Soft Solids

2

Slender objects

Mini-projects ! https://blog.espci.fr/softsolids

1. the sound of the whip
2. the safety band wrapping
3. cutting roast-beef
4. how fast can you hit a golf ball
5. sticky climber competition
6. The solid chain siphon
7. folding by stretching
8. highest pile of cube?
9. self-inflating Kamifusen
10. paper popper
11. the size of a splat ball
12. oak or reed, which one breaks first?
13. How do spider webs stick to walls?

14. maximum height of fall of an M&Ms
15. Reversing an umbrella under strong wind
16. the landing of a snake
17. Why is that so hard to open a packaging ?
18. The mowing of wheat: cutting or bedding?
19. Is it good to have a flexible fishing rod?
20. what is the design a good inverted cap for jumping?
21. What is the bouncing height for water beads?
22. How should captain Haddock shake his finger to debond the band aid?

23… other idea (come and discuss with us)

Thickness h = 50cm

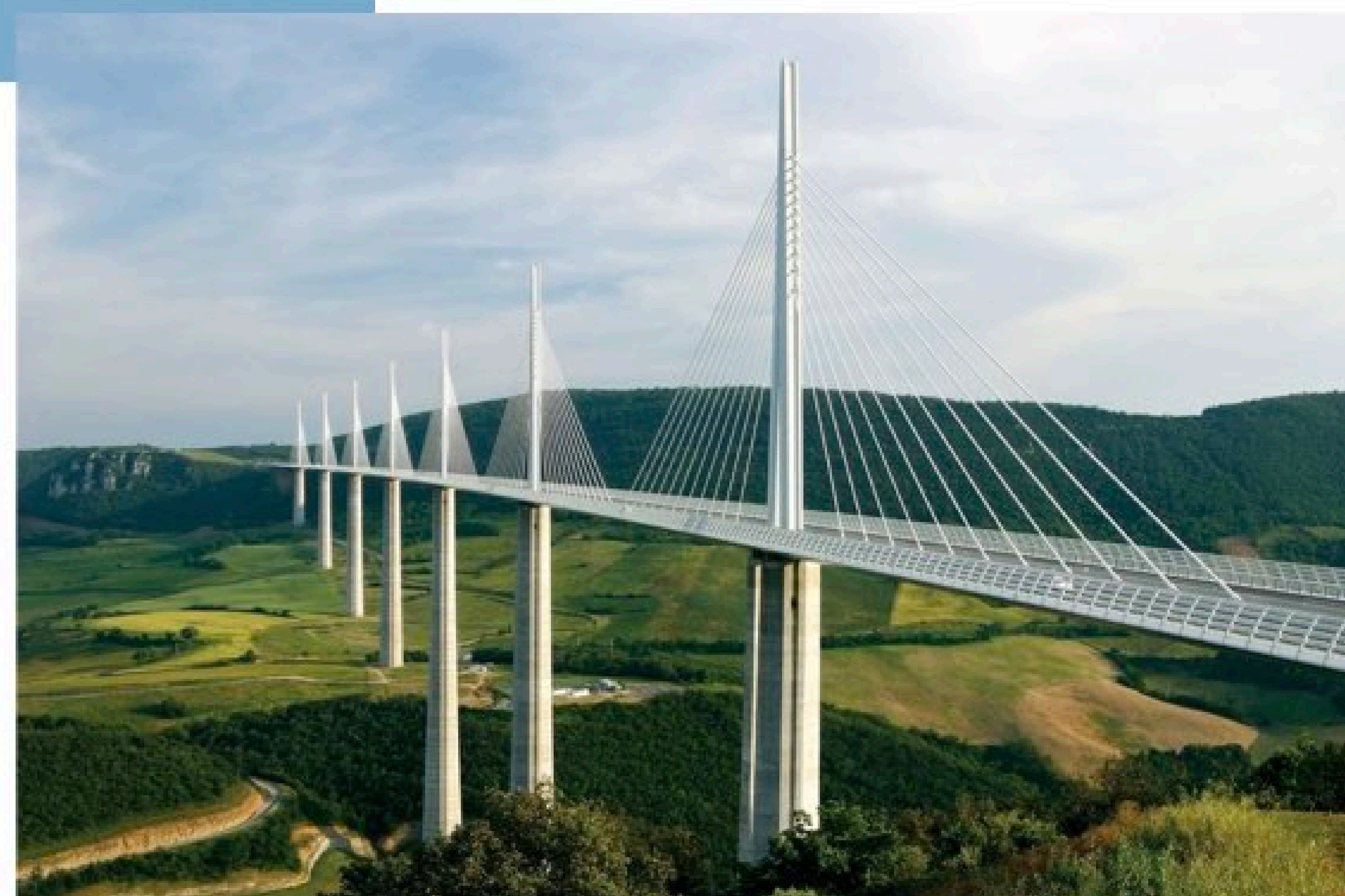
Size L~ 80m

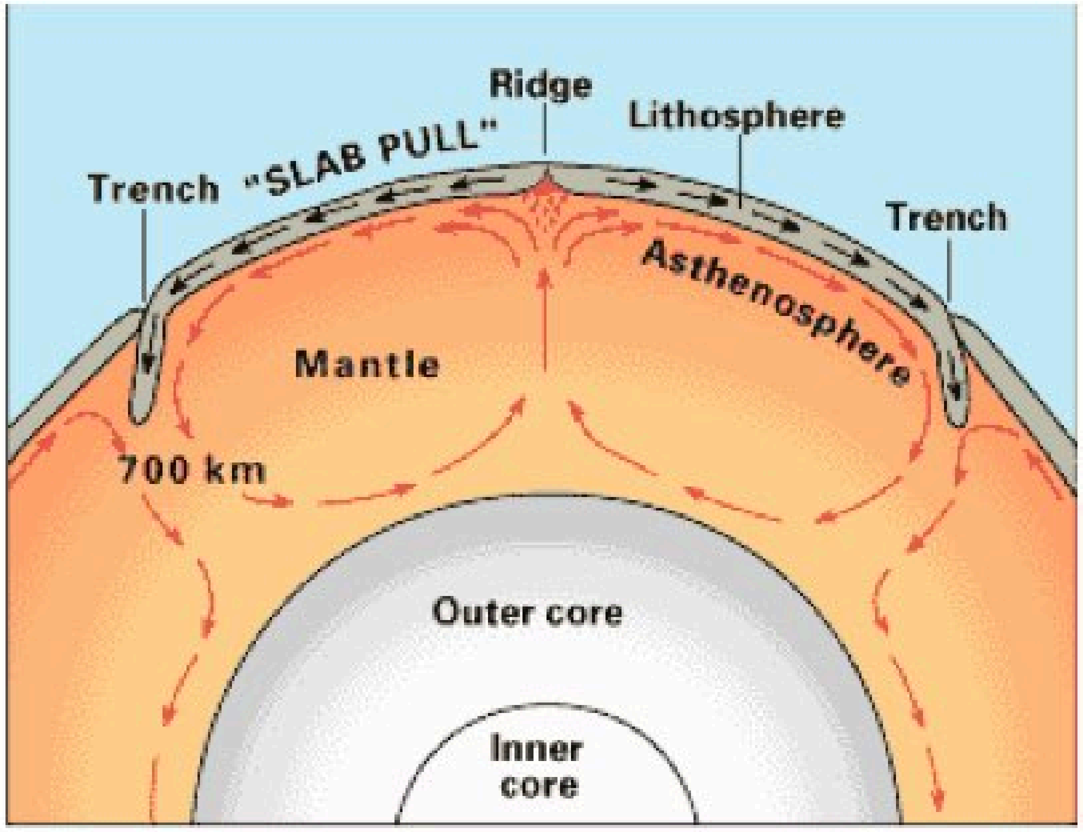
