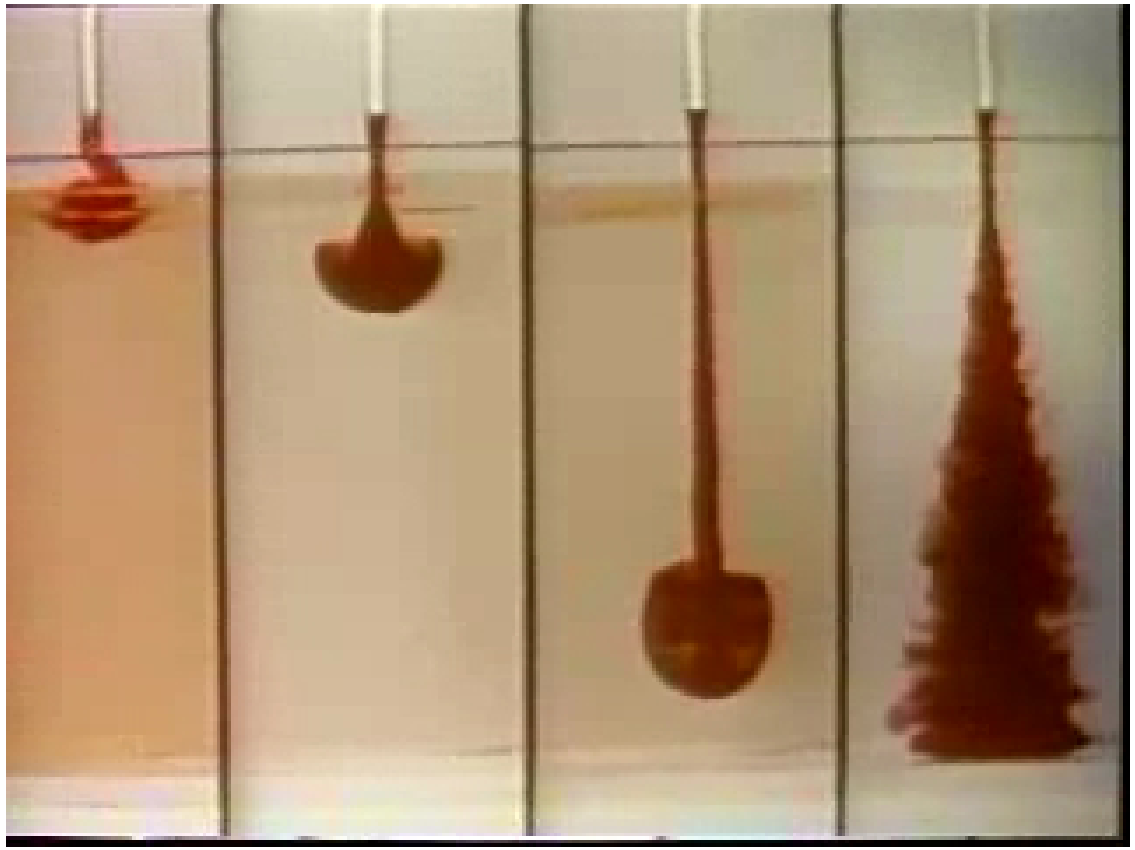
Inertie contre Viscosité

sirop glycérine eau + glycérine eau

Un monde sans inertie



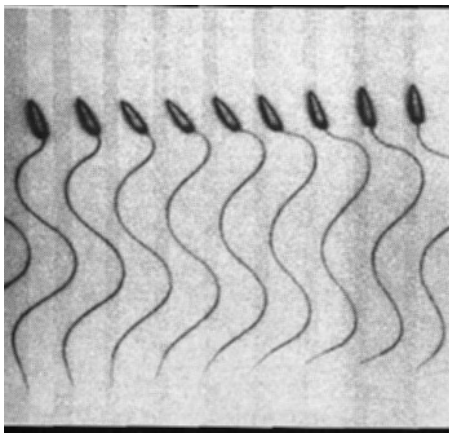
Les écoulements à petit nombre de Reynolds

