Experiments in Fluids 37 (2004) 301–308
DOI 10.1007/s00348-004-0824-0

Velocity fields and streamline patterns of miscible displacements in cylindrical tubes

J. Kuang, P. Petitjeans, T. Maxworthy

Abstract Displacements of a viscous fluid by a miscible fluid of a lesser viscosity and density in cylindrical tubes were investigated experimentally. Details of velocity and Stokes streamline fields in vertical tubes were measured using a DPIV (digital particle image velocimetry) technique. In a reference frame moving with the fingertip, the streamline patterns around the fingertip obtained from the present measurements confirm the hypothesis of Taylor (1961) for the external patterns, and that of Petitjeans and Maxworthy (1996) for the internal patterns. As discussed in these papers, the dependent variable, m, a measure of the volume of viscous fluid left on the tube wall after the passage of the displacing finger, is a parameter that determines the flow pattern. When $m>0.5$ there is one stagnation point at the tip of the finger; when $m<0.5$ there are two stagnation points on the centerline, one at the tip and the other inside the fingertip, and a stagnation ring on the finger surface with a toroidal recirculation in the fingertip between the two stagnation points. The finger profile is obtained from the zero streamline of the streamline pattern.

Keywords Miscible fluids, Finger, Fingering displacement, PIV/DPIV, Velocity field, Streamline, Flow pattern, Experiment

1 Introduction

The displacement of one viscous fluid by another in a porous medium is an important problem with a variety of applications, e.g., the secondary recovery of petroleum in the petrochemical industry. As the pore-level details of such flows in a real porous medium are too complicated to measure experimentally, and to simulate numerically, the flow in simple geometries, such as a Hele-Shaw cell or a capillary tube, has been used to model certain aspects of these flows. One possible model, that of viscous fluid displaced by another fluid of less viscosity in a capillary tube, has been studied in a number of papers, e.g., Taylor (1961), Cox (1962), Reinelt and Saffman (1985), Petitjeans and Maxworthy (1996) [hereinafter PM], and Chen and Meiburg (1996) [CM] among many others. Taylor (1961) studied the displacement of a viscous fluid by another immiscible fluid (air) in a horizontal capillary tube. A diagnostic for the fluid left on the tube wall behind the finger front, i.e., the fraction $m=1-U_m/U_{tip}$, was measured as a function of capillary number, $Ca=U_m\mu_2/\sigma$. Here,U_m is the mean velocity of the Poiseuille flow ahead of the fingertip,U_{tip} is the tip velocity of the finger propagation, μ_2 is the viscosity of the displaced fluid, and $\sigma$is the surface tension. For a large Ca and a value of viscous Atwood number close to 1, Cox (1962) found that the fraction, m, approached a constant value of 0.6 (Taylor's experiment extended only to a relatively small capillary number and gave the value to be 0.56). Here, the Atwood number,$At=(\mu_2-\mu_1)/(\mu_2+\mu_1)$, is a measure of viscosity contrast between the two fluids, with μ_1 and μ_2 being the dynamic viscosities of the displacing and displaced fluids, respectively. The corresponding numerical simulations by Reinelt and Saffman (1985) agreed well with these experiments.

If two fluids are miscible, then the interfacial tension between them tends to zero and the Peclet number (Pe) must replace the capillary number in the immiscible case as one of the controlling parameters. The Peclet number, defined as $Pe=U_m d/D$ here, represents the relative importance of convection and diffusion, where d is the tube inner diameter and D is a suitably averaged diffusion coefficient. PM (1996) extended Taylor (1961) by using two miscible fluids (glycerine and glycerine/water mixtures) in capillary tubes. Kuang, Maxworthy, and Petitjeans (2003) [hereinafter KMP] conducted a similar experiment using silicone oils to even smaller Pe. For large Peclet number, the fraction m was found to depend only on At. When At tended to unity, the fraction m increased to a value of 0.61, which is virtually identical to the value of 0.6 obtained by Cox (1962) for immiscible displacements. For a small Peclet number, PM (1996) found the fraction m depended strongly on At and a gravitational parameter, $F = gd^2\Delta\rho/\nu_2 U_m\rho_2$, which represents the

Received: 6 October 2003 / Accepted: 13 April 2004
Published online: 26 May 2004

J. Kuang (✉), T. Maxworthy
Department of Aerospace and Mechanical Engineering,
University of Southern California, Los Angeles,
California, 90089, USA
E-mail: kjun@usc.edu

P. Petitjeans, T. Maxworthy
Laboratoire de Physique et Mecanique des Milieux Heterogenes,
Ecole Superieure de Physique et de Chimie Industrielles,
10 rue Vue Vauquelin, 75231, Paris Cedex 05, France

This work was supported by the NASA Microgravity Program under grant Number NAG3–2125 to the University of Southern California.

relative importance of the gravitational and viscous forces. Here, g is the gravitational acceleration, $\Delta\rho = \rho_2 - \rho_1$ is the density difference between the two fluids, $\rho_2$and v_2 are the density and kinematic viscosity of the displaced fluid, respectively. The parameter F is negative if gravitational effects stabilize the finger displacement and positive if they destabilize it.

