

Rappel

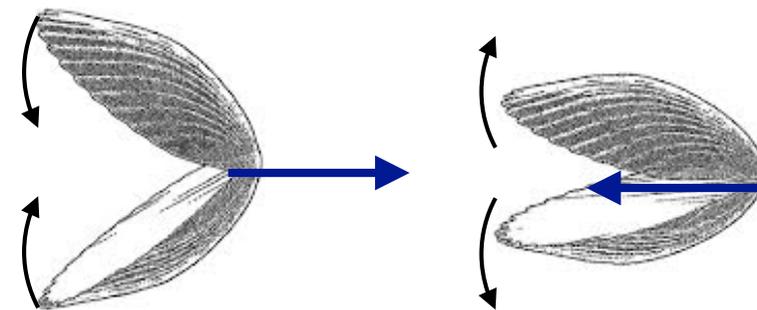
Équation de Navier & Stokes:

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \eta \Delta \mathbf{u} + \rho \mathbf{f}$$

$$Re \ll 1$$

Équation linéaire \Rightarrow solutions additives

Équation réversible dans le temps



“Scallop theorem”

Les écoulements à petit nombre de Reynolds

Pas d'inertie

Navier-Stokes



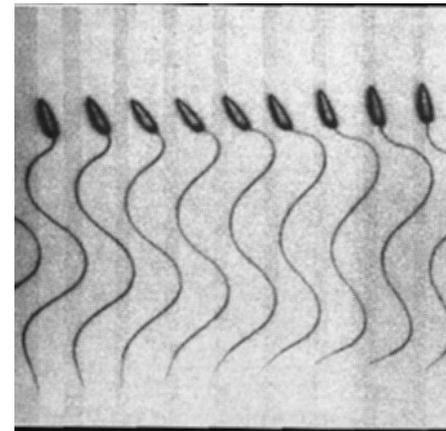
Équation de Stokes

$$\eta \Delta \mathbf{u} = \nabla p$$

Les écoulements en
couche mince et
l'approximation de
lubrification



Les forces sur les objets
solides, la réversibilité
cinématique et la
propulsion à petit
Reynolds