Pas d'inertie — Navier-Stokes → Équation de Stokes

$$\eta \Delta \mathbf{u} = \nabla p$$

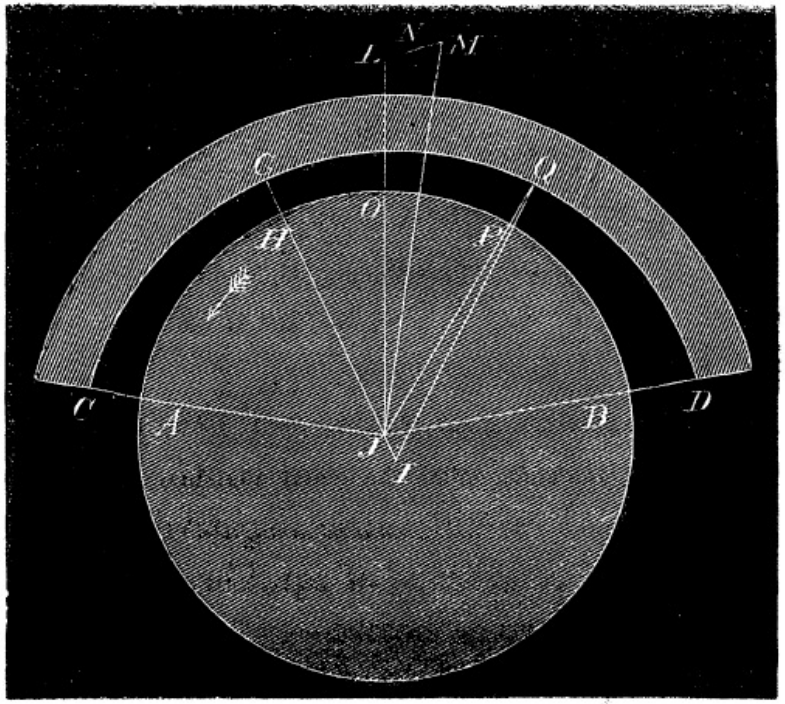
Les écoulements en couche mince et l'approximation de lubrification

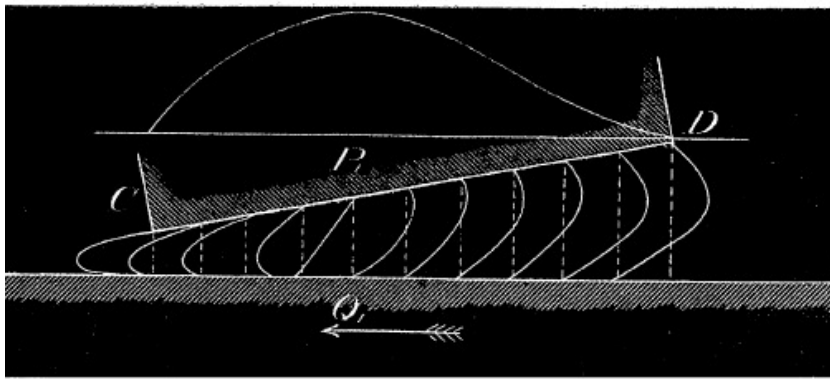
Les forces sur les objets solides, la réversibilité cinématique et la propulsion à petit Reynolds

