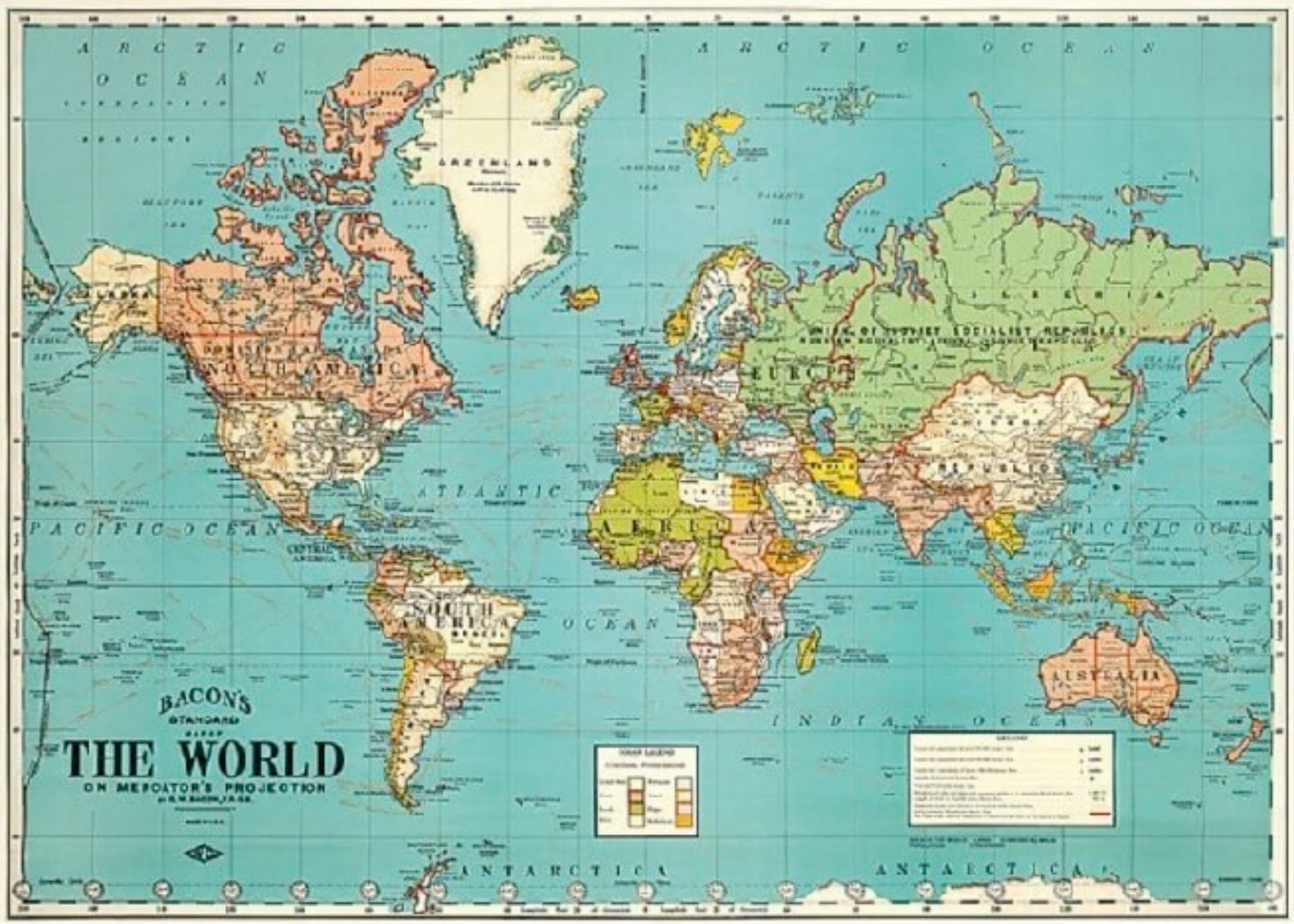
Stefan Kühn, Wikimedia

difficult

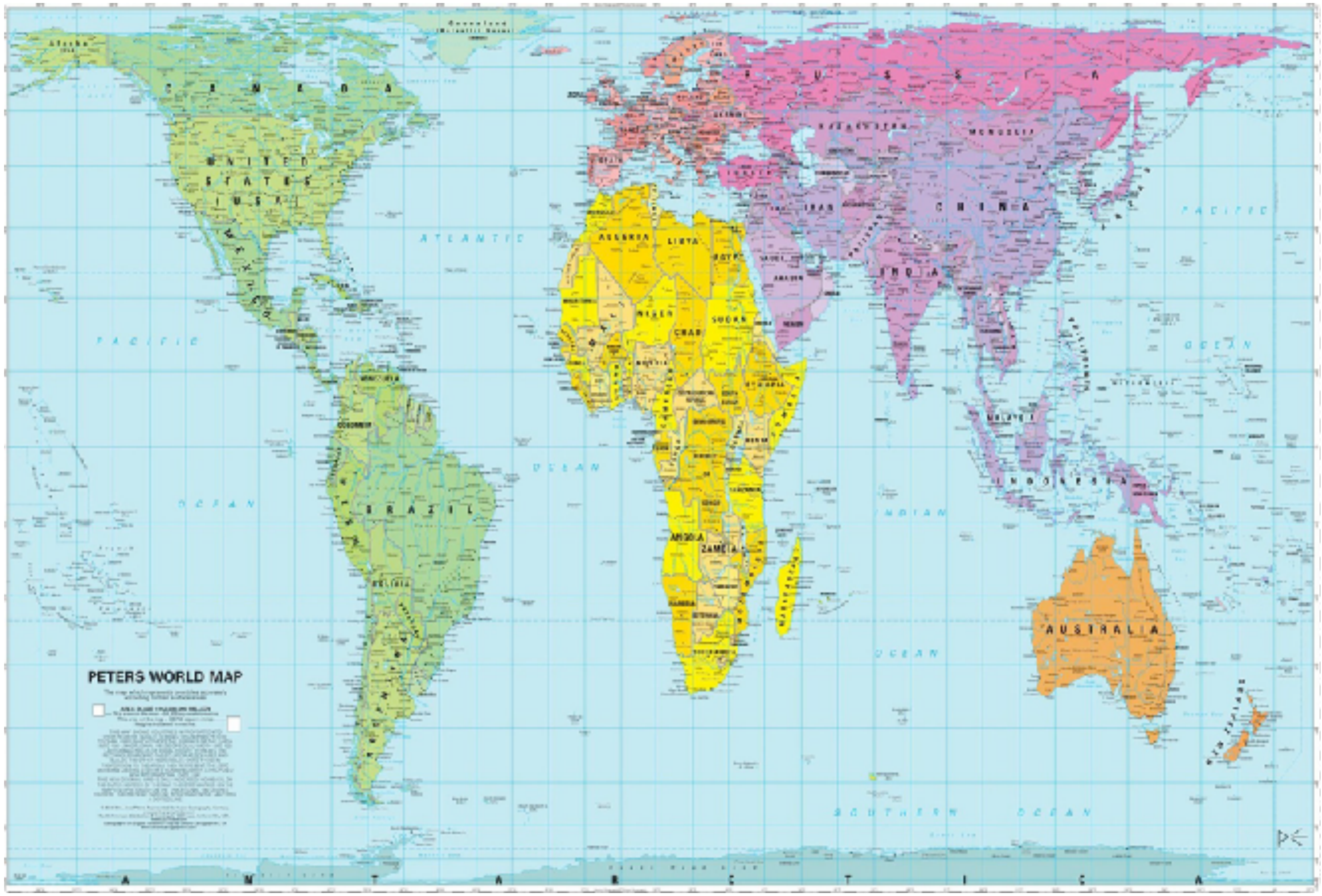


Eric Gaba, Wikimedia

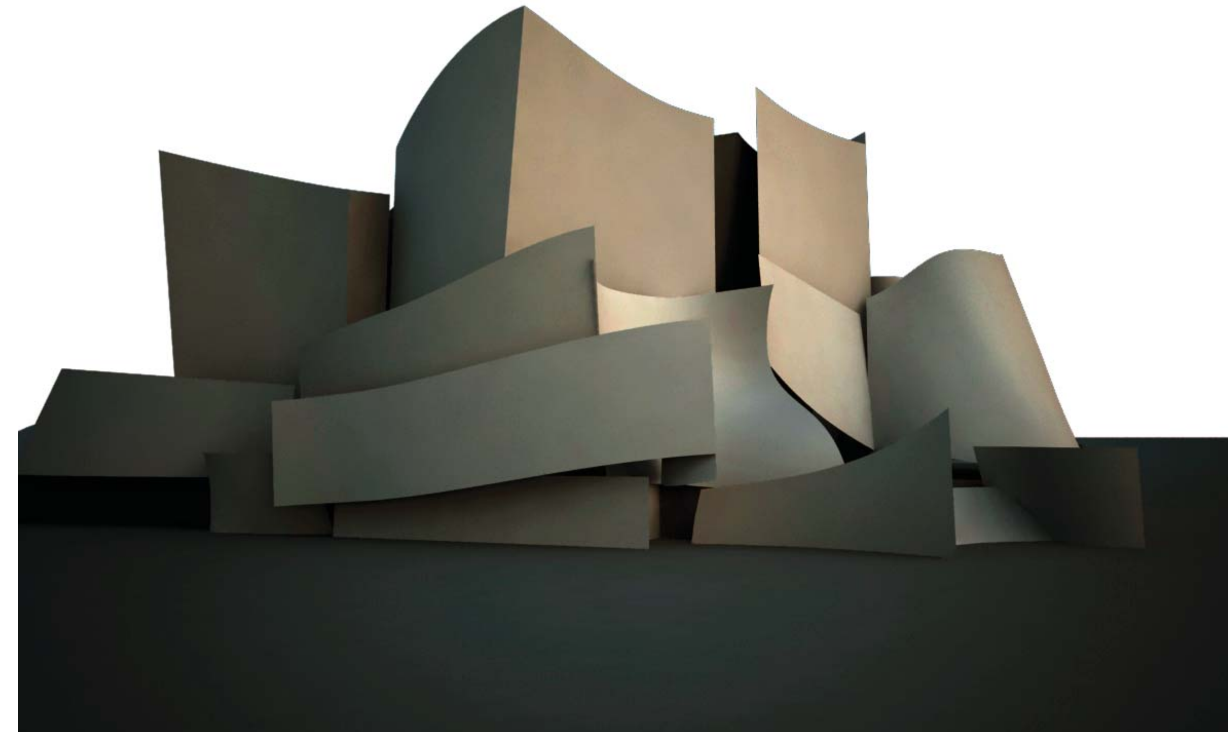
avec les “bons” angles



avec les “bonnes” aires

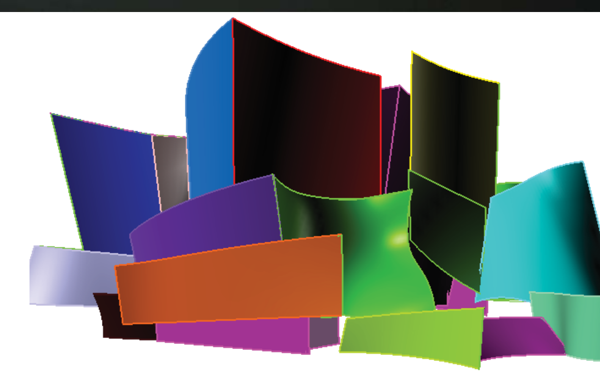


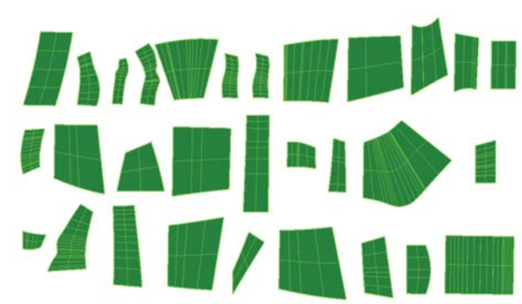
Frank O. Gehry, Walt Disney Concert Hall 1987-2003



Front Entrance Perspective

70 Curves, guiding the surfaces





All Surfaces the unrolled

Unrolled surfaces of the front enveloppe

from D Te kloese

principal strain

$$\mathcal{U}_{\text{stretching}} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ (\mathrm{Tr}\,\varepsilon)^2 - 2(1-\nu)\det\varepsilon \right\} = \frac{1}{2}\frac{Eh}{1-\nu^2}\left\{ \varepsilon_1^2 + 2\nu\varepsilon_1\varepsilon_2 + \varepsilon_2^2 \right\}$$

