## Fano type resonance in Wood anomalies

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Resonant scattering from periodic gratings has been the subject of extensive investigations [1]. The scattering coefficients of any periodic grating are characterized by resonant features, the most remarkable being the manifestations of so-called Wood's anomalies [2,β]. In recent papers [4,5], studies of the polarization properties in spectral transmittance of a nanohole array grating have been reported. The observations have been interpreted in terms of Fano type resonances resulting from the coexistence of the two Wood's anomalies (in [4]; the Fano shape is interpreted in terms of the coherent interference between a discrete and a continuum of states). We present a "study based on modal analysis to quantitatively predict the transmission spectrum of an array, accounting for the polarisation (p- or s- polarisations) and on the grating material. It is shown that the equivalent admittance of the grating can be determined in the weak scattering approximation, by integration of a Riccatti type equation governing this admittance. Then, following Oliner and Hessel [3], we propose analytical expressions of the reflexion coefficients for each interference order (of each mode in terms of modal analysis), that account for the shape and for the composition of the grating. Comparison with direct numerical calculations reveals the accuracy of our prediction (Fig. 1). It is shown that the occurence of Fano shape in the reflectance only occurs under certain circumstances, (for s-polarized wave, see Fig. 1, and corresponding electric field on Fig. 2, 3). This is due to the fact that the first Wood anomaly (often referred as the Rayleigh Wood anomaly) always occurs at the cut off frequencies producing the extinction of all the propagative modes while the second -resonant- Wood anomaly does not happen for all gratings (essentially, this is dependent on the wave polarization and on the grating material).

## References

Abstract

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s-polarized

[1] Potos Issue. Extraordinary Light Hansinssion Finologi Sub-wavelength Structured Surfaces, Express 12, 3618–3706 (2004).
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(A) Wave propagation



(a) Grating A

(D) Analytical results (grating A)

Inspecting the form of z for penetrable

inclusions show

(b) Grating B

equation for the magnetic field H), which

equalitication of the ingest and index to the the second s

(i)  $k_m = 0$ , several discrepension  $\underline{c}_{n \neq m,0} \cong \underline{c}_{n \neq m,0}$  be pointed out

 $\nabla . (\frac{1}{\mu} \nabla E) + \omega^2 \epsilon E = 0$  $\nabla \cdot (\frac{1}{\epsilon} \nabla H) + \omega^2 \mu H = 0$ p- polarized

## (B) Numerical resolution

Coupled wave analysis p, being either E or H.

$$p(x,y) = \sum p_m(x)\varphi_m(y) \qquad \mathbf{p} \equiv (p_m)$$

The modal components satisfy

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix}' = \begin{pmatrix} 0 & \mathsf{E}^{-1} \\ \mathsf{K}^2 + \mathsf{F} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix}$$

with q=Yp, leads to a Ricatti equation

$$\mathsf{Y}' = -\mathsf{Y}\mathsf{E}^{-1}\mathsf{Y} + \mathsf{K}^2 + \mathsf{F}$$

solved using a Magnus scheme to find Y and then, the wavefield p, being either E or H. Also, Y gives the reflection and transmission coefficients.

## (C) Analytical prediction

Weak scattering approximation

 $\begin{array}{l} \begin{array}{c} -\mathbf{z}_{m0} \\ \text{severa} \mathbb{R}\text{discrepencies cal} (\mathbb{B}\text{), pointed out } (\mathbb{B}\text{), pointed$  $\mathsf{R}_{n0}(k) \simeq -\frac{\mathsf{z}_{n0}}{1 + \sum \mathsf{z}_{jj}}$ equation for the magnetic field H), which is confit a dictor  $\mathcal{P}/\mathbb{W}$  it the product of H. seven at time to be the seven as not in the back of the transformation  $\mathcal{A}^{j\neq m}$  is a constrained by the transformation  $\mathcal{A}^{j\neq m}$  is a constrained by the transformation of the transformatio Inspecting the form of strand and raises have that in a property of the difference of the second of the difference of the second always observed and a second anomaly occurs for  $\epsilon > \epsilon_0$ .  $|R_{20}|^B/B_0$  $B/B_0 = 0.5$ Reflection coefficent of the offener wave for singlarized waves  $|R_{00}|$ 0.02 $|R_0^{\rm E}|$ 0.01 0.02/ 0.5250.25 $\epsilon < \epsilon_0$ 0 0.01 0.2521equation for the magnetic field H), which is contradictory with the present here sult. Although several discrepencies can be pointed out (we not  $a_{1}^{\mu_{20}}$  consider  $a_{2}^{\mu_{20}}$  size scattered and not  $2^{20}$ 0.5 errating made of square penetrable scattere reflection gratings), we do not have an expla- $|R_{20}|$  $|R_{00}|$ acticiente Ree tons (Reto)fo 0.51 0.5  $|R_{00}|$  $2\pi$ 0.250Ĵ5 0.250.02 ${}^0{}^{\mathsf{L}}_0$ FIG. 7: Reflection co ) plane wave impinging at store where the store of the st 2  $\overline{2\pi}2$ 0.01(E) Numerical results (grating B) energy penetrable scatterers of side a = |h/10|, mass dense

$$Y = Y_0[1+z], ||z|| \ll 1$$

 $Y_0$  in the absence of grating

mandus 7BR enter tipat company and the second states and the second Here, the resonance of the mode 2 occurs and the solution of the mode 2 occurs and the solution of the solutio FIG. 7: Reflection  $B_0$  such 0.8 n through piene in a star hard braking show one 0.6 <sup>3</sup> show anaillus trations for heavely sead ab method 0.4 siont through penet 0. FIG. 7: Reflection n plane way grating made of square penetraple scatterers of side  $f_{110}$  mass density  $f_{110}$  mass rical results ictures.dia. The ered media. The ission term multimedal me sh seci nomoj turea structur sion , we roug netra fiftigered, med ered structure igwhof nteverationedia nt tætri 19iteototensmissions coustic

transmission enhancementational grating structures of P

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