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The asymmetrical friction mechanism that puts the curl in the curling stone



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ABSTRACT

Curling is an Olympic winter sport in which two teams slide stones across a sheet of ice towards a target area, some 28 m away from the release line. The sport has its name from the fact that the trajectory of a rotating stone becomes slightly curled, a fact used to reach open spots or take out opponent stones behind hindering "guarding" stones, etc. By slowly turning the stone clockwise when it is released, it will curl to the right, and vice versa. The resulting sideward deviation is typically slightly more than a metre. This intriguing tribological phenomenon has so far lacked a satisfactory explanation, although many attempts have been presented. In many of them, the curling motion has been attributed to an asymmetrical distribution of the friction force acting on the sliding stone, such that the friction on the rear of the stone (as seen in the direction of motion) is higher than that on the front. In a recent paper, we could show that no such redistribution of the friction, no matter how extreme, can explain the magnitude of the observed motion of a real curling stone. The present work presents an alternative asymmetrical mechanism that actually is strong enough to account for the observed motion. Further, in contrast to previous models, it satisfies other observed phenomena, including the independence of rotational speed of the stone and the strong dependence of the roughness of the stone. The model is backed up by experimental evidence and is based on the specific tribological conditions presented by the contact between a scratched curling stone and a pebbled ice sheet.

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1. Introduction

1.1. Background to the sport and the curling motion

In the Olympic winter sport of curling, two teams take turns sliding stones weighing close to 20 kg across a sheet of ice towards a target area, some 28 m away from the point of release. Points are awarded in every round to the team resting closest to the centre of the target area after 16 stones have been played; one point for each stone beating the best one from the opponent team.

After being released, the stone is only affected by friction mechanisms, and the only possibility for the players to influence its path is by sweeping the ice just in front of the sliding stone, thus slightly lowering the friction, and elongating the trajectory of the stone.

The sliding stone follows a slightly curved trajectory, deviating sideways as it moves across the ice, rather than following a straight line. This phenomenon is called "curl", and is of great importance to the game, as it can be used to reach open spots behind previously played stones, or take out opponent stones

behind hindering "guarding" stones. The direction of the curl can be controlled by the player, and is determined by the rotation of the stone. If the player gives the stone a clockwise rotation as it is released, it curls to the right, and vice versa. In contrast, it is not possible for the player to influence the amount of curl, and it typically amounts to slightly more than 1 m. As rotation of the stone is necessary for the curl to occur, it is tempting to assume that more rapid rotation would cause more curl. However, as well known among curlers and as confirmed in previously published studies, the amount of curl is actually insensitive to the rotational speed, within a wide range [1–3]. In normal play, the stone completes 1–3 rotations as it covers approximately 28 m from the release line to the centre of the target area.

The bottom of a curling stone is not flat, but curved and hollowed to form a ring-shaped running band, approximately 6 mm wide, with a diameter of 120 mm, to which the contact between stone and ice is restricted. The surface of this running band is intentionally roughened (a process called "scratching"), in order to reach the desired friction level and curl distance. The contact is further restricted to the flattened ice pebbles (0.2 mm high, up to 5 mm wide), formed by sprinkling water on the ice. The pebbles are crucial, as they lower the friction between stone and ice, enabling the stone to be played with a reasonable effort. To ensure that the ice behaves consistently during the

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entire game, the tops of the pebbles are usually removed beforehand, to form flat sliding surfaces. This is done either by repeatedly dragging stones across the ice, or by cutting the tops off using a special tool, a process called nipping [4]. The geometry of the stone and pebbled ice is illustrated in Fig. 1.

1.2. Previous models explaining the curling motion

Even though the behaviour of sliding curling stones is well known, along with the importance of several of its influencing factors, no satisfactory explanation for it have been presented. Several authors have presented suggestions, both in popular and scientific media, but none as of yet has been able to explain all observations. Most models have been immediately discarded upon closer review, whereas others may still be generally regarded as plausible, although yet unproven. A review of common explanations was made by the present authors in a recent publication [3], so only a short and general overview will be given here.