Un monde de très petites forces dans l'eau et l'air

$$Re = \frac{UL}{\nu} \leq 1$$

$$0 = -\nabla p + \eta \Delta \mathbf{u}$$

$$0 \sim \frac{p}{L} + \eta \frac{U}{L^2}$$

$$p \sim \eta \frac{U}{L}$$

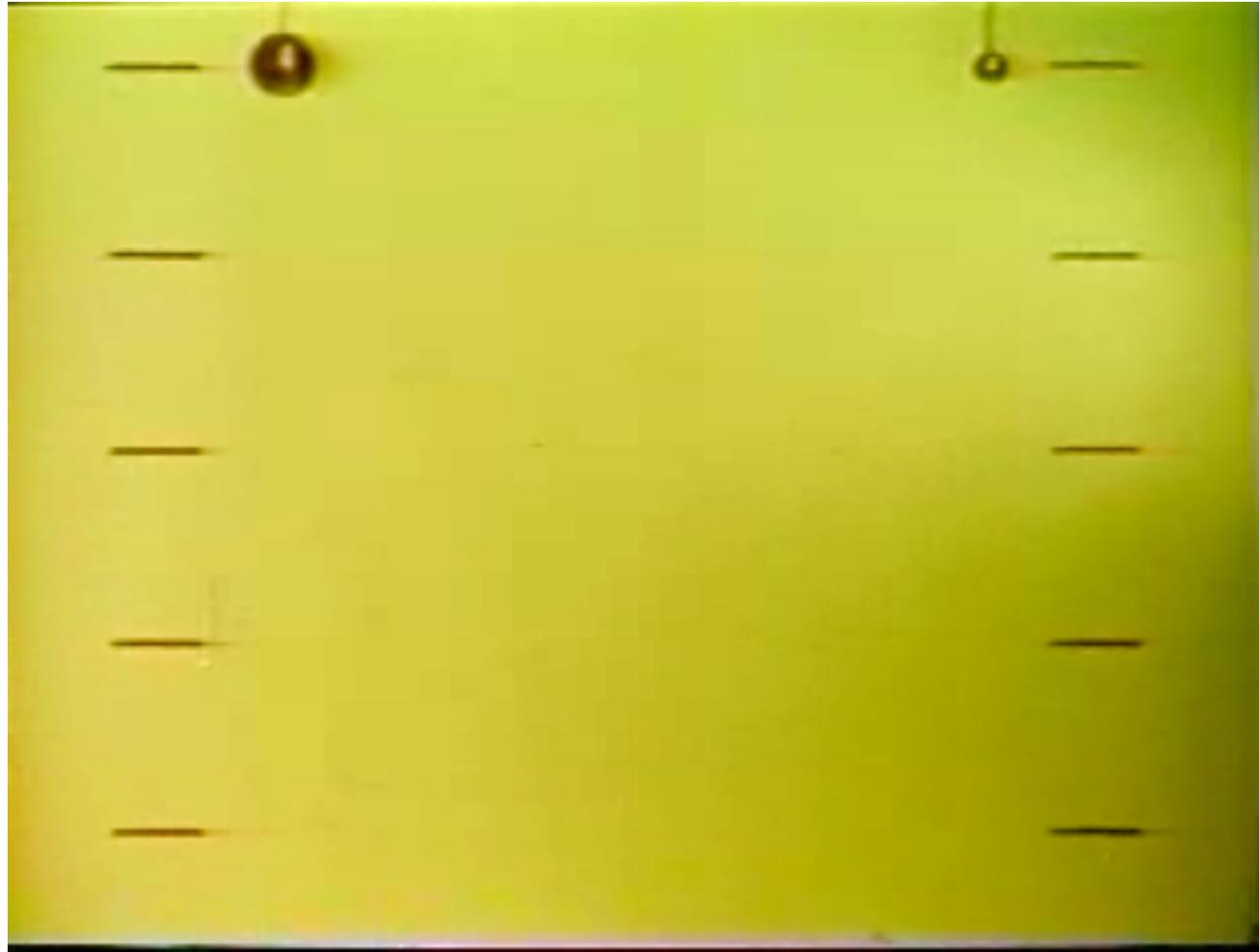
$$F \sim pL^2 \sim \eta U L$$

$$F \leq \eta \nu$$

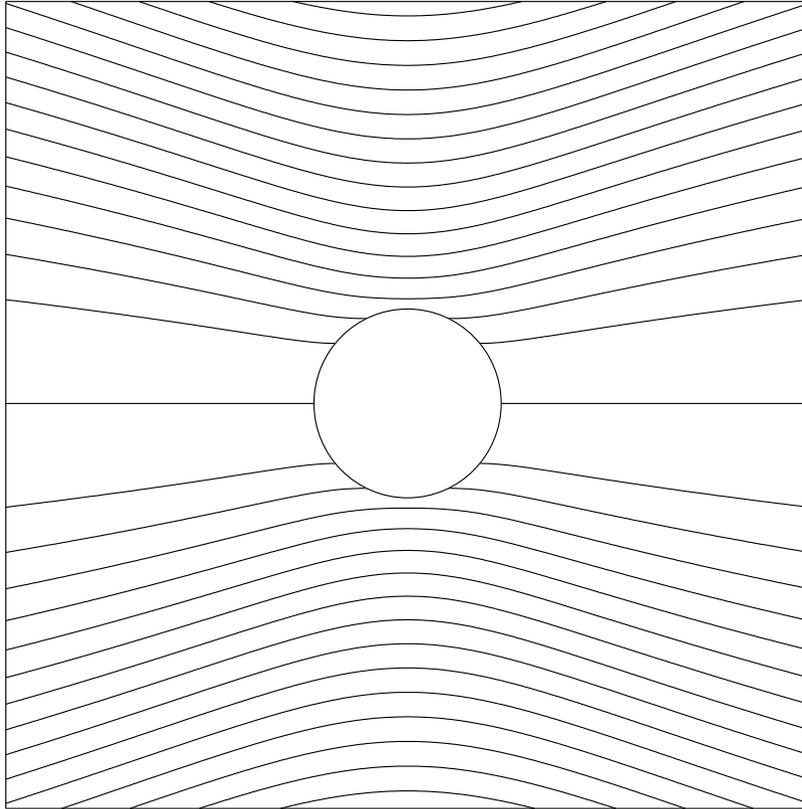
0,000000001 N
1 nN dans l'eau

0,0000000002 N
0,2 nN dans l'air

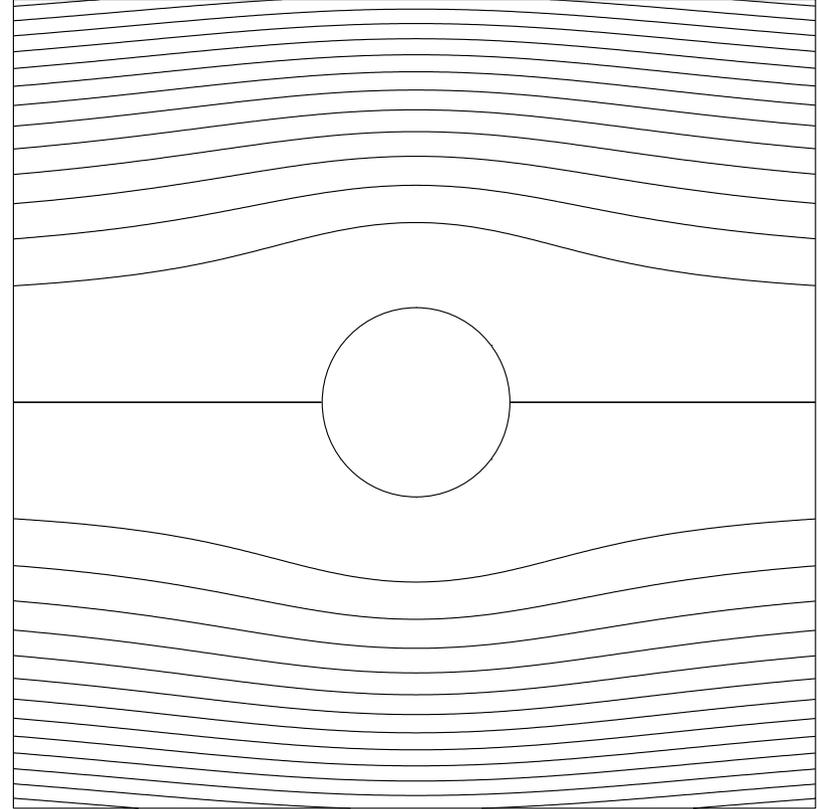
Les forces sur un objet en mouvement : la sphère



