Enhanced transmission through gratings:
Compositional and geometrical effects

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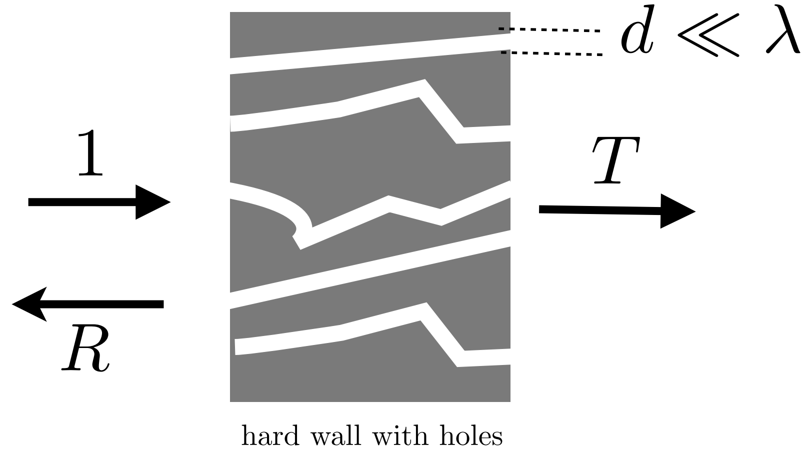
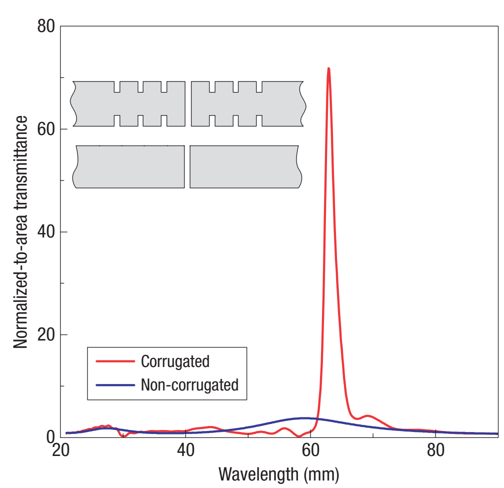
Compositional and geometrical effects

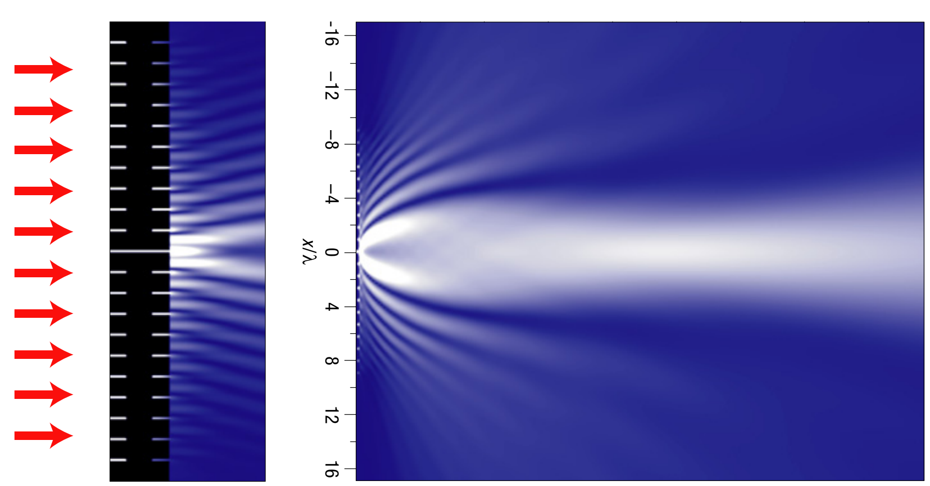


A priori, one expects $T \sim 0$

Plasmon resonance $\rightarrow T \sim 1$

T.W. Ebbesen *et al.*, Nature 391, 667 (1998).