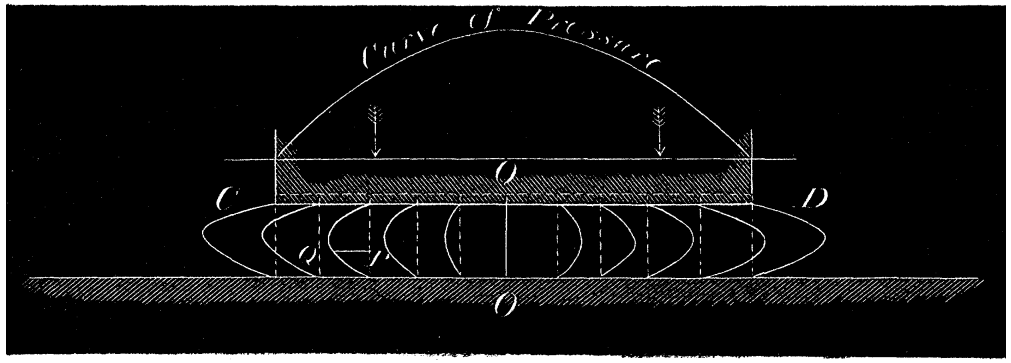


Les écoulements visqueux en couches minces

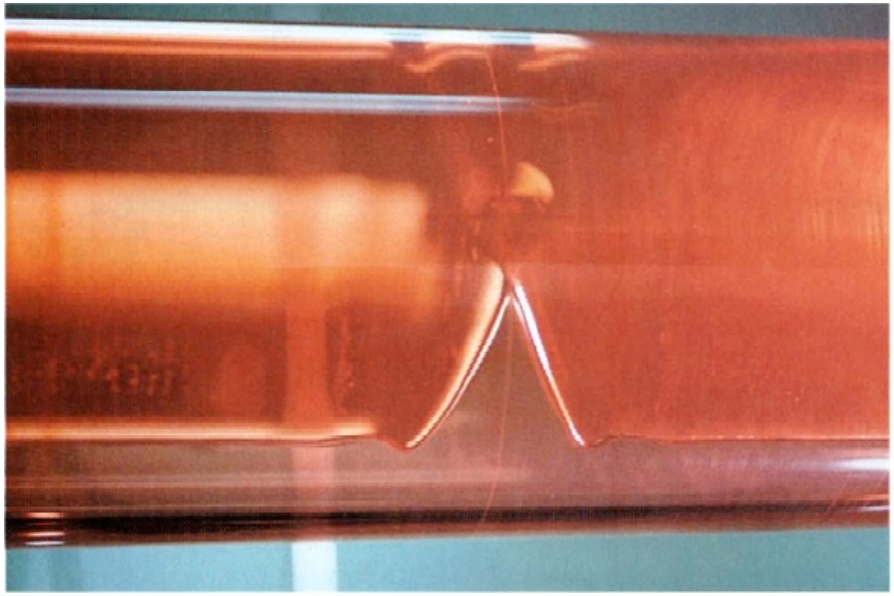
Lubrification (Reynolds 1882)

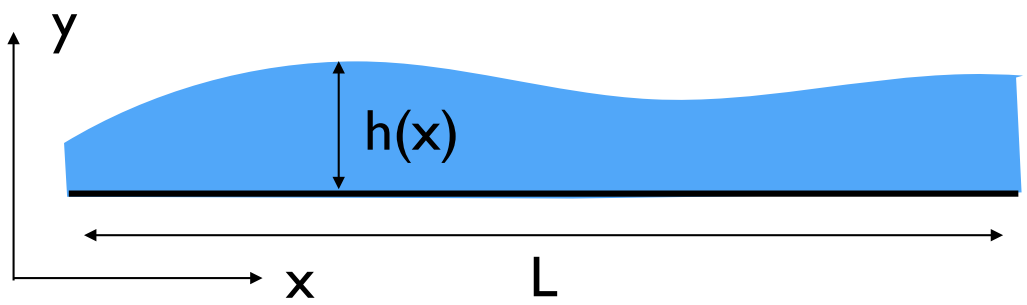






Écoulements en couche mince à surface libre

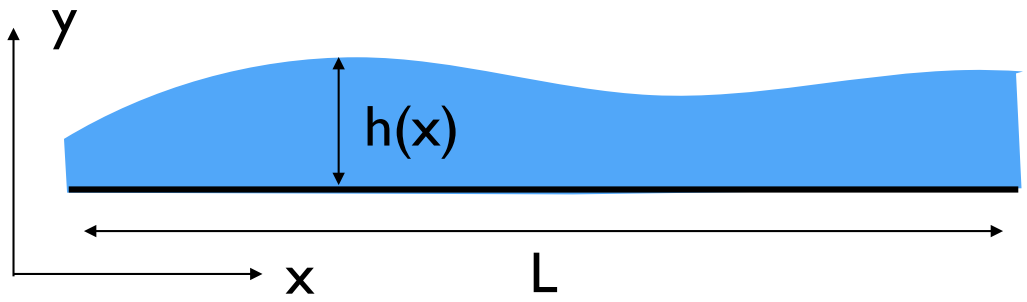




$$h \ll L$$

$$\nabla.\mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \sim \frac{u_x}{L} + \frac{u_y}{h}$$