$$\mathcal{U}_{\text{bending}} = \frac{1}{2}\frac{Eh^3}{(1-\nu^2)12}\left\{ (\mathrm{Tr}\,C)^2 - 2(1-\nu)\det C \right\} = \frac{1}{2} D \left\{ k_1^2 + 2\nu k_1 k_2 + k_2^2 \right\}$$

principal curvature.

. $h \longrightarrow 0$ 1st approximation,
consider $\varepsilon_1 = \varepsilon_2 = 0$ (inextensible)
isometries.

. Plate: isometry $K_G = 0$. ruled surfaces
- but tangent plane is the same along lines

2) crumpling

$h \to 0$

developable surfaces are not enough

crumpling singularities

Ben Amar - Pomeau 1997
Developpable cones

Lobkovsky 1995,1996
ridges

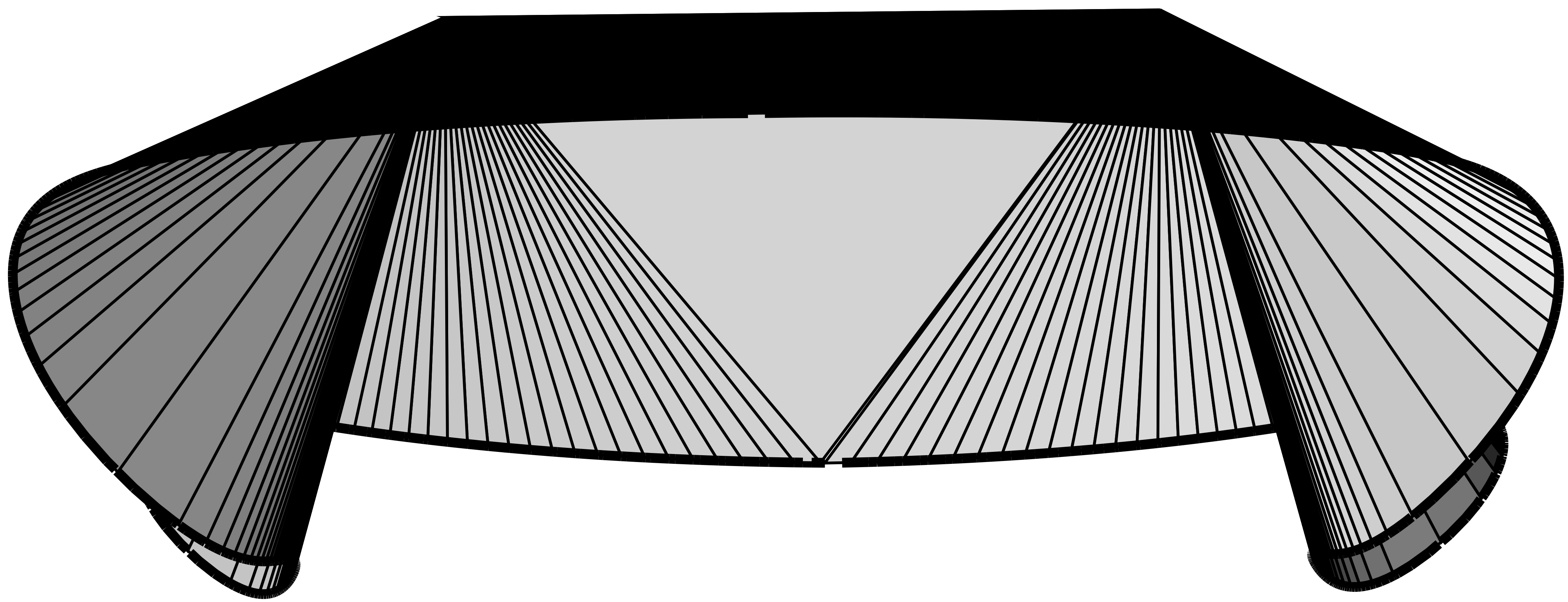
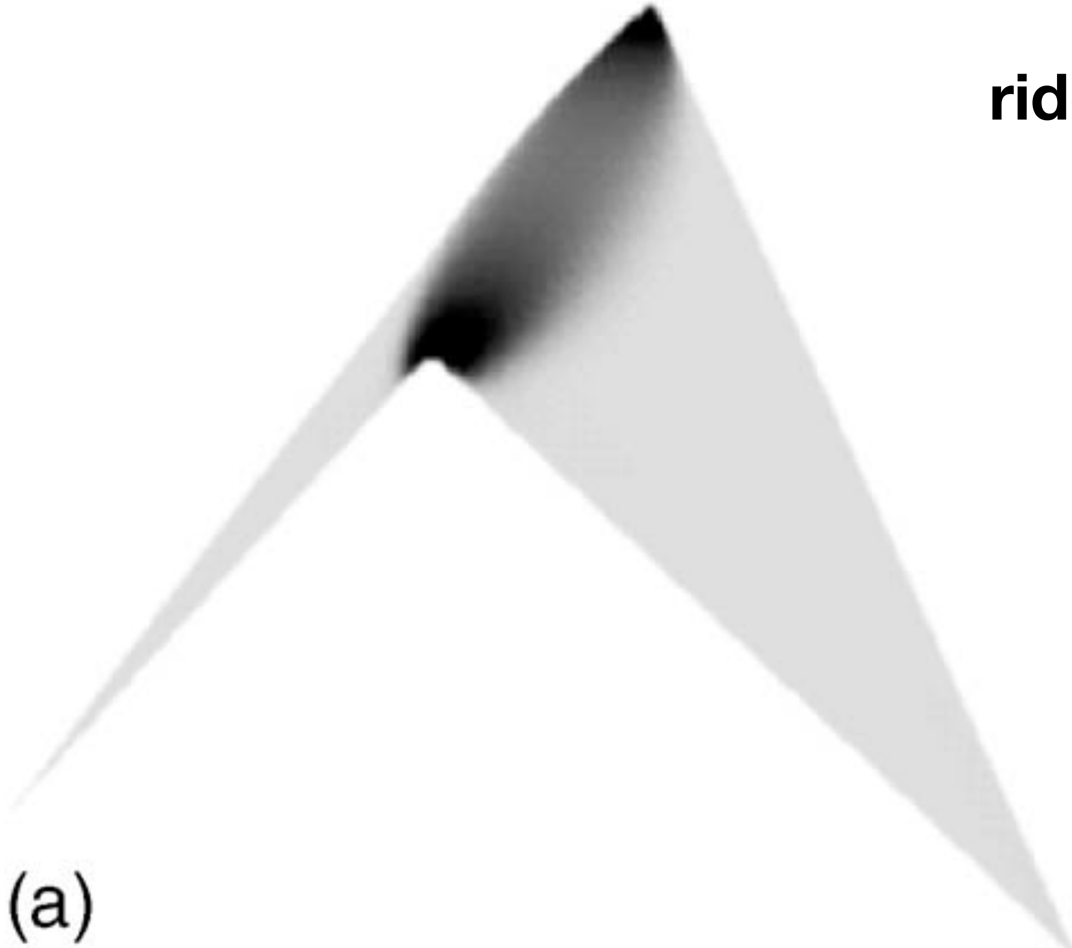
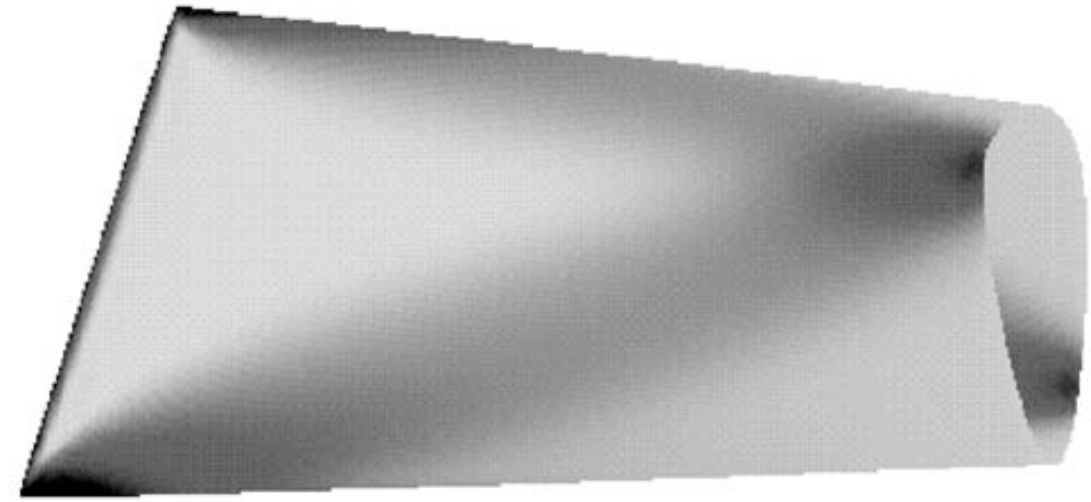


