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Physics Procedia 70 (2015) 863 – 866

Physics Procedia

2015 International Congress on Ultrasonics, 2015 ICU Metz

Propagation of nonlinear waves passing over submerged step

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Abstract

Nonlinear water waves have been studied for decades. However, numeric models have always been validated with punctual measurements. In this study we measure the surface deformation of water waves with the Fourier Transform Profilometry (FTP) technique, obtaining a complete space-time resolved field. This permits to separate free and bound waves in the shallow water region, revealing the near resonant interaction between those components. When we change the absorbing beach by a reflecting wall at the end of the channel, we observe an interesting resonance for fixed frequencies. At the resonant frequencies, the system shows a chaotic behavior.

Peer-review under responsibility of the Scientific Committee of ICU 2015

Keywords: Non linear propagation; water wave; dispersive waves ; bound and free waves; chaotic behaviour

1. Introduction

In this study, we present space time resolved measurements of nonlinear surface wave interactions with variable bathymetry. This is done using Fourier Transform Profilometry (FTP) technique, developed by Cobelli et al. (2009), which allows to extract the field of surface elevation in both time and 2D space. We are able to describe the propagation of nonlinear waves passing over a submerged step, especially identifying the near resonant interaction between the two components, free and bound waves, of the second harmonic (see punctual measurement in Beji and Batjes (1992)).

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Peer-review under responsibility of the Scientific Committee of ICU 2015
doi:10.1016/j.phpro.2015.08.177

In order to obtain nonlinear waves in a controlled way, we put a submerged step that separates a deep and a shallow water regions. Waves are generated in the deep water region and travel through the shallow water region. Finally, a non-reflecting beach is used to ends the shallow water region in order to avoid reflection. For each frequency, a temporal decomposition permits to analyze the propagation of the different harmonics, showing the nonlinear dynamics present in the shallow water region.

Results show clearly the presence of bound waves, with a wavenumber twice the one of the fundamental mode, and free waves, which propagates according to the linear dispersion relation. These bound and free waves have different wavenumbers, which produce a phase mismatch that can be observed in a beating of the second harmonic, as was described by Ohyama et al.(1994). Our results show that the beating length agrees with the linear dispersion relation, in the weakly nonlinear region.

2. Experimental Setup

The experiments were carried out in a laboratory tank designed to measure the deformation of the free surface with Fourier Transform Profilometry (FTP). Waves are generated by a wave maker with frequency f in the range [1, 4] Hz. Waves travel through a deep water region (6.5 cm depth) of 40 cm length. A step separates the deep water region from a shallow water region with depth 2 cm. After 80 cm, a non-reflecting beach of 8% slope ends the tank

3. Results

We first consider the linear dispersion relation or water waves:

$$\omega^2 = gk \tanh\left(kh + \frac{\sigma k^3}{\rho} \right) \tag{1}$$

where $\omega=2\pi f$. With forcing frequency f, the two first harmonics ω and 2ω are associated to the wavenumbers $k_1=k(\omega)$ and $k_2=k(2\omega)$ respectively; this latter wavenumber will correspond to free waves. Also, at the second harmonics, the bound wave is associated to wavenumber $2k_1$. The wavenumber mismatch $\Delta k=k_2-2k_1$ corresponds to the inverse of the beat length of the second harmonics, as derived by Massel (1983):

$$L_b = \frac{2\pi}{k_2 - 2k_1} \tag{2}$$

A snapshot of the FTP measurement of the 2D field at a particular time is shown in Fig. 1(a). We have checked that the beach produces a very weak reflection: the amplitude of the reflected waves is smaller than 5% of the amplitude of the incident wave. On the other hand, lateral walls do not perturb the flatness of the wave front, (see Fig. 1), allowing us to model the phenomenon as 1D in space. The spatiotemporal representation of the surface elevation $S(x, t)$ is showed in Fig. 1(b).

In this representation, the horizontal axis has its origin at $x=0$ where is located the step. As expected the crests have smaller velocity in the shallow water region ($x > 0$). Besides, nonlinear effects can be clearly evaluated by the asymmetry between crests and troughs: whilst the proportion of negative and positive values is similar in the deep water region, the part of positive values becomes narrower in the shallow water region. This difference is due to the cnoidal shape of the waves in shallow water conditions. Importantly, we can notice the presence of two different types of waves, with different velocities, in the shallow water region.

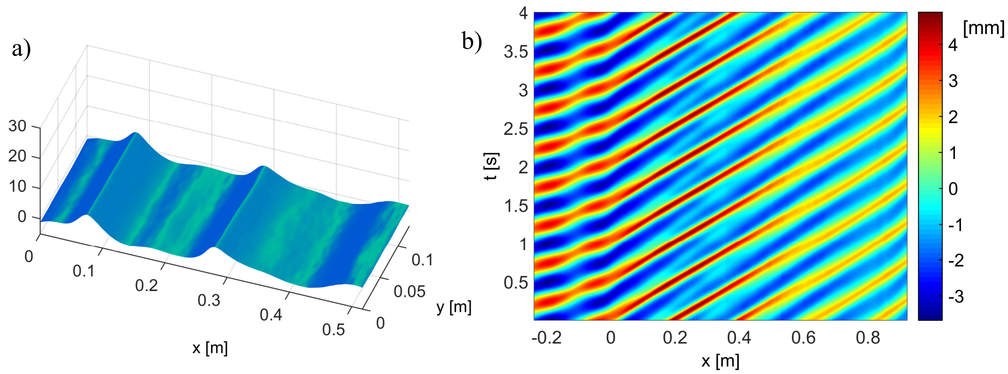

Fig. 1. (a) Surface deformation in the shallow water region; (b) Space-Time graph for f=2 Hz.