Deux sphères de rapport de diamètre égal à 2



Fluide immobile à l'infini



Sphère immobile

Vitesse de chute d'une sphère

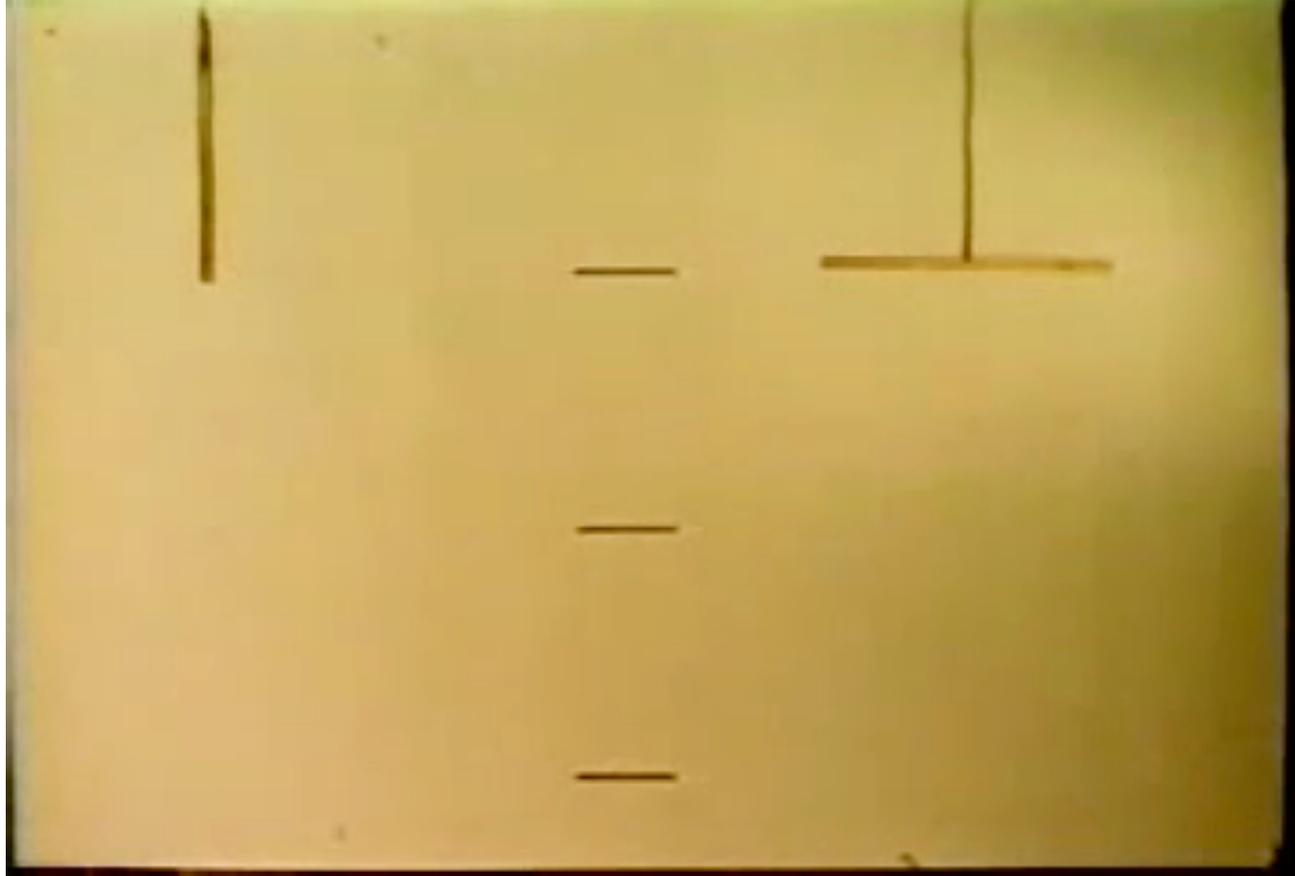
$$F \sim \eta U R$$

$$F = 6\pi\eta U R$$

$$P_{app} = \delta\rho g \frac{4}{3}\pi R^3$$

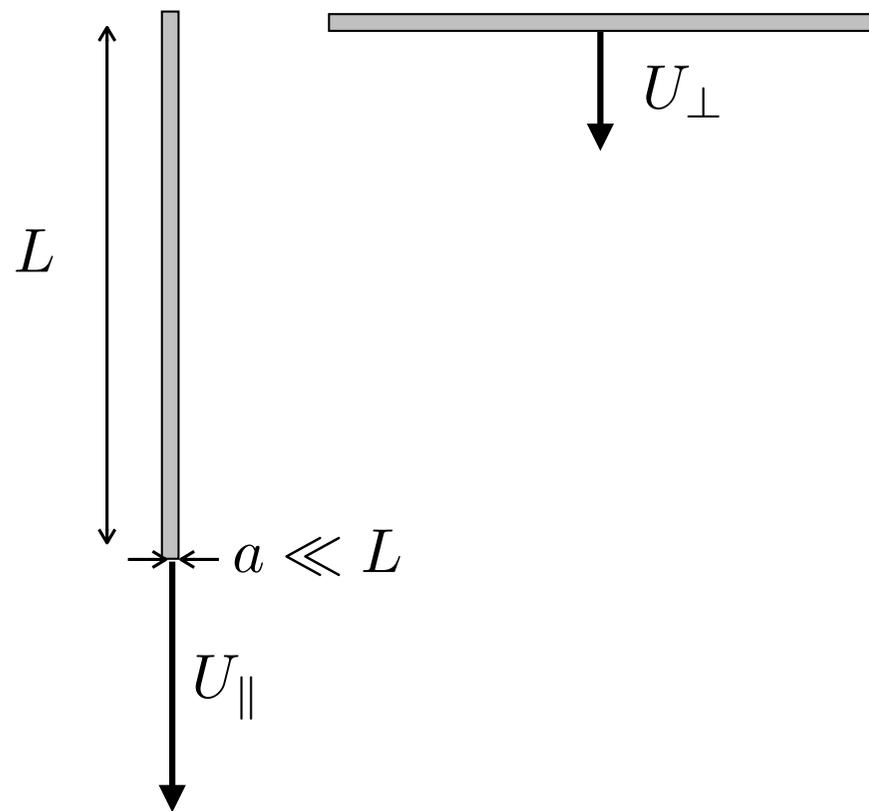
$$U_{sed} = \frac{P_{app}}{6\pi\eta R} = \frac{2}{9} \frac{\delta\rho g R^2}{\eta}$$

Les forces sur un objet en mouvement : un bâtonnet



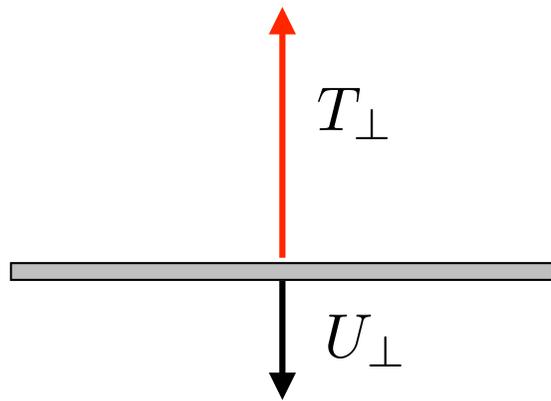
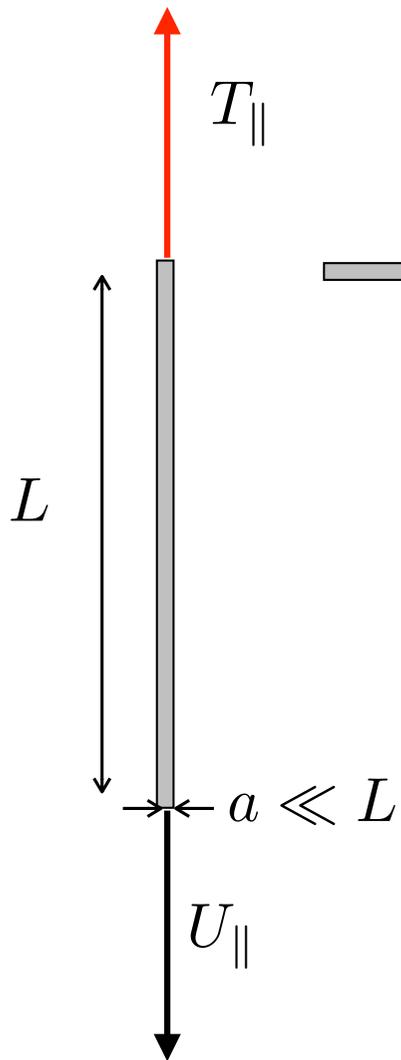
Deux bâtonnets identiques orientés perpendiculairement l'un de l'autre,
puis un bâtonnet incliné

Chute d'une tige



$$U_{\parallel} \simeq 2U_{\perp}$$

Chute d'une tige



$$U_{\parallel} \simeq 2U_{\perp}$$

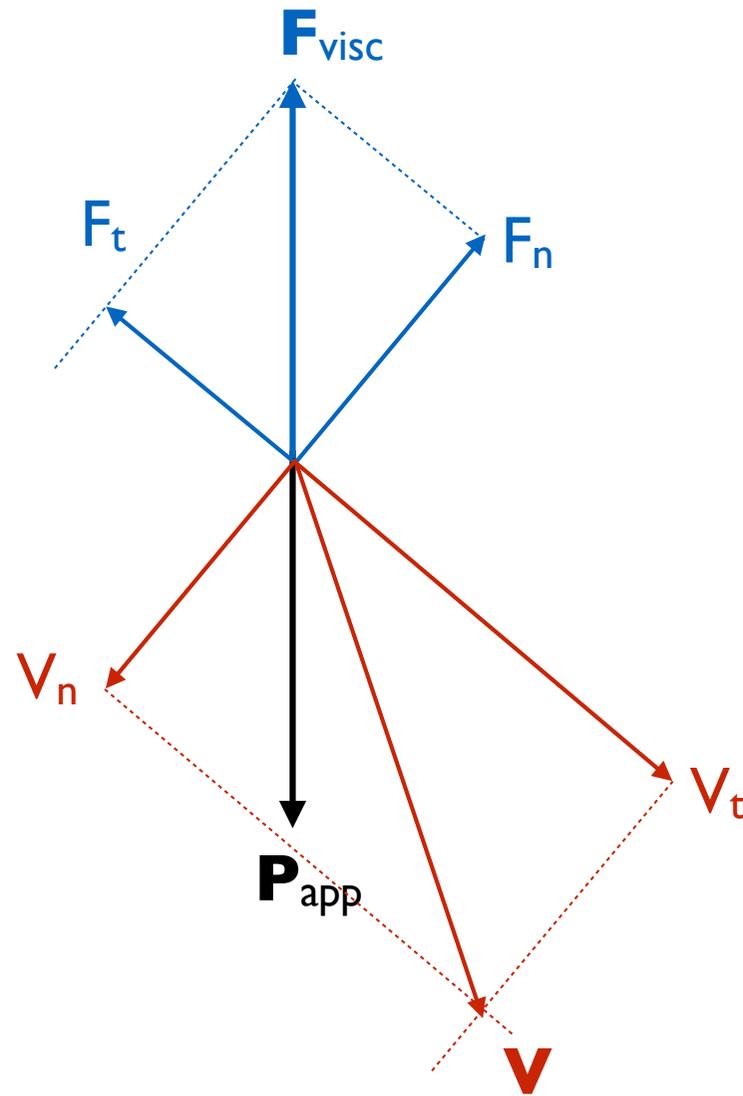
$$T_{\parallel} = -c_{\parallel} \eta L U_{\parallel}$$