The dependence of the finger streamline patterns on the value of m was hypothesized originally by Taylor (1961) for the flow outside the finger and extended by PM (1996) to the flow inside the finger. They suggested two probable streamline patterns in a reference frame moving with the fingertip: when m>0.5, the flow pattern shows only one stagnation point at the tip of the finger; when m<0.5, there are two stagnation points on the centerline, one at the tip of the finger and the other within the fingertip, together with a stagnation ring on the finger surface. For the latter case, PM (1996) argued that a toroidal recirculation must exist inside the fingertip. However, because of the lack of experimental data, the flow patterns have not been confirmed quantitatively. The numerical simulations of CM (1996) for miscible displacements in tubes showed a similar streamline pattern for m>0.5, but a different one for m<0.5. In a frame of reference moving with the c=0.5 contour, where c is the dimensionless concentration, CM's streamline pattern (1996) shows, in the latter case, no stagnation points on the tube centerline so that a continuous flow of less viscous fluid emanates along the tube centerline.

As one of the motivations, an attempt is made to give a quantitative description of the velocity fields and streamline patterns of such miscible displacements. Also, this work is a part of an ongoing project in which space experiments, uncontaminated by gravitational forcing, will be performed to investigate the dynamics of miscible interfaces in micro-gravity. In this paper, three possible cases were studied: (1) the gravity-driven case with no fluid injection, (2) the gravitationally unstable case with fluid injection, and (3) the gravitationally stable case with fluid injection.

2
Experimental setup

Since we were unable to measure the velocity fields in capillary tubes used by PM (1996) and KMP (2003), cylindrical tubes with larger diameters had to be used so that the DPIV (digital particle image velocimetry) technique could be applied. Figure 1 gives a schematic diagram of the experimental setup. A cylindrical tube, with a diameter of 0.7 cm or 1 cm and a length of 16.5 cm, was mounted vertically inside a square glass tube with a cross-section of 2.5 cm×2.5 cm. The space between the two tubes was filled with a glycerine-water mixture (about 98% concentration) with approximately the same refractive index as the glass so that an undistorted view of the flow inside the cylindrical tube could be produced. In the present experiment of fingering displacement, two silicone oils with different viscosities and densities were used. The less viscous (displacing) fluid is 100cs silicone oil with a viscosity,v_1=1 cm^2/s and a densityρ_1=0.964 g/cm^3, and the more viscous (displaced) fluid is 1000cs silicone oil withv_2=10 cm^2/s andρ_2=0.97 g/cm^3. For this fluid pair, the Atwood number is At=0.82 and the diffusion coefficient is D=4.6×10^{-8}cm^2/s. Because the less viscous fluid had a smaller density, to prepare for an experiment, about two thirds of the cylindrical tube was first filled with the more viscous fluid, injected from the bottom, and then the less viscous fluid was added to fill the rest of the tube from the upper end. Consequently, the fluids in the tube were in a gravitationally stable state and a flat interface was generated between them. To perform an upward displacement in the gravitationally unstable state, the tube had to be inverted 180°.

A detailed description of the DPIV technique used in the present experiment has been provided by Fincham and Spedding (1997). The measurement apparatus, as shown in Fig. 1, included a laser and a cylindrical lens to generate a laser sheet of about 1 mm in thickness, a digital video camera to provide images of 768×484 pixels, and an image capture card with the software package, CIVit, installed on a PC for data acquisition and processing. To conduct DPIV measurements, the silicone oils were seeded with tiny particles with an average diameter of 12 μm and an approximate density of 1.1 g/cm^3. The particle concentration in the fluids was very small (of the order of 10^{-3}g/cm^3) so that the influence of the particles on the fluid properties was negligible. By carefully running the experiment and setting parameters from the CIVit interface, the velocity measurement could give approximately ±1% accuracy for the velocity vectors. For example, the time interval between two images is one of key parameters for an accurate DPIV measurement. Generally, it is set so that the particle displacement between images is approximately 5 pixels, with the actual displacement measured to sub-pixel accuracy (Fincham and Spedding 1997).

3
Results of the velocity field measurements

3.1
The gravity-driven unstable case with no fluid injection

Consider an interface in a vertical cylindrical tube with a heavier fluid on the top of a lighter fluid. When there is no fluid injection, the interface is unstable and gravity causes the lighter fluid to rise and the heavier fluid to sink in the tube. The interfacial stability depends strongly on the value of $Pe_g=U_g d/D$, where U_g is a gravity-driven characteristic velocity, as given by Eq. 1 below. Numerical simulations by Fernandez et al. (2002) for two-dimensional flow in a narrow gap show that differences in the initial perturbation (either a sine or a cosine shape) give rise to different finger configurations for a large Pe_g (in their paper, this parameter was designated as the Rayleigh number, but here, the Peclet number is used for consistency). Observation of the present experiment showed a non-symmetric finger to be the result of a non-symmetric perturbation generated when the tube was inverted, as demonstrated in Fig. 2, which is similar to Fig. 15b of Fernandez et al. (2002). Note, however, that Fig. 15 in Fernandez et al. (2002) is the result of simulations for a two-dimensional Hele-Shaw flow rather than a tube flow, so the similarities are only qualitative.