image from Cerda et al, Nature 1999

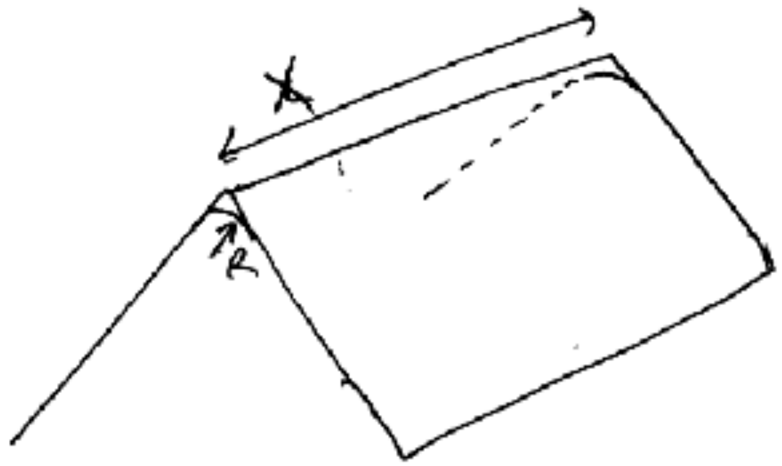
ridges



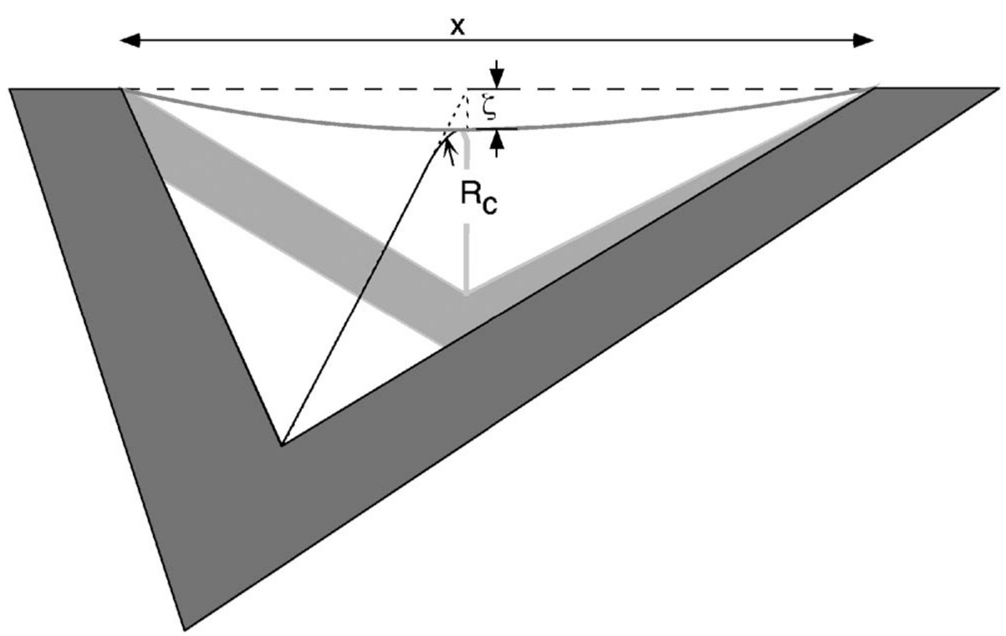
(a)



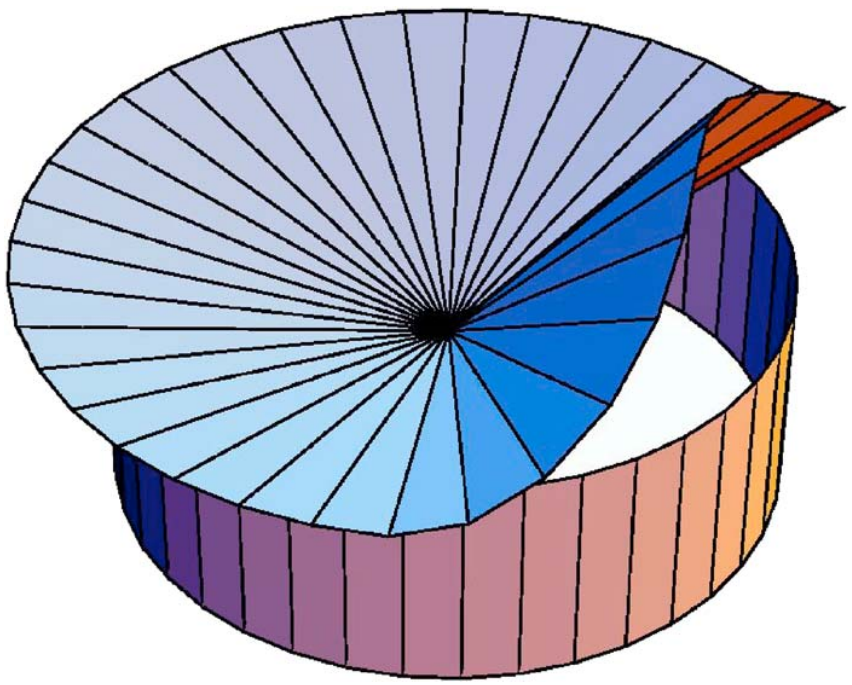
(b)



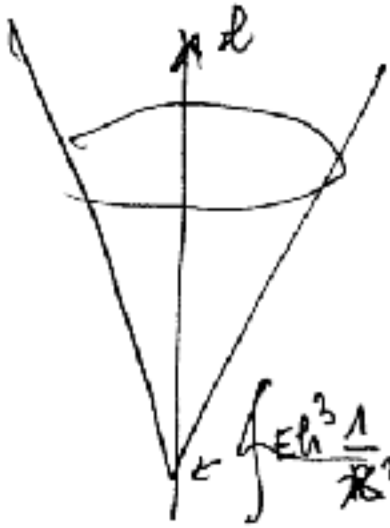
$\mathcal{U}_B \sim \int E h^3 \left[\frac{1}{R^2}\right] R X$ diverges $R \to 0$.



