ONS'13, Capri, Italy, 12-14 September 2013

Coupled Wave Analysis using a Multimodal Admittance

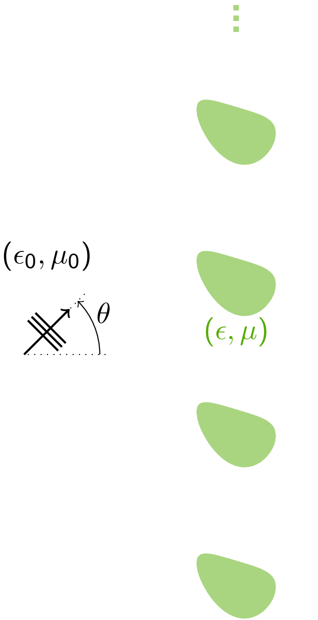
Simon FÉLIX[a] • Jean-François MERCIER[b] • Agnès MAUREL[c]

[a] LAUM, CNRS, Université du Maine, Le Mans, France

[b] Poems, CNRS, ENSTA ParisTech, INRIA, Palaiseau, France

[c] Institut Langevin, CNRS, ESPCI ParisTech, Paris, France

PENETRABLE GRATING



$$\nabla \cdot \left(\frac{1}{a(\mathbf{r})} \nabla u \right) + b(\mathbf{r}) \omega^2 u = 0$$

⇧

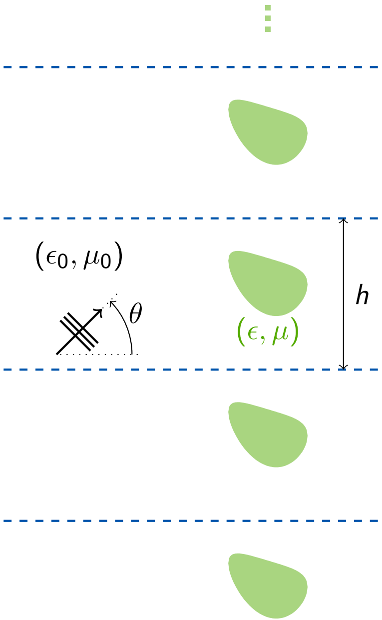
$$\nabla \cdot \left(\frac{1}{\mu(\mathbf{r})} \nabla E \right) + \epsilon(\mathbf{r}) \omega^2 E = 0$$

s-polarized

$$\nabla \cdot \left(\frac{1}{\epsilon(\mathbf{r})} \nabla H \right) + \mu(\mathbf{r}) \omega^2 H = 0$$

p-polarized

PENETRABLE GRATING



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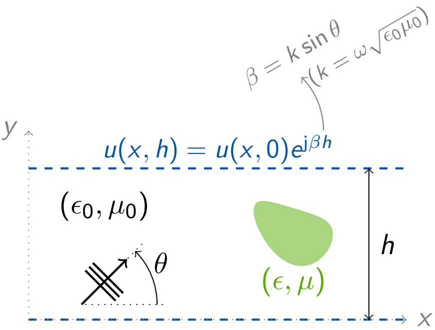

$$\nabla \cdot \left(\frac{1}{a(\mathbf{r})} \nabla u \right) + b(\mathbf{r})\omega^2 u = 0$$

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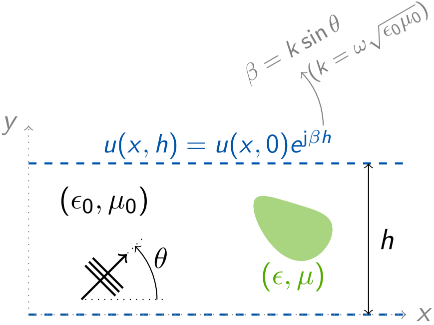