Almost all previously presented models for the motion are based upon redistribution of friction along the contact between stone and ice (i.e. the running band), and differ mainly in the physical background to this asymmetry [1,5–8]. At first, one can note that any redistribution of the friction along the running band must involve higher friction (or possibly higher normal load) on the trailing half of the stone than on the leading, in order to result in motions in the observed direction. Models suggesting differences between the right and left halves of the stone (as seen in the direction of motion) have also been suggested [9], but are simply not plausible, as no sidewards force can result from such redistributions.

As mentioned, redistribution of the friction in such a way that the trailing half of the running experiences a larger friction force than the leading will result in a sidewards force in the observed direction of motion. This is due to the fact that the velocity of the trailing half has a component in the direction opposite to that of the observed curl, so that the friction acting upon it will have a component in said direction. Hence, if the total friction force acting upon the trailing half is greater than that on the leading half, the sidewards force component of the trailing half will dominate. However, as concluded in our recent publication [3],

and as implicitly stated by others [2,8], it is not possible to conceive a strong enough asymmetry to achieve the amounts of curl observed in regular play. Moreover, explanations based on redistributed friction suffer from the deficit that the magnitude of curl in such models is almost directly proportional to the rotational velocity of the stone, violating the observed behaviour. This deficit has been noted by several authors, and attempts at resolving it by letting the redistribution to vary as a function of rotational speed have been made [1,6]. Such models may solve the problem of dependence on rotational speed, but still cannot produce sufficiently strong sidewards force at low rotational speeds to achieve the observed curl.

Perhaps the most elaborate model has been presented by Shegelski et al., and has been described in several publications [1,5,6], slightly evolving over time. The model is based on a frontback friction difference, which is attributed to formation of a liquid film in the contact between stone and ice. This film is said to be dragged around by the rotation of the stone in order to provide low friction at appropriate locations. In later drafts of the model, the friction is said to vary both in magnitude and direction along the running band [1,6]. The direction of the friction force is changed on the leading half, in such a way that its sidewards component is reduced. The change of direction is achieved by letting the friction force be of opposite direction to the entrained liquid film, rather than to the ice underneath the stone. It has been argued in later publications that the existence of liquid water in such quantities that would appear necessary for the proposed mechanism has not been confirmed experimentally. Further details regarding the mechanism of water transport from the trailing to the leading half of the running band are also called for [7]. The authors of the present paper agree with this scepticism, and also find the change of direction of the friction force rather hard to envision from the relatively sketchy description of the mechanisms.

2. Introduction to the new model

We propose a new theory for the curling mechanism that we name curling through scratch-guiding. This theory is based on an

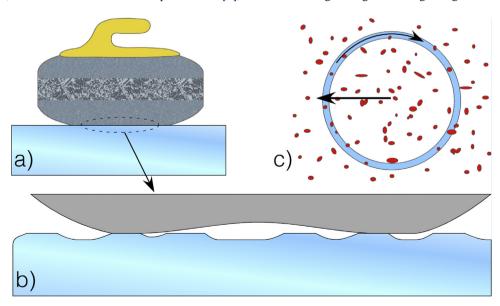


Fig. 1. Due to their topographies, the contact between the curling stone and the ice is strictly limited. It can take place only where the running band on the stone meets the plateaus of the flattened (nipped) ice pebbles. (a) Overview from the side, with the position of the enlarged views indicated. (b) Enlarged view in cross-section, showing the nominally flat running band meeting the flat pebble plateaus. The height dimension is exaggerated to give a clearer view. In reality the heights are very small compared to the lateral dimension. (c) Top view of the sliding interface showing the running band and typically distributed pebble plateaus. Contact can take place only on those pebbles that are momentarily positioned under the running band.

unconventional but relatively simple idea, supported by a number of tribological observations.

During its travel along the ice sheet, the relatively rough surface of the running band will scratch the ice. The asperities on the leading half of the band will cause scratches that point slightly to the left or right, depending on the rotation. Typical crossing angles amount to a few degrees. When the asperities on the trailing half meet these scratches they will have a tendency to follow them, see Fig. 2.