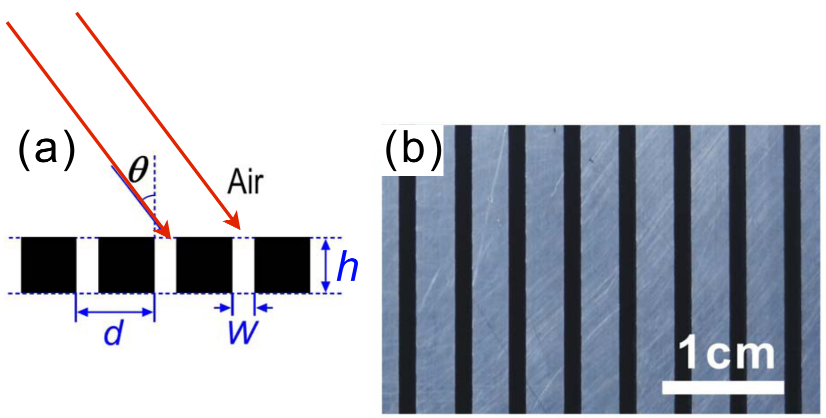


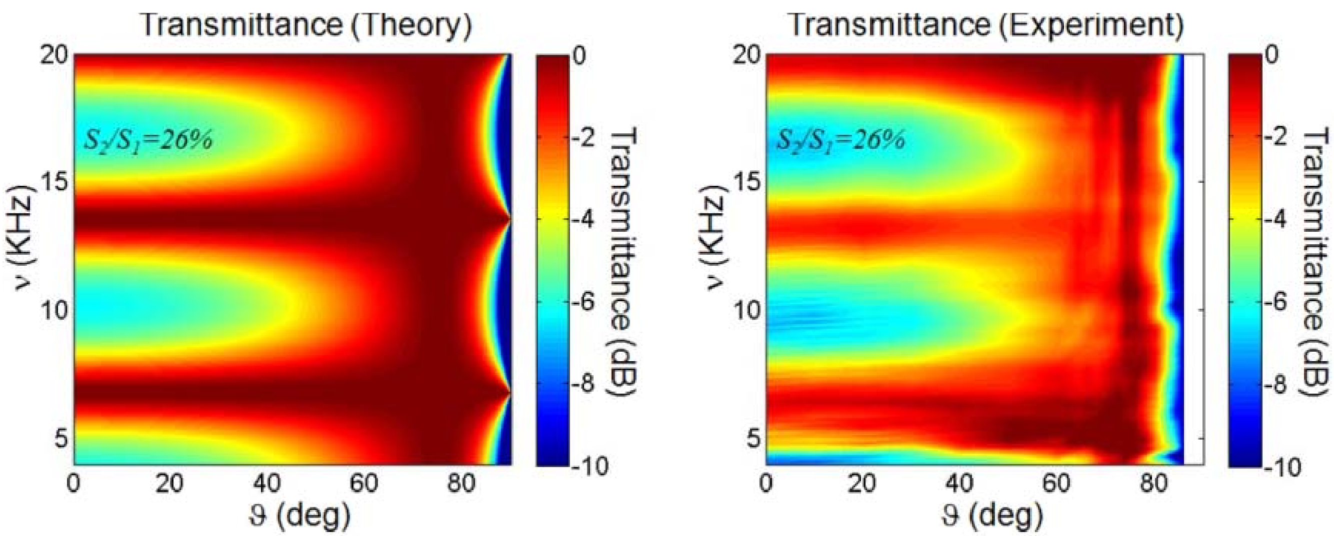


Enhanced transmission through gratings:
Compositional and geometrical effects

Frequency broadband EOT reported first by Huang, Peng and Fan,PRL 105 243901 (2010)

Then extensively studies in a series of papers by Alú and coworkers





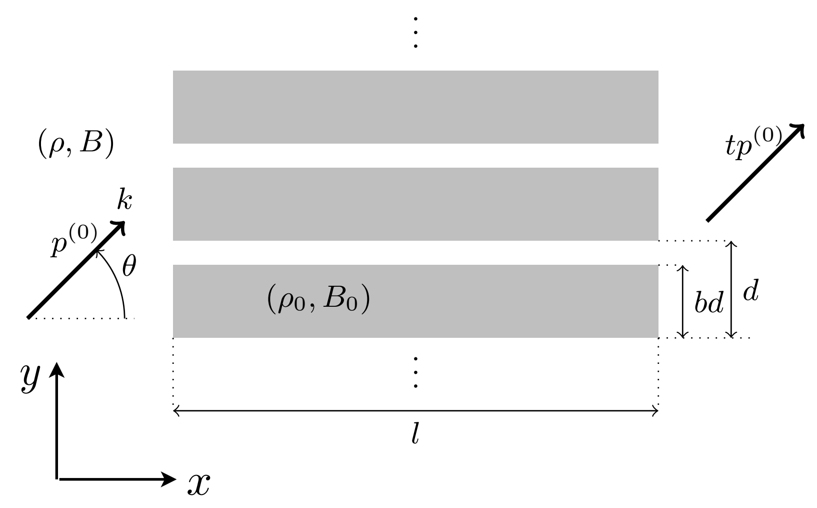
D'Aguanno et al., Scientific Report, 2 340 (2012).

Previous studies: metallic grating
effect of the filling fraction

Our interest: penetrable material
effect of the geometry beyond the filling fraction