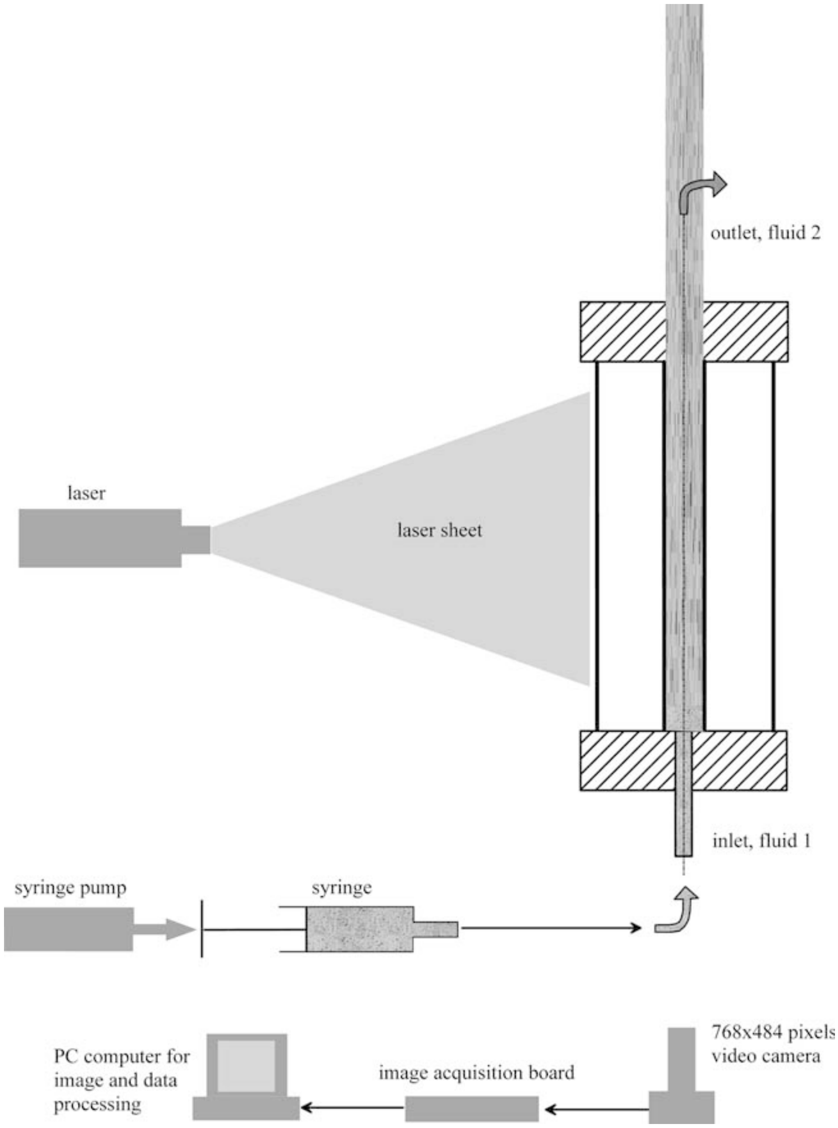

Fig. 1. Schematic diagram of the experimental setup for DPIV measurement of a more viscous fluid 2 displaced by a less viscous fluid 1 in a vertical cylindrical tube

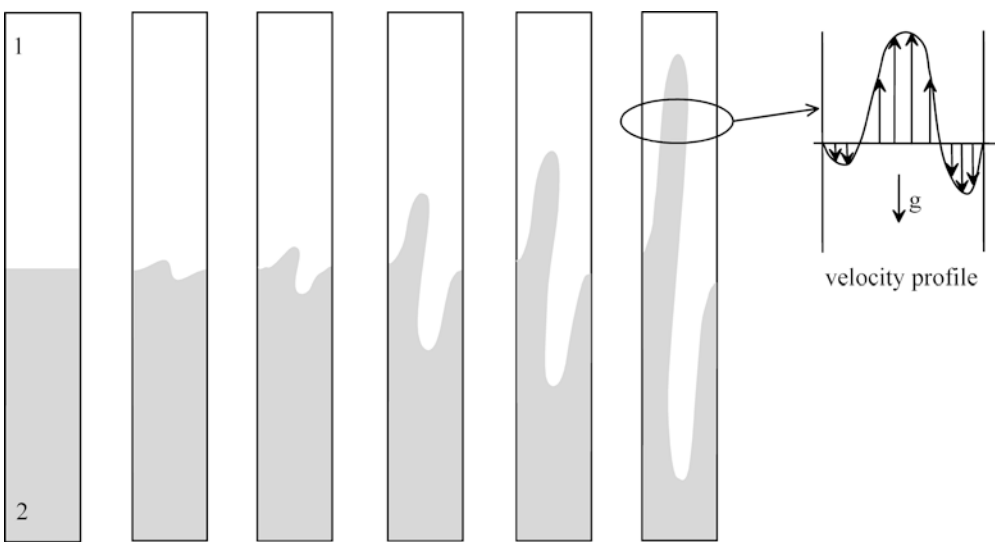

Fig. 2. Sketch of a finger evolution from an unstable interface with no flow injection

DPIV results for the flow field around the upward moving fingertip in a vertical tube of 1 cm diameter with no fluid injection are presented in Fig. 3 for (a) velocity profiles and (b) the streamline pattern. Velocity profiles at values of x from 0 cm to 1.1 cm with intervals of $\Delta x = 0.1$ cm were plotted. The x locations are given in the