This scratch-guiding or track steering mechanism generates the sideway force necessary to cause the curl. Although at first being seemingly simple, the mechanism has several complexities that will be treated in a coming section, following a more detailed description of the tribological conditions.

3. Experimental methods

To enhance the understanding of the detailed mechanisms behind the friction between stone and ice, a number of experiments were performed on a professionally prepared curling ice sheet and using actual curling stones. The environment in the curling hall was regulated, with the aim of keeping the ice temperature at $-3.5\,^{\circ}\text{C}$ and the surrounding air temperature at $6-7\,^{\circ}\text{C}$, with a relative humidity of 55%. The air close to the ice surface is expected to be colder and more humid.

3.1. Topography of pebbled ice and running band of curling

The topography of the ice sheet and the running band of curling stones was replicated using a resin (Heraeus Provil Novo Medium). This type of resin replicates topographical features down to less than a micrometre in size, and sets within 1 h at the ice temperature of about $-4\,^{\circ}\text{C}$. The replicas were imaged by scanning electron microscopy (SEM) and their topographies were quantified by white light optical interferometry (LOI).

3.2. Contact distribution between a curling stone and pebbled ice

In order to study the contact distribution between curling stones and pebbled ice, a pressure sensitive film (Fuji Prescale Super Low) was applied on the stone. The film contains microcapsules with a colour-forming dye. In the local areas where contact is established between the stone and the ice, the pressure breaks the microcapsules and the colour-forming material reacts with a colour developing surrounding material. Red patches appear on the film, revealing the areas of mechanical contact.

3.3. Measurement of friction between sliding stone and ice

A special device was designed to measure the friction force versus sliding speed for a curling stone sliding over the ice, see Fig. 3. A motor driven rotating cylinder was connected to the

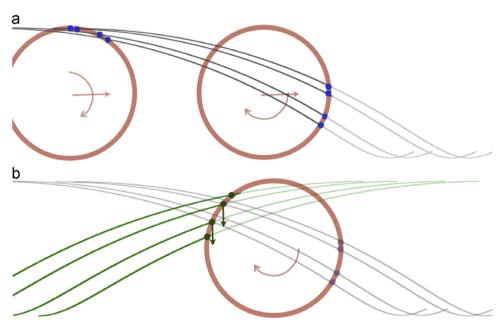


Fig. 2. Schematic drawing showing scratch lines formed by four tips on the rotating and sliding running band. (a) Scratches made by the front edge, during sliding and roughly ¼ rotation. (b) Scratches made by tips on the rear half, two of which are in the position where they cross scratches made by the front half. When passing the scratches these tips experience a sidewards force, as indicated. The sum of all such microscopic "pushes" results in the force that moves the stone sideways. In this simplified illustration, the ice is flat, smooth and free from pebbles.

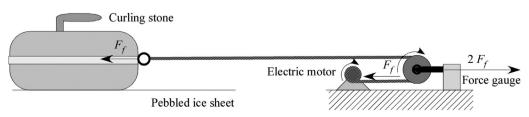


Fig. 3. Principle of friction force measurement of sliding stone.

stone through a thin wire. A low friction tackle facilitated the use of a stationary force gauge. The friction was recorded for an ordinary scratched stone and for a stone where the running band was first polished to drastically reduce the height of protruding asperities.

3.4. Curling on pre-scratched ice

The hypothesis of curl through self-guiding was tested by pre-scratching the ice. Brooms equipped with emery paper were used to create small scratches in the surface of the pebbled ice. The scratches were aligned at a small angle from the original sliding direction of the stone. Areas of about $10 \times 0.5 \, \text{m}^2$ were prepared in this manner. Sliding tests were performed with two different stones: one prepared for play using conventional methods, the other polished to remove most of the topography on the running band. The stones were slid without rotation first over normal ice and then into the prescratched area. The resulting motion was video recorded.