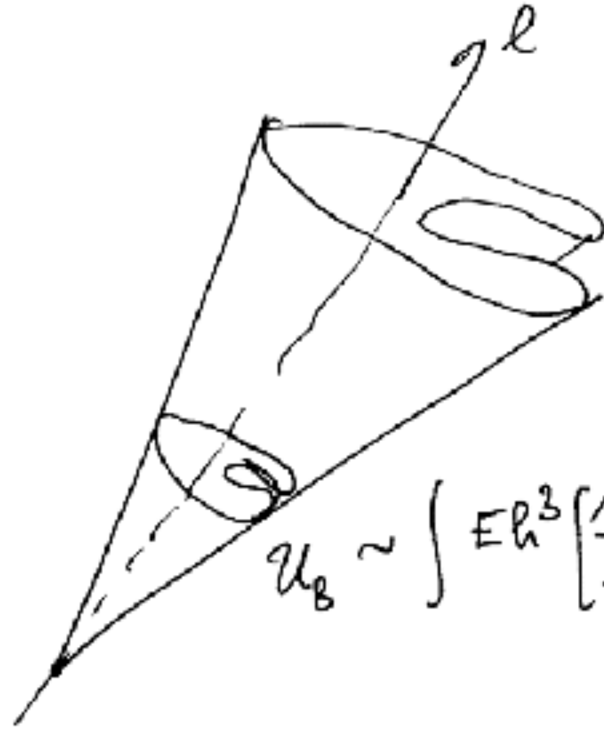
$$R_c \sim X^{2/3} h^{1/3}$$



Developpable cones



$\leftarrow \int E h^3 \frac{1}{R^2} \times x$

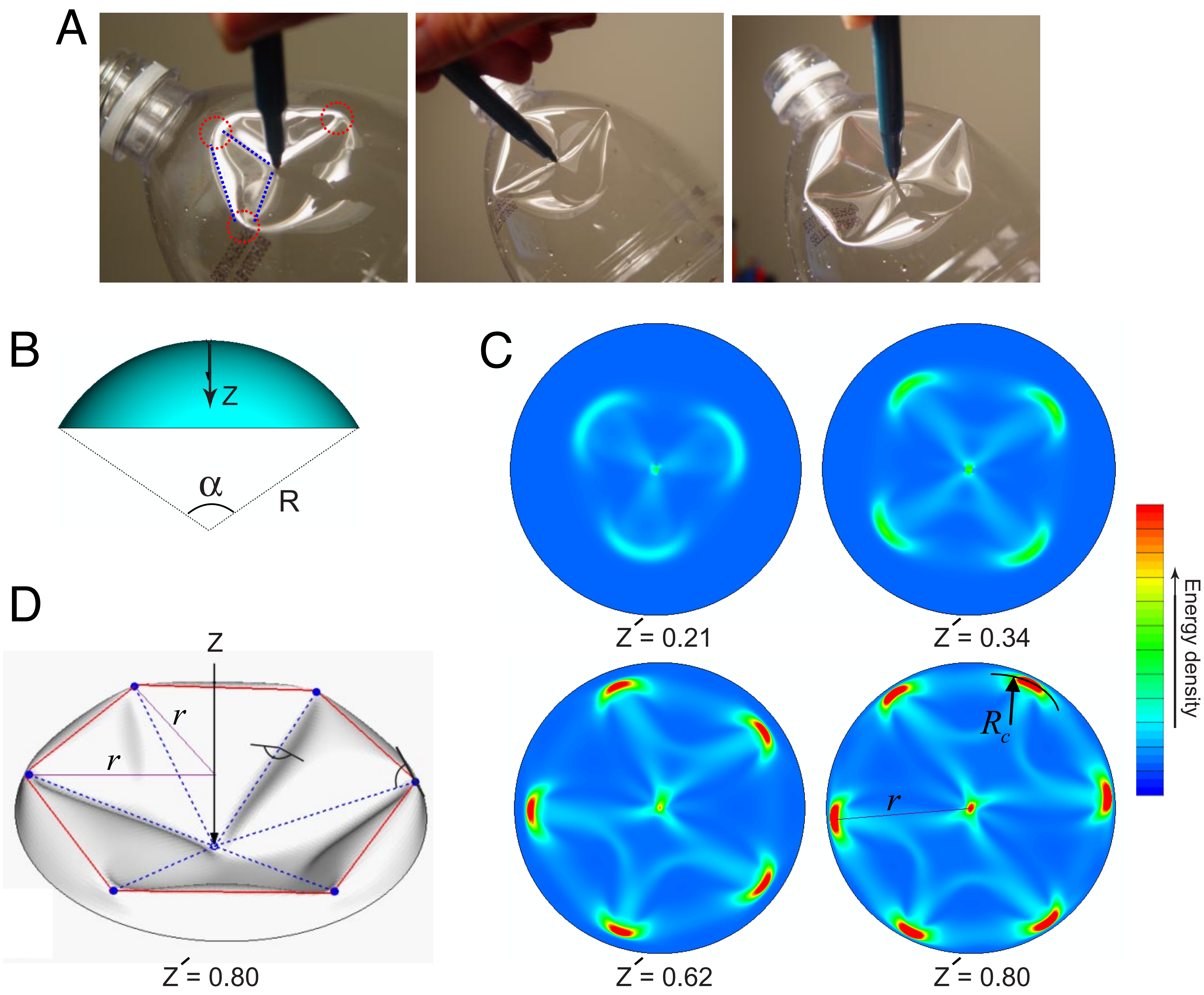


curvature

$U_B \sim \int E h^3 \left[\frac{1}{\ell}\right]^2 \ell \, d\ell \sim \int \frac{1}{\ell} d\ell$

perimeter diverges

core? $\sim h^{1/3} R^{2/3}$.

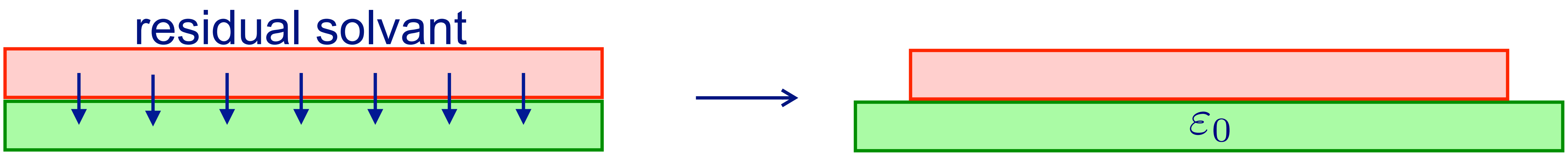


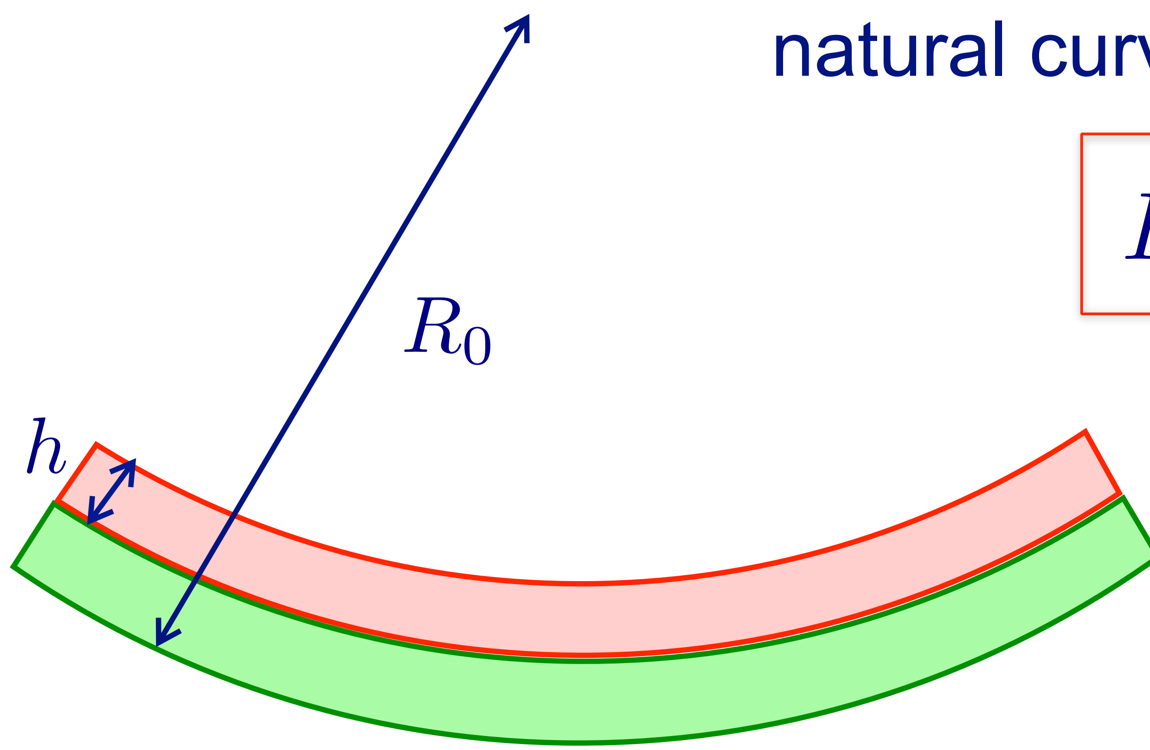
Vaziri, Mahadevan PNAS 2008

3) Actuation of plates

Aurélie Vissac (Amàco)