Grating made of simple layers

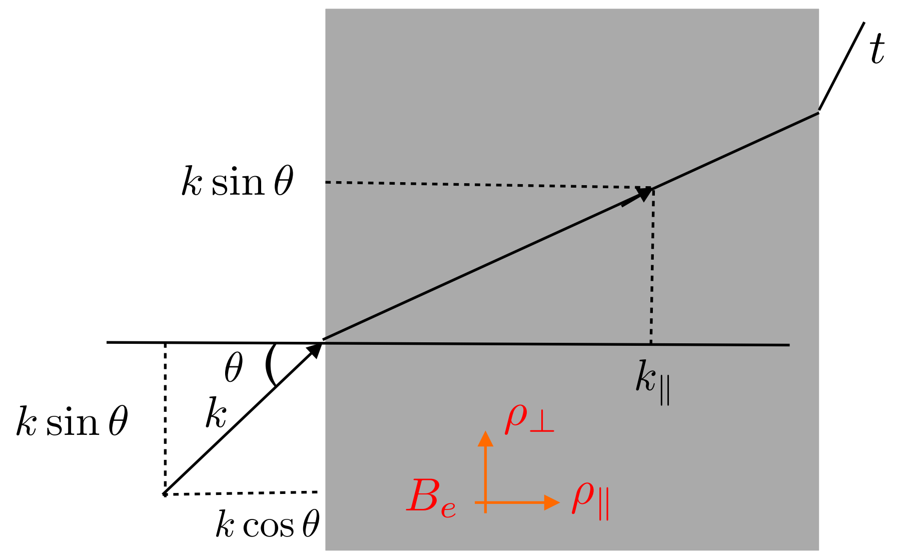
$$\nabla . \left(\frac{1}{\rho(\mathbf{r})} \nabla p(\mathbf{r}) \right) + \frac{\omega^2}{B(\mathbf{r})} p(\mathbf{r}) = 0.$$



Basic tool is homogenization

$$\nabla \cdot \left[\begin{pmatrix} 1/\rho_{||} & 0 \\ 0 & 1/\rho_{\perp} \end{pmatrix} \nabla p \right] + \frac{\omega^2}{B_e} p = 0,$$

with
$$\left\{ \begin{array}{l} \dfrac{1}{\rho_{||}} = \dfrac{b}{\rho_0} + \dfrac{(1-b)}{\rho}, \\ \rho_{\perp} = b\rho_0 + (1-b)\rho, \\ \dfrac{1}{B_e} = \dfrac{b}{B_0} + \dfrac{(1-b)}{B}. \end{array} \right.$$



leading to the expression of the transmission coefficient

$$t = \frac{4u e^{ikl\cos\theta}}{(1+u)^2 e^{-ik_{||}l} - (1-u)^2 e^{ik_{||}l}},$$

$$u \equiv \frac{k\cos\theta}{k_{||}} \frac{\rho_{||}}{\rho} \quad \text{and} \quad k_{||}^2/\rho_{||} + k^2 \sin^2\theta/\rho_{\perp} = \omega^2/B_e.$$

$$\left\{ \begin{array}{l} \dfrac{1}{\rho_{||}} = \dfrac{b}{\rho_0} + \dfrac{(1-b)}{\rho}, \\ \rho_{\perp} = b\rho_0 + (1-b)\rho, \\ \dfrac{1}{B_e} = \dfrac{b}{B_0} + \dfrac{(1-b)}{B}. \end{array} \right.$$

Perfect transmission, $|t| = 1$

$k_{||}l = n\pi \quad \rightarrow \quad$ Fabry-Perot resonances,
$u = 1 \quad \rightarrow \quad$ frequency broadband EOT

Grating made of simple layers

$$t = \frac{4ue^{ikl\cos\theta}}{(1+u)^2 e^{-ik_{\parallel} l} - (1-u)^2 e^{ik_{\parallel} l}},$$

$$u \equiv \frac{k\cos\theta}{k_{||}}\frac{\rho_{||}}{\rho} \qquad \text{and} \qquad k_{||}^2/\rho_{||} + k^2\sin^2\theta/\rho_{\perp} = \omega^2/B_e.$$

$$\begin{cases} \dfrac{1}{\rho_{||}} = \dfrac{b}{\rho_0} + \dfrac{(1-b)}{\rho}, \\ \rho_{\perp} = b\rho_0 + (1-b)\rho, \\ \dfrac{1}{B_e} = \dfrac{b}{B_0} + \dfrac{(1-b)}{B}. \end{cases}$$

Neumann limit $\rho/\rho_0 \to \infty, B/B_0 \to \infty$

$k_{||} \to k$, and $u \to \cos\theta/(1-b)$

FP resonances independent of θ

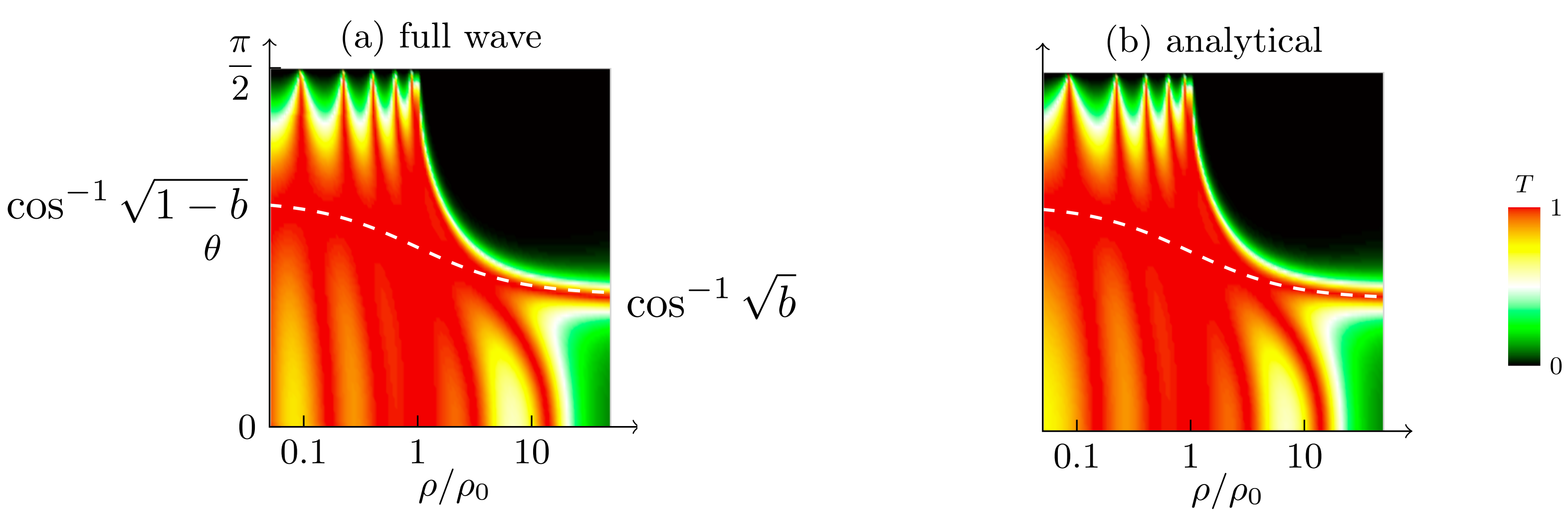
EOT occurs at the Brewster angle $\theta = \cos^{-1}(1-b)$