$\frac{h}{L}$ Slenderness 10^2

Thickness ~4m

Size ~ 2.5km

Slenderness 10^2

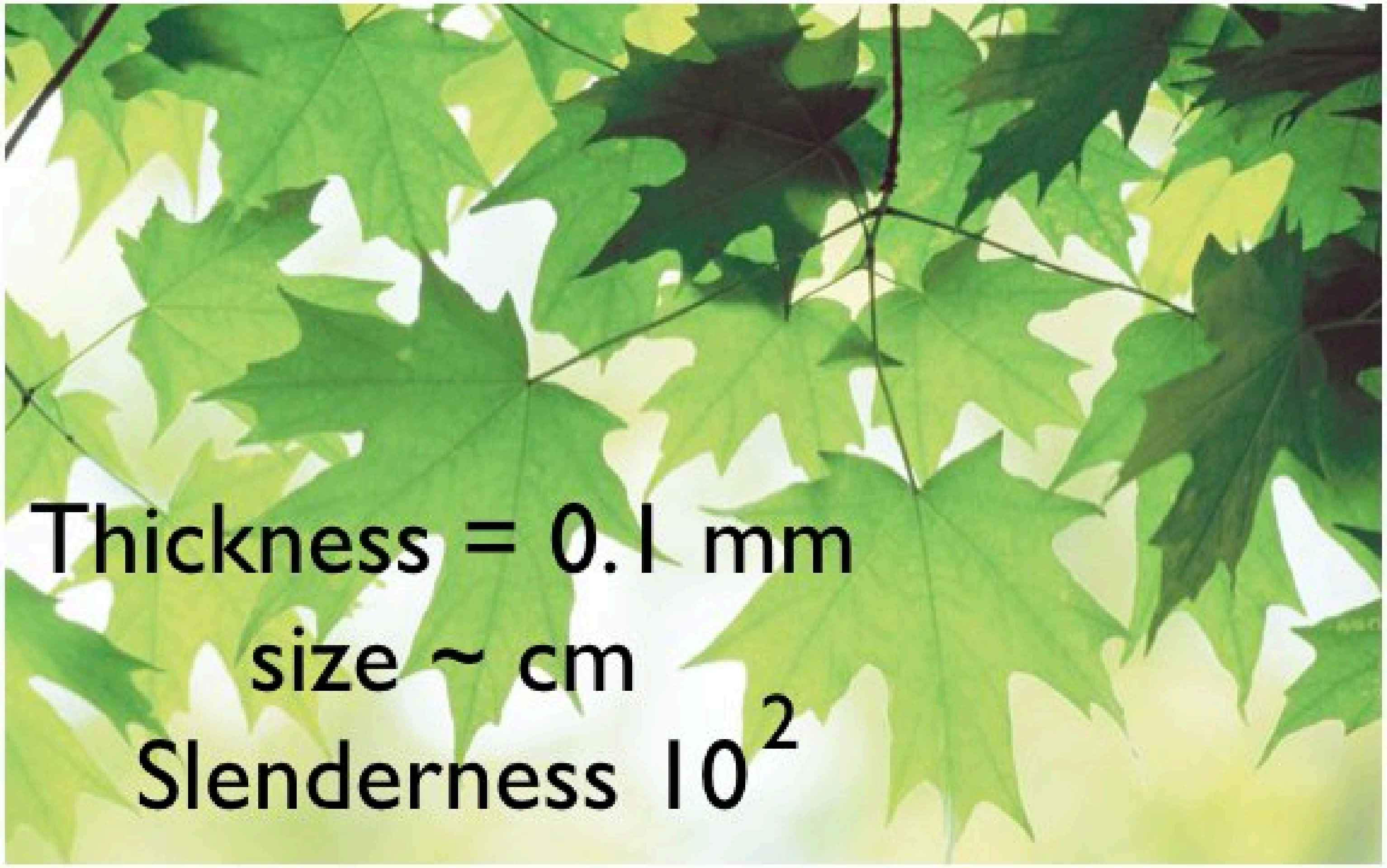


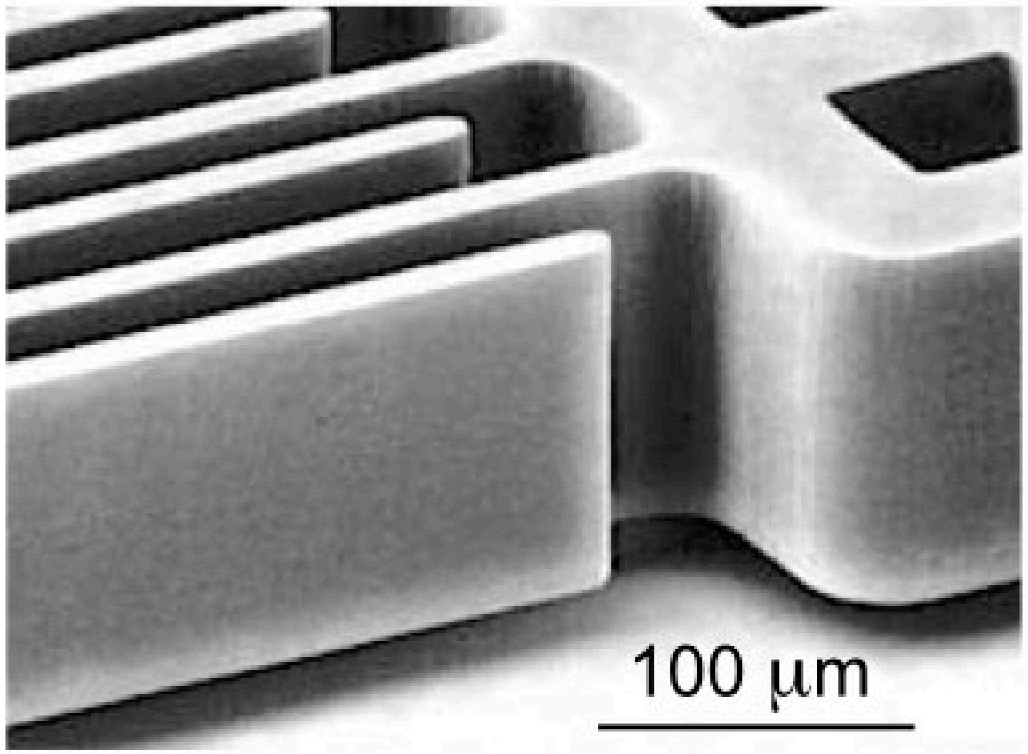


Thickness = 15 - 200 km

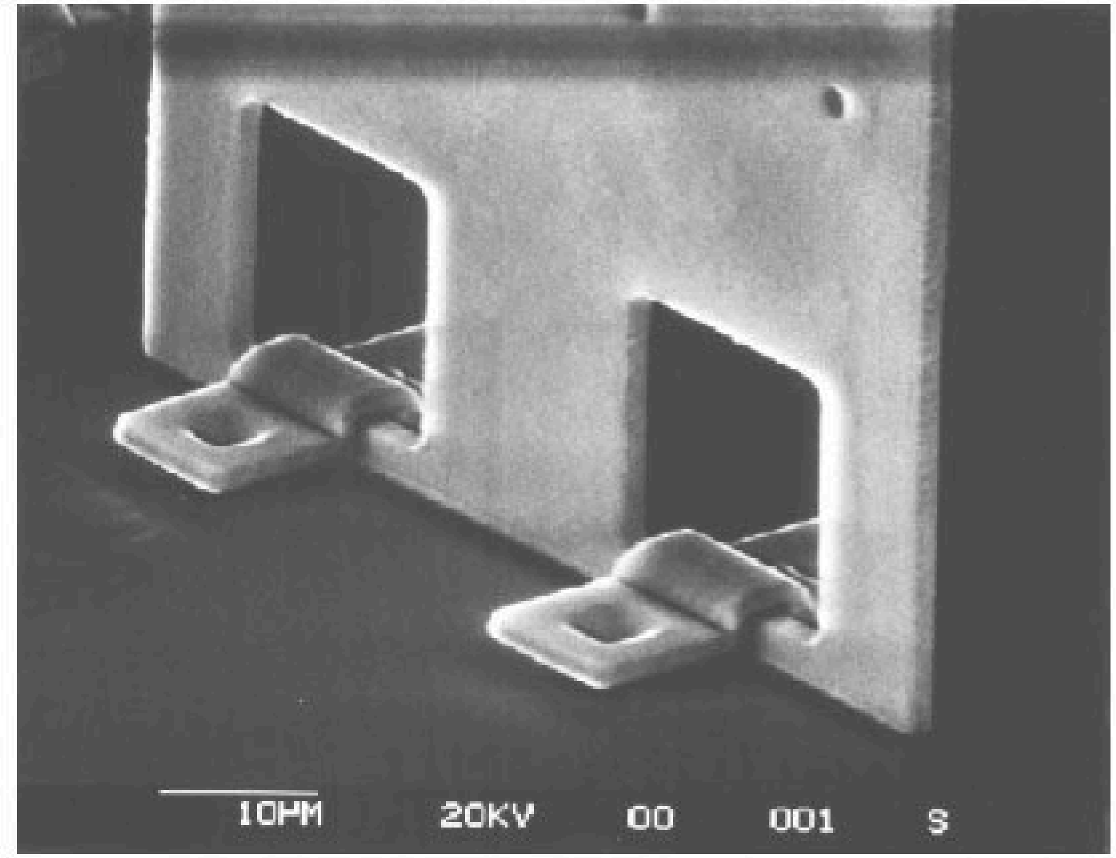
Size ~ 10 000km

Slenderness 10^2

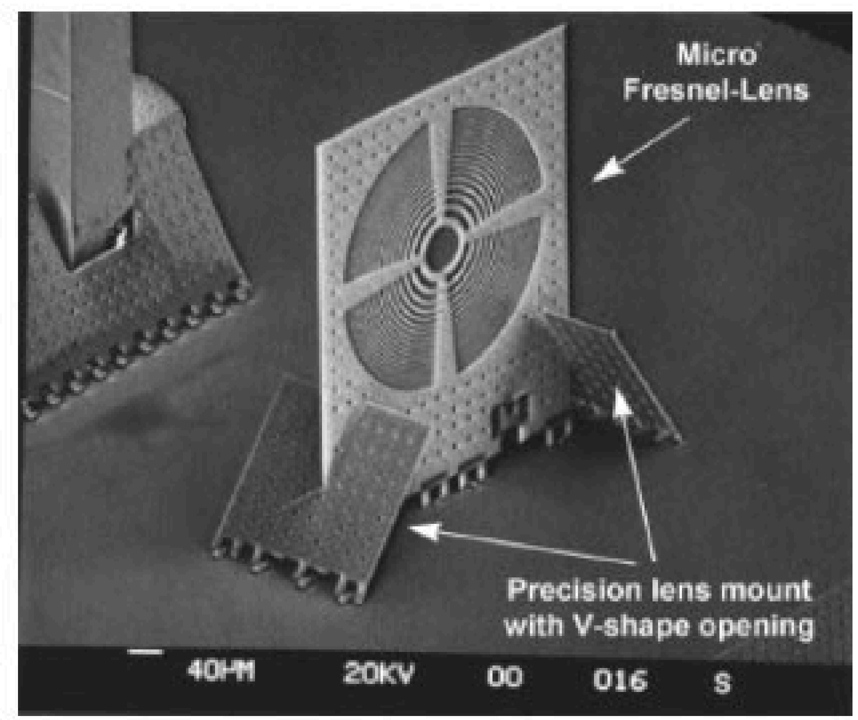

Thickness = 0.1 mm
size ~ cm
Slenderness 10^2



accelerometer

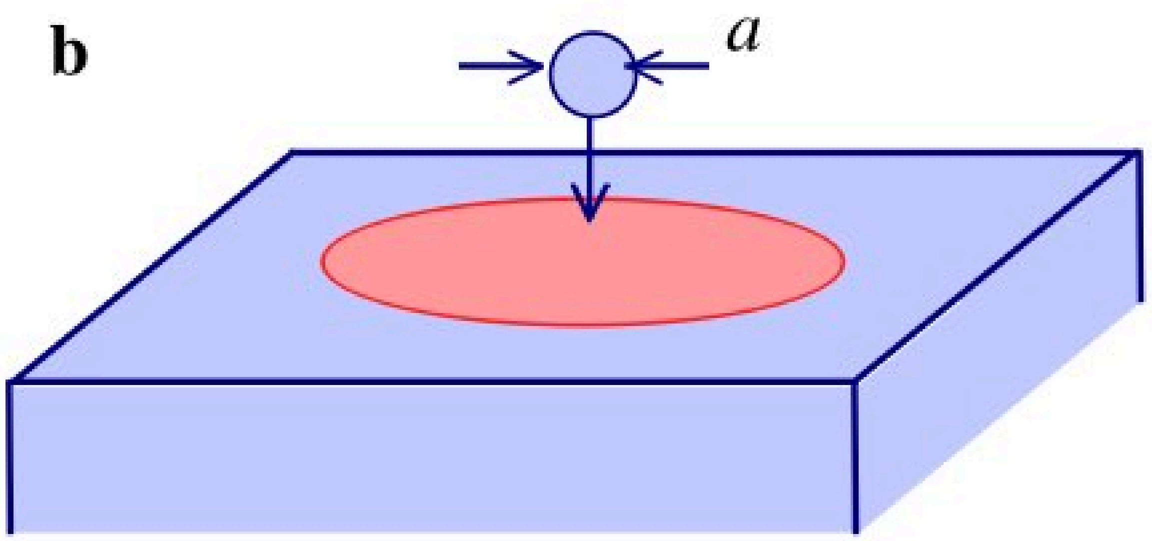


(b)

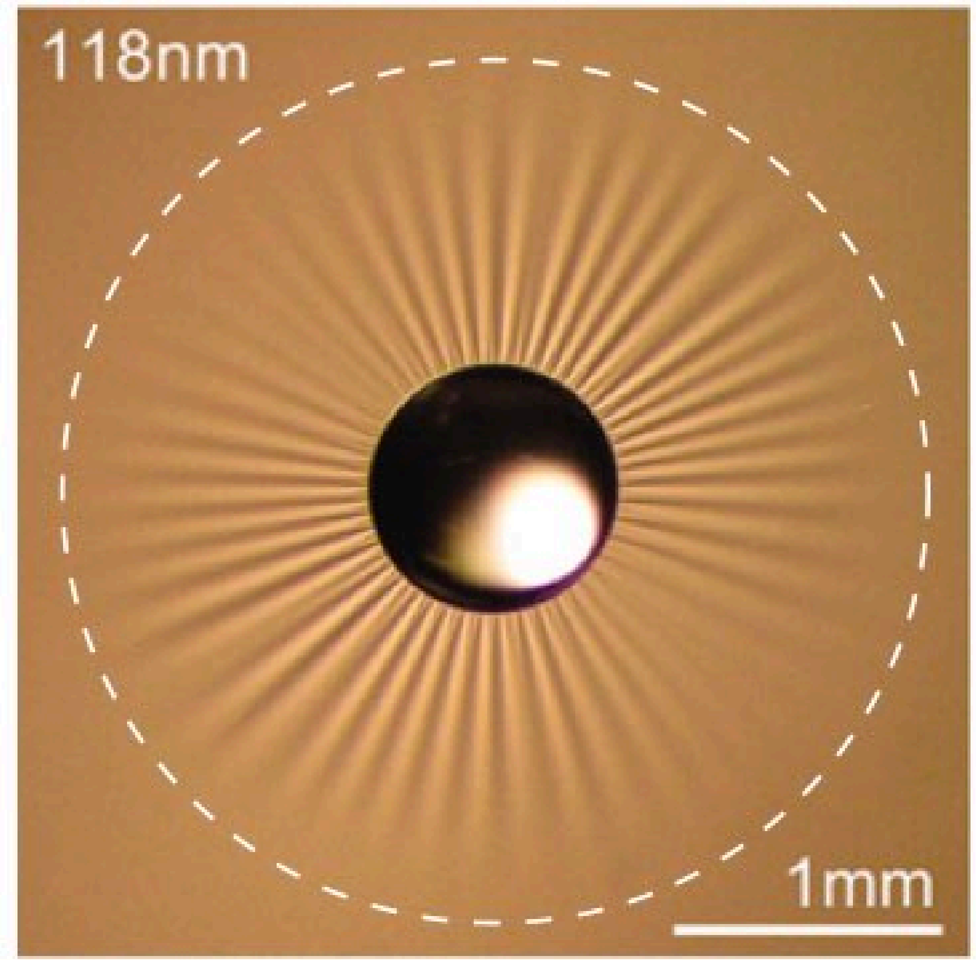


K. S. J. Pister and M. C. Wu

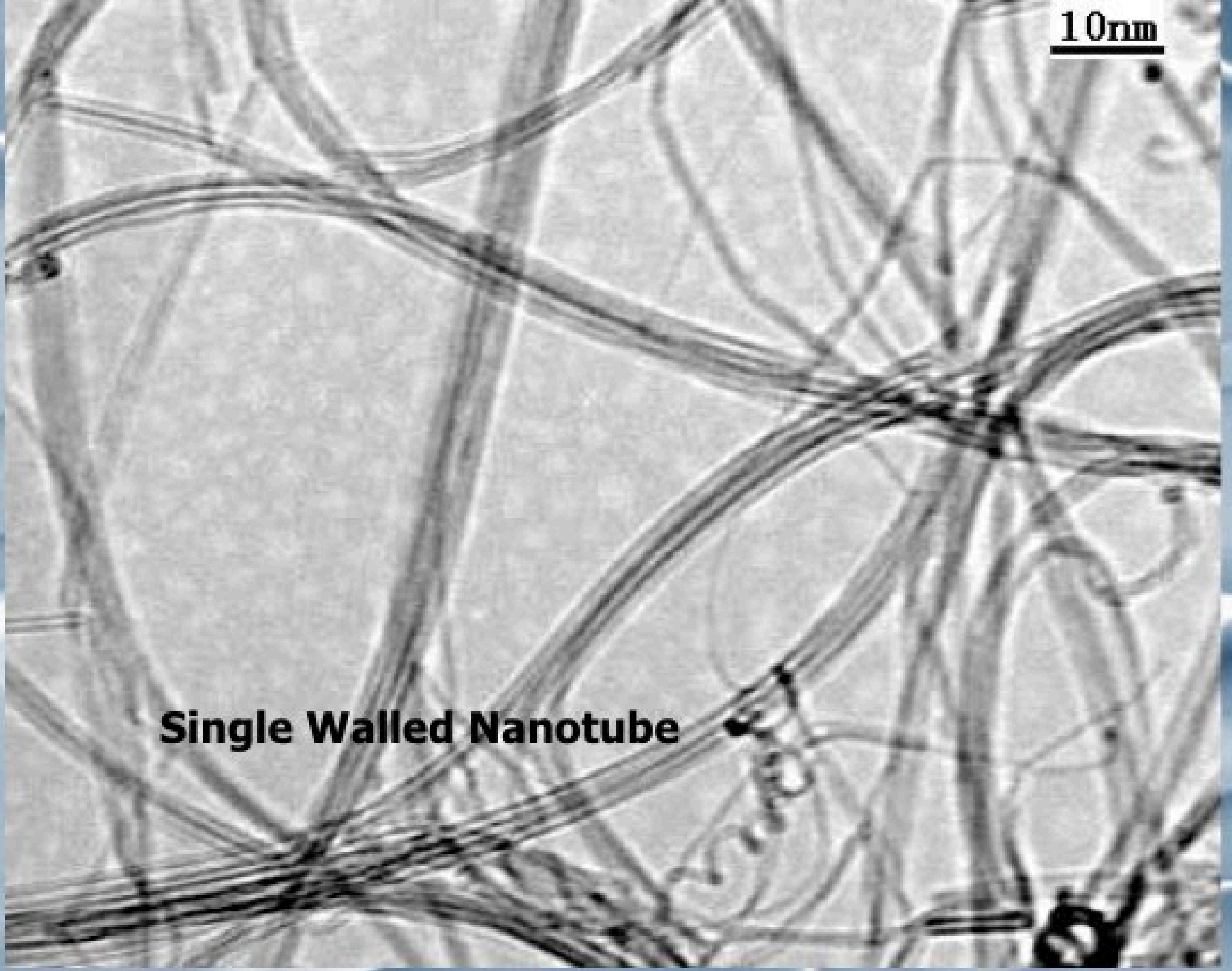
MEMS



Slenderness 10^5



Huang, Juszkiewicz, deJeu, Cerda , Emrick, Menon, Russell *Science 2007*



thickness = 0.3 nm
size ~ cm
Slenderness 10^8

Andre Geim and Kostya Novoselov
Nobel Prize 2010

slenderness $\frac{L}{h} \gg 1$

average quantities through thickness

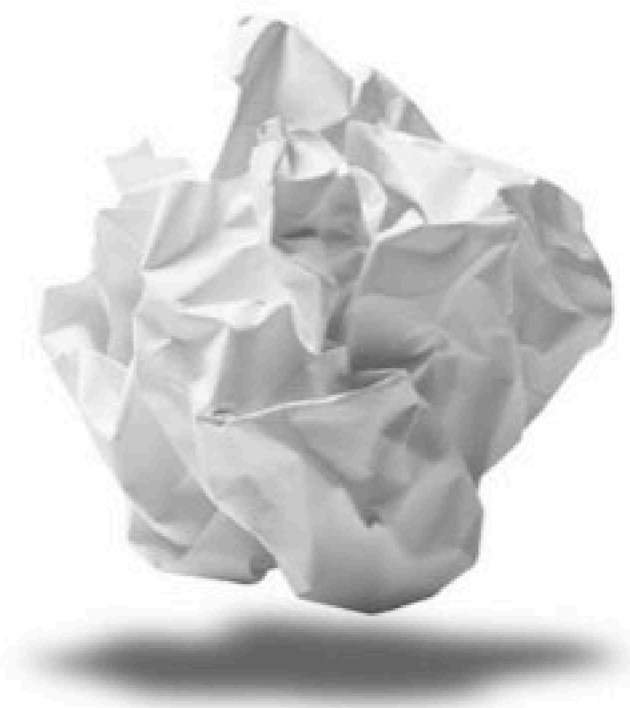
general objectives of the class

basic concepts → current research

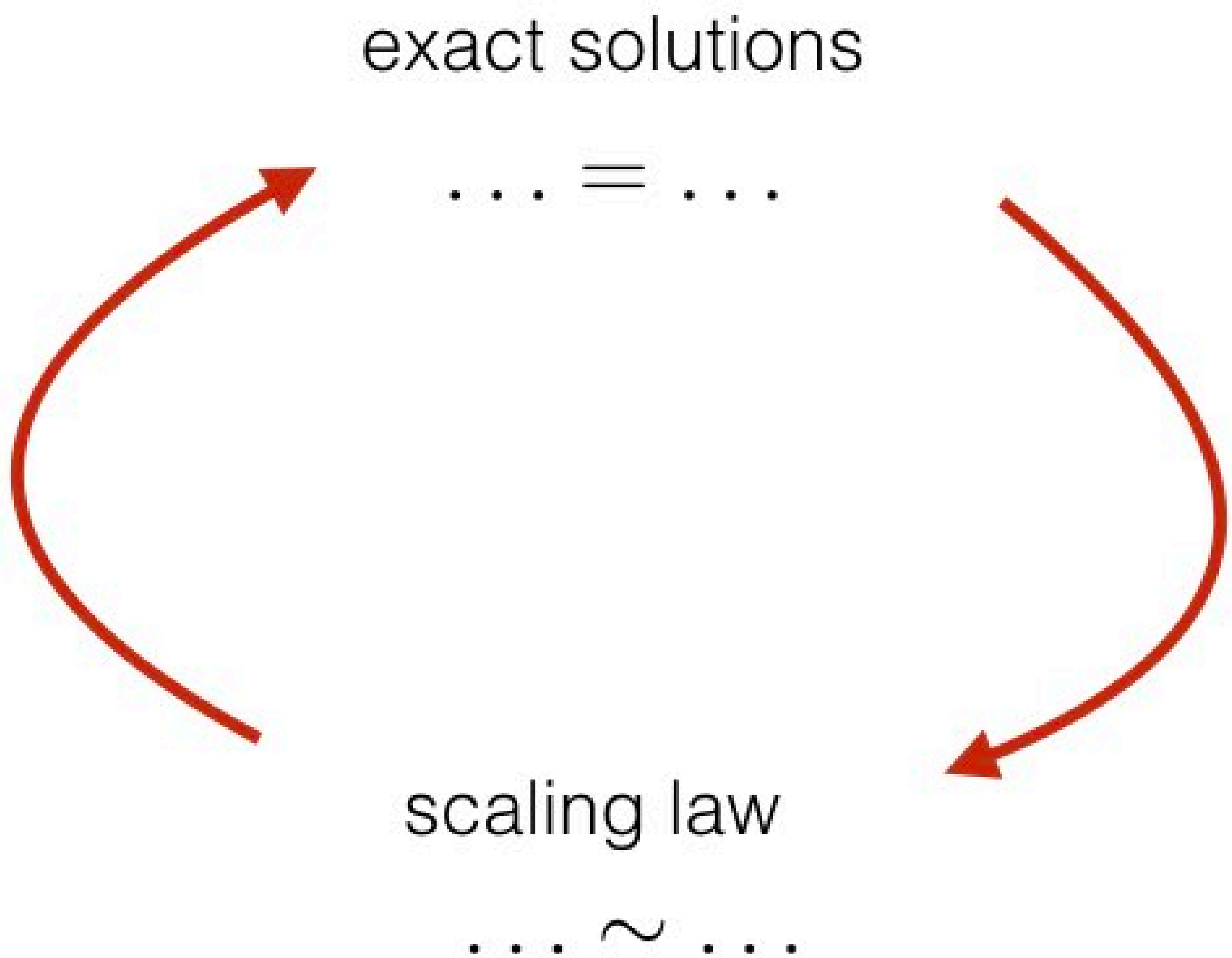
Engineer's point of view

how to design a structure?

curiosity



why does paper crumple?