4. Experimental results

4.1. Topography of pebbled ice and the running band of curling stones

The studied curling ice sheet exhibited distinct topographical features, and the changes imposed by the passage of curling stones were readily distinguished. Examples of the topography of individual pebbles are shown in Fig. 4. Pebbles that have been passed over by curling stones typically display some scratches over the flat top surface and fracture marks along their edges, see Fig. 4b.

Replicas of the topography of the running band were studied by LOI. It was seen that the running band on the stone is macroscopically flat, but shows distinct scratches from the intentional roughening procedure ("scratching"). Typically, the peak to valley distance is on the order of $10~\mu m$. A topographical cross-section of a part of a running band is presented in Fig. 5.

Fig. 5 shows a topographical cross-section view over the situation where the running band of a stone is on top of an ice pebble. Obviously, the running band of the stone is rougher than the initial, as nipped, pebble surface. After being passed over by the stone, the pebble cross-section (as measured perpendicular to the sliding) is similar to that of the stone.

4.2. Contact distribution between curling stone and pebbled ice

The flat running band of a curling stone is about 6 mm wide, with a diameter of ca. 120 mm, resulting in an area of ca. 22 cm². At any instant, however, the contact with the ice sheet is limited to a small number of spots, on top of the pebbles, distributed around the running band. Two experimentally estimated contact spot distributions are given in Fig. 6. As seen, the area of real contact increases significantly as the ice is worn, due to larger plateaus on top of the pebbles.

4.3. Friction between a sliding stone and ice

The friction between a normally scratched curling stone and the ice showed a continuous decrease with increasing velocity, at typical curling stone sliding speeds, similar to what have been reported by others [2], see Fig. 7. For higher speeds the friction

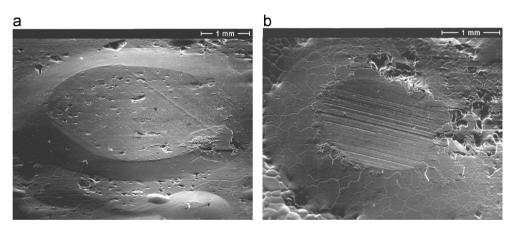


Fig. 4. Individual pebbles on a curling ice sheet. The web-like pattern outside the flattened pebbles depicts grain boundaries in the ice. Replicas imaged in SEM, and turned up-side down to "restore" the convex shape. (a) A "new" pebble, directly after nipping. (b) Pebble appearance after passage of one curling stone. Note the small scratches in the flat surface.

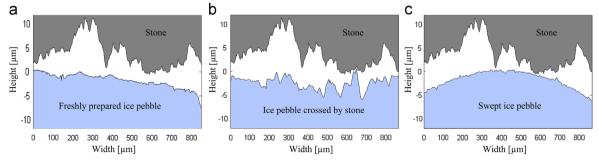


Fig. 5. Typical roughness of the two counter surfaces, illustrating the contact situation between a curling stone and pebble plateau on the ice, as measured on replicas using white light interferometry. The upper part of the images shows the topography of the running band of the stone while the lower part shows the topography of ice pebbles in a freshly prepared state (a), after being crossed by a stone (b) and after being swept with a curling brush (c). Please note that the surface topography has been strongly exaggerated by the use of different vertical and horizontal scales.

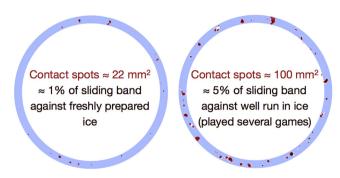


Fig. 6. Contact area distributions between the running band (indicated by the blue rings) and ice pebbles, experimentally estimated using pressure sensitive film. The distribution in (a) was found on freshly prepared ice, after pebbling and nipping. The distribution in (b) was found on an ice that was used for several games after the preparation. The larger plateaus formed due to wear from stones and sweeping during the games had resulted in considerably larger plateaus on the pebbles. The red spots indicate the areas of real contact. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

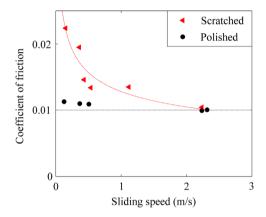


Fig. 7. Friction coefficient versus sliding velocity for a normal (scratched) curling stone (triangles) and a curling stone where the running band was polished (dots).

coefficient levels out at around 0.010. However, for the polished stone, the friction remains at the low level throughout the tested speed interval, with just a slight increase from 0.010 at 2.3 m/s to < 0.012 at 0.1 m/s. The lowest level of 0.010 is the same for the scratched and the polished stone, but for the normally scratched stone, the friction coefficient increases to over 0.02 as the velocity approaches zero.