$$\frac{u_x}{u_y} \sim \frac{L}{h} \gg 1$$



$$\frac{u_x}{u_y} \sim \frac{L}{h} \gg 1$$

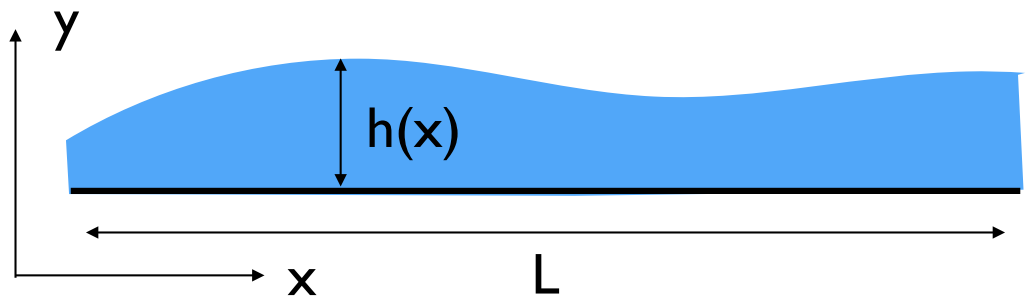
$$0 = -\frac{\partial p}{\partial x} + \eta \left(\cancel{\frac{\partial^2 u_x}{\partial x^2}} + \frac{\partial^2 u_x}{\partial y^2} \right)$$

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u_x}{\partial y^2}$$

$$\frac{U}{L^2} \qquad \frac{U}{h^2}$$

$$0 = -\frac{\partial p}{\partial y} + \eta \left(\cancel{\frac{\partial^2 u_y}{\partial x^2}} + \cancel{\frac{\partial^2 u_y}{\partial y^2}} \right)$$

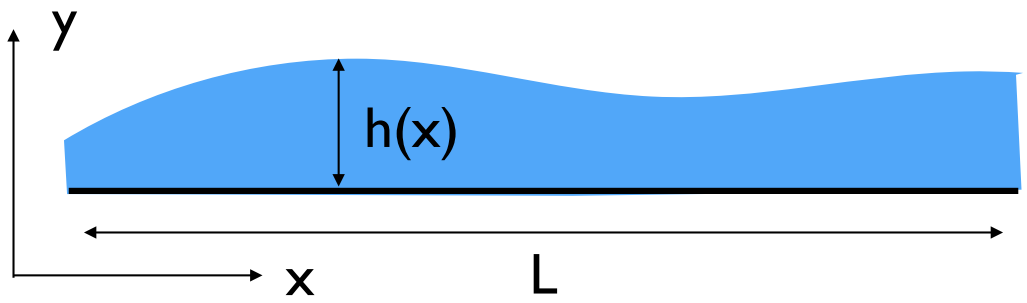
$$\frac{\partial p}{\partial y} \sim 0$$



$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u_x}{\partial y^2} \qquad \frac{p}{L} \sim \eta \frac{U}{h^2} \qquad F_y \sim pL \sim \eta U \frac{L^2}{h^2}$$

$$\sigma_{xy} = \eta \frac{\partial u_x}{\partial y} \sim \eta \frac{U}{h} \qquad F_x \sim \sigma_{xy} L \sim \eta U \frac{L}{h}$$