From the surface elevation $S(x,t)$, the spatial evolution of the Fourier Coefficients $S_n(x)$ of each mode are calculated as:

$$S_n(x) = \frac{1}{T}\int_0^T S(x,t)e^{-i2\pi ft}dt. \tag{3}$$

Next, a spatial Fourier transform is computed:

$$\hat{S}_n(k) = \frac{1}{L}\int_0^L S_n(x)e^{-ikx}dx, \tag{4}$$

with L the total length of the shallow water region.

The Fig. 3(a) shows a typical behavior of $|S_2(k)|$ for $f = 2.4$ Hz. The two maxima corresponding to the free and bound waves are visible, with k_1 and k_2 previously defined. In the whole range of explored frequencies, we have checked that the k_1 and k_2 values roughly agree with the expected values $k(2\omega)$ and $2k(\omega)$ as given by the linear dispersion relation in Fig. 3(b).

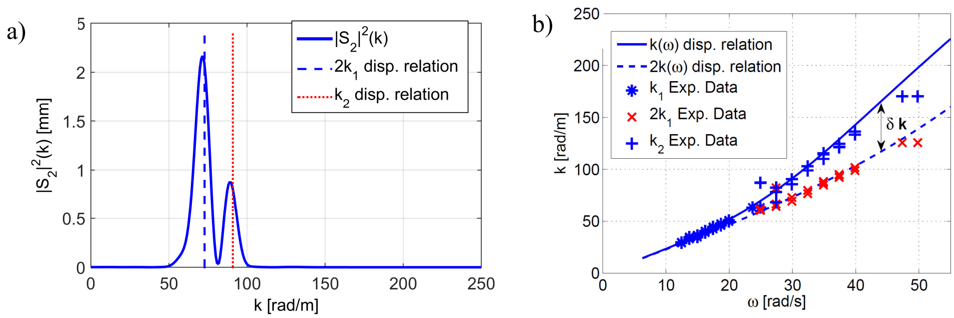

Fig. 3. (a) Spatial spectrum for the second harmonic signal. (b) Dispersion relation of water waves and experimental points: (*) wavenumber of first harmonic, (+) wavenumber of second harmonic free wave, (x) wavenumber of second harmonic bound wave.

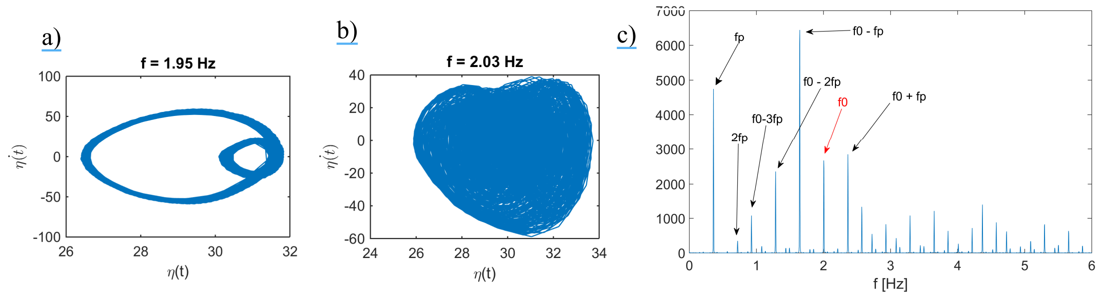

Fig. 4. (a) Phase plane of a non-resonant frequency. (b) Phase plane of a resonant frequency with the typical form of a chaotic behavior. (c) Frequency spectrum of the chaotic case (b). Different peaks of the rich spectrum are identified: f_p is the periodicity frequency, f_0 is the forcing frequency.

4. Border effect with a reflecting wall.

Currently, new experiments are carried out in order to study strong reflection effects, by adding a vertical wall instead of the non-reflecting beach. Preliminary results show an interesting dynamics with rich frequency spectra, revealing the presence of many sub-harmonics, as we observe in Fig. 4(b,c). In Fig. 4(a) we observe a phase plane of a stable case, were there is no generation of sub-harmonics. For comparison, we present in Fig. 4(b) a phase plane generated when the forcing frequency is slightly tuned. Whilst the stable case shows a periodic trajectory, the non-stable case fills the plane showing a typical chaotic behavior. This case can be compared with horizontally forced basins, which show similar responses in the literature. However, the presence of nonlinearities, as was explained in the previous section, turns the problem more complex. The resonance cannot be attributed to one eigen mode, because several wavenumbers are generated in the shallow water part, owing to the interaction between free and bound waves.

5. Conclusions.

We reported new measurements of nonlinear waves passing a submerged shelf. With the FTP technique we are able to separate directly bound and free waves, explaining the contribution of each one in the near resonant interaction of higher harmonics. The measured free waves agree with the linear dispersion relation, also the measured bound waves follow the predicted curve. At the second harmonic, the near resonant interaction between bound and free waves produces a beating that is well predicted by Massel (1983).

When a reflecting wall is imposed at the end of the waveguide, an interesting resonance appears for fixed frequencies. The presence of nonlinearities and the generation of harmonics, propose interesting questions that will be the subject of further works.

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