Fano type resonance in Wood anomalies

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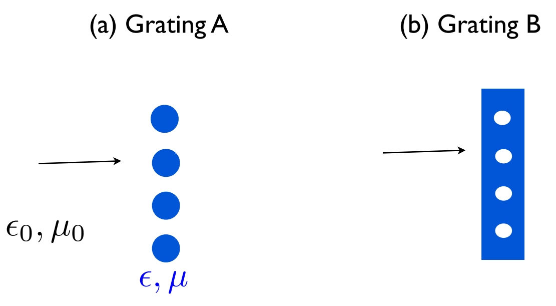
Abstract
Resonant scattering from periodic gratings has been the subject of extensive investigations [1]. The scattering coefficients of any periodic grating are characterized by resonant features, the most remarkable being the manifestations of so-called Wood's anomalies [2,3]. In recent papers [4,5], studies of the polarization properties in spectral transmittance of a nanohole array grating have been reported. The observations have been interpreted in terms of Fano-type resonnances resulting from the coexistence of the two Wood's anomalies (in [4], the Fano shape is interpretd in terms of the coherent interference between a discrete and a continuum of states). We present a study based on modal analysis to quantitatively predict the transmission spectrum of an array, accounting for the polarisation (p- or s- polarisations) and on the grating material. It is shown that the equivalent admittance of the grating can be determined in the weak scattering approximation, by integration of a Riccati type equation governing this admittance. Then, following Oliner and Hessel [3], we propose analytical expressions of the reflexion coefficients for each interference order (of each mode in terms of modal analysis), that account for the shape and for the composition of the grating. Comparison with direct numerical calculations reveals the accuracy of our prediction (Fig. 1). It is shown that the occurence of Fano shape in the reflectance only occurs under certain circumstances, (for s-polarized wave, see Fig. 1, and corresponding electric field on Fig. 2, 3). This is due to the fact that the first Wood anomaly (often referred as the Rayleigh Wood anomaly) always occurs at the cut off frequencies producing the extinction of all the propagative modes while the second -resonant- Wood anomaly does not happen for all gratings (essentially, this is dependent on the wave polarization and on the grating material).



References
[1] Focus Issue: "Extraordinary Light Transmission Through Sub-Wavelength Structured Surfaces," Opt. Express **12**, 3618–3706 (2004).
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[4] K. Tetz, V. Lomakin, M. P. Nezhad, L. Pang and Y. Fainman, J. Opt. Soc. Am. A **27**(4), 911-917 (2010).
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(A) Wave propagation

s-polarized
$$\nabla.(\frac{1}{\mu}\nabla E) + \omega^2 \epsilon E = 0$$

p- polarized
$$\nabla.(\frac{1}{\epsilon}\nabla H) + \omega^2 \mu H = 0$$

(B) Numerical resolution

Coupled wave analysis p, being either E or H.

$$p(x,y) = \sum p_m(x)\varphi_m(y) \qquad \mathbf{p} \equiv (p_m)$$

The modal components satisfy

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix}' = \begin{pmatrix} 0 & \mathsf{E}^{-1} \\ \mathsf{K}^2 + \mathsf{F} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{p} \\ \mathbf{q} \end{pmatrix}$$

with q=Yp, leads to a Ricatti equation

$$\mathsf{Y}' = -\mathsf{Y}\mathsf{E}^{-1}\mathsf{Y} + \mathsf{K}^2 + \mathsf{F},$$

solved using a Magnus scheme to find Y and then, the wavefield p, being either E or H.
Also, Y gives the reflection and transmission coefficients.

(C) Analytical prediction

Weak scattering approximation

$$Y = Y_0[1+z], ||z|| \ll 1$$

Y_0 in the absence of grating

(D) Analytical results (grating A)

Inspecting the form of z for penetrable inclusions show

$$\mathsf{R}_{n0}(k) \simeq -\frac{\mathsf{z}_{n0}}{1 + \sum_j \mathsf{z}_{jj}},$$

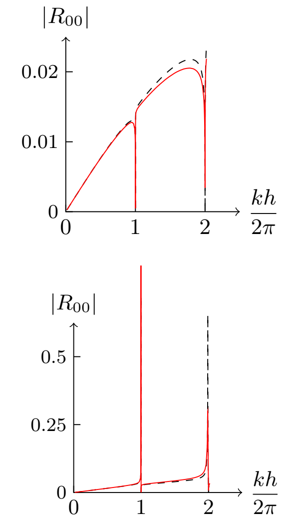
$$\begin{cases} \text{(i)} \quad k_m = 0, & \mathsf{R}_{n\neq m,0} \simeq \frac{-\mathsf{z}_{n0}}{\mathsf{z}_{mm}} \to 0, \\ & \mathsf{R}_{m0} \simeq \frac{-\mathsf{z}_{m0}}{\mathsf{z}_{mm}} \to \mathcal{O}(1), \\ \text{(ii)} \quad 1 + \mathsf{z}_{mm} = 0, & \mathsf{R}_{n0} \simeq -\frac{-\mathsf{z}_{n0}}{\sum_{j\neq m} \mathsf{z}_{jj}} \to \mathcal{O}(1) \end{cases}$$

Inspecting the form of z_{n0} and z_{nn} shows that no anomalies occur in s-polarized configuration. For p-polarized waves, the Rayleigh Wood anomaly ($R_{00} \to 0$ at $k = 2n\pi/d$) is always observed and a second anomaly occurs for $\epsilon > \epsilon_0$.

Reflection coeffiicent of the plane wave for s-polarized waves

$\epsilon < \epsilon_0$

$\epsilon > \epsilon_0$



(E) Numerical results (grating B)

Here, the resonance of the mode 2 occurs below the cut off frequency.