Non magnetic material $B/B_0 = 1$

FP resonances do depend on θ

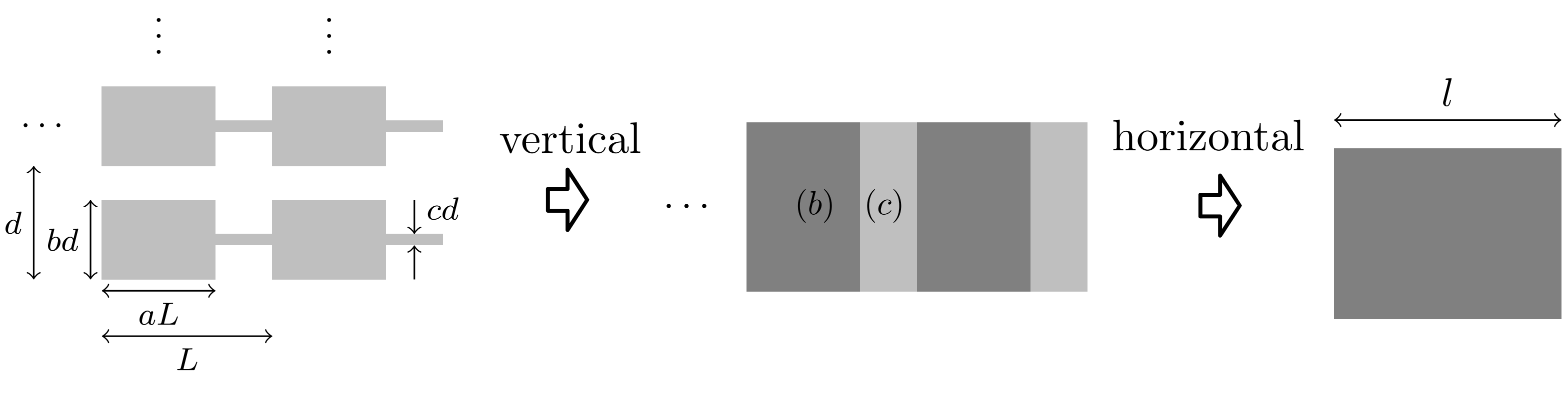
EOT occurs at an optimal angle $\theta^{\text{opt}} = \cos^{-1}\sqrt{\dfrac{(1-b)\rho_0 + b\rho}{\rho_0 + \rho}},$

Comparaison with full wave calculations



Grating with double layers

Two-step homogenization



$$\left\{\begin{aligned} \frac{1}{\rho_{||}^{(b)}} &= \frac{b}{\rho_0} + \frac{(1-b)}{\rho}, \\ \rho_{\perp}^{(b)} &= b\rho_0 + (1-b)\rho, \\ \frac{1}{B_e^{(b)}} &= \frac{b}{B_0} + \frac{(1-b)}{B}. \end{aligned}\right.
\qquad
\left\{\begin{aligned} \rho_{||} &= a\rho_{||}^{(b)} + (1-a)\rho_{||}^{(c)}, \\ \frac{1}{\rho_{\perp}} &= \frac{a}{\rho_{\perp}^{(b)}} + \frac{(1-a)}{\rho_{\perp}^{(c)}}, \\ \frac{1}{B_e} &= \frac{a}{B_e^{(b)}} + \frac{(1-a)}{B_e^{(c)}}, \end{aligned}\right.$$

Again...