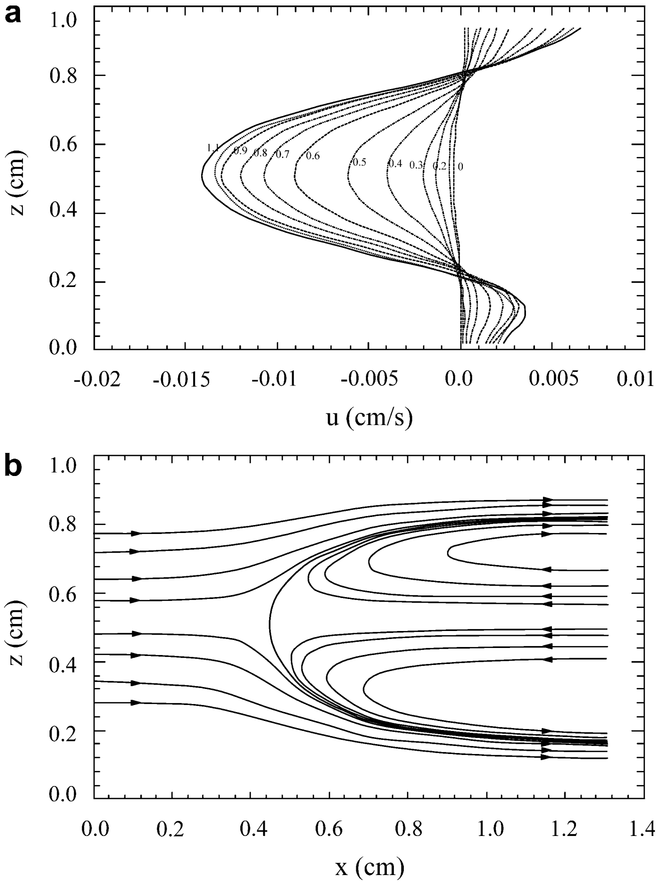

Fig. 3a, b. The velocity field around the fingertip of an upward displacement in a tube of 1 cm diameter with no fluid injection. (**a**) Velocity profiles with x location of each profile numbered. (**b**) Streamline pattern with the zero streamline highlighted

abscissa of Fig. 3b. At the moment of measurement, the tip of the finger was about 3 cm from the initial interface location. The velocity profiles show that the flow field of this displacement is slightly non-symmetric. The tip velocity for this case was found to be 5.63×10^{-3}cm/s. In a reference frame moving with the fingertip, the streamline pattern, as shown in Fig. 3b, reveals a non-symmetric flow field both internal and external to the finger. The method used to obtain the tip velocity and the streamline pattern is outlined in Sect. 3.2. The highlighted zero streamline gives the finger profile, which was found also to be non-symmetric. This leads to a hypothesis, as demonstrated by the present experiment in Sect. 3.2 and the numerical simulation by Payr et al. (2003), that a minimum injected flow is needed to produce an axisymmetric finger. However, the minimum injection velocity or Peclet number to achieve this state has not been determined. In the present experiment, it was always much larger than this minimum value. For the present case of no fluid injection, the mean velocity, U_m, is zero so, strictly speaking, Pe=0 and $F \rightarrow \infty$, based on U_m. To define an equivalent Peclet number, Pe^*, and the gravitational parameter, F^*, to be consistent with the following sections, an equivalent mean velocity, defined by $U_m^* = (1-m)U_{tip}$, was introduced. This equivalent mean velocity can be treated as a mean velocity for the upward moving finger. Here, the fraction $m=1-(d_f/d)^2$ was obtained using the measured finger diameter,d_f, and the tube diameter, d, as defined by PM (1996). The finger diameter was obtained far from the fingertip. However, as shown later in Fig. 5, the finger had nearly a constant diameter after a distance of about $1.5d_f$ from the fingertip. Then, the zero streamline profile at x=1.3 cm in Fig. 3b gives the finger diameter d_f=0.63 cm, or the fraction m=0.6, which leads to the equivalent mean velocity $U_m^* = 2.25 \times 10^{-3}$ cm/s and the corresponding Peclet number $Pe^*=4.9\times10^4$, and F^*=269, based on U_m^*.

Clift et al. (1978) gives a buoyancy velocity,U_g, for an axisymmetric, immiscible bubble rising through a viscous fluid in a tube, which does not depend on the value of the surface tension and is similar to the buoyancy velocity given by Fernandez et al. (2002) for miscible fluids in a Hele-Shaw cell, as:

$$U_g = \frac{(g\Delta\rho/\rho_2)d^2}{102\nu_2}. \tag{1}$$

Extending the use of Eq. 1 to miscible fluids, we have the Peclet number induced by the gravity forcing alone, based on the finger propagating velocity, to be:

$$Pe_g = \frac{(g\Delta\rho/\rho_2)d^3}{102D\nu_2}. \tag{2}$$

For a miscible displacement of 1000cs by 100cs silicone oil in a tube with d=1 cm, using g=980 cm/s^2, $\Delta\rho = 0.006$ g/cm^3,ρ_2=0.97 g/cm^3, andν_2=10 cm^2/s in Eq. 1, we have a gravity-driven rising finger velocity,$U_g=5.94\times10^{-3}$cm/s. This value is very close to the tip velocity of 5.63×10^{-3}cm/s, obtained using the DPIV measurement, which implies that Eq. 1 may give a good estimate of the propagation velocity of a miscible finger driven by gravity only. The Peclet number based on U_g is $Pe_g=1.29\times10^5$.

3.2 The gravitationally unstable case with fluid injection

In this case, as in Sect. 3.1, a heavier and more viscous fluid (1000cs silicone oil) was on the top of a lighter and less viscous fluid (100cs silicone oil) with a finger penetrating upwards into the heavier fluid (F>0). The measurement of the velocity field around the tip of the rising finger in a tube of 1 cm diameter is presented in Fig. 4 with a mean velocity $U_m=3.5\times10^{-3}$cm/s, $Pe=7.7\times10^4$, and F=171.4. The fingertip at the moment of measurement was 3.2 cm above the initial interface location, as in the case in Sect. 3.1. Figure 4a shows velocity profiles at locations from x=0 cm to 1.1 cm and at intervals of $\Delta x = 0.1$ cm, with the x locations given by the abscissa of Fig. 4d. The velocity profiles show an axisymmetric flow field. The maximum velocities in the profiles, as shown in Fig. 4a, increased with x to a constant value of 0.028 cm/s at about 0.5 cm behind the fingertip. Reversal of flow near the tube wall is shown in Fig. 4a by the velocity profiles for x>0.6 cm or about 0.2 cm behind the fingertip, which indicates that the heavier fluid moved downwards under gravity.