$$\boxed{\frac{F_y}{F_x} \sim \frac{L}{h} \gg 1}$$



$$\frac{dh}{dx} \ll 1$$

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u_x}{\partial y^2}$$

Intégration suivant y à h fixé
avec conditions aux limites en y =0 et y=h

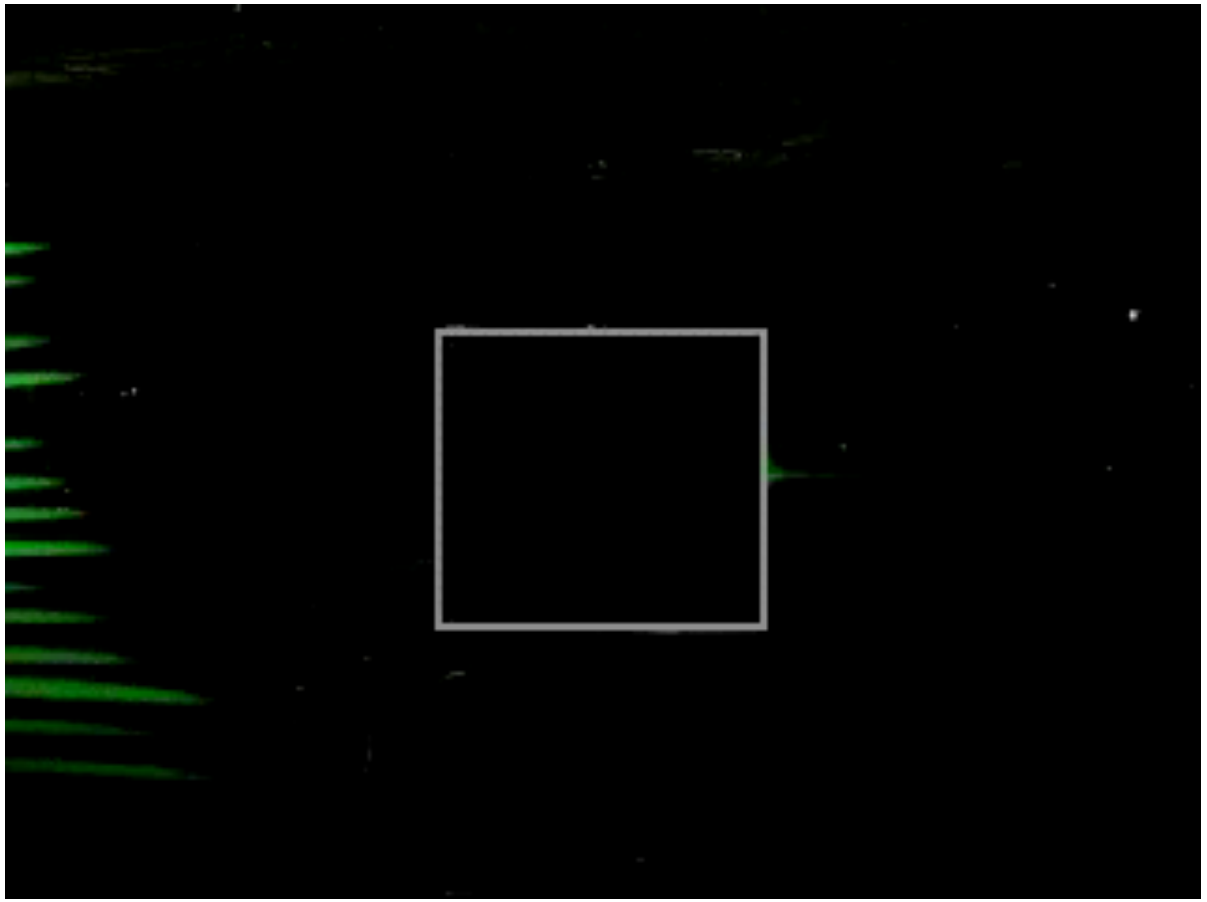
↓

Distribution de pression suivant x

↓

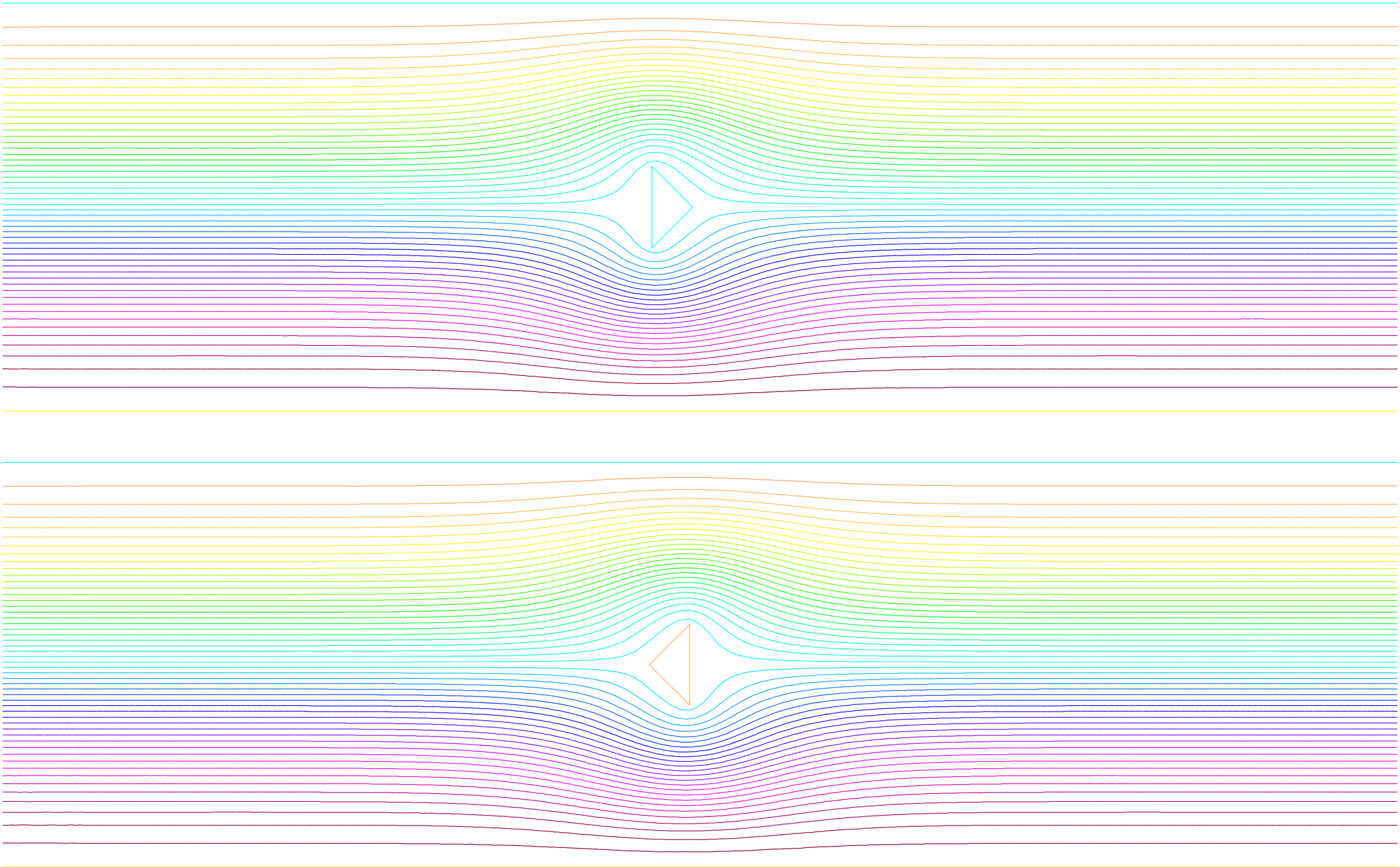
Intégration suivant x → Forces globales

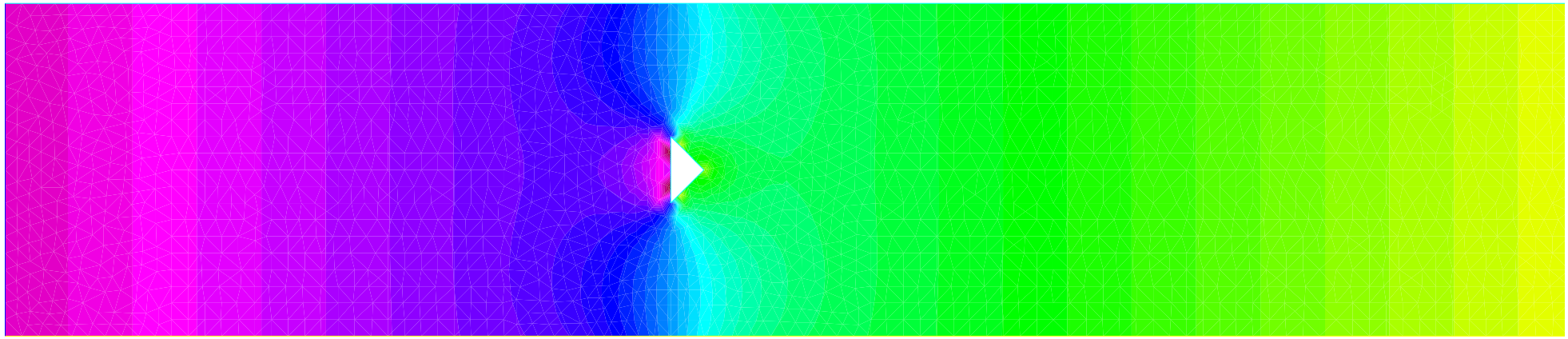
Propriétés de symétrie des écoulements de Stokes