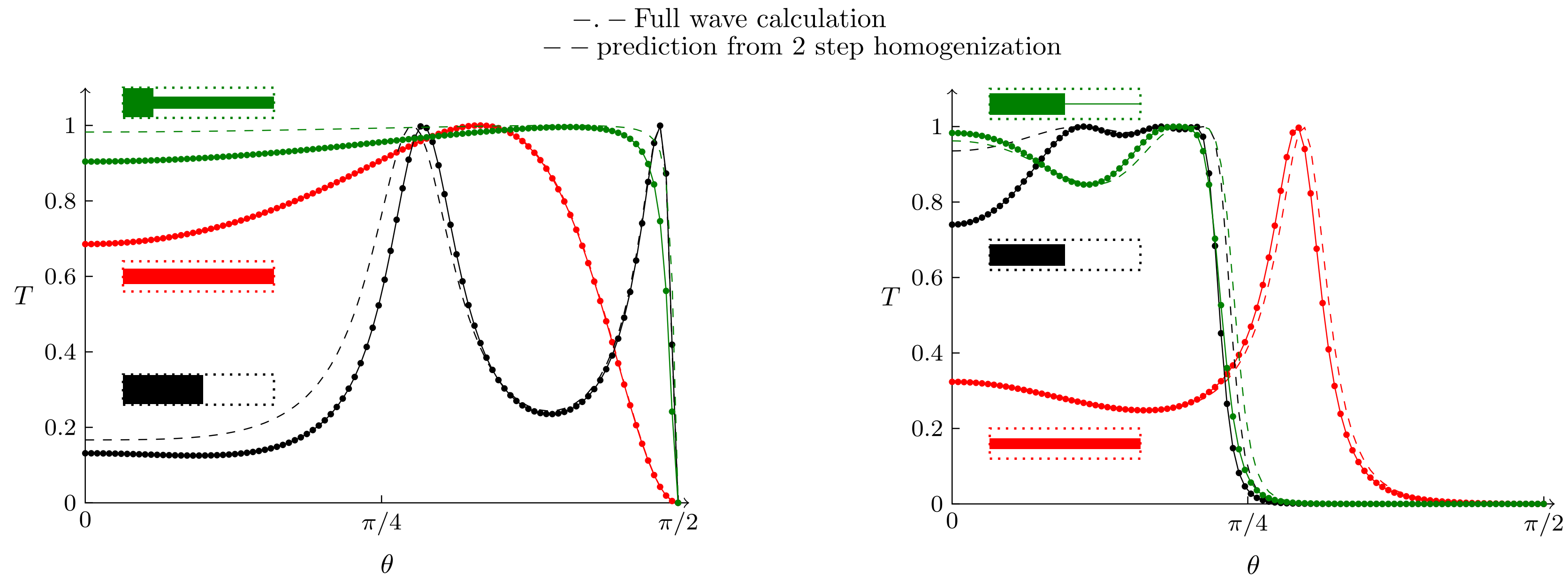
$$t = \frac{4ue^{ikl\cos\theta}}{(1+u)^2 e^{-ik_{\|}l} - (1-u)^2 e^{ik_{\|}l}}, \qquad u \equiv \frac{k\cos\theta}{k_{||}}\frac{\rho_{||}}{\rho} \quad \text{and} \quad k_{||}^2/\rho_{||} + k^2\sin^2\theta/\rho_{\perp} = \omega^2/B_e.$$

with now

$$\left\{\begin{aligned} \frac{\rho_{\|}}{\rho} &= \frac{a}{b\rho/\rho_0 + 1 - b} + \frac{1-a}{c\rho/\rho_0 + 1 - c}, \\ \frac{\rho}{\rho_{\perp}} &= \frac{a}{b\rho_0/\rho + 1 - b} + \frac{1-a}{c\rho_0/\rho + 1 - c}, \\ \frac{B}{B_e} &= [ab + (1-a)c]\,\frac{B}{B_0} + [1 - ab - (1-a)c]\,, \end{aligned}\right.$$