$$\simeq 2\pi + \text{correction}(a/L)$$

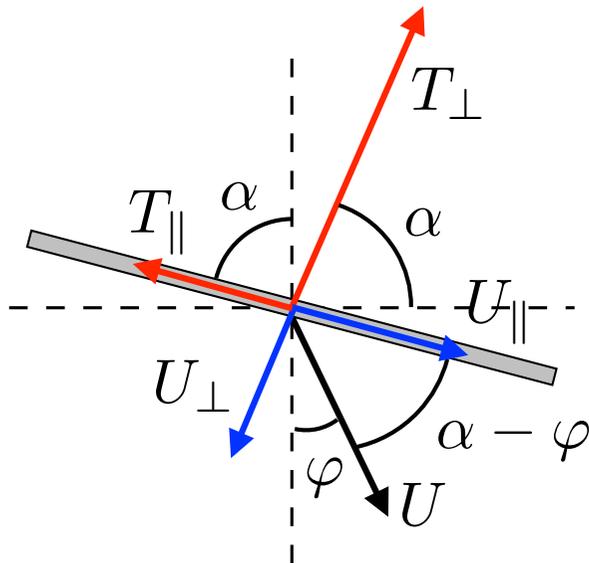
$$T_{\perp} = -c_{\perp} \eta L U_{\perp}$$

$$c_{\perp} \simeq 2c_{\parallel}$$

Forces sur un bâtonnet incliné par rapport au mouvement



Chute d'une tige inclinée (calcul en bonus)



équilibre forces horizontales

$$T_{\parallel} \sin \alpha = T_{\perp} \cos \alpha$$

$$\tan \alpha = \frac{T_{\perp}}{T_{\parallel}} = \frac{c_{\perp} \eta U_{\perp} L}{c_{\parallel} \eta U_{\parallel} L} = 2 \frac{U_{\perp}}{U_{\parallel}}$$

$$\frac{U_{\perp}}{U_{\parallel}} = \tan(\alpha - \varphi)$$

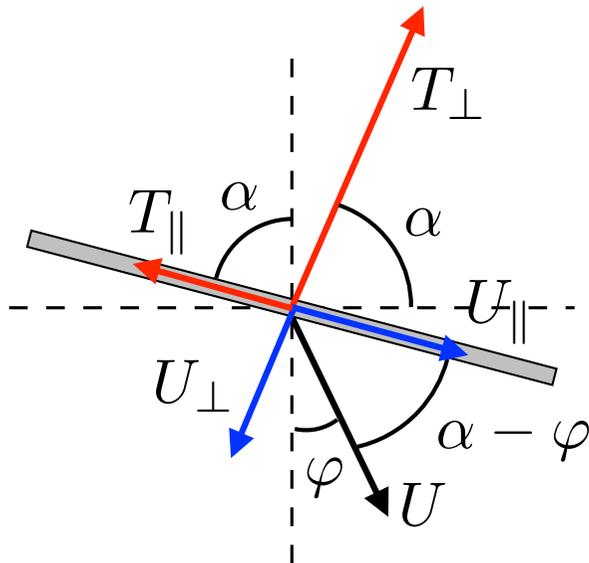
$$\tan(\alpha) = 2 \tan(\alpha - \varphi) = 2 \frac{\tan(\alpha) - \tan(\varphi)}{1 + \tan(\alpha) \tan(\varphi)}$$

$$\tan(\varphi) = \frac{\tan \alpha}{2 + \tan^2 \alpha}$$

$$\varphi_{max} = 19.5^{\circ} \text{ pour } \alpha_m = 54.7^{\circ}$$

Chute d'une tige inclinée (calcul en bonus)

Vitesse de chute ?



équilibre forces verticales

$$T_{\parallel} \cos \alpha + T_{\perp} \sin \alpha = \pi a^2 \Delta \rho L g$$

$$T_{\parallel} \cos \alpha \left(1 + \frac{T_{\perp}}{T_{\parallel}} \tan \alpha \right) = \pi a^2 \Delta \rho L g$$
$$\tan \alpha$$

$$T_{\parallel} \frac{1}{\cos \alpha} = \pi a^2 \Delta \rho L g$$

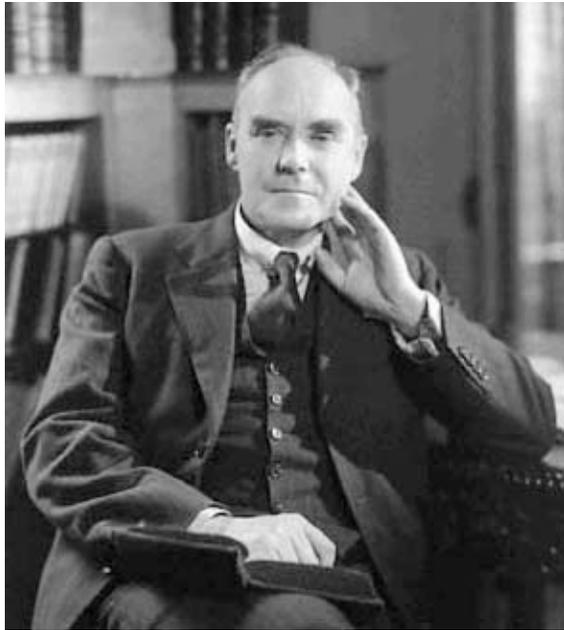
$$c_{\parallel} \eta L U_{\parallel} = \pi a^2 \Delta \rho L g \cos \alpha$$