Periodic transverse functions

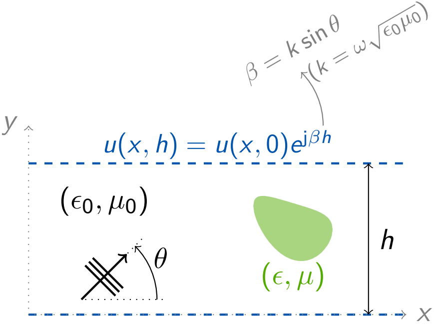
$$\varphi_n(y) \equiv \frac{1}{\sqrt{h}} e^{j(\beta + 2n\pi/h)y}$$

$$\nabla \cdot \left(\frac{1}{a(\mathbf{r})} \nabla u \right) + b(\mathbf{r})\omega^2 u = 0$$

Multimodal formulation

$$u = \sum_{n \in \mathbb{Z}} u_n(x)\varphi_n(y)$$

PENETRABLE GRATING



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⇩

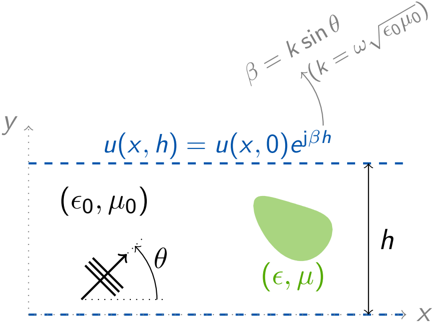
Multimodal formulation

$$u = \sum_{n\in\mathbb{Z}} u_n(x)\varphi_n(y)$$

⇩

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}' = \begin{pmatrix} 0 & \mathsf{E}^{-1} \\ \mathsf{K}^2 + \mathsf{F} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

PENETRABLE GRATING



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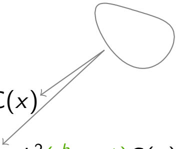
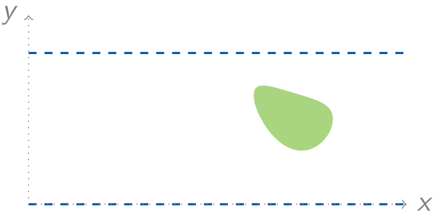


$\mathbf{v} \equiv \mathsf{E}\mathbf{u}'$

$\mathsf{E}(x) \equiv \mathsf{Id} + (\frac{a_0}{a} - 1)\mathsf{C}(x)$

$\mathsf{F}(x) \equiv (\frac{a_0}{a} - 1)\mathsf{D}(x) - k^2(\frac{b}{b_0} - 1)\mathsf{C}(x)$

$\mathsf{K}_{mn} \equiv jk_n\delta_{mn} \quad k_n^2 = k^2 - (\beta + 2n\pi/h)^2$



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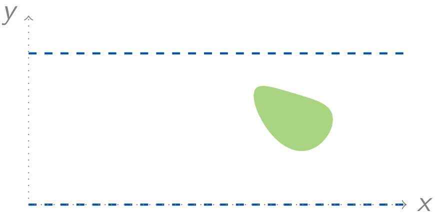
MULTIMODAL ADMITTANCE

$$\mathbf{v} = \mathsf{Y}\mathbf{u}$$



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MULTIMODAL ADMITTANCE

$$\mathbf{v} = \mathsf{Y}\mathbf{u}$$

$$\mathsf{Y}' = -\mathsf{Y}\mathsf{E}^{-1}\mathsf{Y} + \mathsf{K}^2 + \mathsf{F}$$

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}' = \begin{pmatrix} 0 & \mathsf{E}^{-1} \\ \mathsf{K}^2 + \mathsf{F} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

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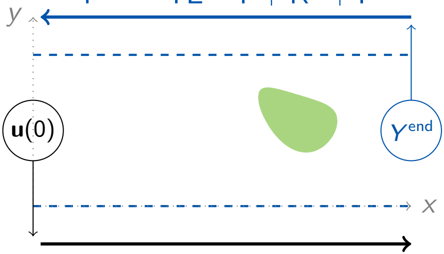

$$\mathbf{u}' = \mathsf{E}^{-1}\mathsf{Y}\mathbf{u}$$

⇩

$$\mathbf{u}(x)$$

WAVEFIELD

$$\begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}' = \begin{pmatrix} 0 & \mathsf{E}^{-1} \\ \mathsf{K}^2 + \mathsf{F} & 0 \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \mathbf{v} \end{pmatrix}$$