Figure 4b shows six velocity distributions along the tube centerline with time intervals of 10 s. The uniform displacement of the centerline velocity profiles indicates

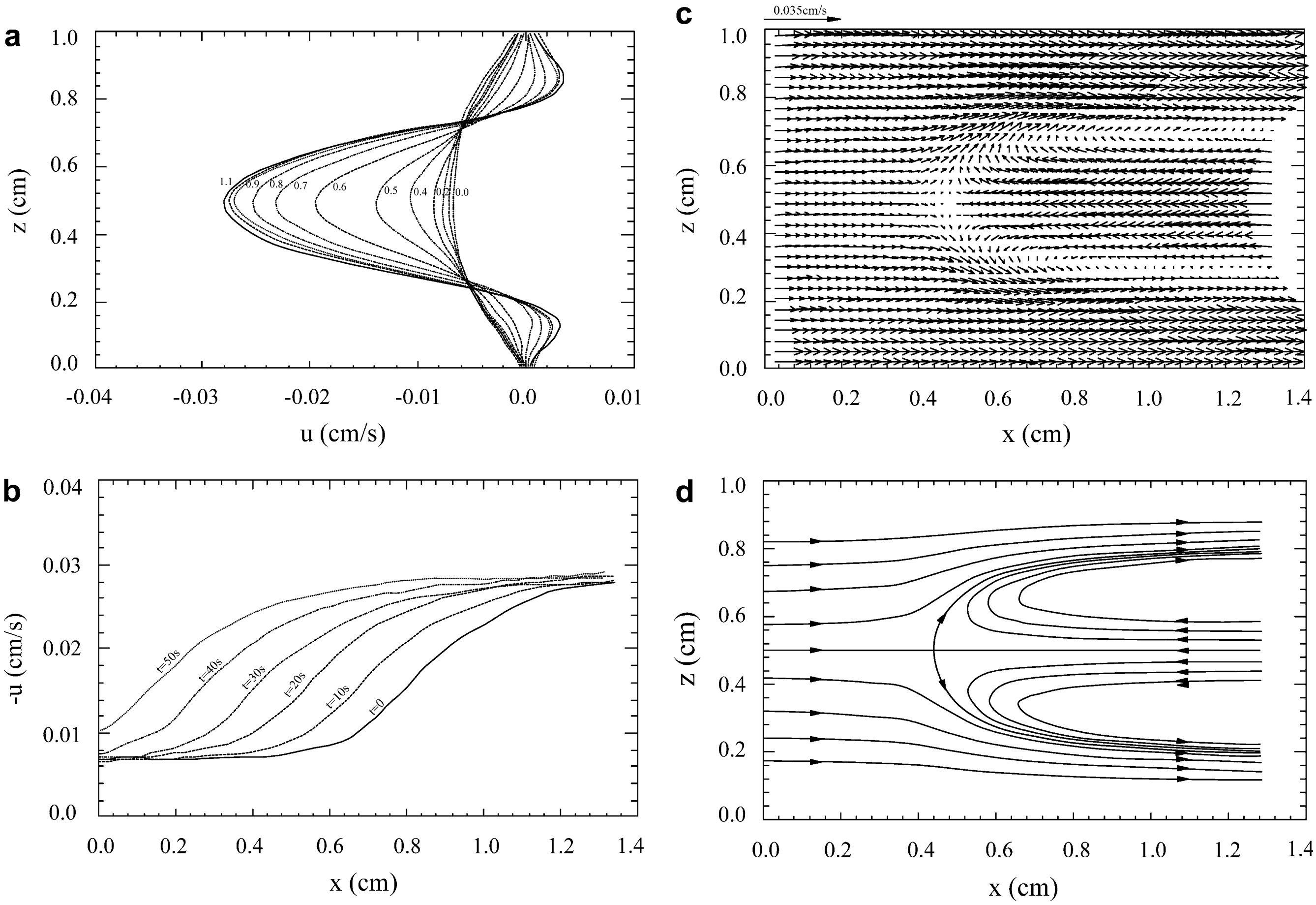

Fig. 4a–d. The velocity field around the fingertip of an upward displacement of 100cs into 1000cs silicone oil in a tube of 1 cm diameter with Pe=7.7×10^4. (**a**) Velocity profiles with x location of each profile numbered. (**b**) Velocity distributions along the tube centerline at 10 s intervals. (**c**) Velocity vectors field. (**d**) Streamline pattern with the zero streamline (finger profile) highlighted

the propagation of the finger at a constant velocity, i.e., the tip velocity, which was calculated to be U_{tip}=13.37×10^{-3}cm/s. The fraction m, calculated using the mean and the tip velocities, was m=0.74. By subtracting the tip velocity from the unsteady flow field, the velocity vector field in a frame of reference moving with the fingertip was obtained, as shown in Fig. 4c. The flow in this frame cannot be precisely steady for miscible fluid displacements due to molecular diffusion, but is quasi-steady for large Pe. The quasi-steady state velocity vector field shows the finger front (a stagnation point), but not the finger profile. The finger profile was obtained by calculating the instantaneous, equivalent steady-state Stokes streamline patterns, as shown in Fig. 4d. The zero streamline, as highlighted in the figure, gives the finger profile and a stagnation point at the tip of the finger is confirmed. The streamline pattern is identical to that hypothesized by Taylor (1961) and PM (1996), and proposed by CM (1996) for m>0.5. Figure 5a shows a photograph of the finger for the same condition. A comparison between this profile and the zero streamline of Fig. 4d is presented in Fig. 5b, where the open circles represent the image profile and the solid line the zero streamline. The correspondence between the two is satisfying. Also, the zero streamline profile has been fitted to an exponential function, as described by Rakotomalala et al. (1997). By analyzing the constants in the fitting exponential function, the equation of the finger profile was obtained:

$$z_1 = \pm\frac{d_f}{2} \mp (1-m)\exp\left[-\frac{\pi x_1}{2(1-m)}\right],\ z_1 \gtrless 0, \qquad (3)$$

where x_1,z_1 are the coordinates with the origin at the fingertip. The fitted result, for d_f=0.6 cm and m=0.74, is also plotted in Fig. 5b, as shown by the dashed line.