Grating with double layers

Validation



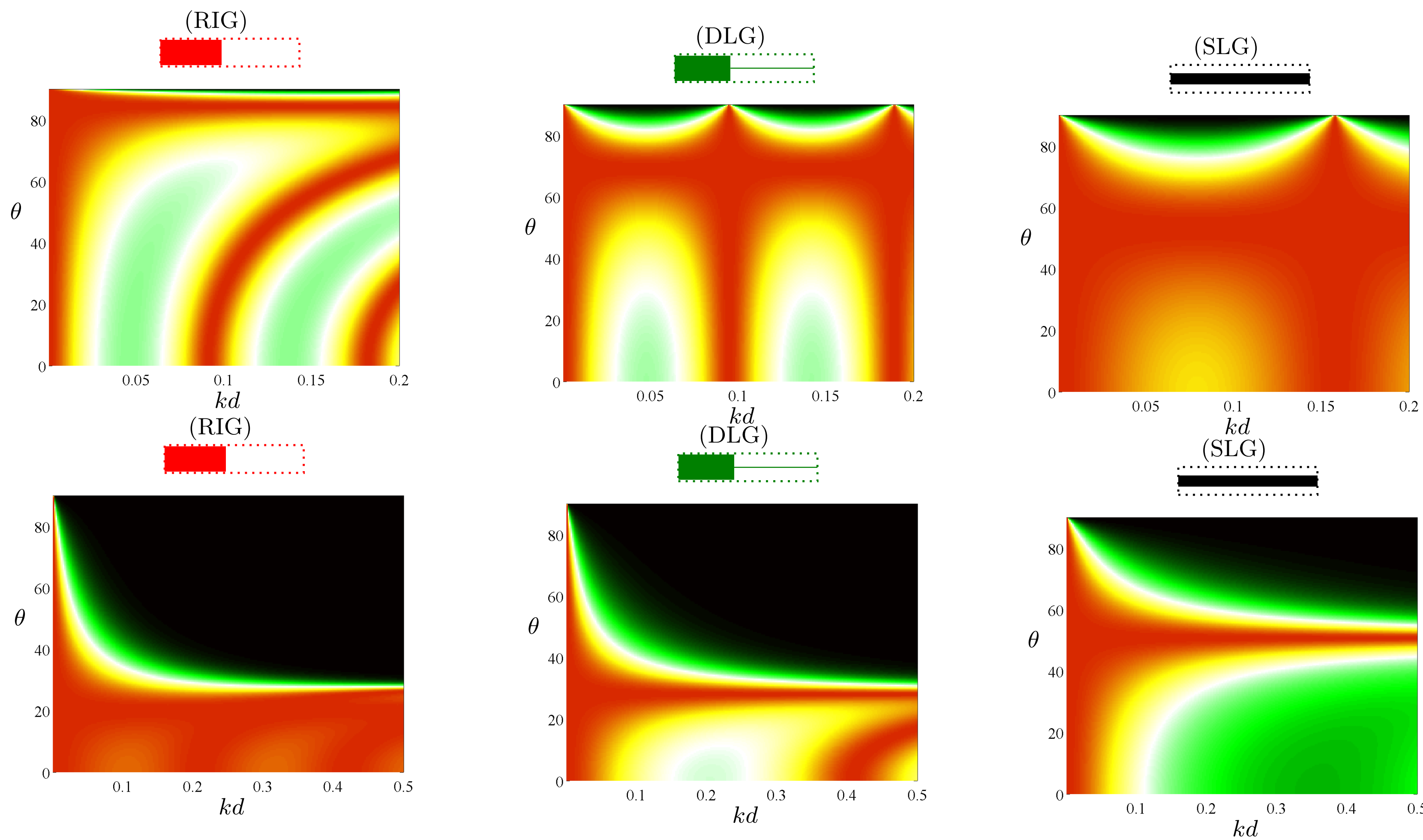
Sound hard material

non magnetic material

Comparison of the spectra for 3 gratings with the same filling fraction

Sound hard material

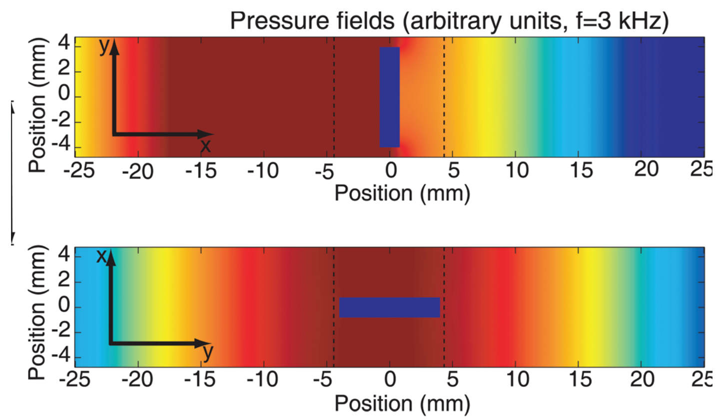
non magnetic material



Perspectives

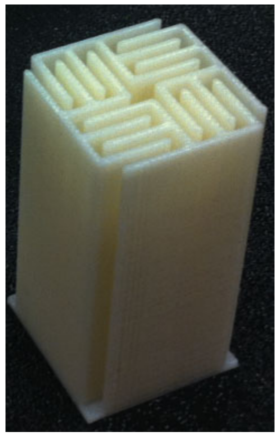
Inspect the limit of this simple homogenization
(notably the case of the squares ;-)

Compare our prediction on the effective parameters to the inversion proposed by
V. Fokin, M. Ambati, C. Sun, and X. Zhang, Phys. Rev. B 76, 144302 (2007).



PRB 84, 024305 (2011)

What do we do with more complexe structures ?.



ANR ProCoMedia
ANR-10-INTB-0914
http://blog.espci.fr/procomedia/accueil/

Experiments

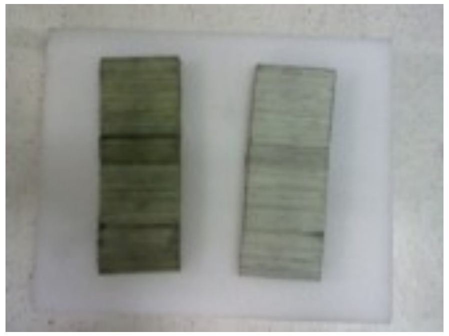
Electromagnetism (A. Ourir)
Acoustics (Y. Auregan, S. Felix,O. Richoux)
Water waves (A. Maurel, V. Pagneux, P. Petitjeans)