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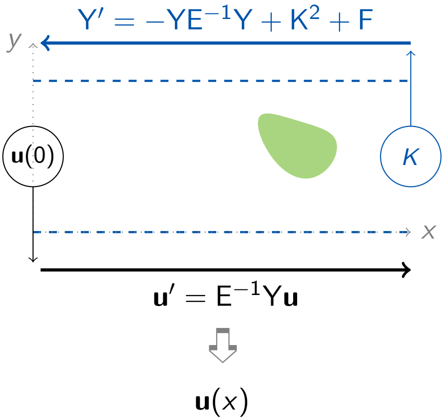
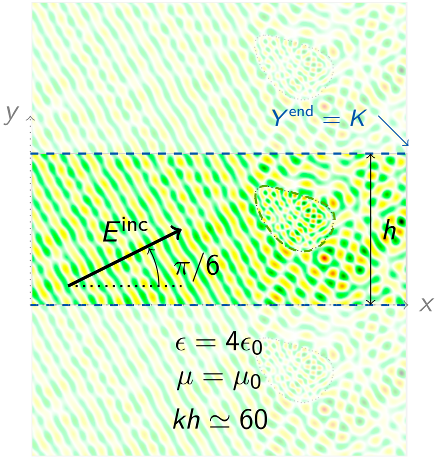
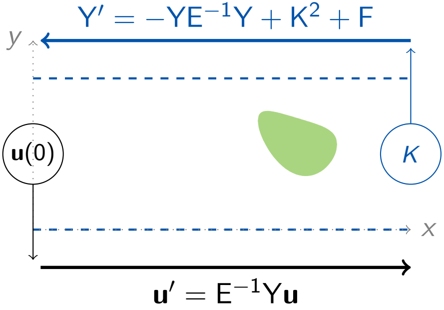




WAVEFIELD

MULTIMODAL ADMITTANCE

$$\mathbf{v} = \mathsf{Y}\mathbf{u}$$

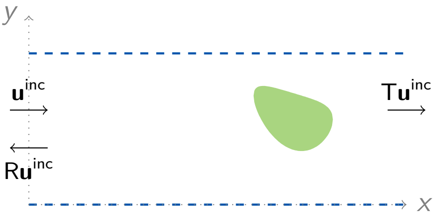


$\Downarrow$

$$\mathbf{u}(x)$$

WAVEFIELD

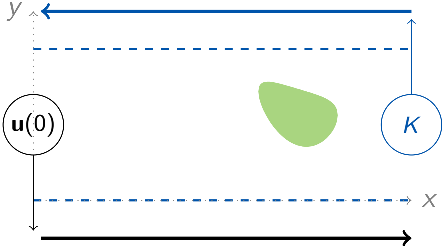
SCATTERING PROPERTIES



MULTIMODAL ADMITTANCE

$$\mathbf{v} = \mathsf{Y}\mathbf{u}$$

$$\mathsf{Y}' = -\mathsf{Y}\mathsf{E}^{-1}\mathsf{Y} + \mathsf{K}^2 + \mathsf{F}$$