3.3
The gravitationally stable case

In the gravitationally stable case (F<0), the lighter and less viscous fluid was injected into the heavier and more viscous fluid in a downward direction. PM (1996) and KMP (2003) predicted that the fraction m should be greater than 0.5 for a high-speed displacement when the absolute value of F tended to zero. In that case, the streamline pattern should be similar to that presented in Sect. 3.2. When the absolute value of F was large and the fraction m became less than 0.5, gravity made the interface at the fingertip flatter and a different flow pattern could be expected. Figure 6 shows the results of injecting 100cs into 1000cs silicone oil in a tube of 1 cm diameter at a mean velocity

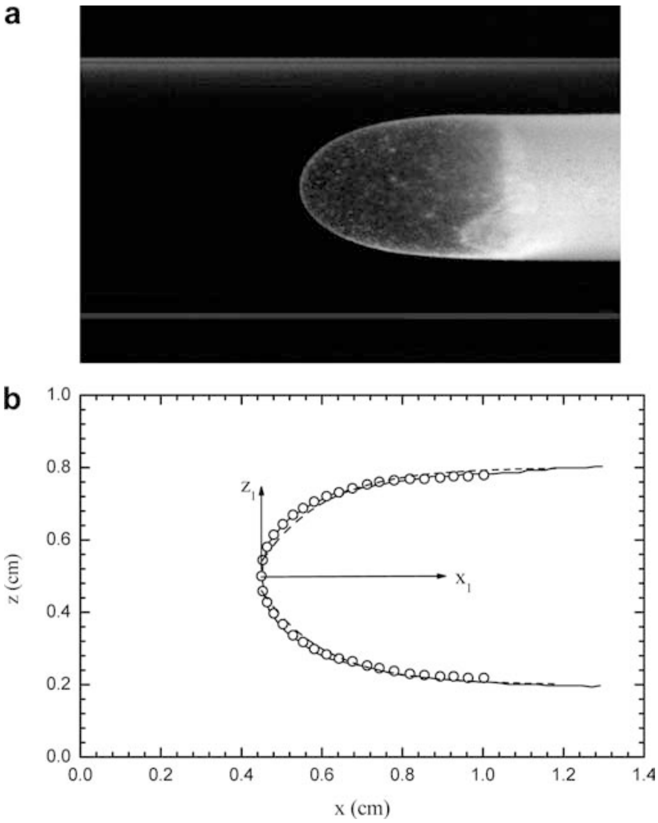

Fig. 5a, b. Finger profile of an upward displacement (**a**) image profile and (**b**) finger profile from zero streamline in Fig. 4d. The *dashed line* is a fitting of zero streamline using Eq. 3

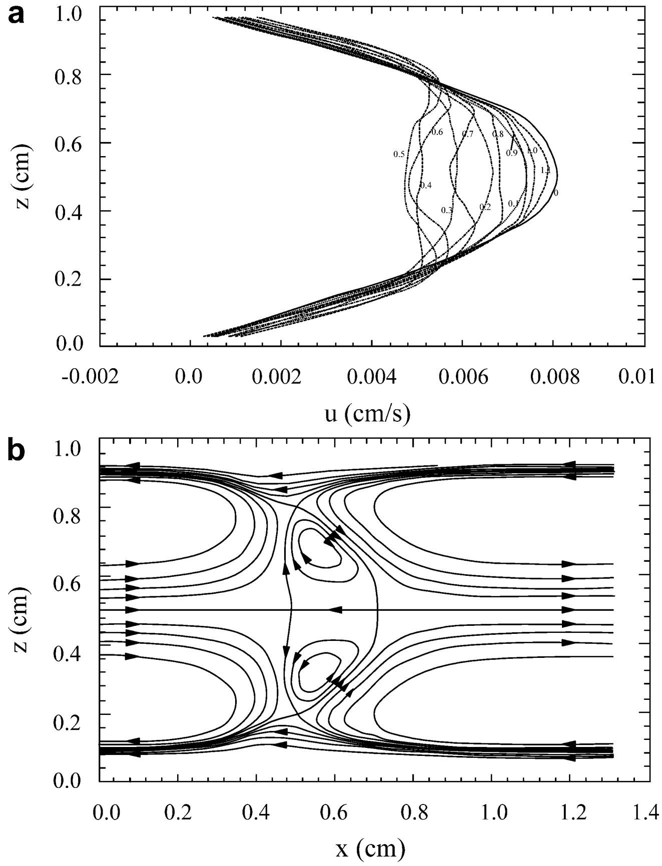

Fig. 6a, b. The velocity field around the fingertip of a downward displacement of 100cs into 1000cs silicone oil in a tube of 1 cm diameter with $Pe=9\times10^4$. (**a**) Velocity profiles with *x* location of each profile numbered, and (**b**) streamline pattern with a zero streamline (finger profile) highlighted