Swelling bilayers



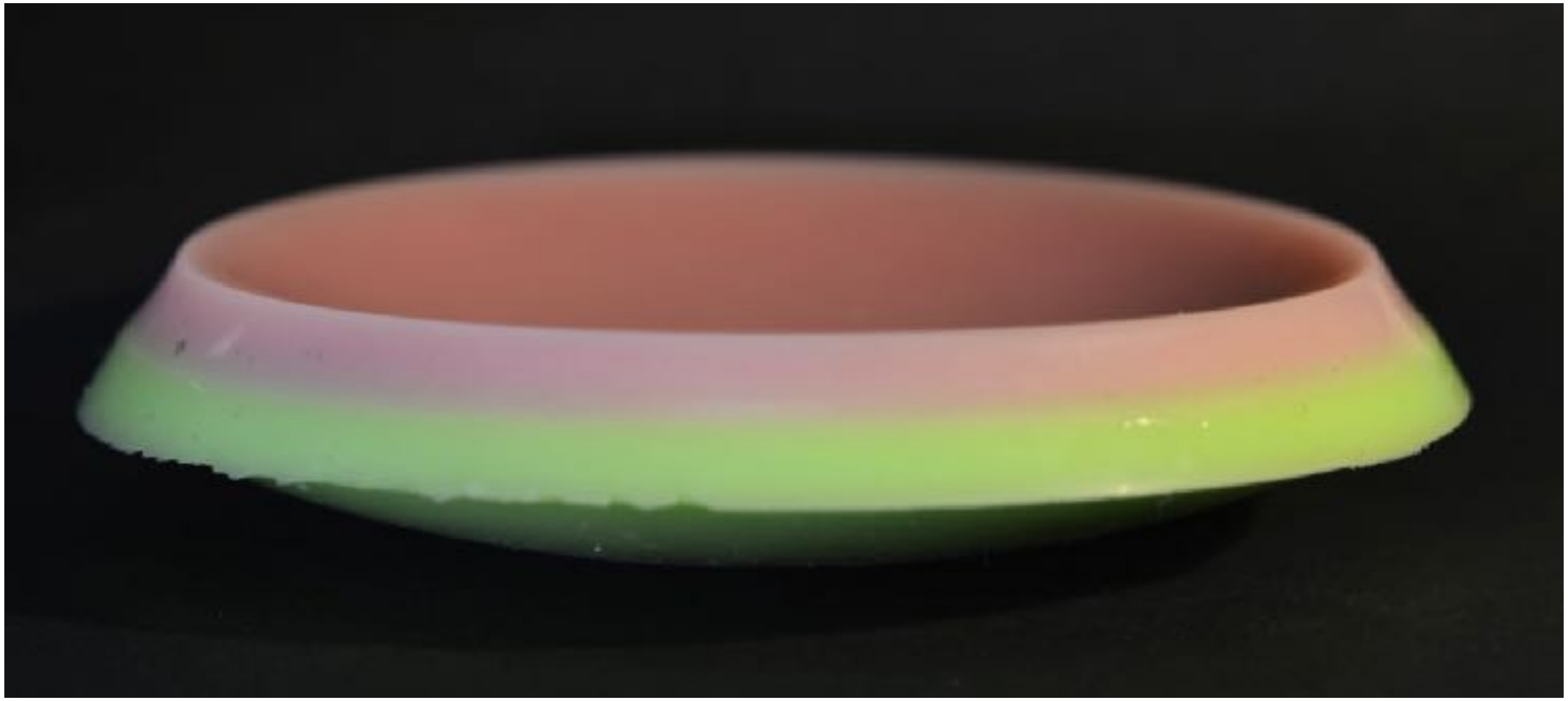


natural curvature:

$$\boxed{R_0 \sim h/\varepsilon_0}$$

in 2D ?

Thick bilayer



Gaussian curvature

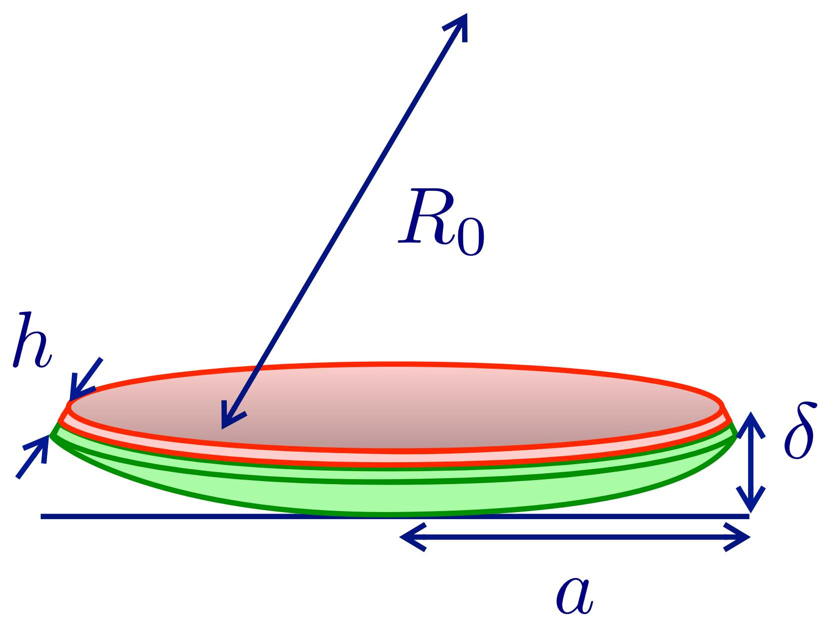
Pezzula et al 2016

Thin bilayer: frustrated rolling

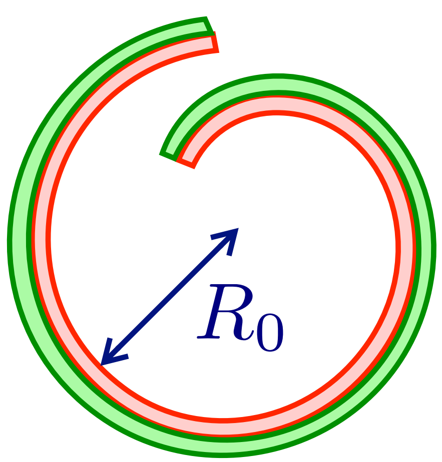
M.Pezzulla, P. Nardinocchi & D.Holmes (2016)

Transition to rolling

Two scenarios



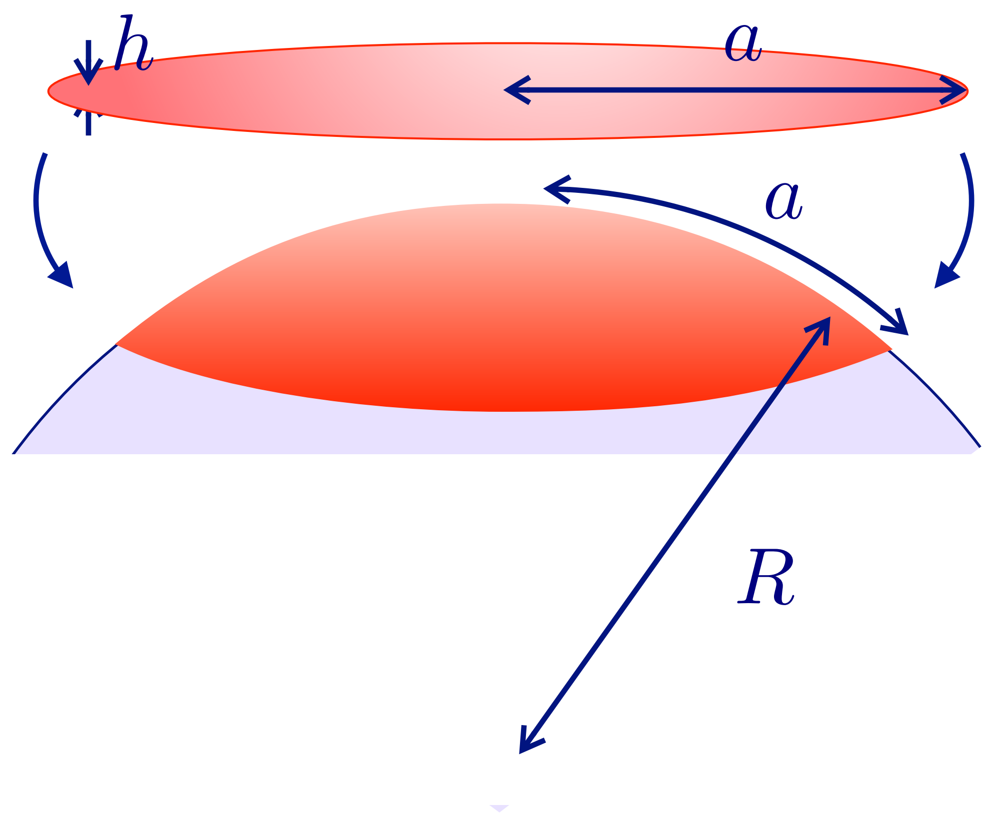
stretching/compression : ?