4.4. Curling stone motion on pre-scratched ice

The experiments where curling stones were slid without rotation over pre-scratched ice showed very interesting results. When the ordinary curling stone entered the pre-scratched area it showed a distinct deviation along the scratch direction. It was also possible to have it moving in a zigzag pattern by preparing two pre-scratched zones with alternating scratch directions. When repeating the experiment several times over the same area, the guiding effect became gradually weaker, indicating that the scratches were worn off by the stone.

When performing the same experiment with the curling stone where the running band had been polished, to remove all large asperities, passing the pre-scratched zones had no effect at all.

In order to confirm an already known fact, we also compared the motion of the ordinary and polished stones when sliding over ordinary, non-prepared ice. While the ordinary stone showed the expected sidewards motion depending on chosen direction of rotation, the polished stone travelled straight ahead, irrespective of the rotation. This experiment gives a strong evidence for the importance of the roughness of the stone, which surprisingly is not treated by any of the published curling models.

5. Details of the proposed model

The basic idea behind the proposed model was introduced in a previous section. Here, some more details regarding it will be presented, based upon the experimental observations above. The implications of the proposed model will also be treated, in terms of the ability of the model to explain observed properties of the curling motion.

5.1. Magnitude of scratching forces

In order to estimate the possible impact of the scratch-guiding effect, the magnitude of the forces involved in scratching is first approximated. The presented experiments show a coefficient of friction of 0.01 at a sliding speed of 2 m/s. As this was the result for both the normal (scratched) stone and for that with a polished running band, it is reasonable to believe that the friction at this sliding speed includes no ploughing contribution at all. As the sliding speed is lowered to 0.5 m/s, the friction increased to approximately 0.02 for the scratched stone, while remaining largely unchanged for the polished stone. We therefore assume that the increase in friction for the scratched stone as the speed is lowered is entirely due to ploughing, i.e. that the ploughing contribution is 0.01, or approximately 2 N of force.

Secondly, let us estimate the number of asperities involved in the ploughing friction, and that may thus contribute to the scratch-guiding. From the study of the topography of a scratched ice pebble, we may estimate the typical size of the scratches to be $3 \times 50 \,\mu\text{m}^2$, with triangular cross-sections. Using the hardness of ice at relevant temperatures (which has been reported in literature as 35 MPa [10]) as an approximation of the pressure opposing the scratching motion, the force acting upon a single scratching tip can be estimated as 2.6 mN (calculated as pressure multiplied by scratch cross-section). In order to account for the estimated total ploughing force, it is therefore necessary for an average of approximately 800 scratching tips to be in simultaneous contact. Judging from the experimentally estimated number of pebbles in simultaneous contact (Fig. 6), around 40 for the run in ice, each pebble would have to be scratched by some 20 tips. This seems to be in good agreement with the presented image of a scratched pebble (Fig. 4b).

The resulting sidewards force necessary for the observed motion can be estimated by simple mechanics, and amounts to about 0.07 N for a typical curl distance of 1.2 m. Out of the 800 estimated scratching tips, half of them (those on the trailing half) may be assumed to contribute to the sidewards force. This means that an average of 0.18 mN of sidewards force is needed for each tip, corresponding to roughly 7% of the total force.

Of course it must be noted that the scratching tips are highly irregularly formed rather than conical, and correspondingly that the scratch geometries therefore become irregular rather than triangular. This will however not affect the general reasoning, but only make more realistic geometrical calculations much trickier.