$$U_{\parallel} = \frac{\pi a^2 \Delta \rho g \cos \alpha}{c_{\parallel} \eta}$$

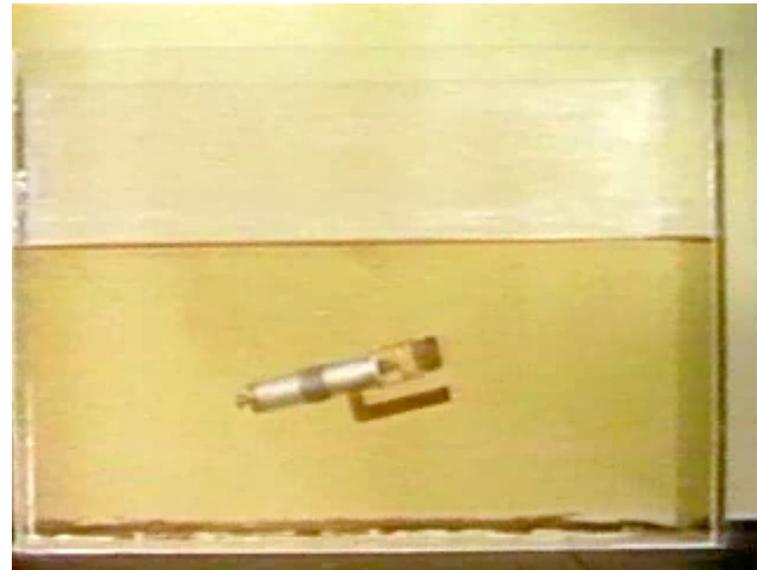
$$U_{\perp} = \tan(\alpha - \varphi) U_{\parallel} = \frac{\tan \alpha}{2} U_{\parallel}$$

$$U = \left(1 + \frac{\tan^2 \alpha}{4} \right)^{1/2} \frac{\pi a^2 \Delta \rho g \cos \alpha}{c_{\parallel} \eta}$$

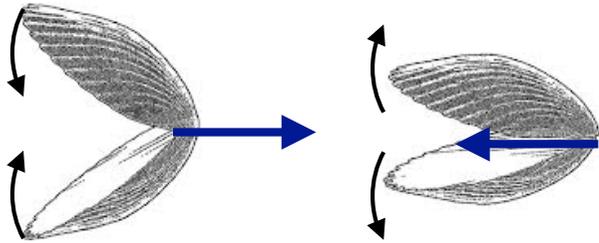
Réversibilité cinématique et propulsion



G. I. Taylor



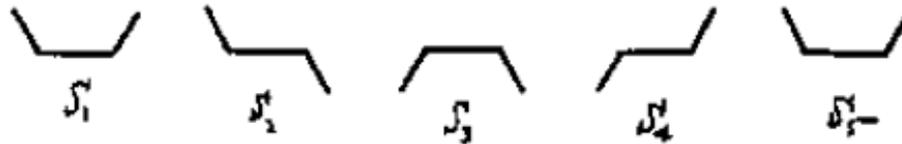
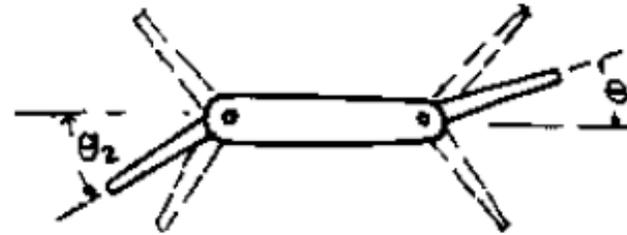
Le théorème de la coquille St Jacques



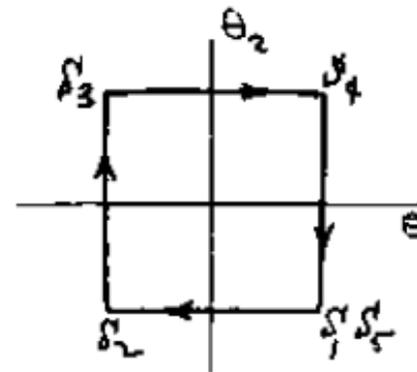
“Scallop theorem”

Qui et al., *Nature Communications* (2014)

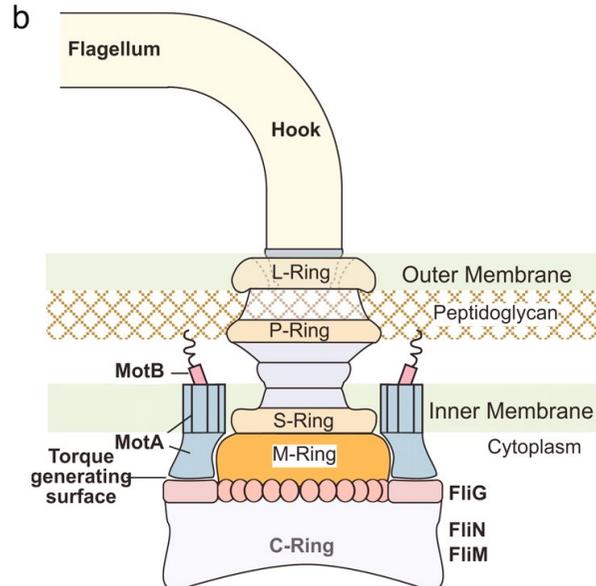
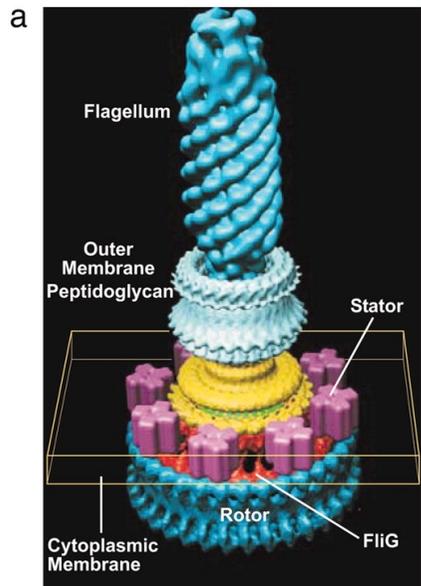
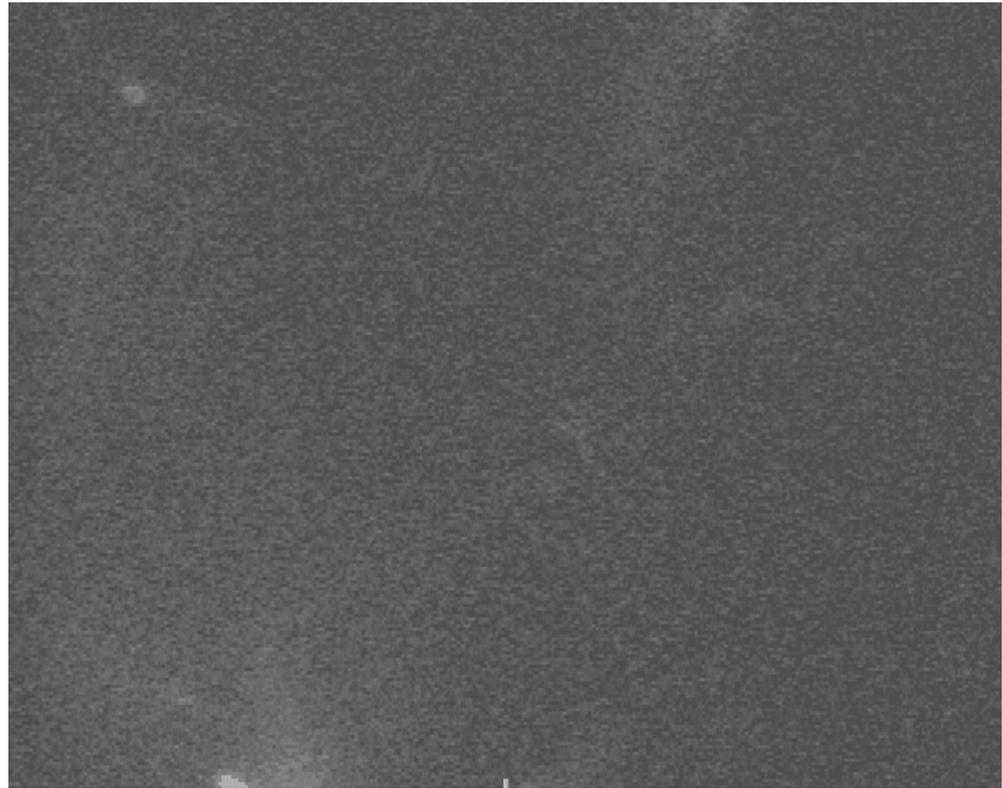
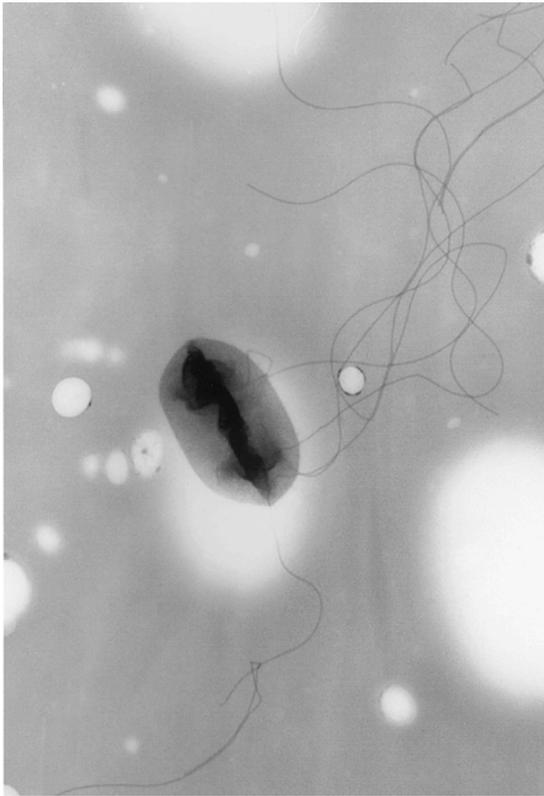
1 seul degré de liberté :
réversibilité imposée