No stretching/compression,
but bending lost in one direction:

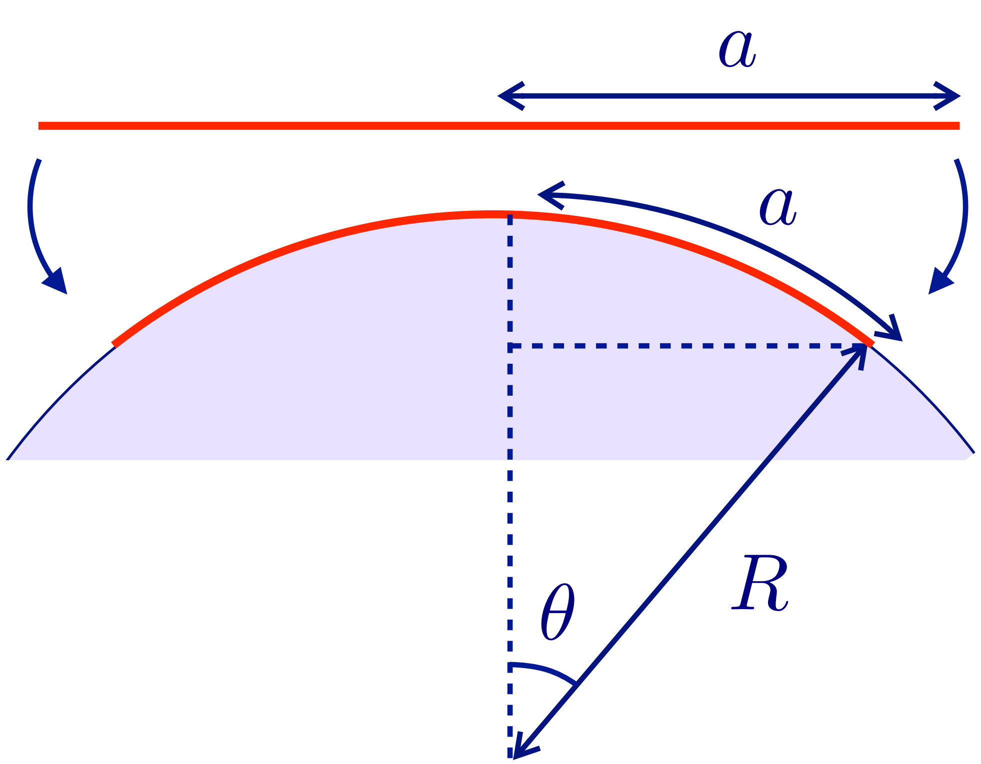
$$\mathcal{U}_b \sim Eh^3 \frac{a^2}{R^2} \sim Eha^2\varepsilon_0^2$$

from a plane to a sphere : stretching



if radius preserved, perimeter in excess

from a plane to a sphere : stretching



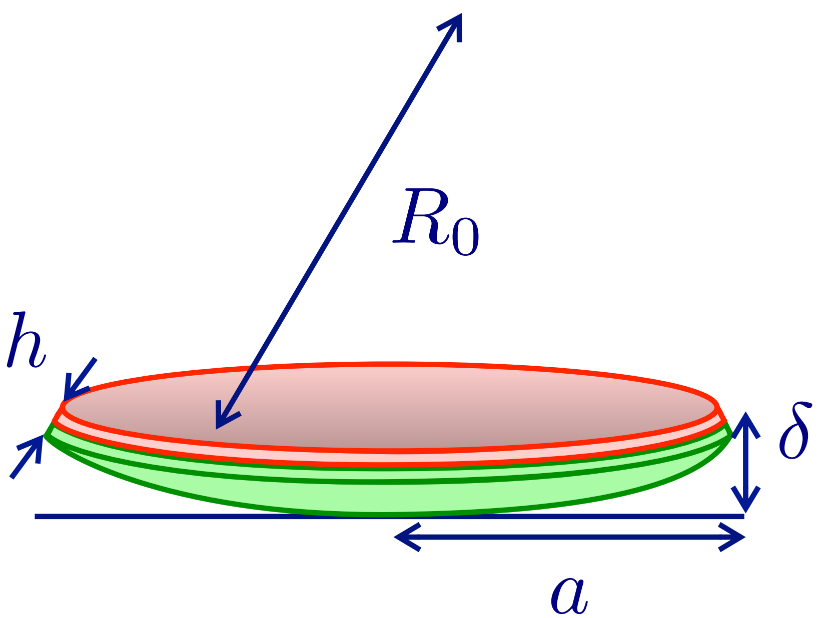
relative excess

$$\varepsilon \sim \frac{a - R\sin\theta}{a} \sim 1 - \frac{\sin\theta}{\theta} \sim \theta^2$$

$$\boxed{\varepsilon \sim \left(\frac{a}{R}\right)^2}$$

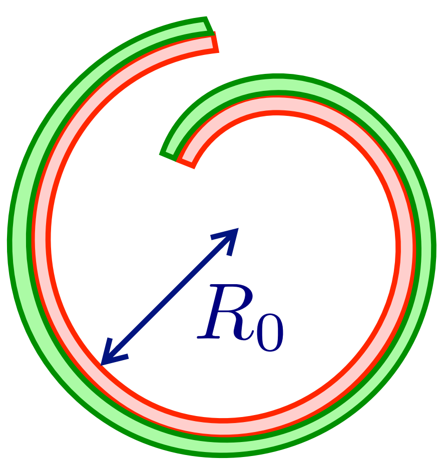
Transition to rolling

Two scenarios



Bending OK, but stretching/compression:

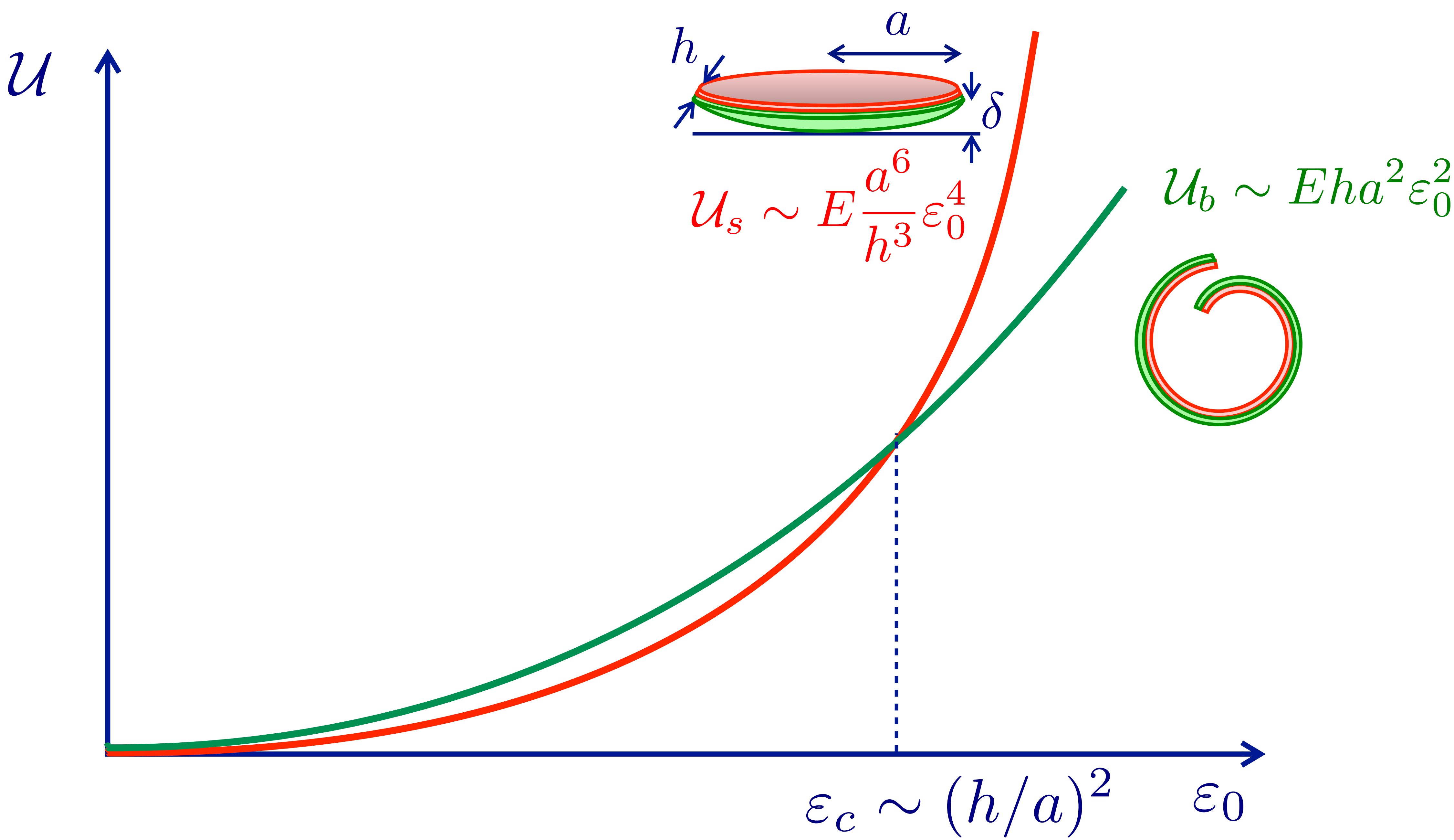
$$\mathcal{U}_s \sim Ea^2h\left(\frac{a}{R}\right)^4 \sim E\frac{a^6}{h^3}\varepsilon_0^4$$