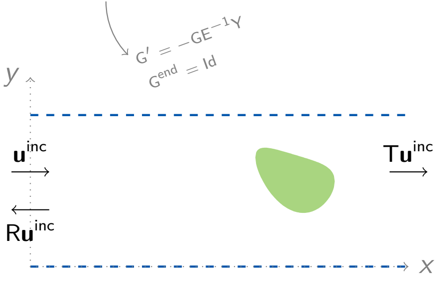
$$\mathbf{u}' = \mathsf{E}^{-1}\mathsf{Y}\mathbf{u}$$

$$\mathbf{u}(x)$$

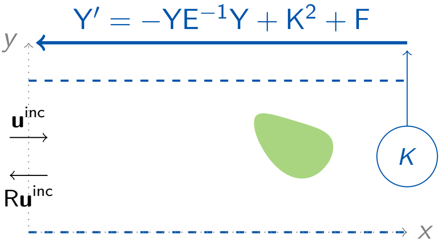
WAVEFIELD

SCATTERING PROPERTIES

$$\mathsf{R} = [\mathsf{K} + \mathsf{Y}(0)]^{-1}[\mathsf{K} - \mathsf{Y}(0)]$$
$$\mathsf{T} = \mathsf{G}(0)[\mathsf{Id} + \mathsf{R}]$$



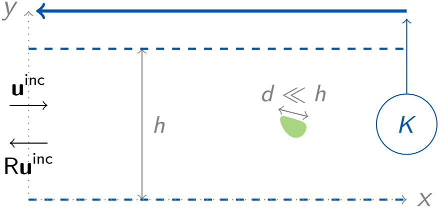
- No storage of Y, G
- No need to compute **u** in a particular configuration



$$R = [K + Y(0)]^{-1}[K - Y(0)]$$

WEAK SCATTERING

$$\mathsf{Y}' = -\mathsf{Y}\mathsf{E}^{-1}\mathsf{Y} + \mathsf{K}^2 + \mathsf{F}$$



$$\mathsf{R} = [\mathsf{K} + \mathsf{Y}(0)]^{-1}[\mathsf{K} - \mathsf{Y}(0)]$$

$$\mathsf{E}(x) \equiv \mathsf{Id} + (\tfrac{a_0}{a} - 1)\mathsf{C}(x)$$

$$\mathsf{F}(x) \equiv (\tfrac{a_0}{a} - 1)\mathsf{D}(x) - k^2(\tfrac{b}{b_0} - 1)\mathsf{C}(x)$$

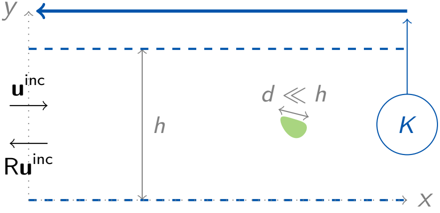
$$\mathsf{K}_{mn} \equiv \mathrm{j}k_n\delta_{mn} \quad k_n^2 = k^2 - (\beta + 2n\pi/h)^2$$

WEAK SCATTERING

$$\mathsf{Y} = \mathsf{K} + \mathsf{y}$$

$\|\mathsf{y}\| \ll \|\mathsf{K}\|$

$$\mathsf{Y}' = -\mathsf{Y}\mathsf{E}^{-1}\mathsf{Y} + \mathsf{K}^2 + \mathsf{F}$$



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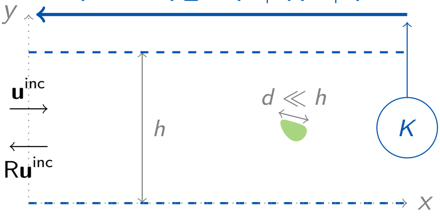
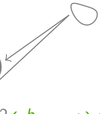


$$\mathsf{R} = [\mathsf{K} + \mathsf{Y}(0)]^{-1}[\mathsf{K} - \mathsf{Y}(0)]$$

plane wave at normal incidence

$$\mathsf{R}_{m0} \simeq -\frac{\mathsf{z}_{m0}}{1 + \sum_i \mathsf{z}_{ii}}$$

$\mathsf{z}_{mn} = \mathsf{y}_{mn}/2\mathrm{j}k_m$



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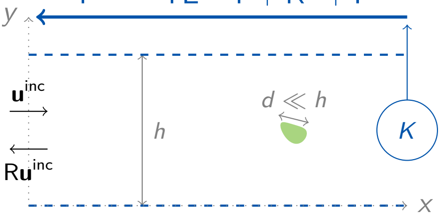
WEAK SCATTERING