exact solutions
$\ldots = \ldots$
scaling law
$\ldots \sim \ldots$

objective today


DISCORSI
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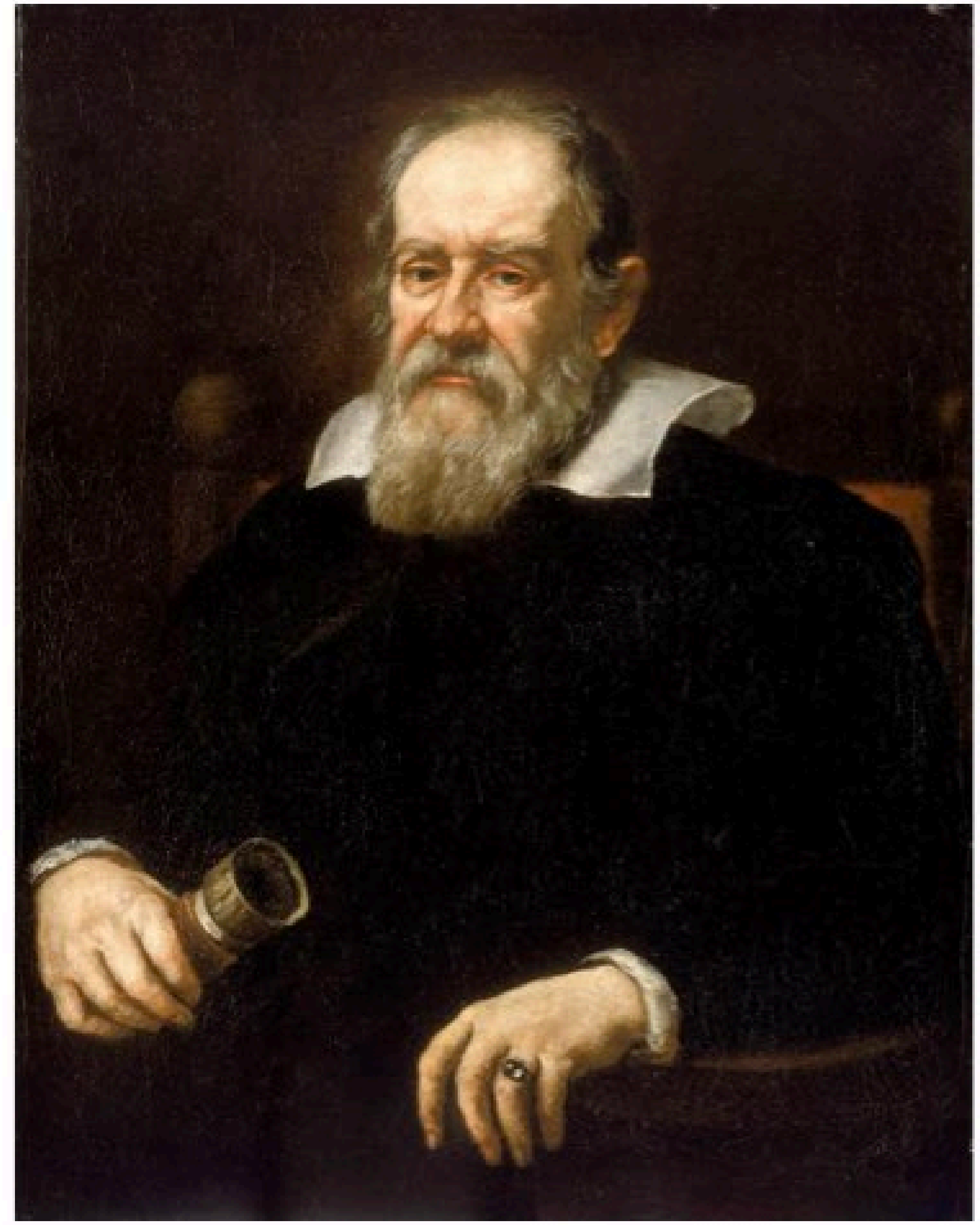
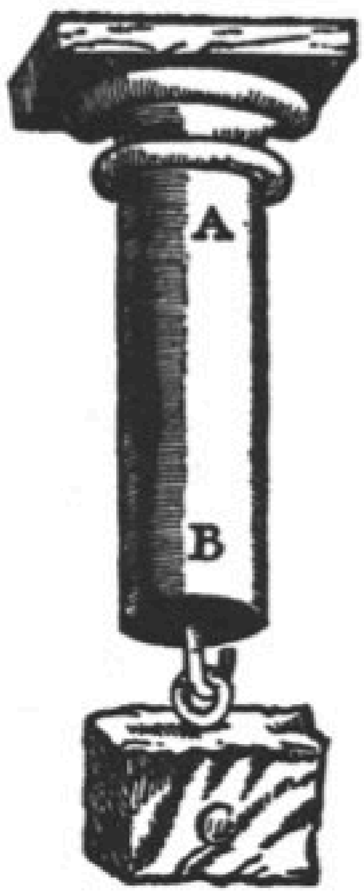
1638

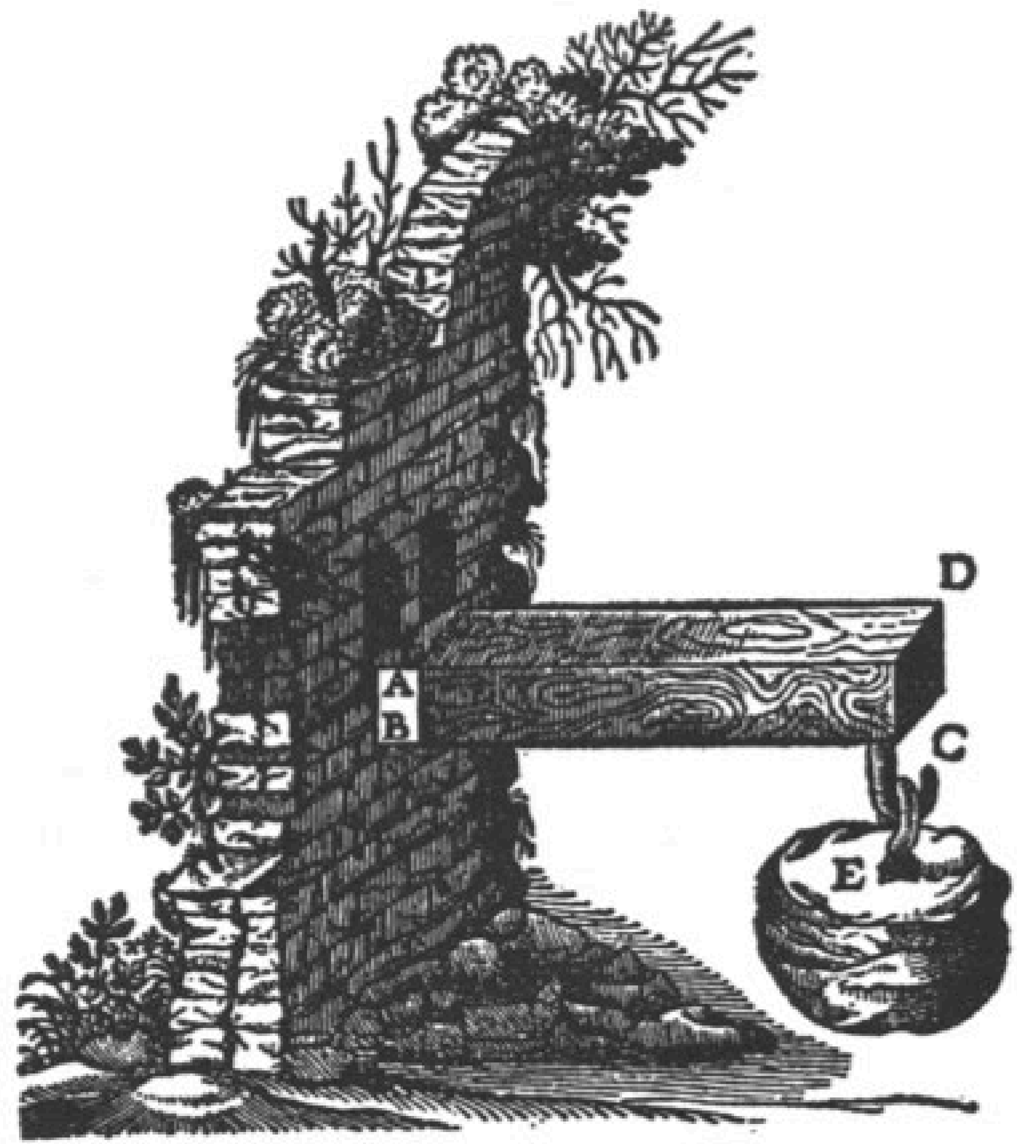


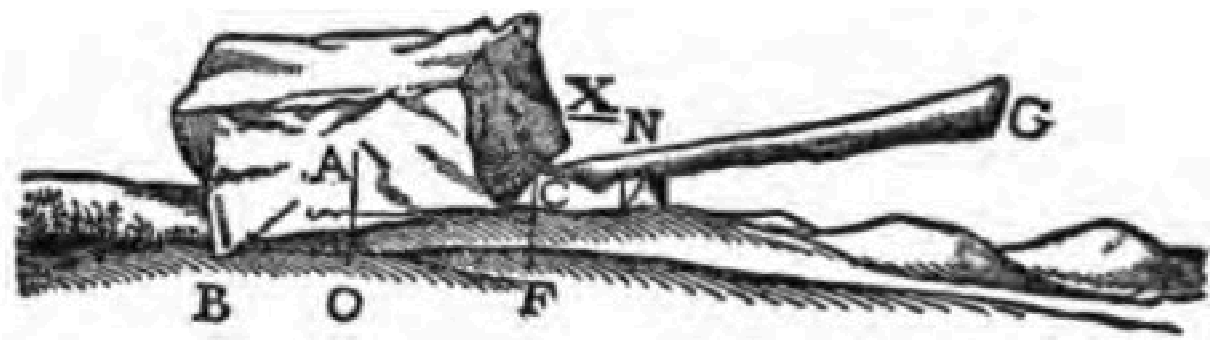
Galileo (1564-1642)

best way to break a rod ?

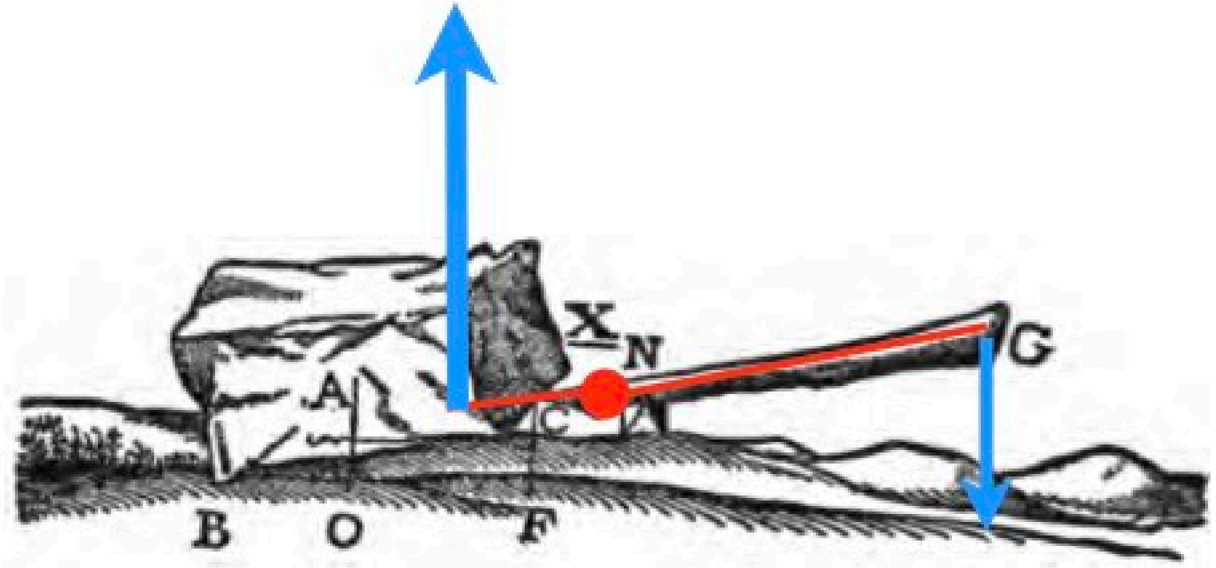
«...Though the resistance [max. force] *is very great in the case of a direct* ***pull,*** *it is found, as a rule, to be less in the case of* ***bending*** *force...»*



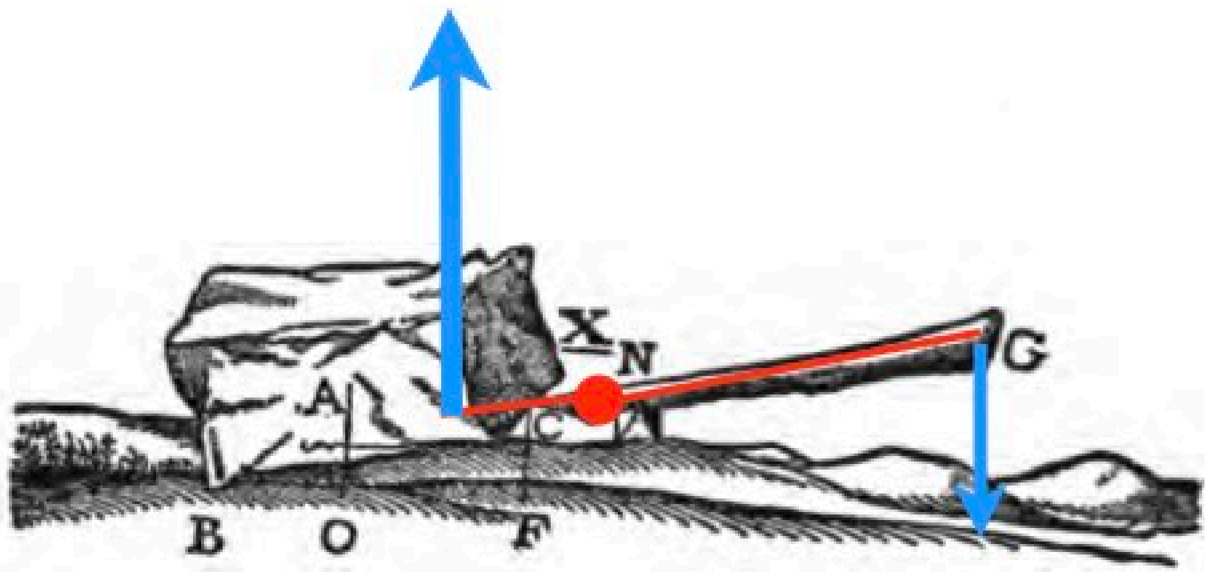




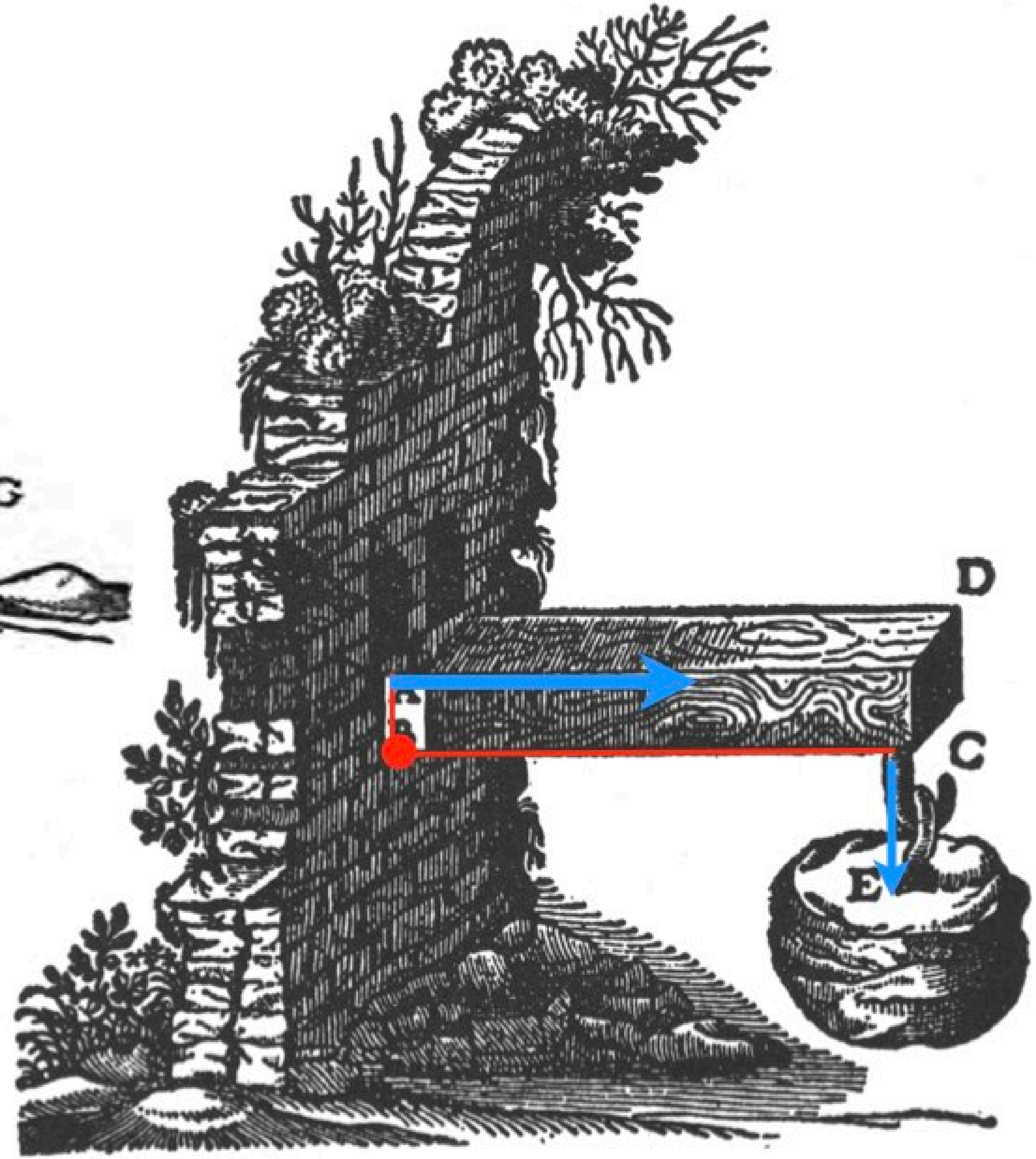
Lever effect (Aristote)