No stretching/compression,
but bending lost in one direction:

$$\mathcal{U}_b \sim Eh^3\frac{a^2}{R^2} \sim Eha^2\varepsilon_0^2$$

Transition to rolling



deflection $\delta_c \sim h$

A general result : avoiding stretching dominates when large deflection

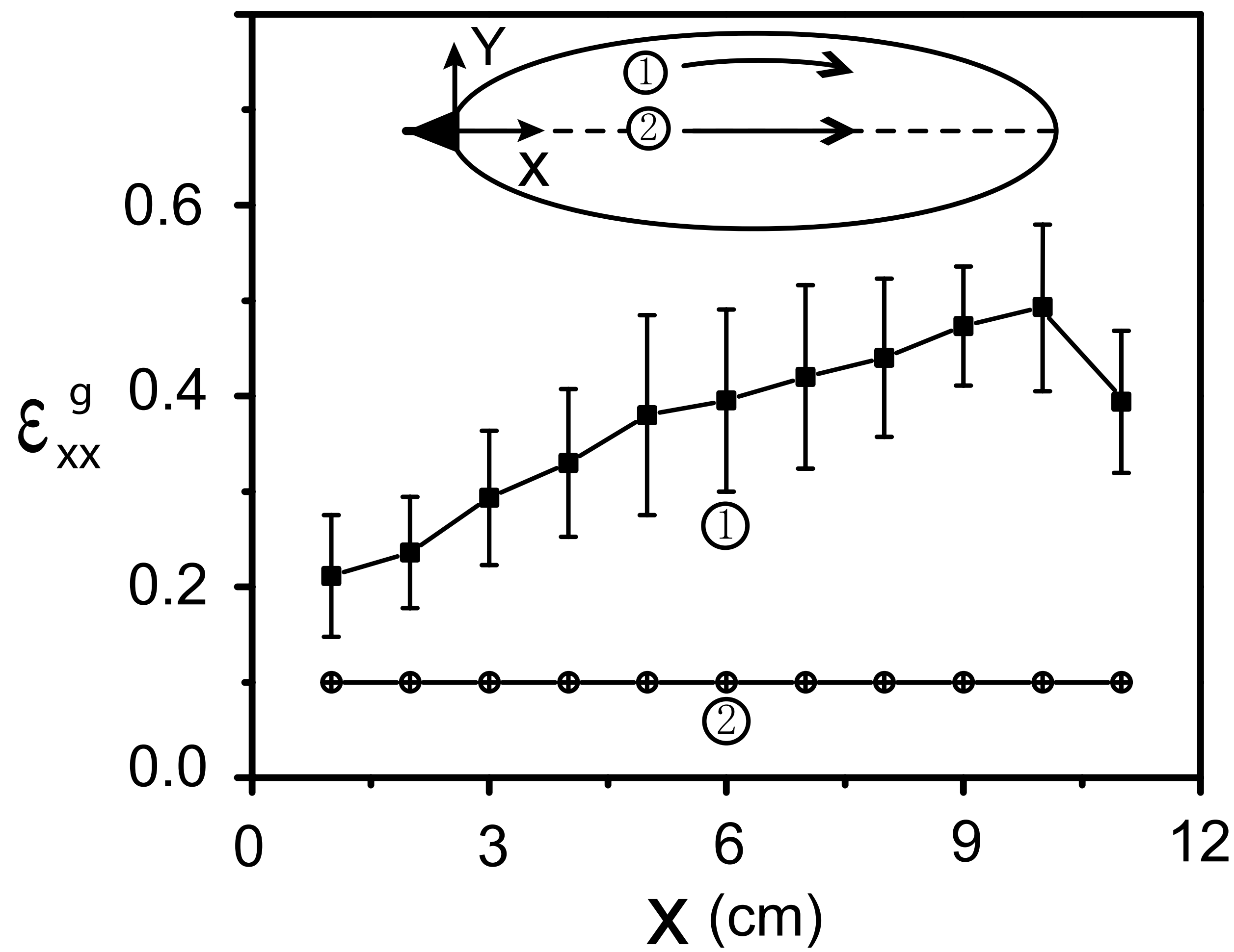
bilayer actuation : only cylinders !

going further?

blooming

courbure G modifiée = croissance différentielle

in-plane differential growth leads to curvature

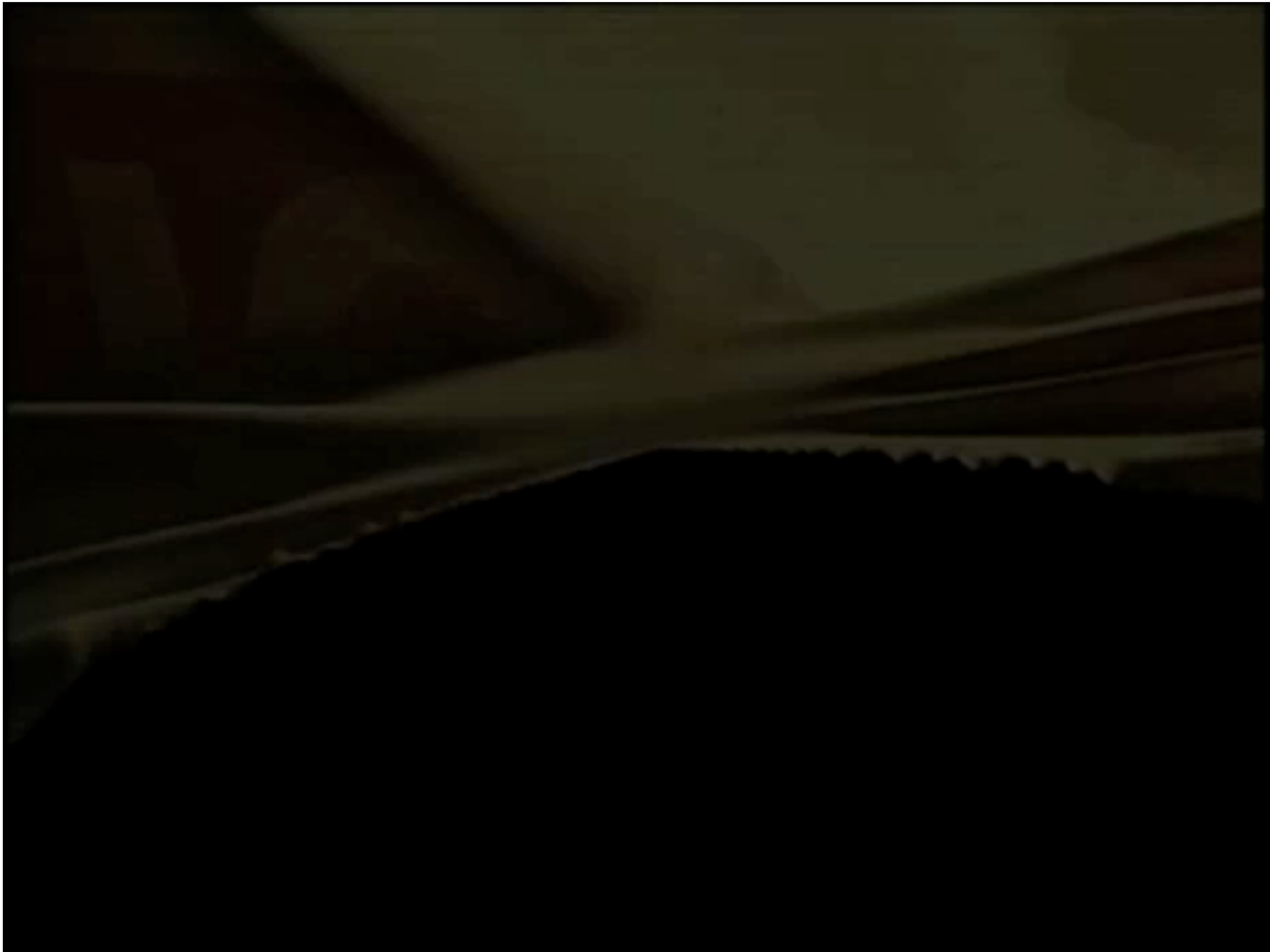


Liang *et al* PNAS 2011

Drying slice of fresh wood

vimeo.com/144342962

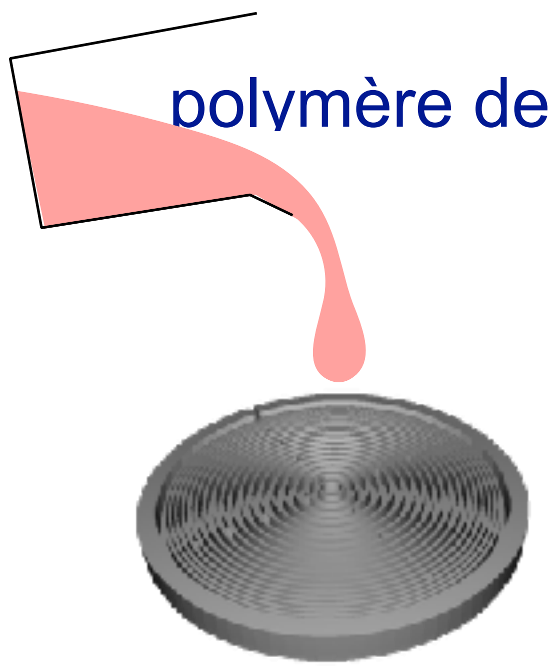
Aurélie Vissac (Amàco) & Pascal Oudet



going further?

mess up with the in-plane distances (metric) !