$U_m=4.14\times10^{-3}$cm/s with $Pe=9\times10^4$and $F=-146.4$. The fingertip was located at about 2 cm from the initial interface at the time of measurement. Figure 6a shows the velocity profiles at *x* locations from $x=0$ cm to 1.1 cm with intervals of $\Delta x = 0.1$ cm, where the *x* locations correspond to the abscissa of Fig. 6b of the streamline pattern. The velocity profile downstream of the fingertip, e.g., at $x=1.1$ cm, is a parabola with a maximum velocity of around 8.1×10^{-3}cm/s, close to the calculated maximum velocity of 8.28×10^{-3}cm/s for a Poiseuille profile with the applied volume flux. When approaching the tip, the profile exhibits a decrease in the maximum velocity. At the location $x=0.4$ cm, the velocity has a nearly flat profile inside the finger and then twin-peaked profiles at $x=0.5$, 0.6, and 0.7 cm. The twin-peaked profile represents a reversed flow inside the fingertip. Ahead of the fingertip at $x=0.8$ cm, the twin-peaked profile becomes flat and, eventually, evolves to a Poiseuille profile again.

The velocity of the finger propagation was found, using the method described in Sect. 3.2, to be $U_{tip}=5.06\times10^{-3}$cm/s, from which the fraction *m* was calculated to be 0.18. In a reference frame moving at the tip velocity, the steady-state streamline pattern was obtained as shown in Fig. 6b. The highlighted zero streamline gives the finger profile. The streamline pattern clearly shows the flow field around the fingertip. There are two stagnation points with one at the tip of the finger and the other inside the finger on the axis. On the finger surface off the tube axis, there is a stagnation ring. A toroidal recirculation, which reflects the twin-peaked velocity profile in the fingertip, is between the two stagnation points and the stagnation ring. Again, this streamline pattern agrees well with the streamline patterns hypothesized by Taylor (1961) for immiscible displacement and PM (1996) for miscible displacement at $m<0.5$. The similarity of flow patterns for immiscible and miscible flows for both *m* greater or less than 0.5 raises a question: why do immiscible flows with surface tension look so much like miscible flows with zero surface tension and when does this similarity break down? In fact, this was one of the major reasons for studying such miscible flows in the first place. In the region with great concentration gradient, additional, so-called Korteweg (1901), stresses can potentially be important, as discussed by Joseph and Renardy (1993) and Chen and Meiburg (2002). Based on a comparison with Taylor (1961) for immiscible fluids, PM (1996) obtained an ‘effective’ surface tension for miscible fluids which was considered to be a result of Korteweg stresses.

Observations by PM (1996), Lajeunesse et al. (1999), as well as the present experiment, showed a leakage of the less viscous fluid into the more viscous miscible fluid through the fingertip for $m<0.5$. Figure 7, for 100cs into 1000cs silicone oil downwards in a tube of 1 cm diameter with $Pe=9\times10^4$, shows that less viscous fluid continuously leaked from the finger tip to form another thinner finger, which has

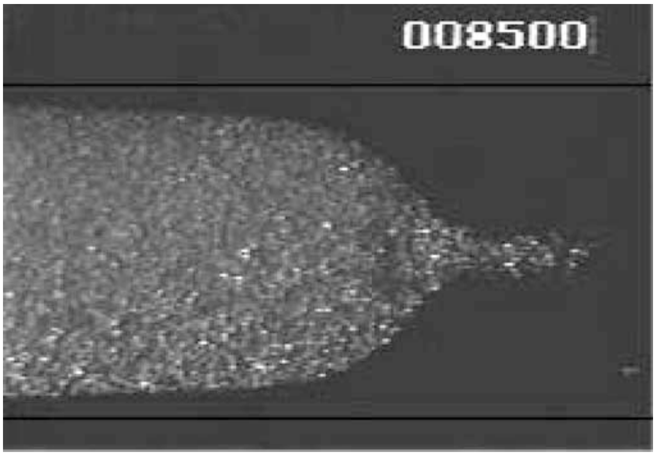

Fig. 7. Image of a spike on the fingertip for downward displacement of 100cs into 1000cs silicone oil in a tube of 1 cm diameter at Pe=9×10[4]

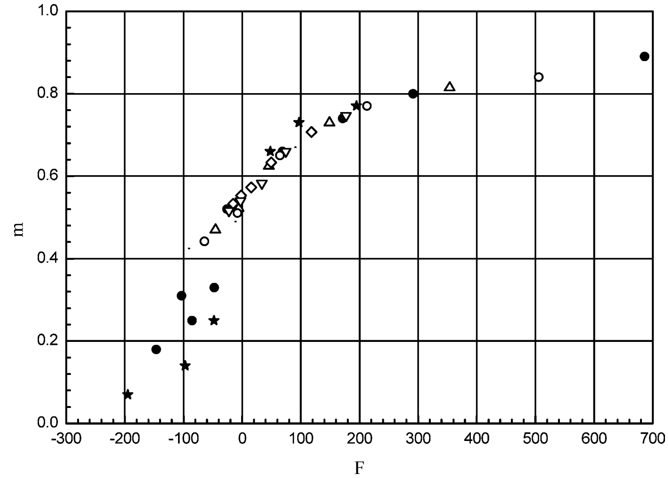

Fig. 8. Fraction m as a function of F. ● data of the present DPIV measurements; ○, △, ▽, ◇ data of KMP (2003); ★ data of PM (1996)

been called a "spike". This type of spike cannot exist in an immiscible displacement because the interfacial tension between two immiscible fluids prevents the interface from deforming. However, surface forces of a miscible displacement are not large enough to prevent this leakage. Thus, the spike is generated if the fluids are miscible and the maximum velocity of the Poiseuille profile is larger than the tip velocity of the finger, i.e., $m<0.5$. The leakage of the less viscous fluid from the tip of the finger means that such flows are never steady, even in a reference frame moving with the underlying fingertip. The spike in Fig. 7 has a diameter of about one tenth of the principal finger diameter. It propagates at a higher speed until it reaches the maximum velocity of the Poiseuille profile ahead of the fingertip. The unsteady development of the spike finger cannot be detected by the present velocity measurements since the streamline pattern in Fig. 6b represents only the instantaneous values of a slowly evolving velocity field. To detect the spike, a measure of the species concentration is required, as shown in the numerical simulations of CM (1996).