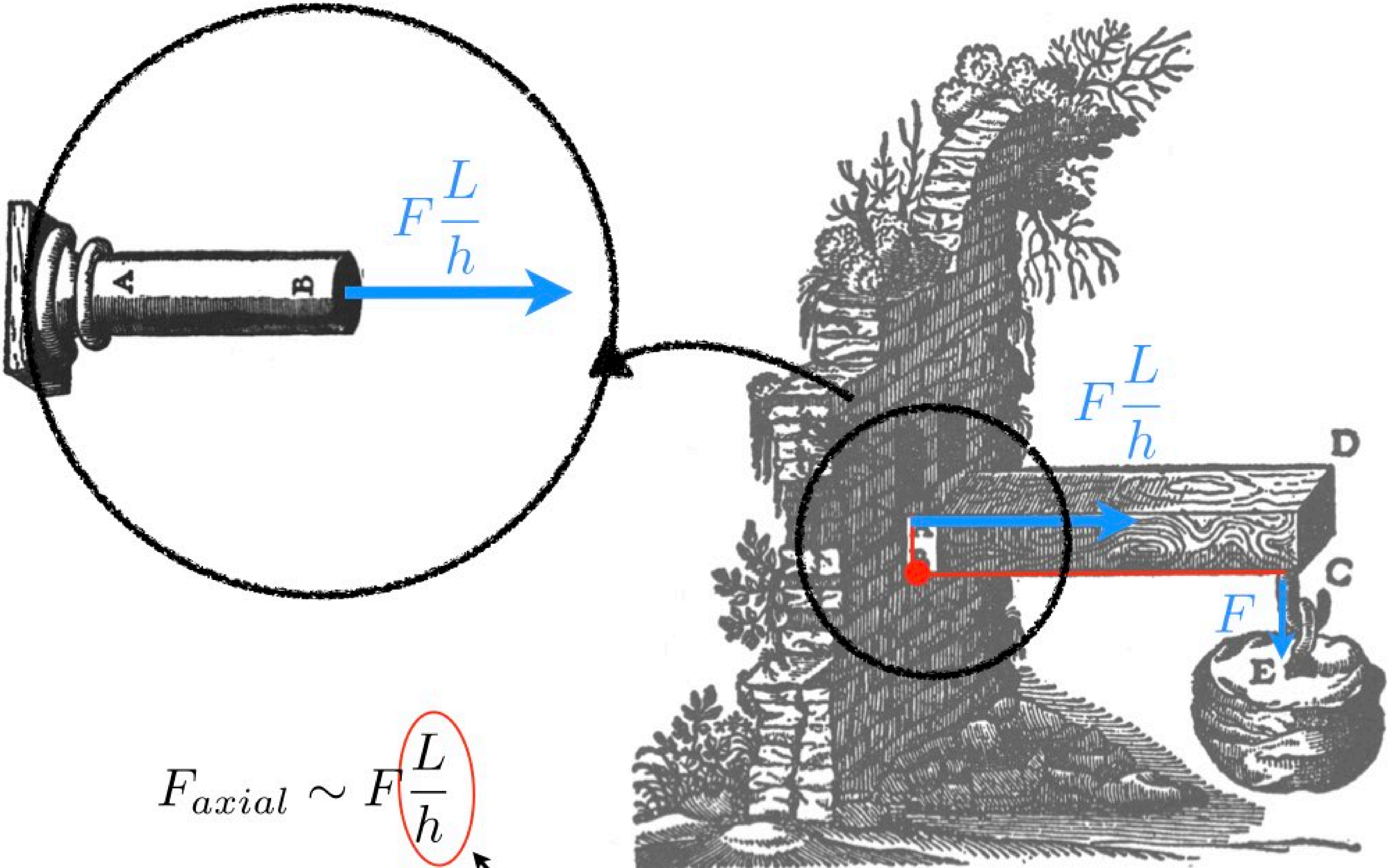


Lever effect (Aristote)



Lever effect (Aristote)