$$\mathsf{Y} = \mathsf{K} + \mathsf{y} \qquad \|\mathsf{y}\| \ll \|\mathsf{K}\|$$

$$\mathsf{Y}' = -\mathsf{Y}\mathsf{E}^{-1}\mathsf{Y} + \mathsf{K}^2 + \mathsf{F}$$



$$\mathsf{R} = [\mathsf{K} + \mathsf{Y}(0)]^{-1}[\mathsf{K} - \mathsf{Y}(0)]$$

plane wave at ⇩ normal incidence

$$\mathsf{R}_{m0} \simeq -\frac{\mathsf{z}_{m0}}{1 + \sum_i \mathsf{z}_{ii}}$$

$\mathsf{z}_{mn} = \mathsf{y}_{mn}/2\mathrm{j}k_m$

R is not small if

(i) $k_n = 0$

$$\mathsf{R}_{m\neq n,0} \simeq -\frac{\mathsf{z}_{m0}}{\mathsf{z}_{nn}} \to 0$$

$$\mathsf{R}_{n0} \simeq -\frac{\mathsf{z}_{n0}}{\mathsf{z}_{nn}} \to \mathcal{O}(1)$$

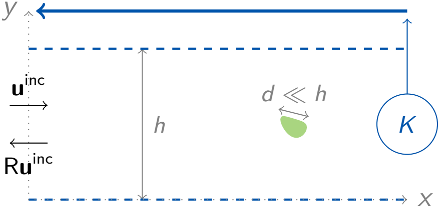
Wood anomaly associated with the Rayleigh wavenumber k_n

WEAK SCATTERING

$$Y = K + y$$

$\|y\| \ll \|K\|$

$$Y' = -YE^{-1}Y + K^2 + F$$



$$R = [K + Y(0)]^{-1}[K - Y(0)]$$

plane wave at normal incidence

$$R_{m0} \simeq -\frac{z_{m0}}{1 + \sum_i z_{ii}}$$

$z_{mn} = y_{mn}/2jk_m$

R is not small if

(i) $k_n = 0$

$$R_{m\neq n,0} \simeq -\frac{z_{m0}}{z_{nn}} \to 0$$

$$R_{n0} \simeq -\frac{z_{n0}}{z_{nn}} \to \mathcal{O}(1)$$

Wood anomaly associated with the Rayleigh wavenumber k_n

(ii) $1 + z_{nn} = 0$

$$R_{m\neq n,0} \simeq -\frac{z_{m0}}{\sum_{i\neq n} z_{ii}} \to \mathcal{O}(1)$$

Wood anomaly associated with a resonance wavenumber

WEAK SCATTERING

$$Y = K + y \qquad \|y\| \ll \|K\|$$

$$Y' = -YE^{-1}Y + K^2 + F$$

$$R = [K + Y(0)]^{-1}[K - Y(0)]$$

plane wave at ⇩ normal incidence

$$R_{m0} \simeq -\frac{z_{m0}}{1 + \sum_i z_{ii}}$$

$z_{mn} = y_{mn}/2jk_m$

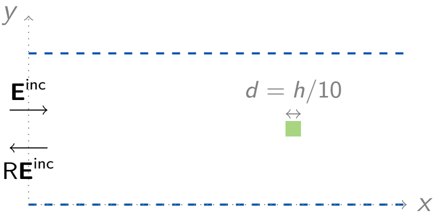
Inspecting the form of z_{m0} and z_{mm} shows that