Perspectives

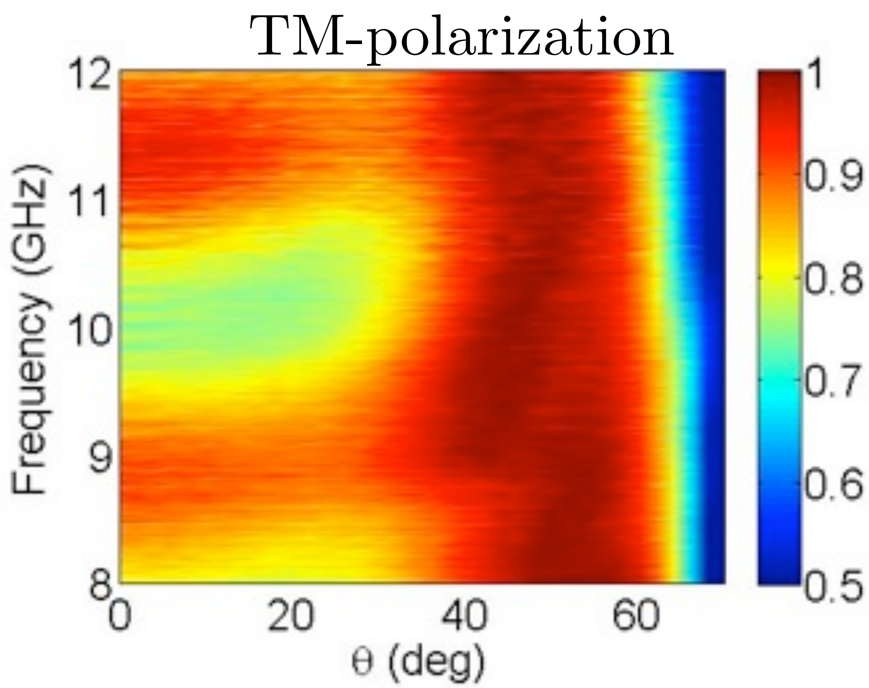
Electromagnetic waves

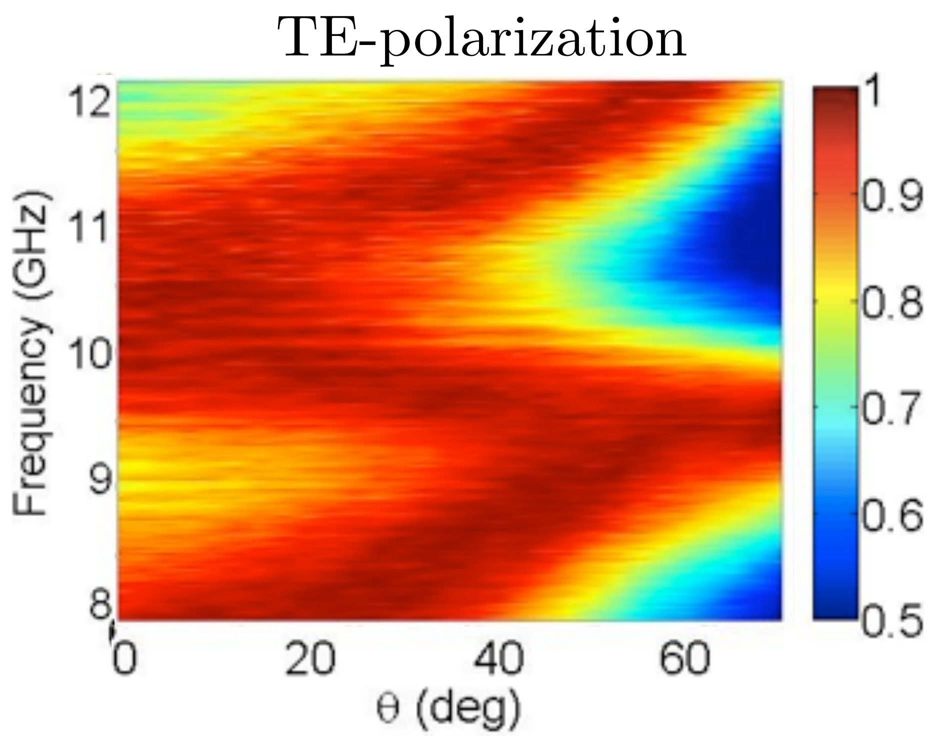
$$\begin{cases} \nabla . \left(\frac{1}{\epsilon(\mathbf{r})} \nabla H(\mathbf{r}) \right) + \mu(\mathbf{r}) \omega^2 H(\mathbf{r}) = 0, \\ \nabla . \left(\frac{1}{\mu(\mathbf{r})} \nabla E(\mathbf{r}) \right) + \epsilon(\mathbf{r}) \omega^2 E(\mathbf{r}) = 0, \end{cases}$$