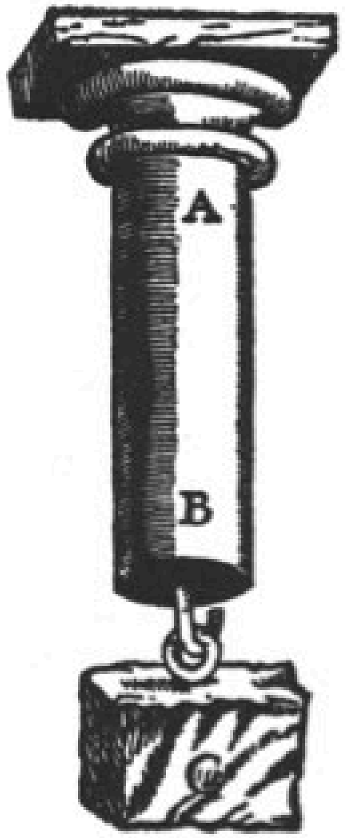




$$F_{axial} \sim F\frac{L}{h}$$

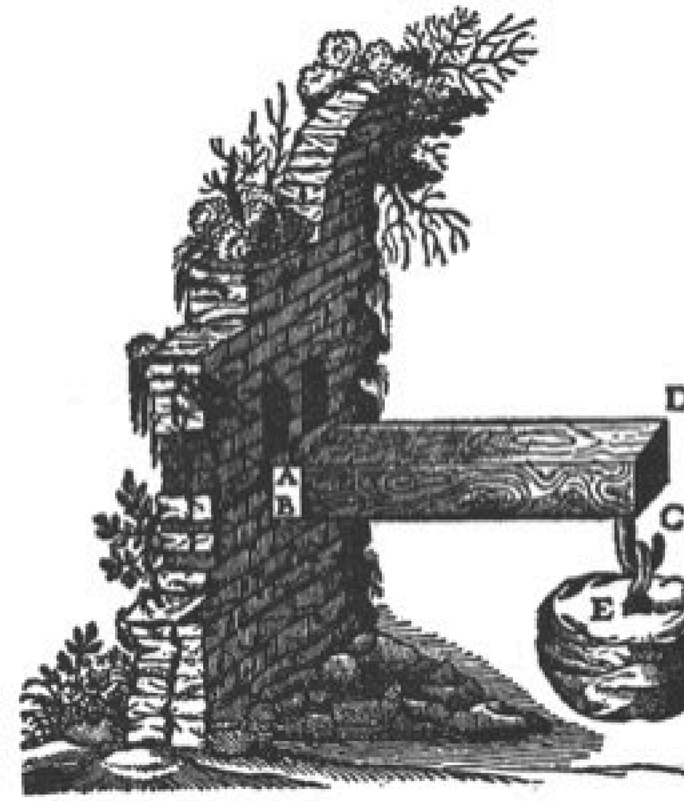
Slenderness constant

Comparing geometrically similar structures : scaling laws



Stretching

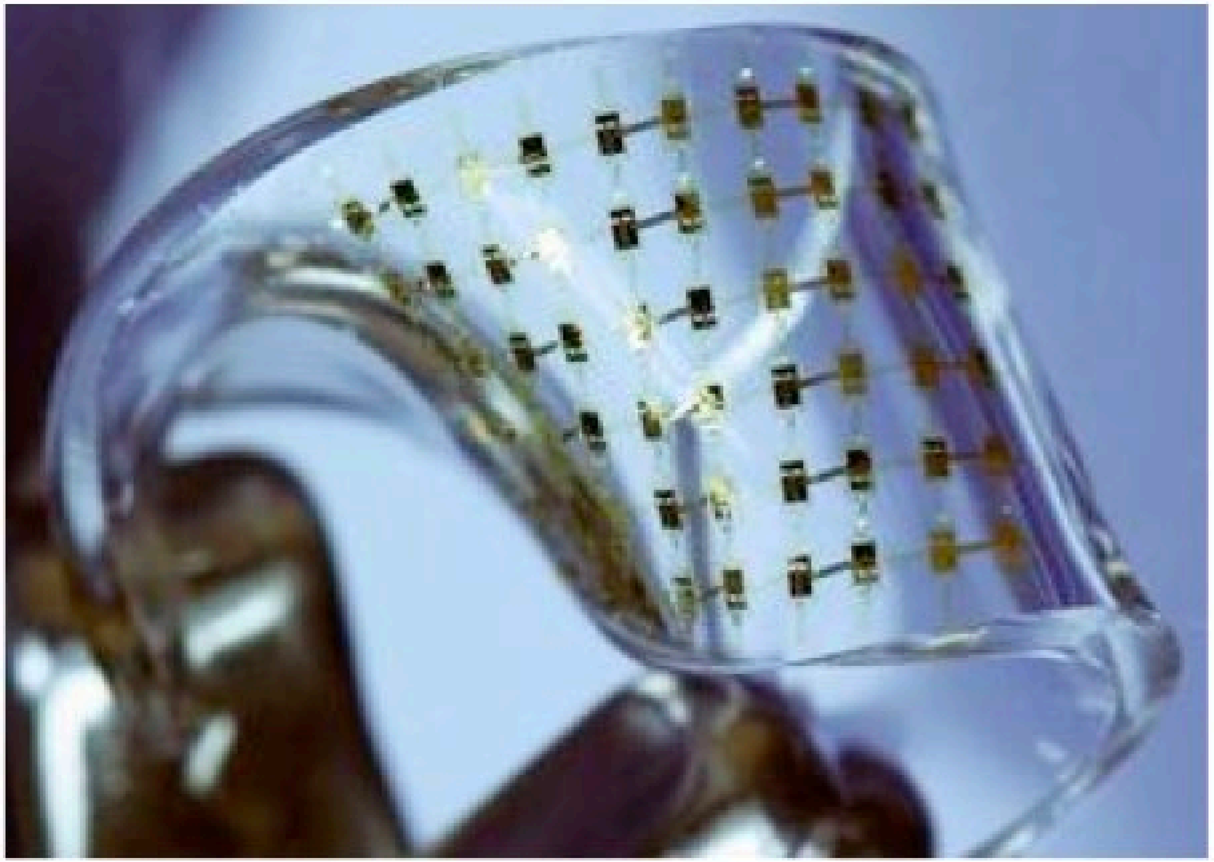
F_{break}



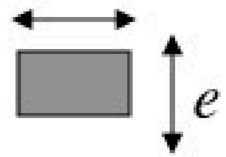
Bending

$F_{break} \sim \frac{h}{L}$ smaller

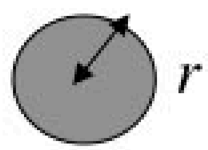
Stretchable electronics



J.Rogers, Urbana Champaign

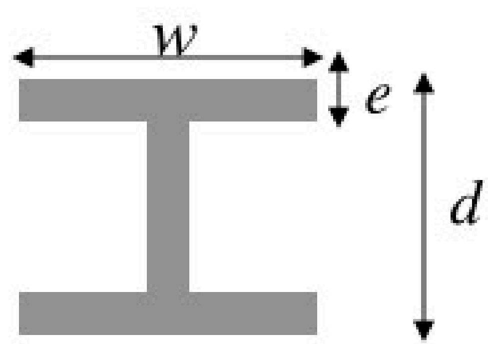


$$I = e^3 w/12$$



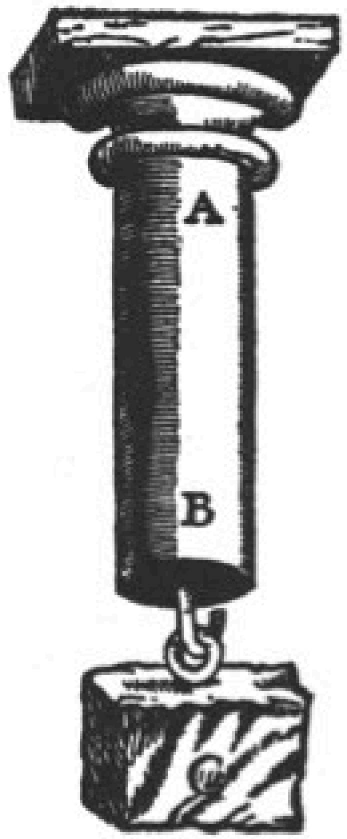
$$I = \pi r^4/4$$

$$I = e^3 w/12(1-\nu^2)$$



$$I = d^2 e w/4$$

Comparing geometrically similar structures : scaling laws



Stretching

$F_{\max} \sim \sigma_u r^2$

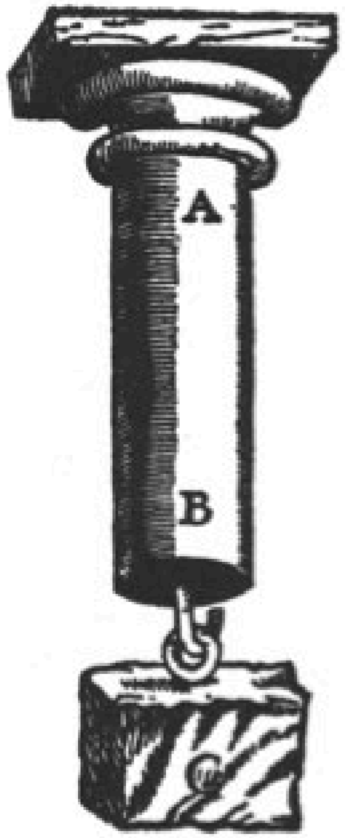
F_{break}

Bending

$F_{\max} \sim \sigma_u r^2 (r/L)$

$F_{break} \sim \frac{h}{L}$ smaller

Comparing geometrically similar structures : scaling laws



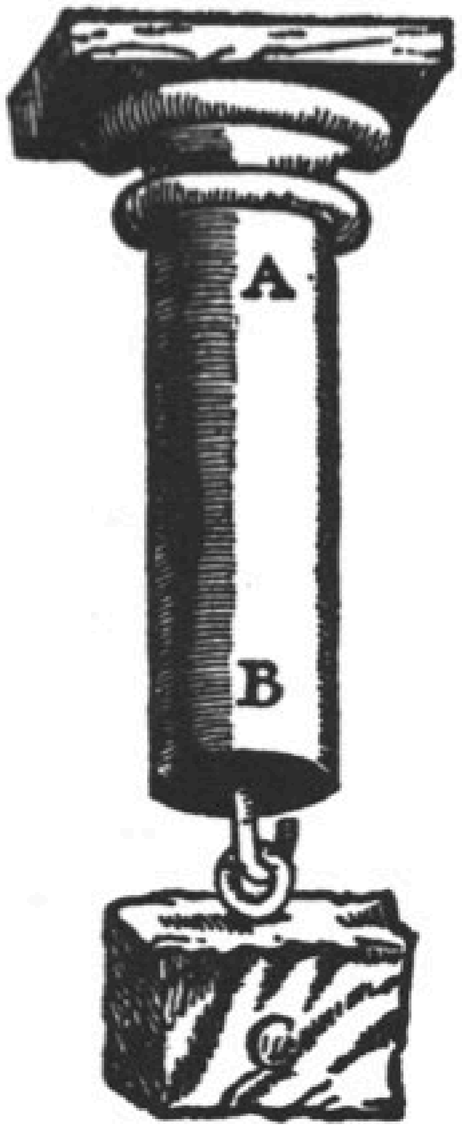
Stretching

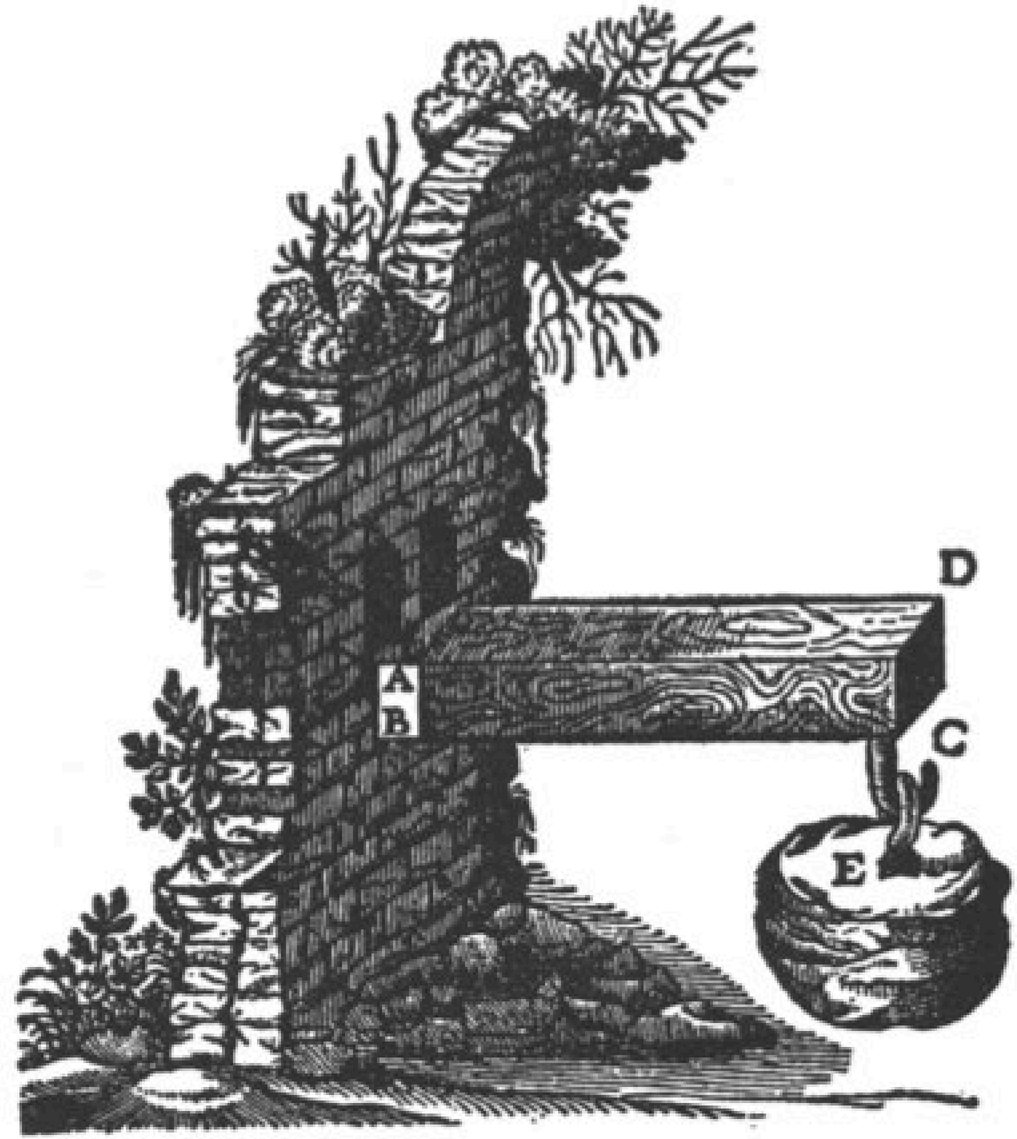
$$F_{\max} \sim \sigma_u r^2 \sim L^2$$

Bending

$$F_{\max} \sim \sigma_u r^2 (r/L) \sim L^2$$

weight $\sim L^3$



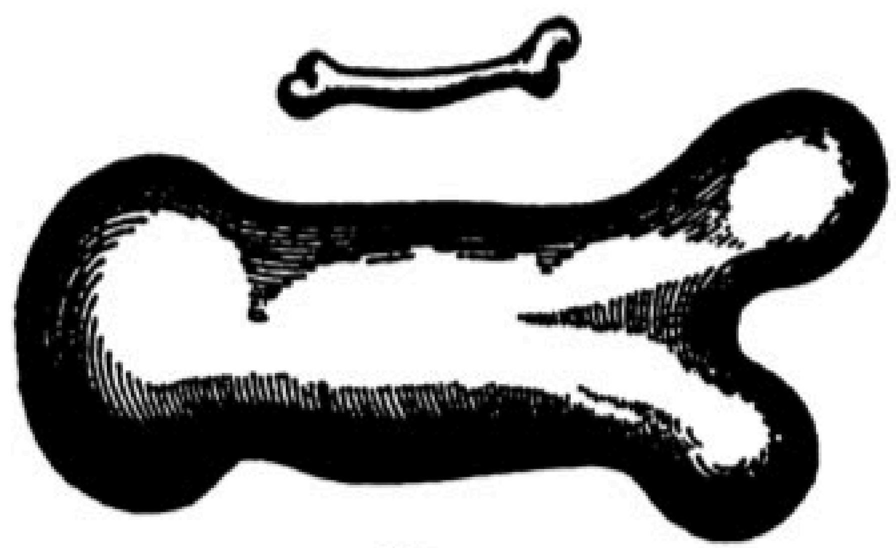


Stretching

Bending

larger structure = weaker (by the same proportion)

«...You can see the impossibility of increasing the size of structures to vast dimension; building ships, palaces, temples of enormous size;
nor can Nature produce trees of extraordinary size because branches would break under their own weight;
it would be impossible to build up the bony structure of men, horses or other animals so as to hold together and perform their normal functions if these animals were to be increased enormously in height; for this increase in height can be accomplished only by employing a material which is stronger than usual, or by enlarging the size of bones thus changing their shape until the form of the animals suggest a monstruosity»



«..a small dog could probably carry on his back two or three dogs of his own size, but i believe that a horse could not carry even one of his own size.»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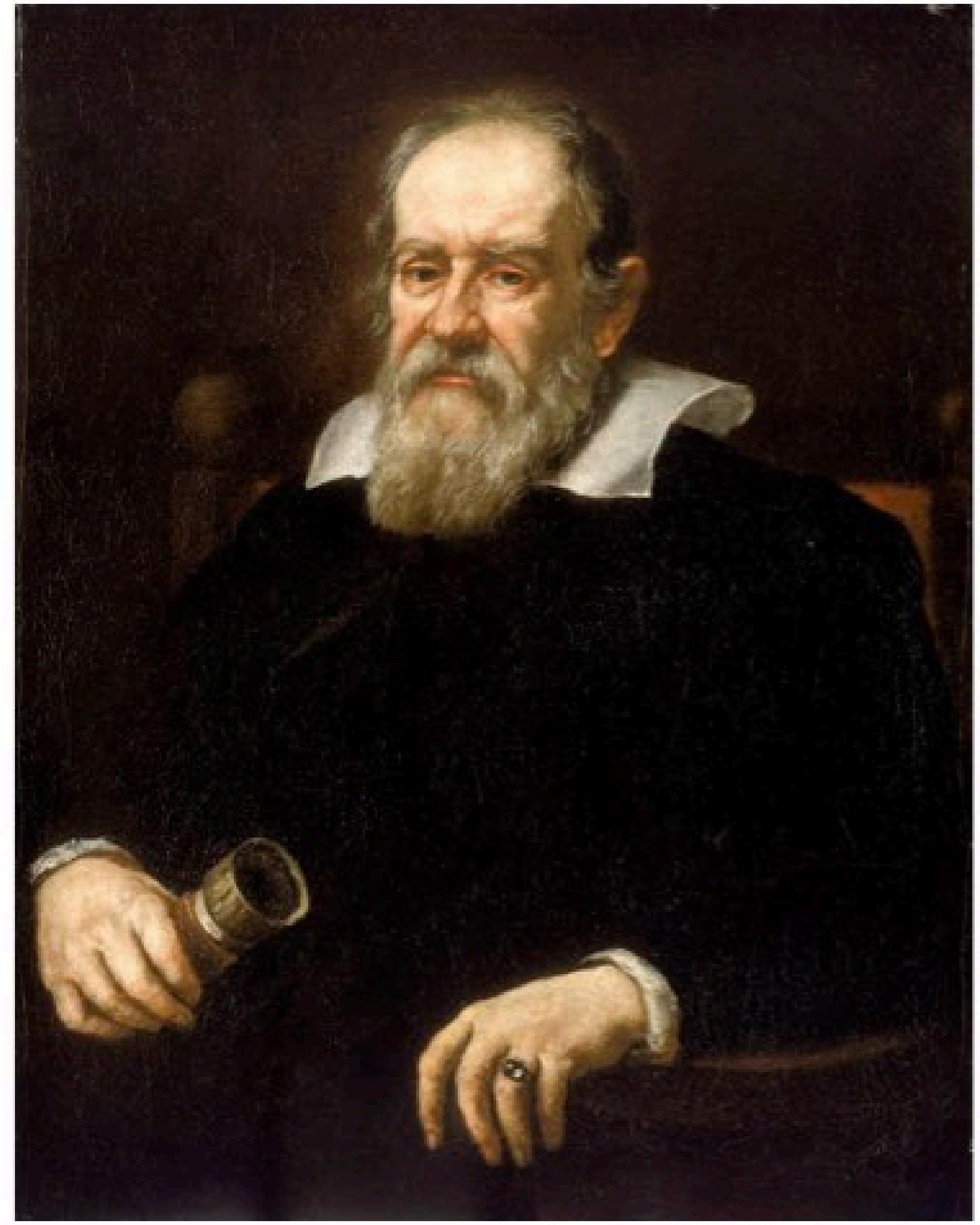


Galileo (1564-1642)

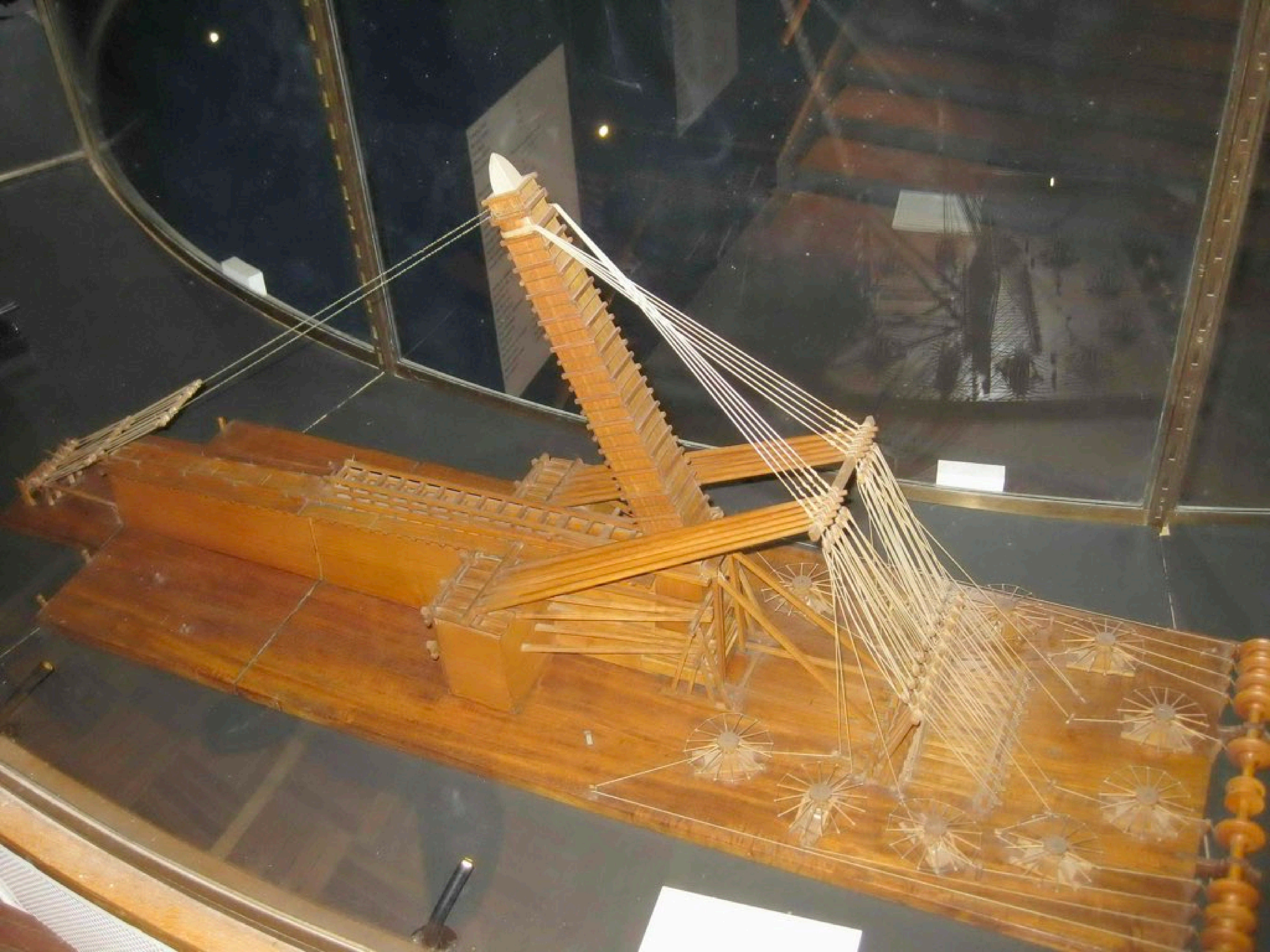
«...a small obelisk or column can certainly laid down or set up without danger of breaking, while the very large ones will go to pieces under the slightest provocation...»

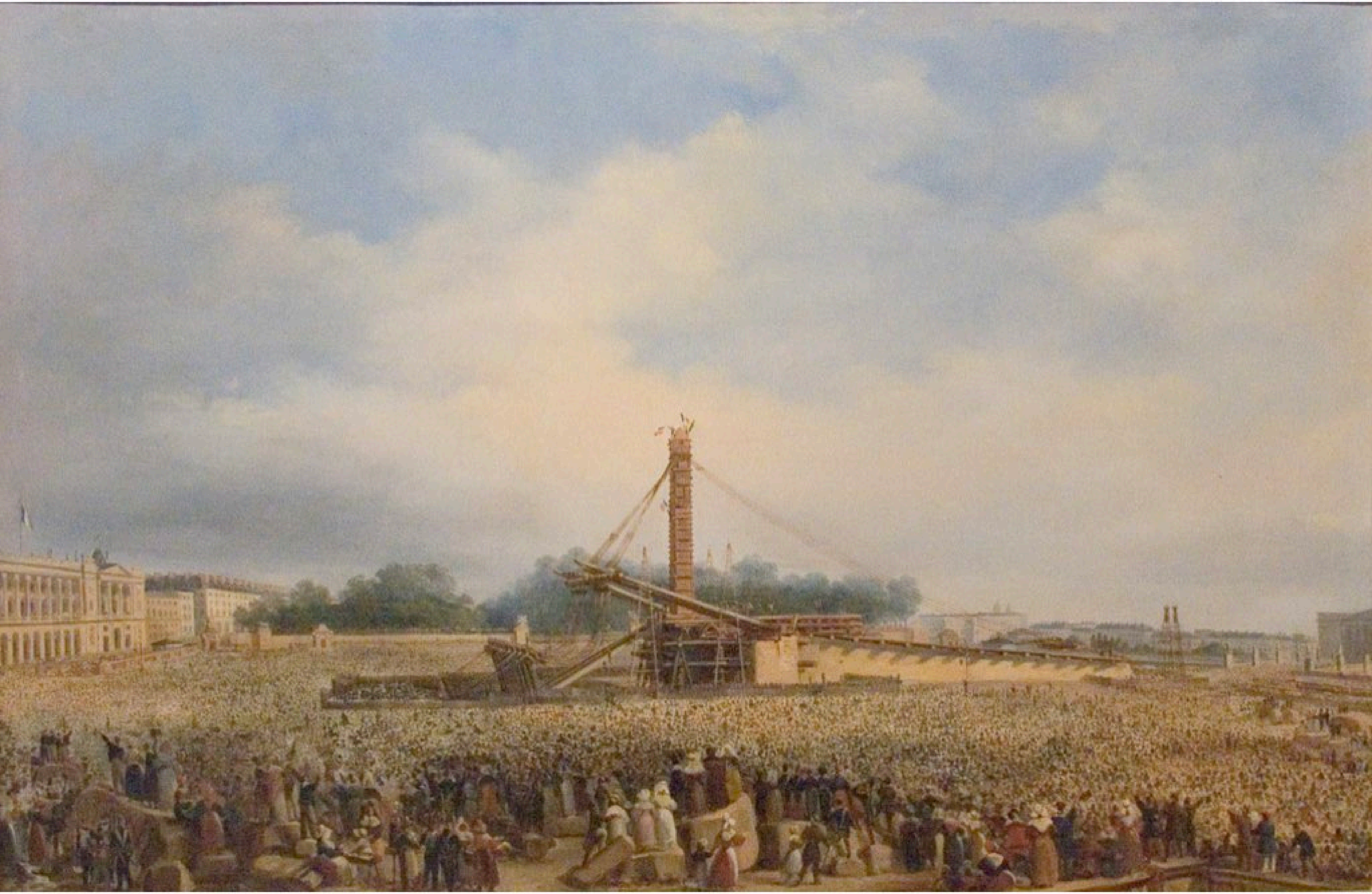
Appolinaire Lebas (1797-1873)