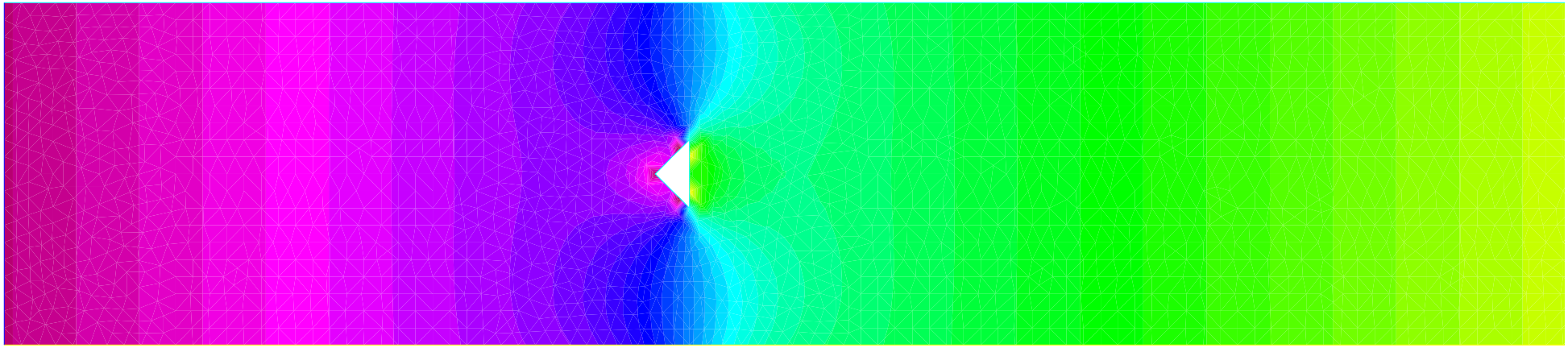


$$\eta \Delta \mathbf{u} = \nabla p$$

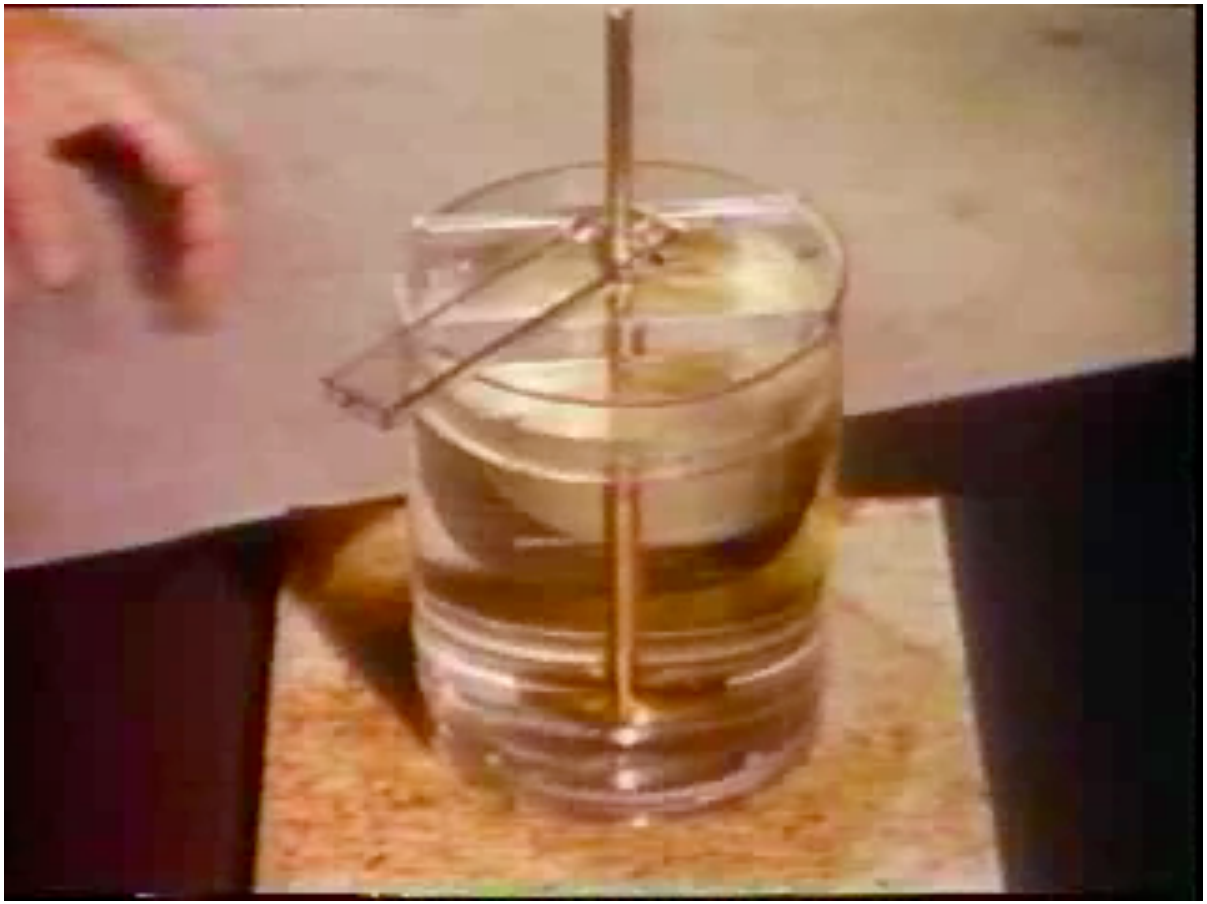
$$\nabla p \to -\nabla p \qquad\qquad \mathbf{u} \to -\mathbf{u}$$