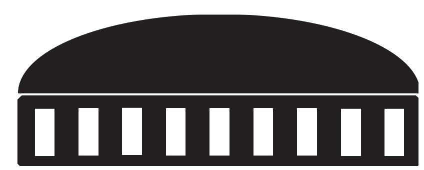
Baromorphs



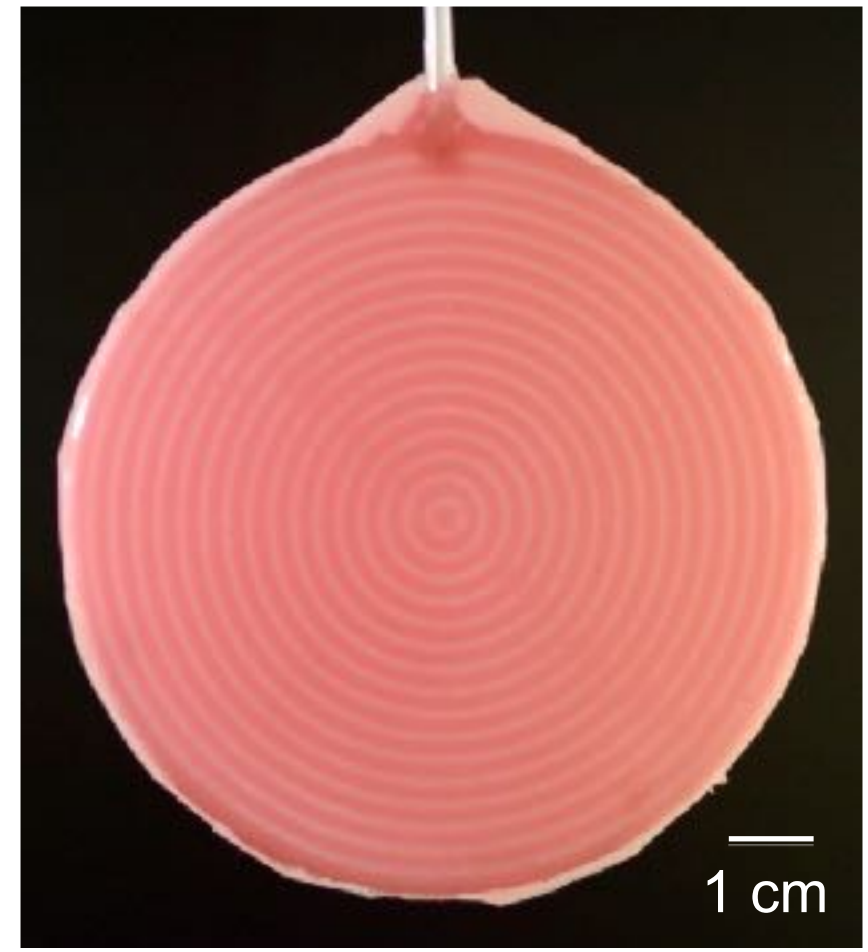
moule

Canaux

dessus



côté

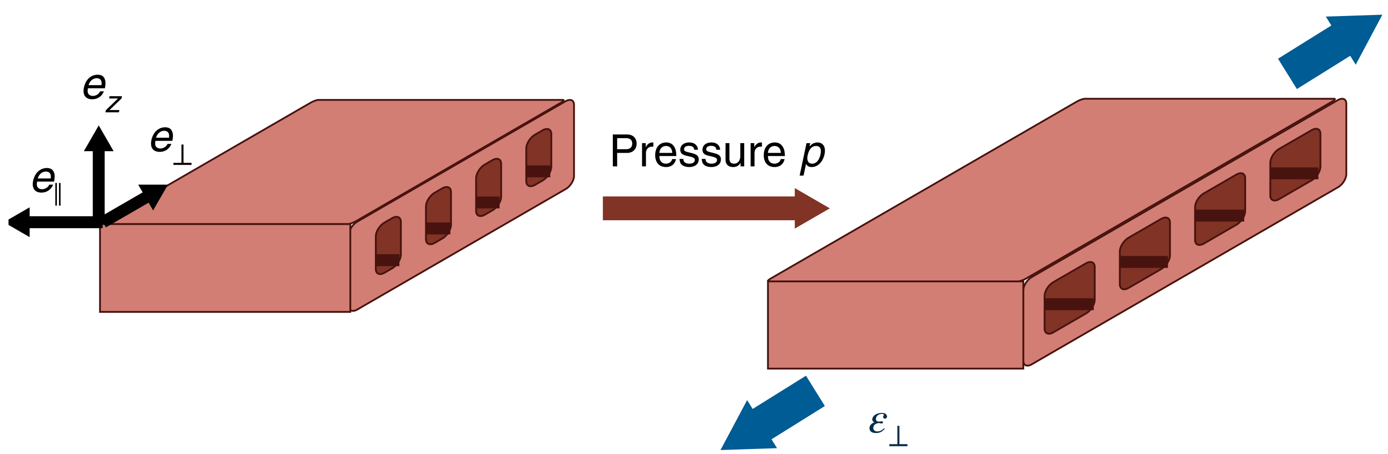


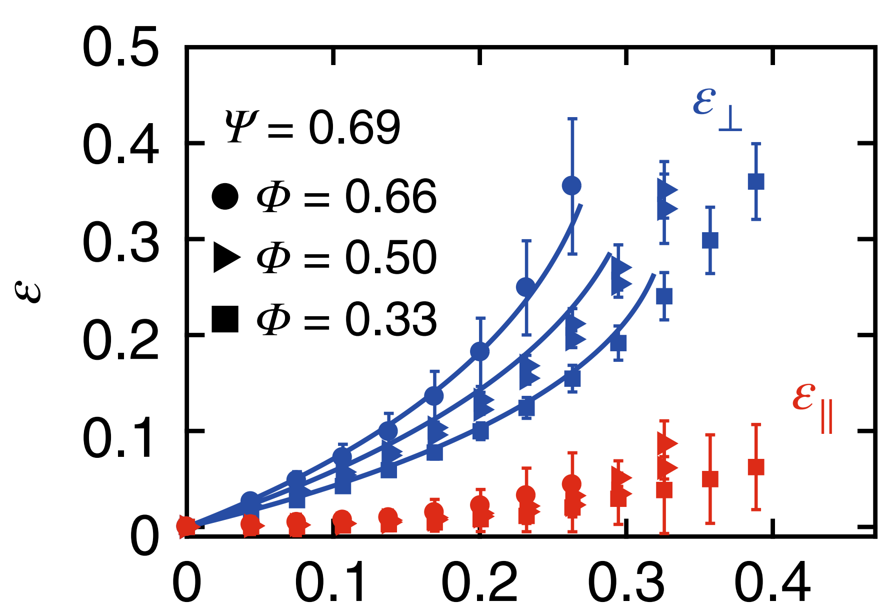
Can we overcome Gauss' theorem ?

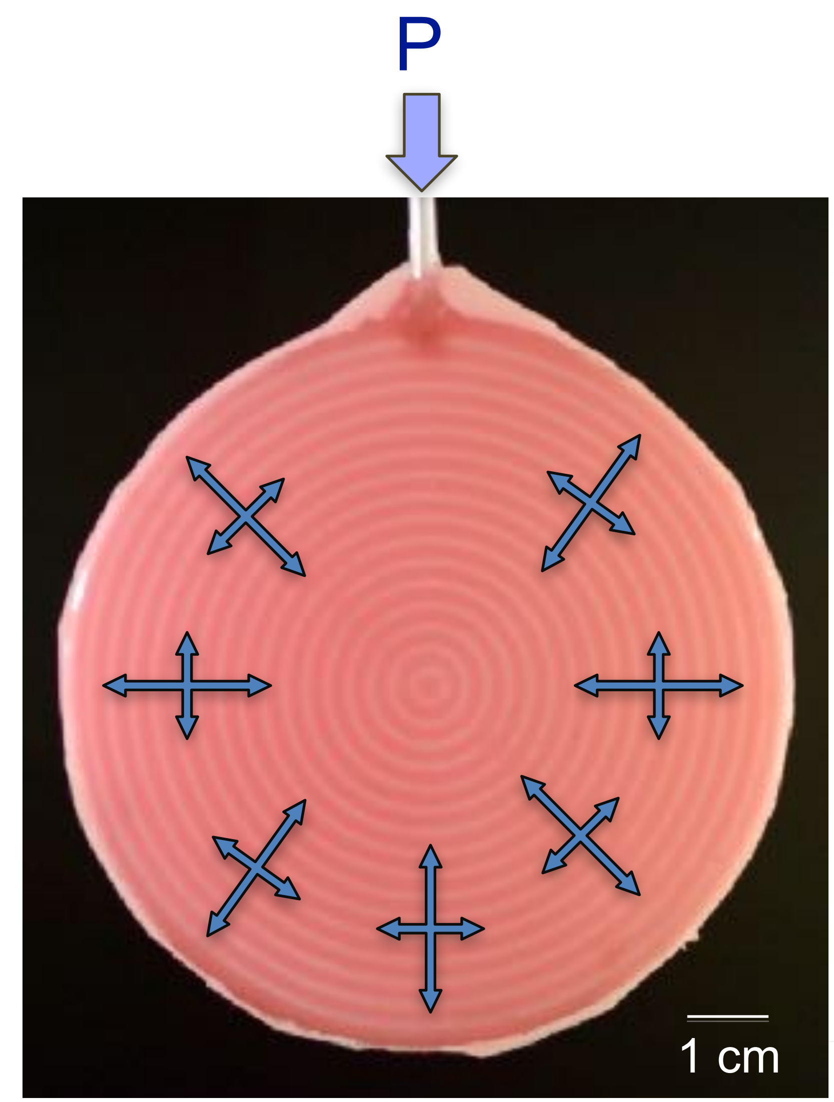
for shape morphing : change distances !

Concentric channels

non-isotropic expansion







perimeter < 2pi R !!

Application : soft robotics

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2) Bending energy
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1) what to expect (small slope eq. for beams)
2) plate equations
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$\boxed{\text{GEOMETRY}}$