Values of m obtained in the present experiment using DPIV, as well as measurements of the fraction m in capillary tubes presented in KMP (2003) and the results by PM (1996), are plotted in Fig. 8 as a function of F. All of these data show good agreement. In particular, when F becomes large and negative, m approaches zero, as hypothesized earlier for the very stable case. On the other hand, as F becomes large and positive, m appears to be approaching unity, or at least a value close to this. This suggests that the finger becomes thinner and thinner as F increases and may, in fact, approach some asymptotic state very similar to the spike observed when m is less than 0.5.

4 Conclusions

Velocity fields of fingering displacement were measured using a DPIV technique in vertical cylindrical tubes. In the present experiment, three cases of measurements, including (1) the gravitationally unstable case with no fluid injection (gravity-driven only), (2) the gravitationally unstable case with fluid injection, and (3) the gravitationally stable case with fluid injection, were performed. A non-symmetric finger flow field was found for case 1 and an axisymmetric finger for case 2. This leads to the hypotheses that a minimum injection of fluid is needed to generate an axisymmetric finger in a cylindrical tube in the gravitationally unstable case, and that a non-symmetric finger will be generated if the injected flow is less than the minimum value. Future investigations will be necessary to determine the minimum injection velocity needed to produce a symmetry finger. Velocity profiles show symmetric flow fields around the fingertip in cases 2 and 3 with fluid injection. Instantaneous steady-state streamline patterns in a frame moving with the fingertip are consistent with the hypotheses of Taylor (1961) and their simple extension by PM (1996) for both m less and greater than 0.5. For the gravitationally unstable case ($F>0$), the fraction m was found to be greater than 0.5 and the streamline patterns show one stagnation point at the tip. For a gravitationally stable case ($F<0$) with $m<0.5$, the streamline pattern shows two stagnation points, one at the tip of the finger and another inside the fingertip, and a stagnation ring on the finger surface. A toroidal recirculation exists inside the fingertip between the two stagnation points and the stagnation ring, which results in a twin-peaked velocity profile in this region. The finger profile was obtained using the zero streamline from the streamline patterns. By fitting the zero streamline for the finger with $m>0.5$, an exponential function, similar to that given by Rakotomalala et al. (1997) for a finger in a Hele-Shaw cell, was found to closely represent the finger profile. Finally, the values of the fraction m from the velocity measurements were in good agreement with previous data obtained by PM (1996) and KMP (2003) in small diameter capillary tubes.

References

Chen CY, Meiburg E (1996) Miscible displacements in a capillary tube. Part II. Numerical simulations. J Fluid Mech 326:57–90

Chen CY, Meiburg E (2002) Miscible displacements in a capillary tubes: influence of Korteweg stresses and divergence effects. Phys Fluids 14(7):2052–2058

Clift R, Grace JR, Weber ME (1978) Bubbles, drops, and particles. Academic Press, New York
Cox BG (1962) On driving a viscous fluid out of a tube. J Fluid Mech 14:81–96
Fernandez J, Kurowski P, Petitjeans P, Meiburg E (2002) Density-driven unstable flows of miscible fluids in a Hele-Shaw cell. J Fluid Mech 451:239–260
Fincham AM, Spedding GR (1997) Low cost, high resolution DPIV for measurement of turbulent fluid flow. Exp Fluids 23:449–462
Joseph D, Renardy Y (1993) Fundamentals of two-fluid dynamics. Part II. Lubricated transport, drops and miscible liquids. Springer, Berlin Heidelberg New York
Korteweg DJ (1901) Sur la forme que prennant les equations du mouvement des fluides si l'on tient compte des forces capillaires causees par des variations de densite. Arch Neerl Sci Ex Nat (ii), 6, 1
Kuang J, Maxworthy T, Petitjeans P (2003) Miscible displacement between silicone oils in capillary tubes. Eur J Mech B–Fluid 22:271–277
Lajeunesse E, Martin J, Rakotomalala N, Salin D, Yortsos YC (1999) Miscible displacement in a Hele-Shaw cell at high rates. J Fluid Mech 398:299–319
Payr M, Vanaparthy SH, Meiburg E (2003) Influence of variable viscosity on density-driven instabilities in capillary tubes. To appear in J Fluid Mech
Petitjeans P, Maxworthy T (1996) Miscible displacements in a capillary tube. Part 1. Experiments. J Fluid Mech 326:37–56
Rakotomalala N, Salin D, Watzky P (1997) Miscible displacement between two parallel plates: BGK lattices gas simulations. J Fluid Mech 338:277–297
Reinelt DA, Saffman PG (1985) The penetration of a finger into a viscous fluid in a channel and tube. SIAM J Sci Stat Comp 6:542–561
Taylor GI (1961) Deposition of a viscous fluid on the wall of a tube. J Fluid Mech 10:161–165