5.2. Origin of the sidewards force

The origin of the scratch-guiding force has been touched upon previously, but requires a more thorough explanation. The basis of the idea is that, due to the rotation, the leading and trailing halves of the stone will have slight sidewards, and opposite, directions, as compared to the total velocity of the stone. This means that the scratches formed on top of the ice pebbles by the trailing half will have some small angle (typically a few degrees) towards those formed by the leading edge, as it crosses them. The sidewards force is due to each scratching tip meeting a resistance as it enters the wall of the scratch it is passing. This force has a sidewards resultant as long as the tip is still partly in the scratch it is passing, as this requires more material to be deformed by one of the sides of the tip; see Fig. 8 for a clarification of the geometry.

The magnitude of the momentary force is assumed to be proportional to the cross-section area of the scratch track, as projected in the sidewards direction. This projected area varies as the tip enters the unscratched ice. For the tip geometry assumed above, the maximum magnitude of the sidewards force can be estimated from the largest possible projected sidewards area as 1.3 mN. This is almost an order of magnitude larger than the necessary average force for each tip mentioned above. The average force on the tip, as it crosses the scratch, will of course be significantly lower than this estimated maximum value. Secondly, not all tips on the trailing half will be involved in crossing other scratches at every instant, further reducing the total sidewards force. An interesting note is that the maximum sidewards force acting on the scratching tip is actually not dependent on the scratch crossing angle (for angles below 45°).

A question may now arise among observant readers; only what happens when an asperity leaves a scratch, thus penetrating a "wall" of ice that has been described. As the asperity enters the next scratch, by breaking through a similar wall, the opposite situation as the one described will occur, but with otherwise identical geometry. If the ice showed ideal behaviour, this should lead to an equal sidewards force, in the opposite direction as the one when the asperity left the previous scratch. As yet, there is no finished answer to why this does not happen, but it is the firm belief of the current authors that it is so, perhaps most strongly based on the observations from the experiment of sliding over pre-scratched ice. We may suggest two different mechanisms that would cause the asymmetry needed to explain the net effect.

Firstly, one may speculate that crack propagation in the ice plays a role. An asperity entering a scratch would easily cause a brittle collapse of the wall, requiring relatively little energy, while when leaving the scratch the side wall will have better support; cracks will not as readily form so the ice must deform plastically, which requires considerably higher energy.

Secondly, another type of geometric asymmetry may be important. Each scratching asperity will elastically compress the ice under it. However, each time an asperity enters a crossing

scratch, the normal load is temporarily at least partly relieved, so the ice will recover elastically. This local lift of the ice towards the stone will have the same effect as if the stone "fell down into the scratch". For a crossing asperity, the scratch will appear to be deeper when exiting than it did when entering. Correspondingly, there will be an asymmetry in the resulting sidewards force.

5.3. Influence of rotational speed

The almost complete independence of the curl on the rotational speed of the stone is one of its most surprising properties, and also the one that has caused the most trouble to those who have sought an explanation for it. In the model presented here, the magnitude of curl depends on several opposing factors, in a far from obvious way. Much effort has been spent in investigating the combined effect of these, but the presentation here will include only the conclusions we believe to have found, rather than the complete solution. The reasoning can be summarised as follows:

- The number of scratches crossed per unit time increases linearly with the rotational speed.
- The average sidewards force on a tip, as it crosses a scratch, seems to be almost independent of the scratch crossing angle, and thus rotational speed, at the angles used in practical curling.
- The sliding distance with net sidewards force (i.e. the distance required for a tip to go from touching the scratch wall to fully entering it) varies as the cotangent of the angle between crossing scratches making it close to inversely proportional to the rotational speed.

It is the combined effect of the above three factors: (i) the number of scratches crossed per unit time, (ii) the average sidewards force on an asperity as it crosses a scratch and (iii) the duration of this force (i.e. the time it takes to cross each scratch) that determines the total force acting upon the stone. It appears that these three factors vary in such a way as to result in almost no dependence on the rotational speed.