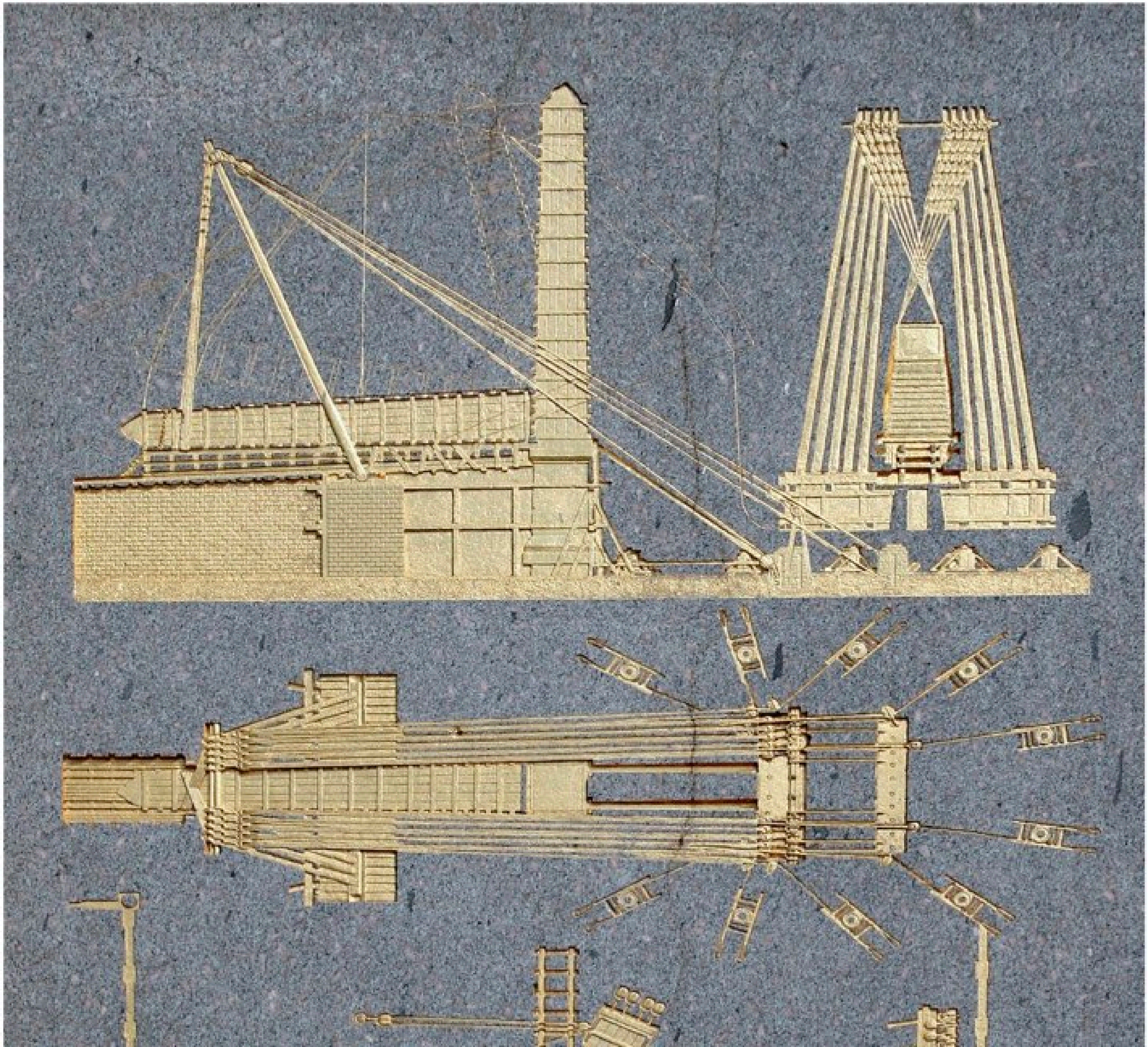
150 000 spectators
4h work
350 workers

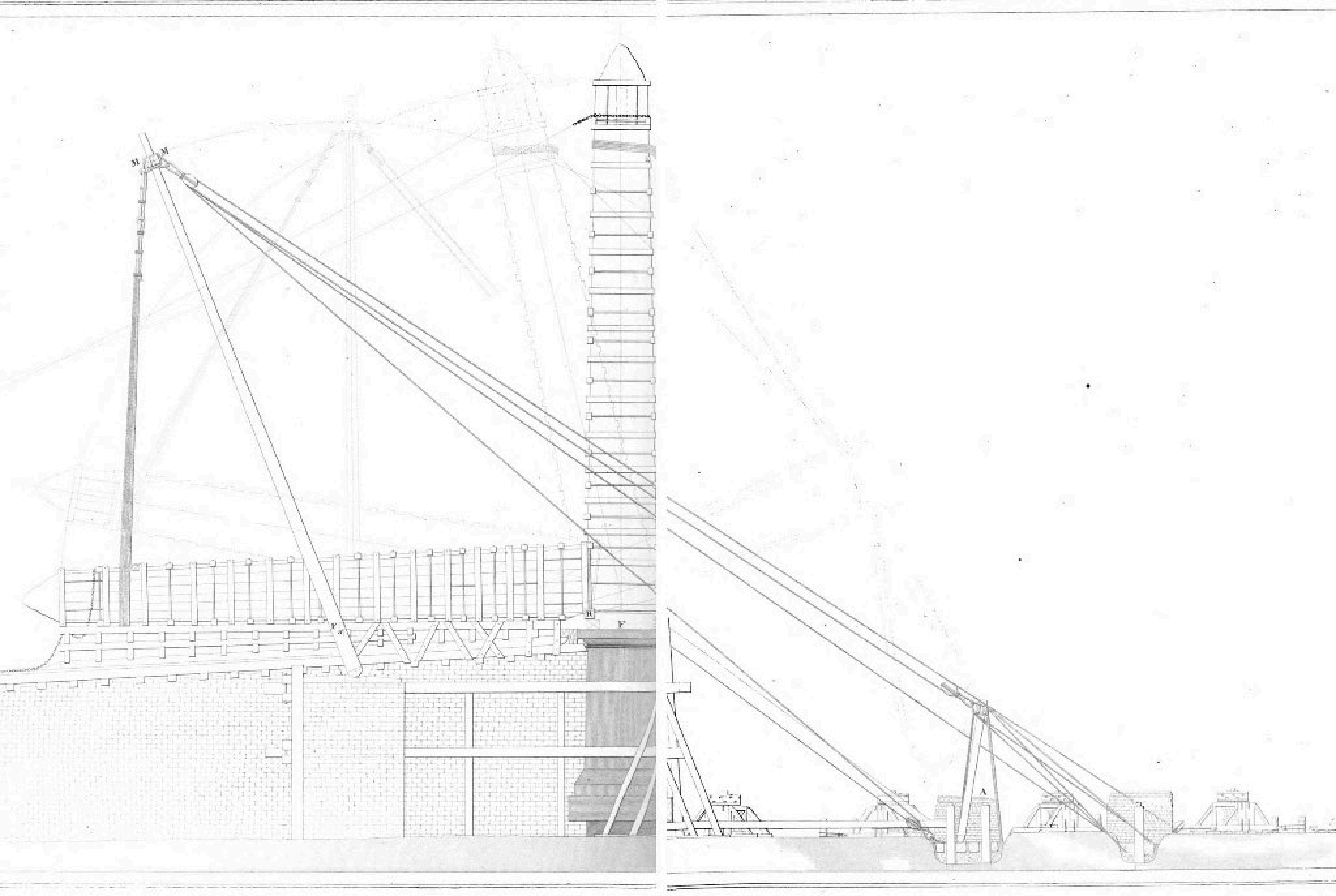
222 tons
23m height

25th of october 1836

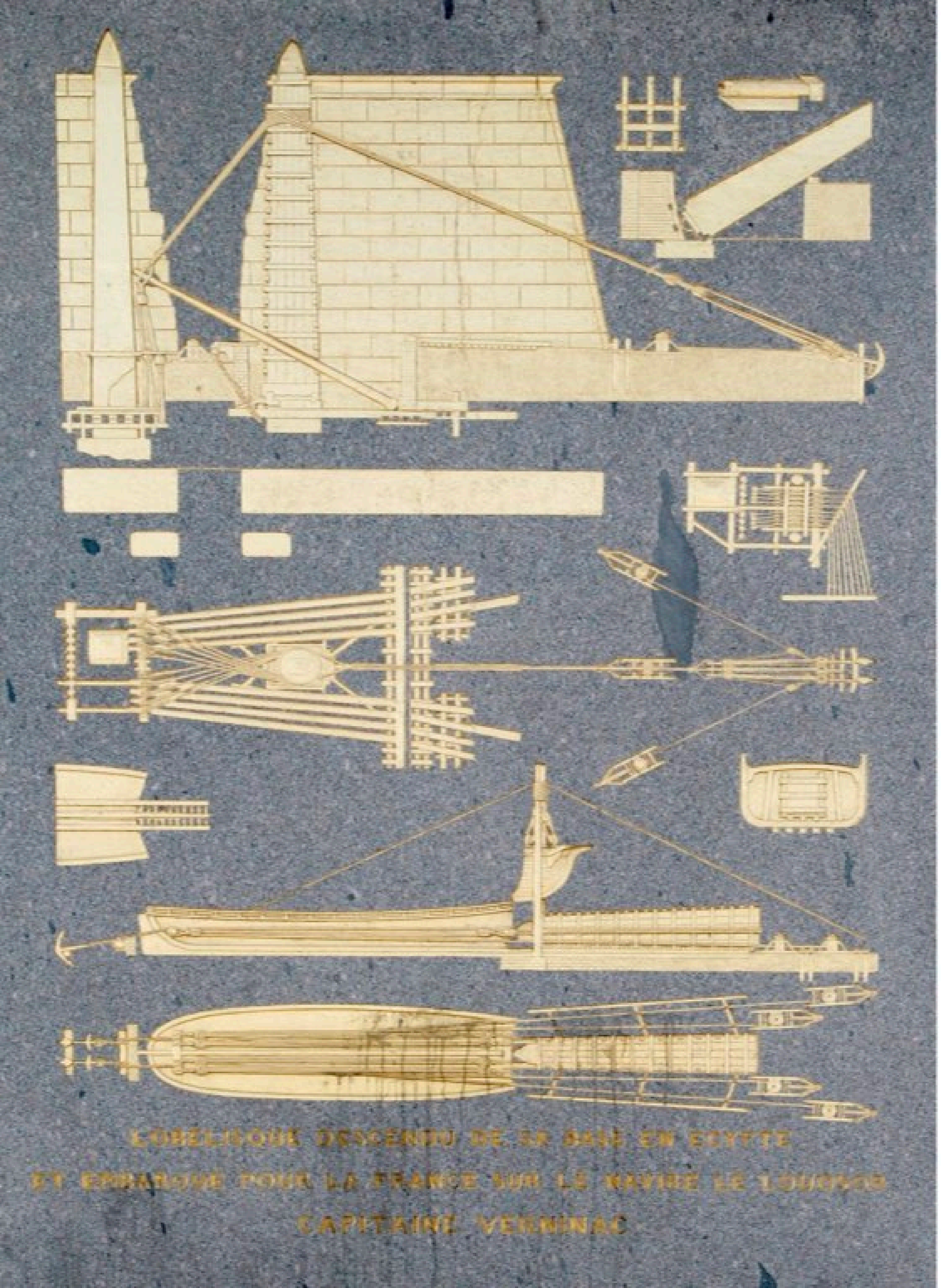




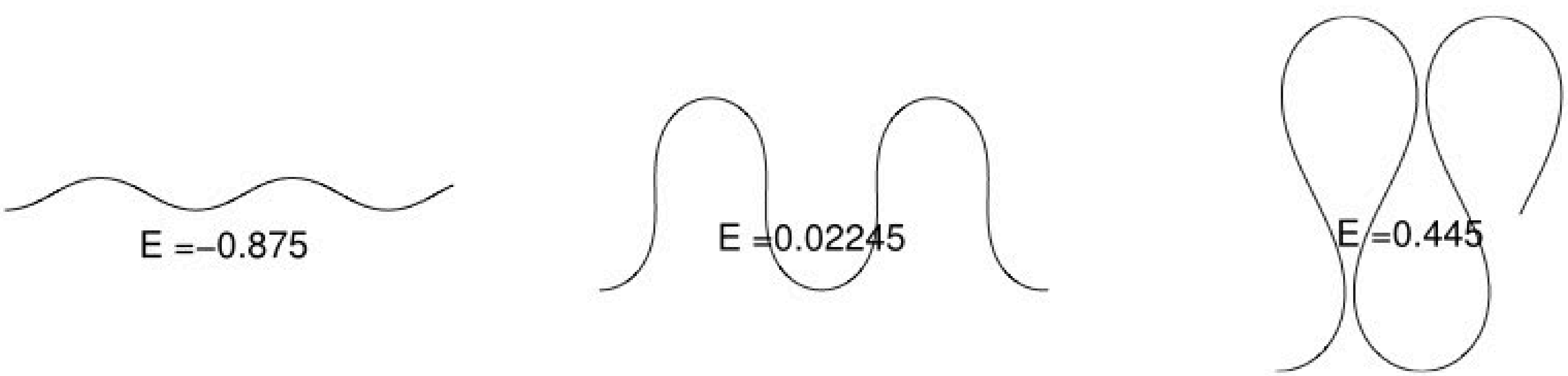




Paris, Carilian-Gœury et Vve Dalmont éditeurs


L'OBELISQUE DESCENDU DE SA BASE EN EGYPTE
ET EMBARQUE POUR LA FRANCE SUR LE NAVIRE LE LOUQSOR
CAPITAINE VERNINAC

$$\frac{d^2\theta}{ds^2} = -\sin\theta, \qquad E = \frac{1}{2}\dot{\theta}^2 - \cos\theta$$