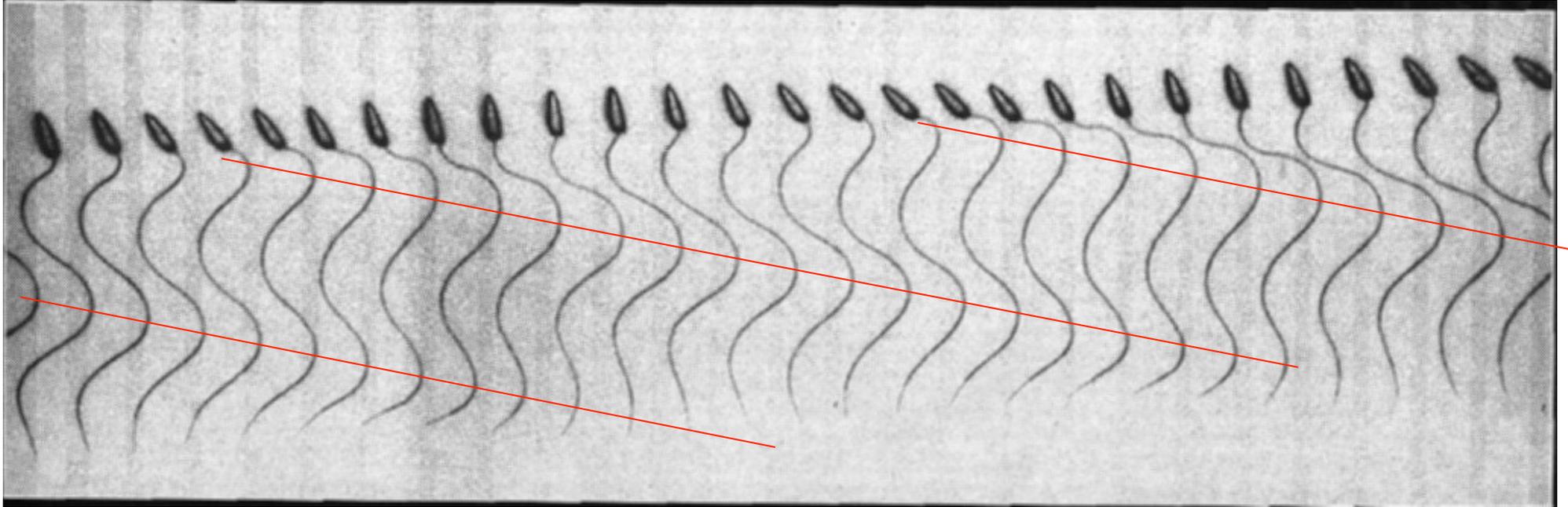
2 degrés de liberté :
possibilité de briser la
symétrie +t/-t