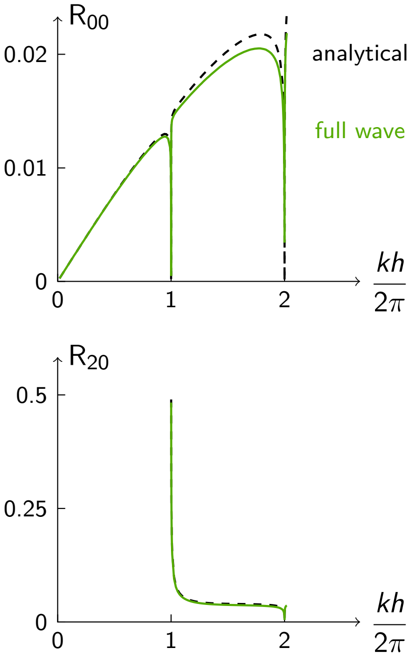
- for a contrast in a only, no anomalies are expected ;
- for a contrast in b only,
  - if $b < b_0$: Rayleigh anomaly only ;
  - if $b > b_0$: resonant anomaly, preceding the Rayleigh anomaly → max-min Fano type resonance

WOOD ANOMALIES

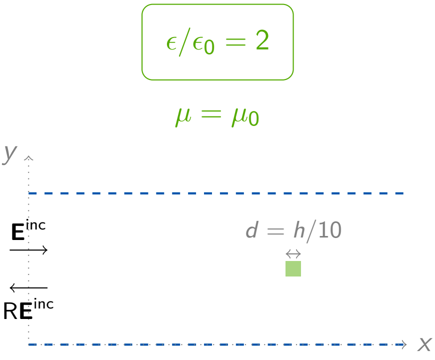
$\epsilon/\epsilon_0 = 0.5$

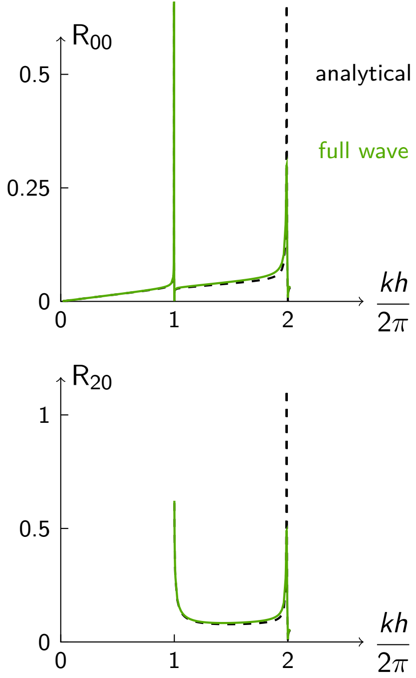
$\mu = \mu_0$





WOOD ANOMALIES





Today, at 12:45

ONS'13, Capri, Italy, 12-14 September 2013

Enhanced transmission through gratings: Compositional and geometrical effects

Agnès MAUREL[a] • Jean-François MERCIER[b] • Simon FÉLIX[c]

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Coupled Wave Analysis using a Multimodal Admittance

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ProCoMedia

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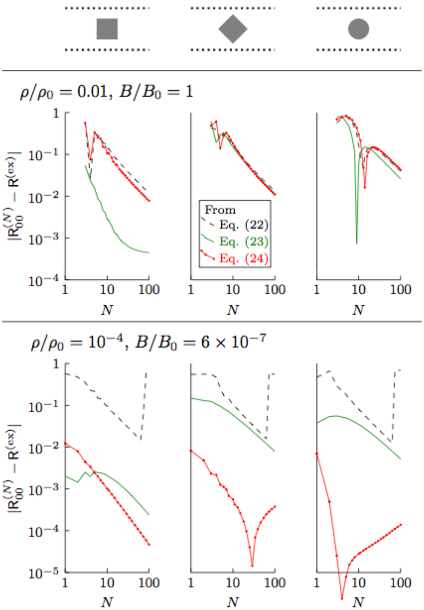
http://blog.espci.fr/procomedia/accueil/



$\rho/\rho_0 = 0.01$, $B/B_0 = 1$
$|\mathsf{R}_{00}^{(N)} - \mathsf{R}^{(\mathrm{ex})}|$
N
From
Eq. (22)
Eq. (23)
Eq. (24)
$\rho/\rho_0 = 10^{-4}$, $B/B_0 = 6 \times 10^{-7}$