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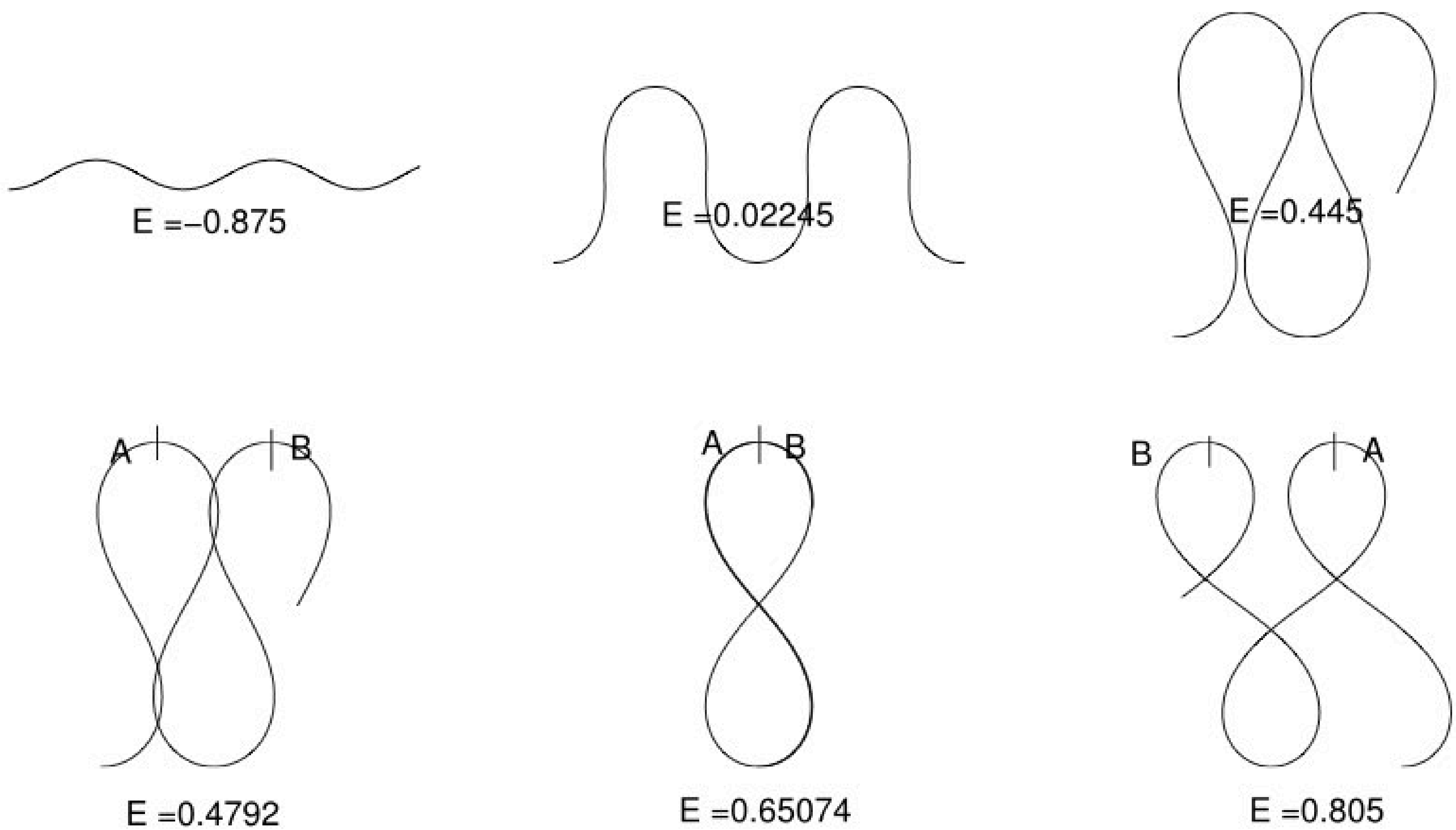
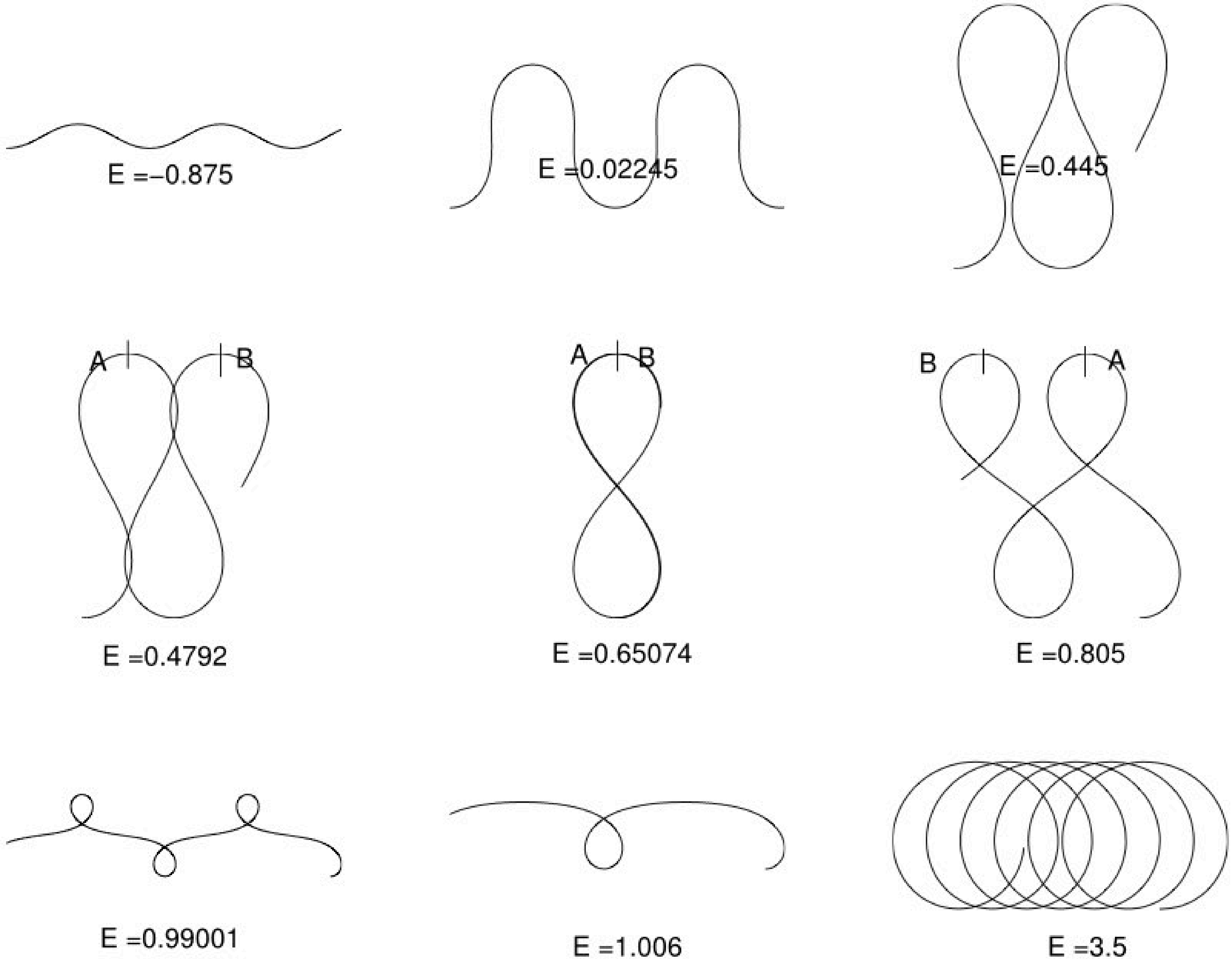
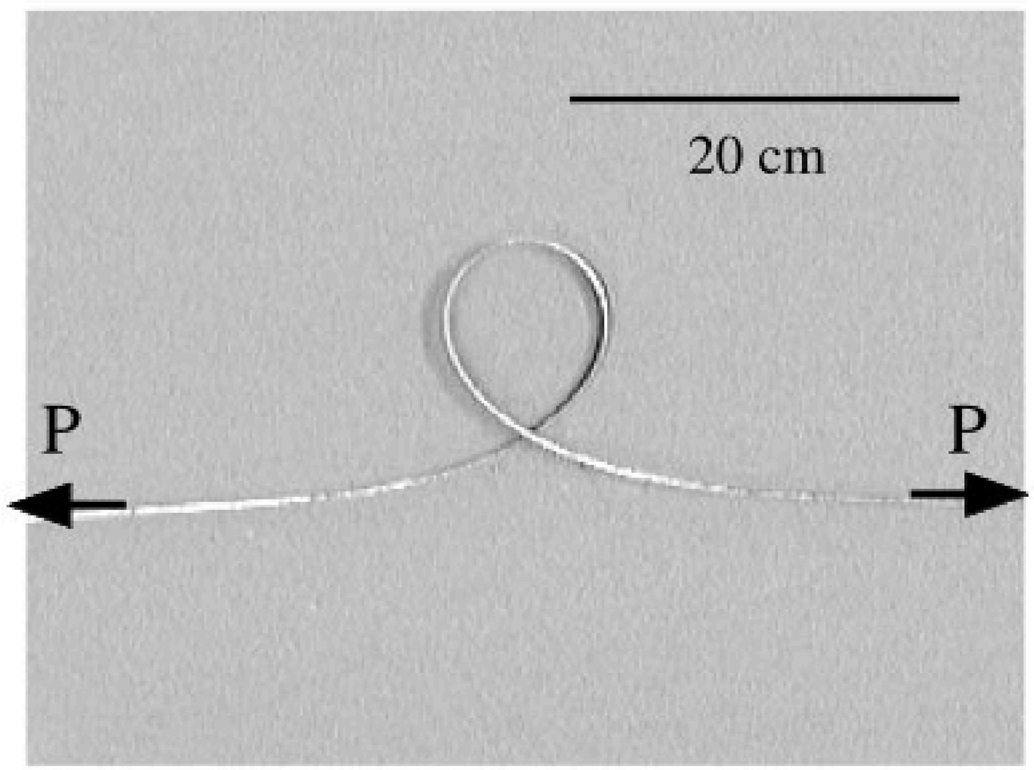
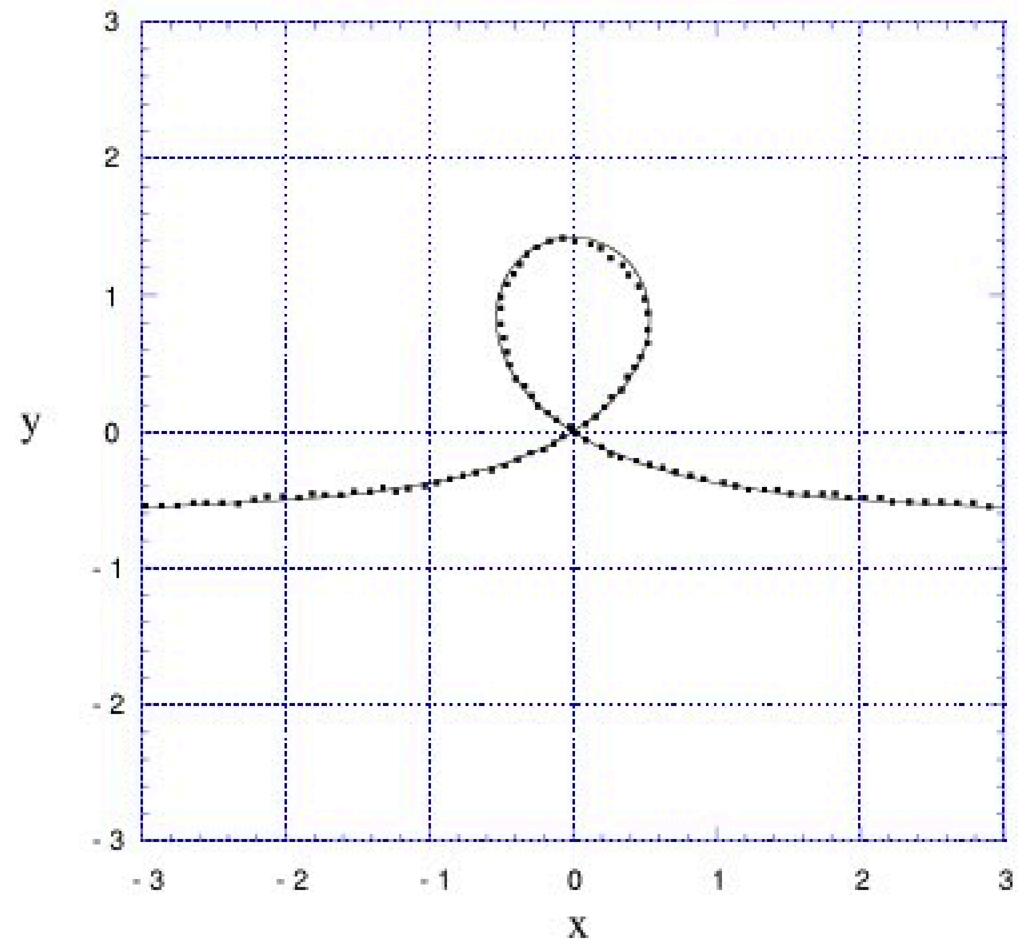


$$\frac{d^2\theta}{ds^2} = -\sin\theta, \qquad E = \frac{1}{2}\dot{\theta}^2 - \cos\theta$$




20 cm
P
P


y
x
3
2
1
0
- 1
- 2
- 3
- 3
- 2
- 1
0
1
2
3

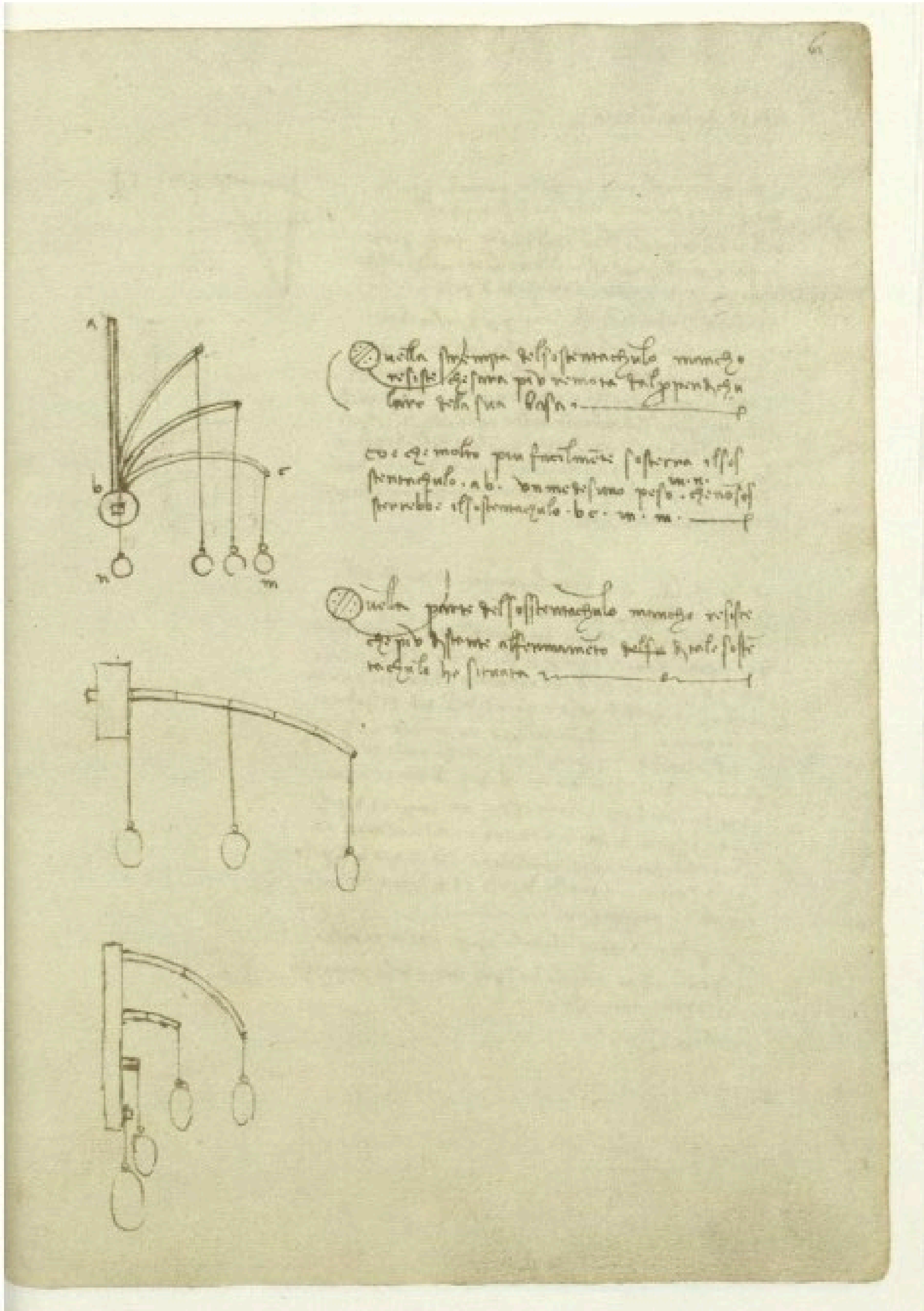
Elastic lines

Leonhard Paul Euler 1707- 1783

by Johann Georg Brucker

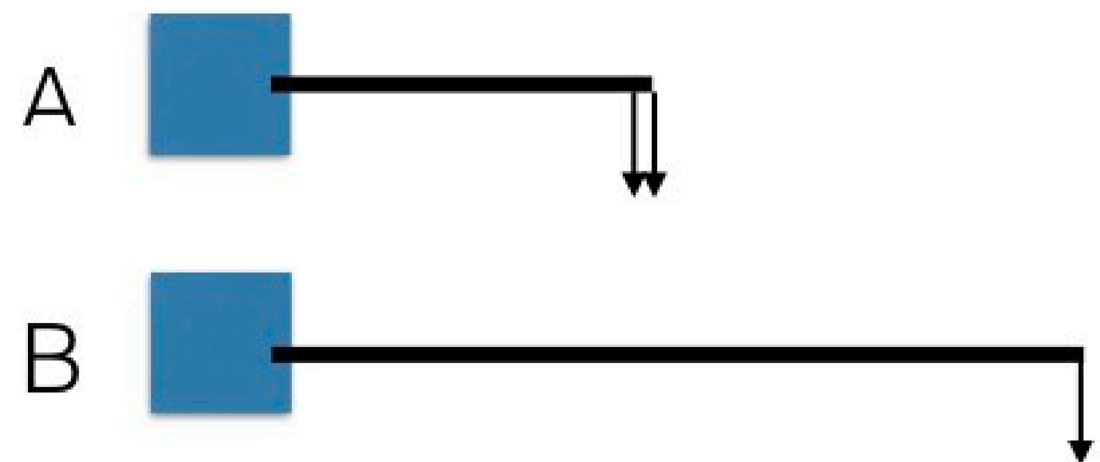
Since the fabric of the universe is most perfect [..], every effect in the universe can be explained as satisfactorily by the aid of the method of maxima and minima, as it can from the effective causes themselves (forces)
[...]
Not only one solution is greatly strengthen by the otHer, but, more than that, from agreement between the two solutions, we secure the very highest satisfaction

L. Da Vinci (Codex Madrid, 1490 -1499)



Quiz !