6. Conclusive discussion

In this article, previous models attempting to explain the motion of curling stones have been briefly discussed. It was concluded that none of the published models are able to correctly describe the observed behaviour, in several regards. Firstly, no

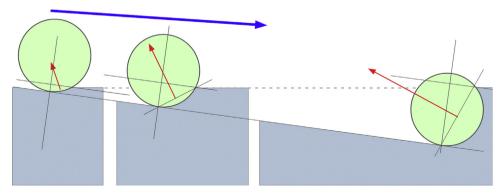


Fig. 8. Geometry and forces explaining the situation when a scratching tip (green circle) enters the side wall (blue) of an already present scratch at a small angle. Three stages of penetration are shown, with varying resulting magnitude and angle of the reactive force. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

previous model has explained the large dependence on the topography of the stone surface. In fact, to the knowledge of the current authors, no other model has ever included it in the mechanism. Therefore, previous models cannot explain why, for instance, polished stones do not curl, since roughness would not change their results. Secondly, as shown in our recent publication [3], no other model has presented plausible evidence for a strong enough effect to explain the observed motion. As mentioned, simple front–back redistribution of friction is not sufficient, and as this is the basis of several previous models, they cannot be regarded as viable. Finally, no other model has given a clear explanation to the almost complete independence of the curl on the rotational speed of the stone.

It is our view that all these points may be satisfactorily explained by the here presented theory of scratch-guided motion, even though several details remain to be presented more thoroughly. The proposed model has a very simple basis, and thereby serves to demystify the curling movement. However, the detailed analysis of the origin of the resulting guiding forces and calculations of their magnitudes for the complex contact shapes between rough stones and rough ice are less straightforward, and will require further studies. In particular, the unclear mechanism behind the asymmetry of "leaving" and "entering" a crossing scratch must be further touched upon.

However, the combination of

- observations showing that (rotation less) sliding on pre-scratched ice leads to sidewards motion for stones with the typical rough running band, while stones with polished running bands are totally unaffected,
- the simple mechanical reasoning,
- experimental evidence clearly showing that scratches of sufficient size are actually formed by the stone in the ice surface,
- a good understanding of the actual contact conditions between the stone and the ice,
- estimates showing that the proposed sidewards mechanism is sufficiently strong, and

• calculations showing that the strength of the mechanism is independent of the rotational speed,

allows us to conclude that the curling movement of rotating curling stones is due to the pattern of slightly inclined scratches produced by the asperities on the front half of the sliding band guiding the asperities on the rear half. This scratch-guiding or track steering mechanism generates the sideway force necessary to cause the curl. Interestingly, the scratch-guiding mechanism generates a force that, in contrast to ordinary friction mechanisms, is not opposite to the local sliding motion.

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References

- E.T. Jensen, M.R.A. Shegelski, The motion of curling rocks: experimental investigation and semi-phenomenological description, Canadian Journal of Physics 82 (2004) 791–809.
- [2] R. Penner, The physics of sliding cylinders and curling rocks, American Journal of Physics 69 (3) (2001) 332–339.
- [3] H. Nyberg, S. Hogmark, S. Jacobson, Calculated trajectories of curling stones sliding under asymmetrical friction, in: Proceedings of the 15th Nordic Symposium on Tribology, 2012.
- [4] L. Öhman. Curling Ice Explained, The World Curling Federation, 2004.
- [5] M.R.A. Shegelski, R. Niebergall, M.A. Walton, The motion of a curling rock, Canadian Journal of Physics 74 (1996) 663–670.
- [6] M.R.A. Shegelski, R. Niebergall, The motion of rapidly rotating curling rocks, Australian Journal of Physics 52 (1999) 1025–1038.
- [7] N. Maeno, Curl mechanism for a curling stone on ice pebbles, Bulletin of Glaciological Research 28 (2010) 1–6.
- [8] M. Denny, Curling rock dynamics: towards a realistic model, Canadian Journal of Physics 80 (2002) 1005–1014.
- [9] M. Denny, Curling rock dynamics, Canadian Journal of Physics 76 (1998)
- [10] P.V. Hobbs, Ice Physics, Oxford, U.P., London, 1974.