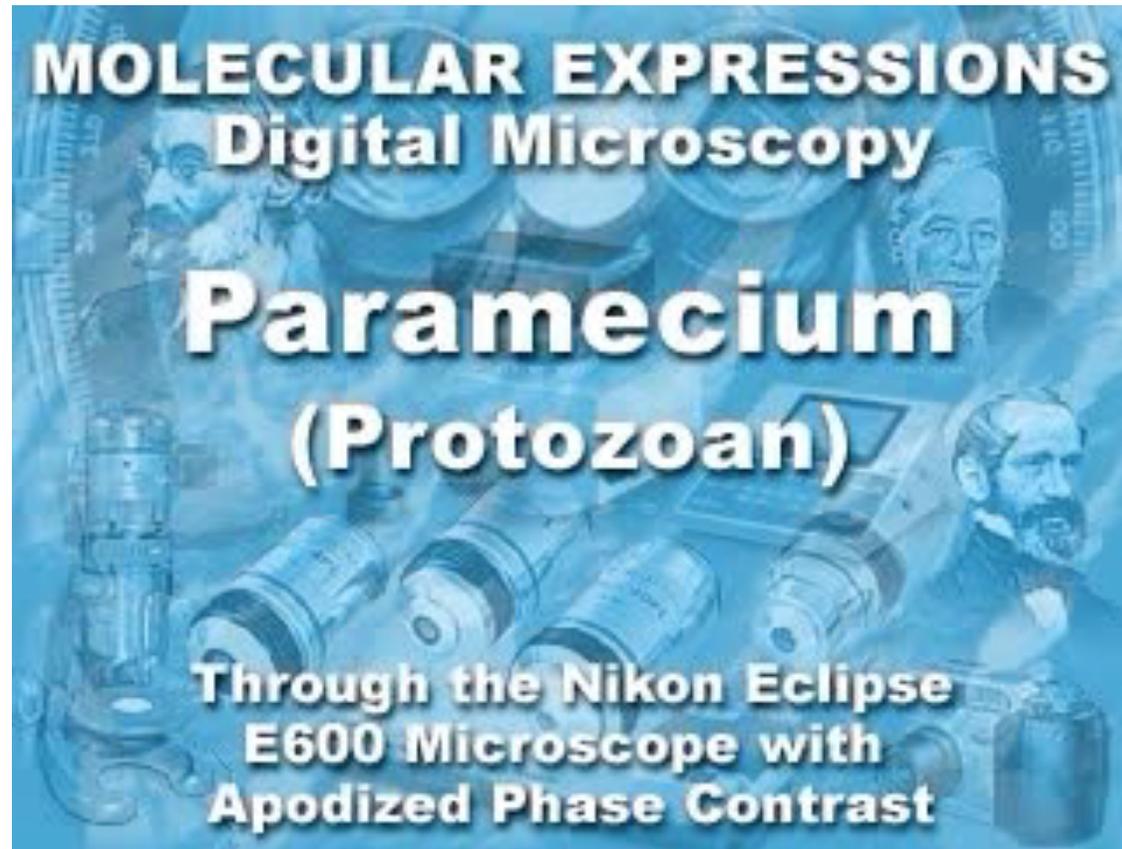
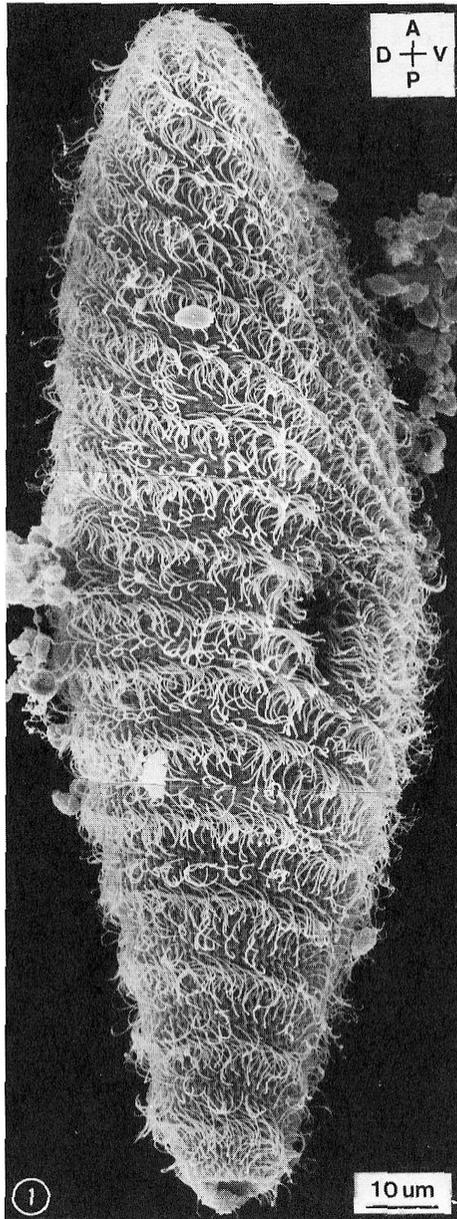
E. Purcell, life at low
Reynolds number



Escherischia Coli



Mouvement collectif des cils



A l'autre bout du spectre en nombre de Reynolds

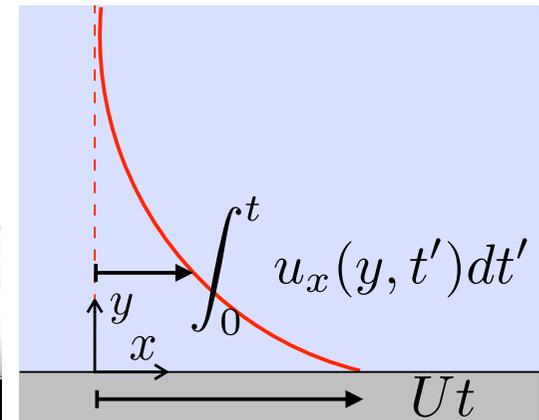
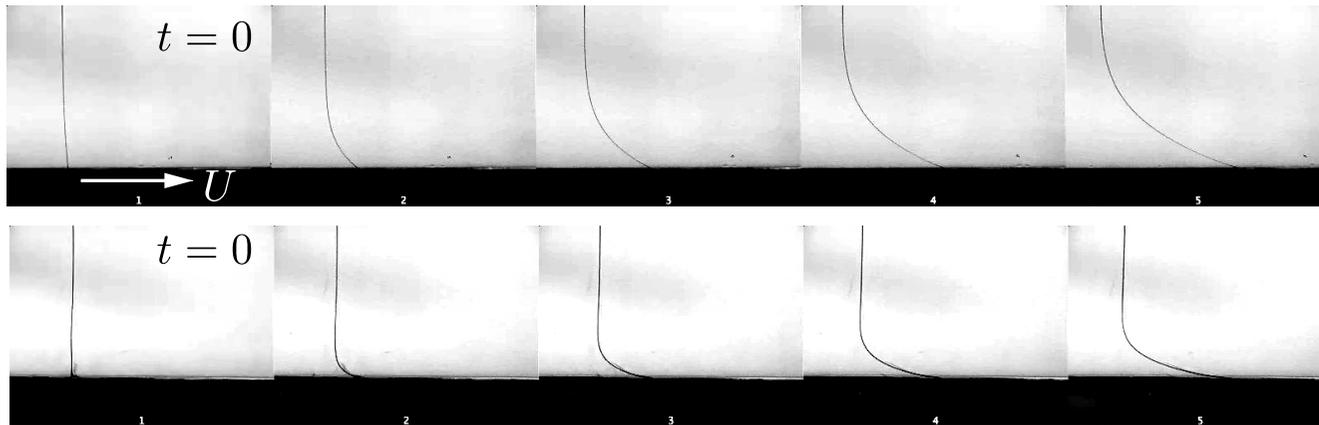
La notion de couche limite