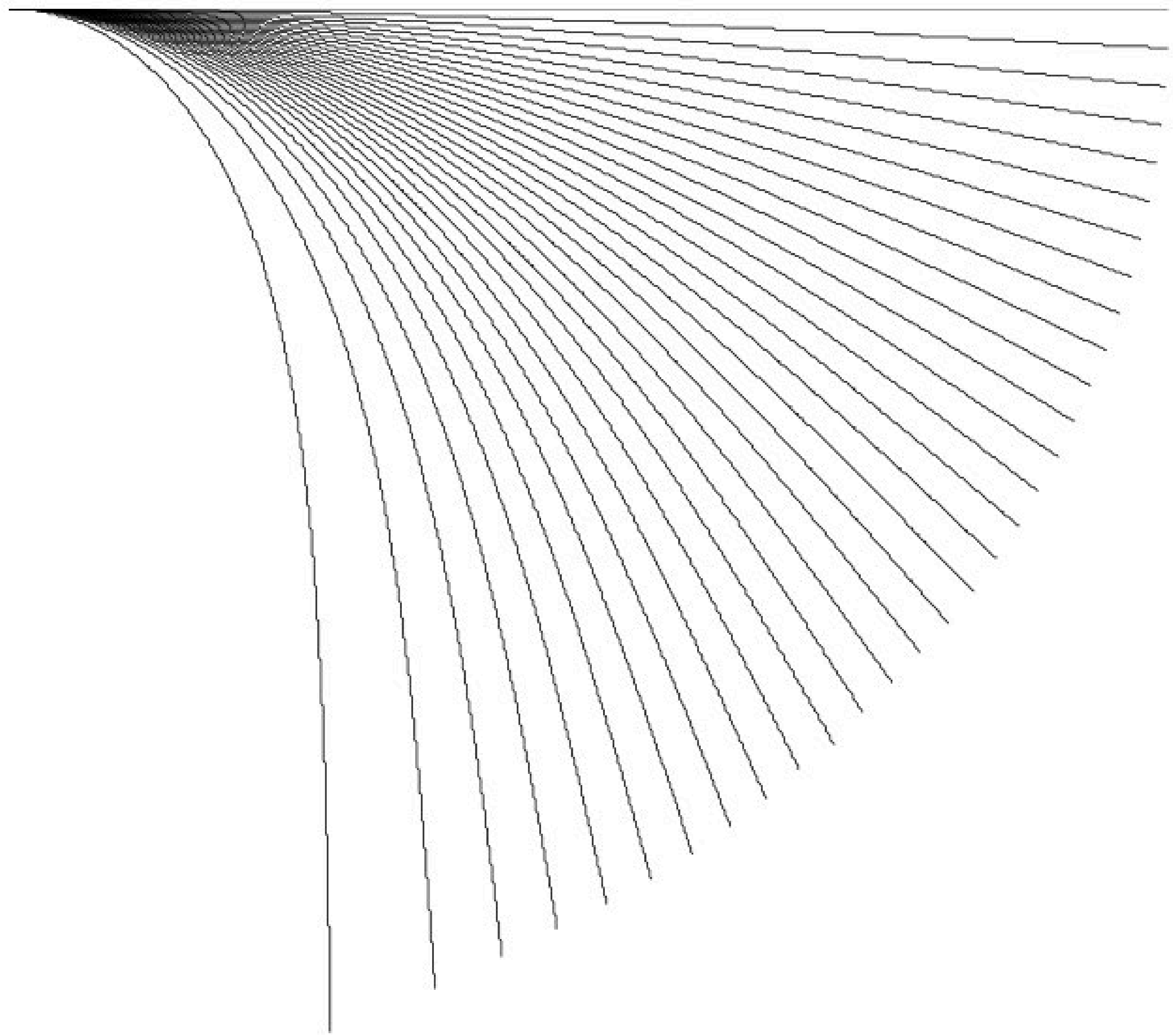
double load on on twice shorter rod



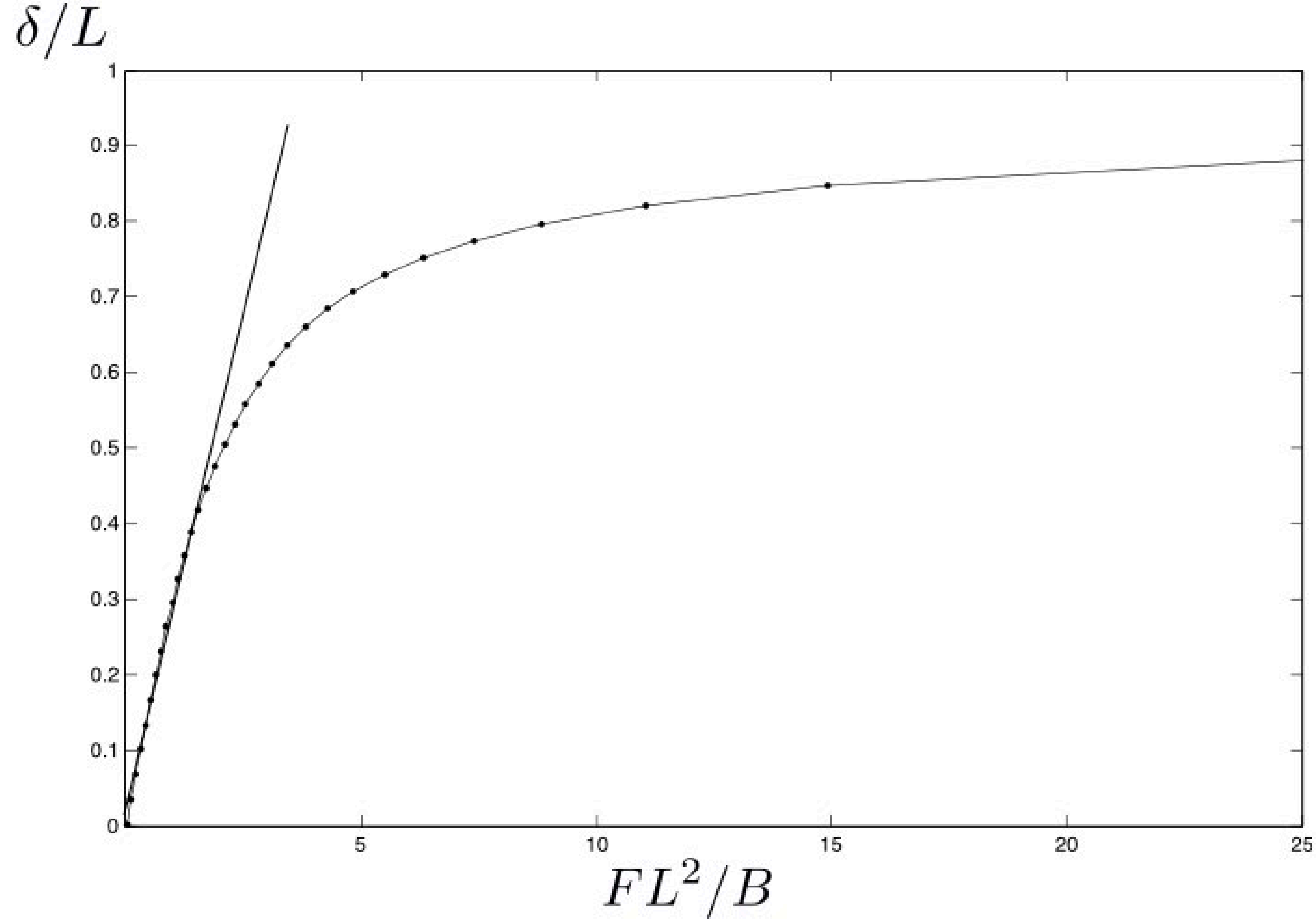
which one deflects more ?

Cantilever (end force)

$$B\frac{d^2\theta}{ds^2} - q\cos\theta = 0$$

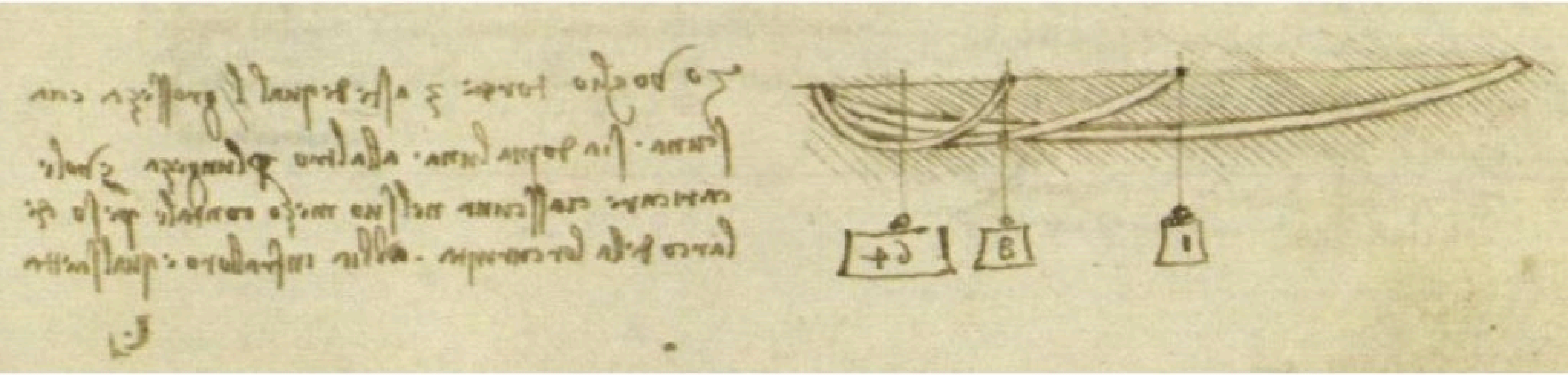


Cantilever (end force)



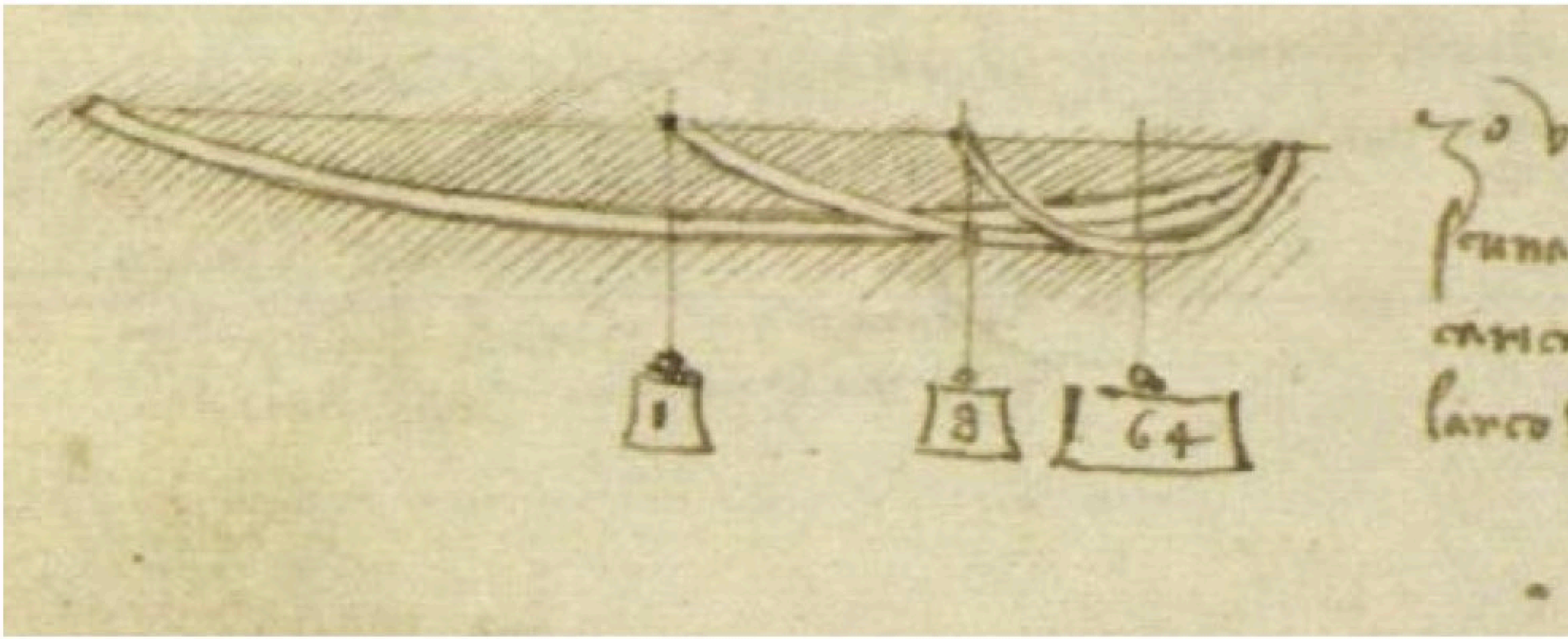
$$\delta/L \sim (FL^2/B)$$

L Da Vinci (Codex Madrid, 1490 -1499)



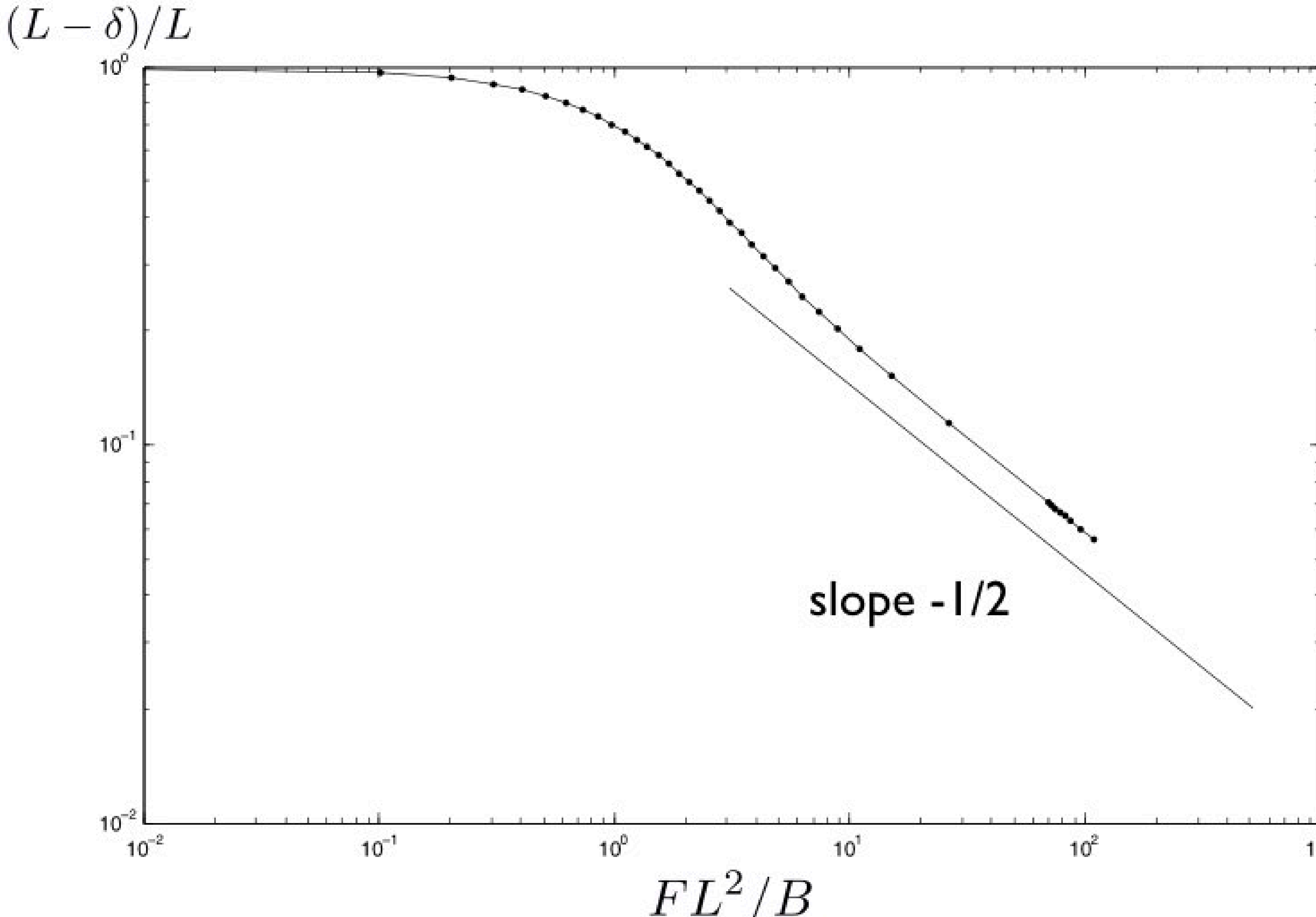
same deflection for different loads

L Da Vinci (Codex Madrid, 1490 -1499)



$$\delta \sim FL^3$$

Cantilever (end force)



$$(L-\delta)/L \sim (B/FL^2)^{-1/2}$$