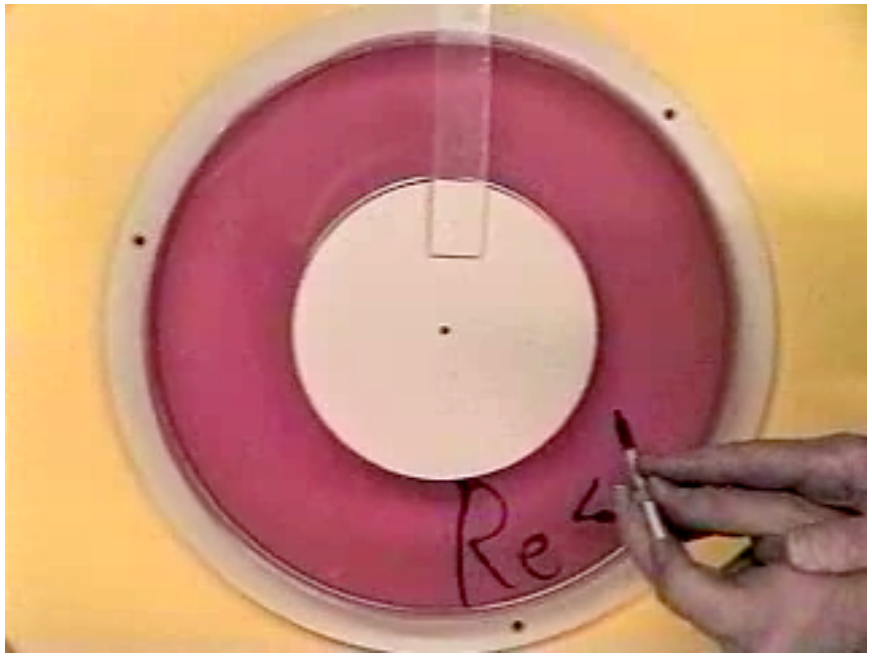


Réversibilité cinématique



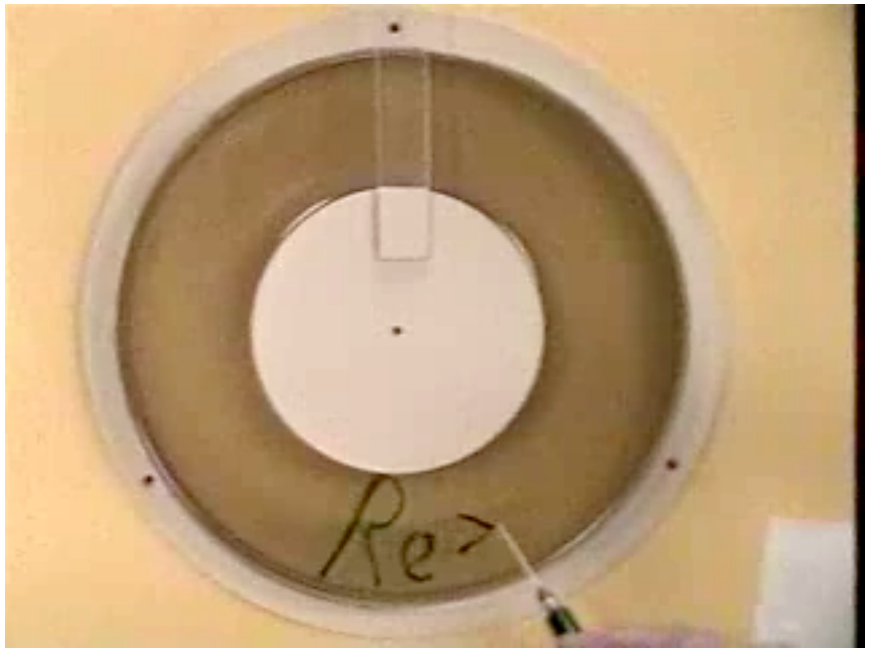
Issu de notre « MOOC » favori par G.I. Taylor:

https://www.youtube.com/watch?v=51-6QCJTAjU



$Re \ll 1$

Réversibilité cinématique



$Re \gg 1$

Irréversibilité cinématique