Écoulements rotationnels et irrotationnels

Écoulements potentiels et loi de Bernoulli



Développement d'une couche limite sur une plaque plane mise en mouvement



Développement d'une couche limite sur une plaque plane mise en mouvement

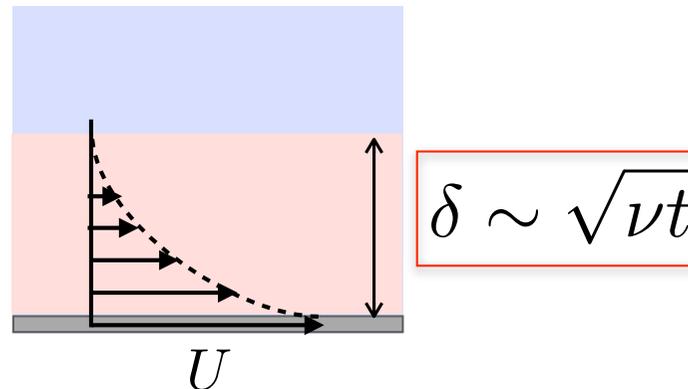
Navier Stokes

$$\rho \frac{\partial u_x}{\partial t} = \eta \frac{\partial^2 u_x}{\partial y^2}$$

$$\rho \frac{U}{t} \sim \eta \frac{U}{\delta^2}$$

Épaisseur de couche de diffusion de la quantité de mouvement

$$\delta \sim \sqrt{\frac{\eta}{\rho} t}$$



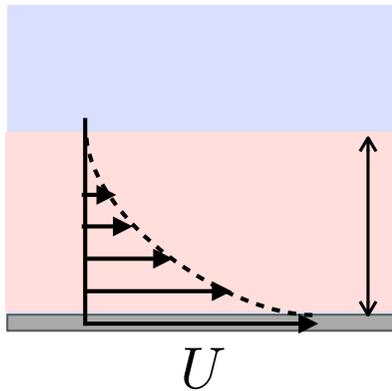
La même expérience du point de vue de la vorticit 

$$\boldsymbol{\omega} = \nabla \wedge \mathbf{u}$$

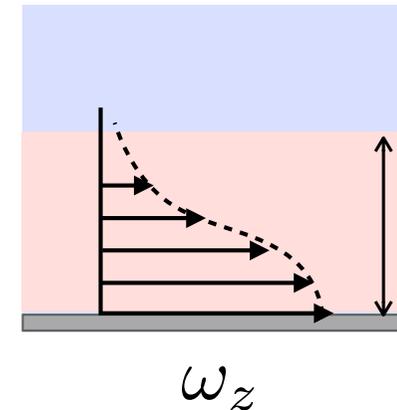
$$\omega_z = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}$$

Navier Stokes (ici)

$$\frac{\partial \omega_z}{\partial t} = \nu \frac{\partial^2 \omega_z}{\partial y^2}$$



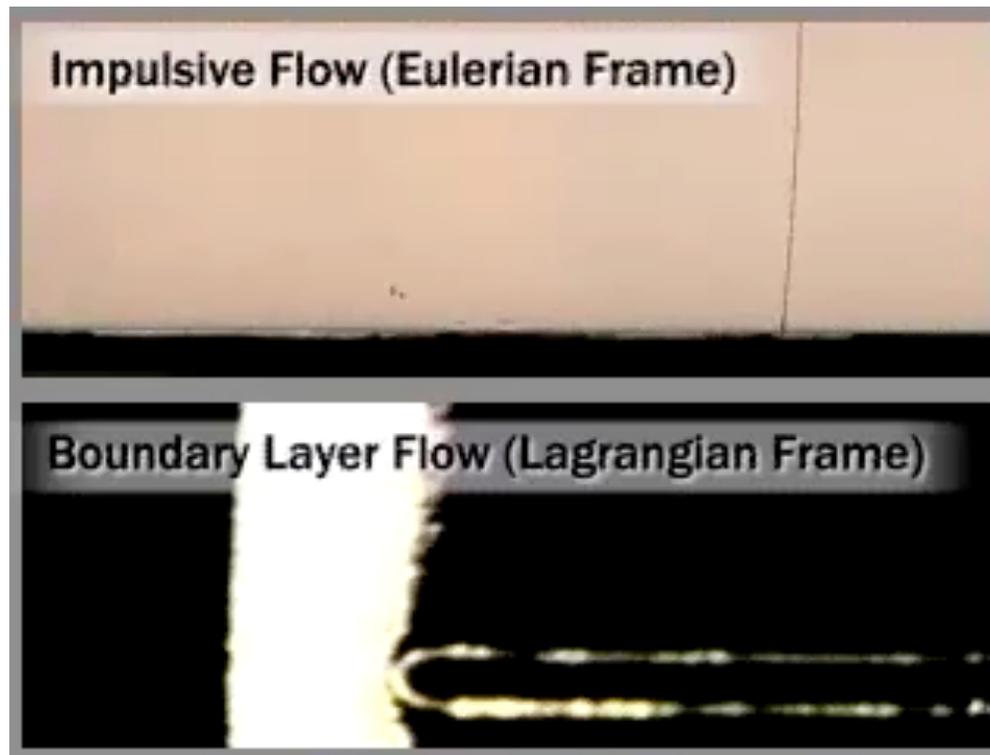
$$\delta \sim \sqrt{\nu t}$$



irrotationnel

$$\omega_z \sim 0$$

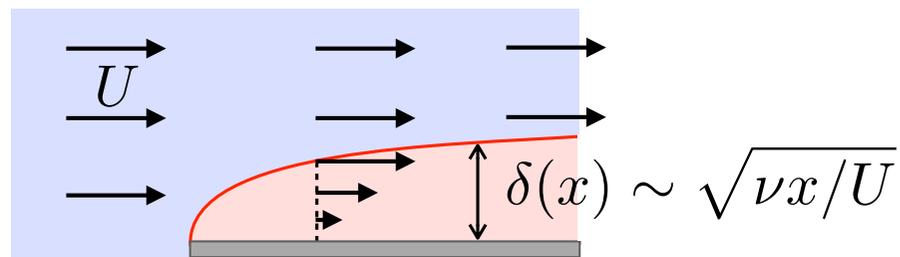
rotationnel

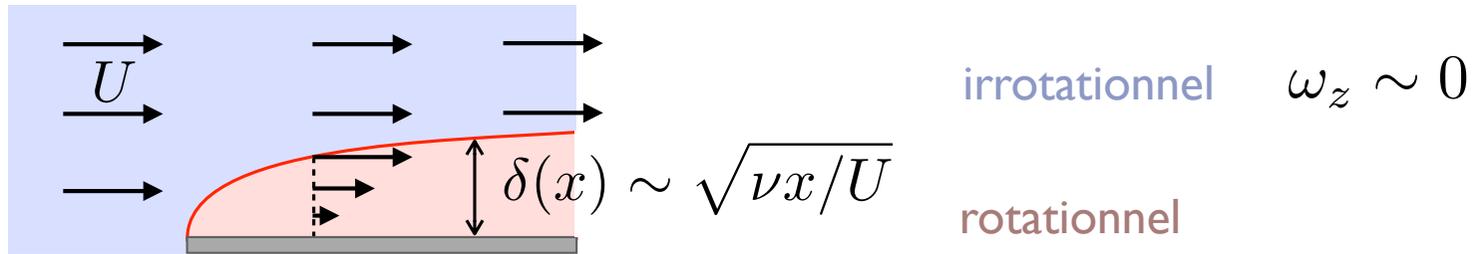


Développement d'une couche limite sur une plaque plane

$$t_d \sim \frac{\delta^2}{\nu}$$

$$t_c \sim \frac{x}{U}$$





$$\delta(L) \sim \sqrt{\frac{\nu L}{U}} = L \sqrt{\frac{\nu}{UL}} = L Re_L^{-1/2}$$

$$Re_L \gg 1 \rightarrow \frac{\delta(L)}{L} \ll 1$$

En dehors de la couche limite :
la viscosité ne joue pas de rôle, l'écoulement est irrotationnel

Dans la couche limite :
la viscosité joue un rôle, l'écoulement est rotationnel