Abdelwaheb Ourir, Ahmed Akarid



Simple layers: Teflon/Air and Teflon/epoxy gratings





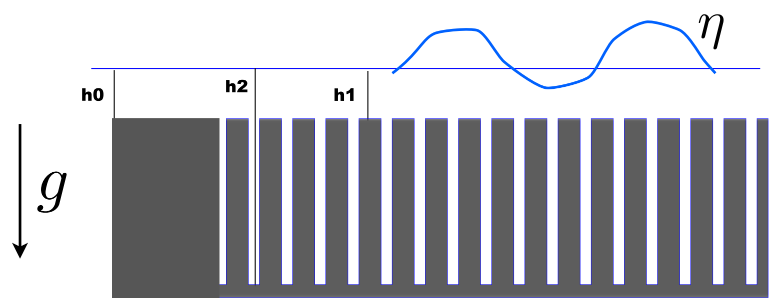
Perspectives

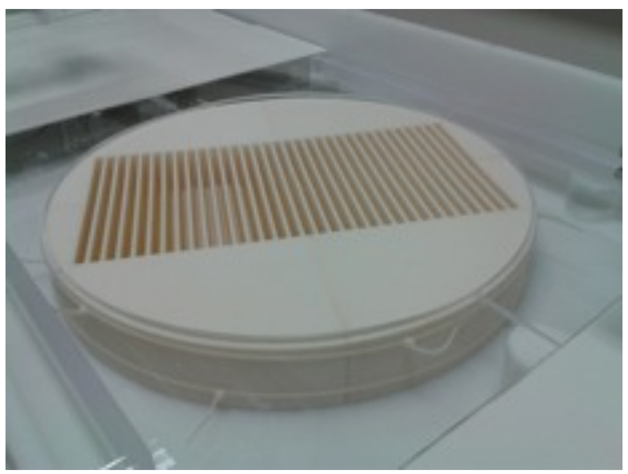
Water waves

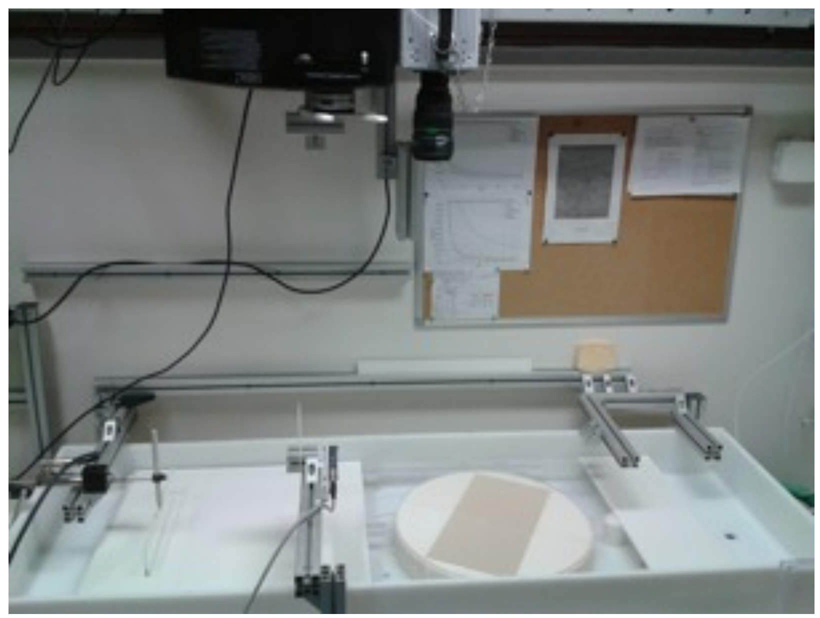
$$\nabla.\left(h(\mathbf{r})\nabla\eta(\mathbf{r})\right)+\frac{\omega^2}{g}\eta(\mathbf{r})=0,$$

A. Maurel, V. Pagneux & P. Petitjeans

G. Favraud & E. Monsalve

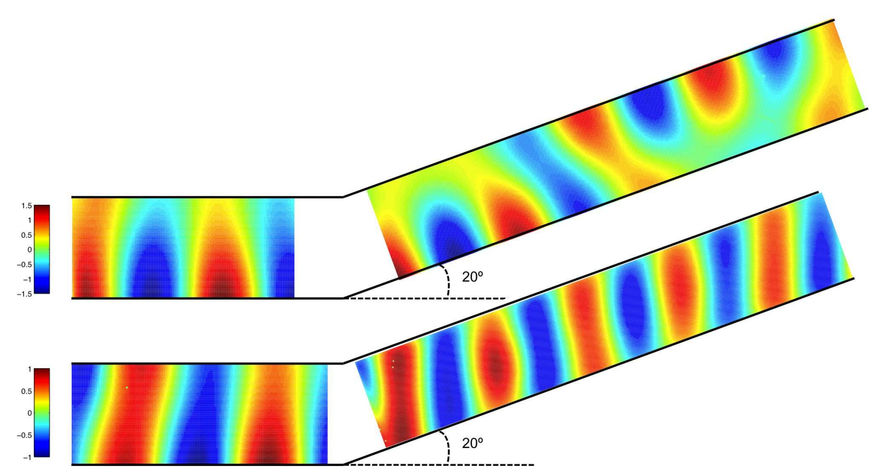






Measurements of the surface wave elevation $\eta(\mathbf{r})$

Effect of a layered medium on the water wave propagation



submitted to PRB