Écoulements irrotationnels (potentiels) en dehors des couches limites

$$\boxed{\nabla \wedge \mathbf{u} = 0 \Leftrightarrow \mathbf{u} = \nabla \phi}$$

Le champ de vitesse dérive d'un potentiel

Loi de Bernoulli :

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \cancel{\eta \Delta \mathbf{u}} + \rho \mathbf{f}$$

$$\nabla \left(\rho \frac{\partial \phi}{\partial t} \right)$$

Loi de Bernoulli :

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \cancel{\eta\Delta\mathbf{u}} + \rho\mathbf{f}$$

$$\nabla\left(\rho\frac{\partial \phi}{\partial t}\right)$$

$$\nabla\left(\frac{1}{2}\rho\mathbf{u}^2\right) - \rho\mathbf{u}\wedge\left(\cancel{\nabla\wedge\mathbf{u}}\right)$$

Loi de Bernoulli :

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \cancel{\eta\Delta\mathbf{u}} + \rho\mathbf{f}$$

$$\nabla\left(\rho\frac{\partial\phi}{\partial t}\right) \qquad \nabla\left(\frac{1}{2}\rho\mathbf{u}^2\right) - \rho\mathbf{u}\wedge\cancel{(\nabla\wedge\mathbf{u})} \qquad \nabla\left(-\rho g z\right)$$

Loi de Bernoulli :

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \cancel{\eta\Delta\mathbf{u}} + \rho\mathbf{f}$$

$$\nabla\left(\rho\frac{\partial\phi}{\partial t}\right) \qquad \nabla\left(\frac{1}{2}\rho\mathbf{u}^2\right) - \rho\mathbf{u}\wedge(\cancel{\nabla\wedge\mathbf{u}}) \qquad \nabla\left(-\rho g z\right)$$

$$\nabla\left(\rho\frac{\partial\phi}{\partial t} + \frac{1}{2}\rho u^2 + p + \rho g z\right) = 0$$

Loi de Bernoulli :

$$\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \cancel{\eta\Delta\mathbf{u}} + \rho\mathbf{f}$$

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$$\nabla\left(\rho\frac{\partial \phi}{\partial t} + \frac{1}{2}\rho u^2 + p + \rho g z\right) = 0$$

$$\boxed{\rho\frac{\partial \phi}{\partial t} + \frac{1}{2}\rho u^2 + p + \rho g z = cte(t)}$$

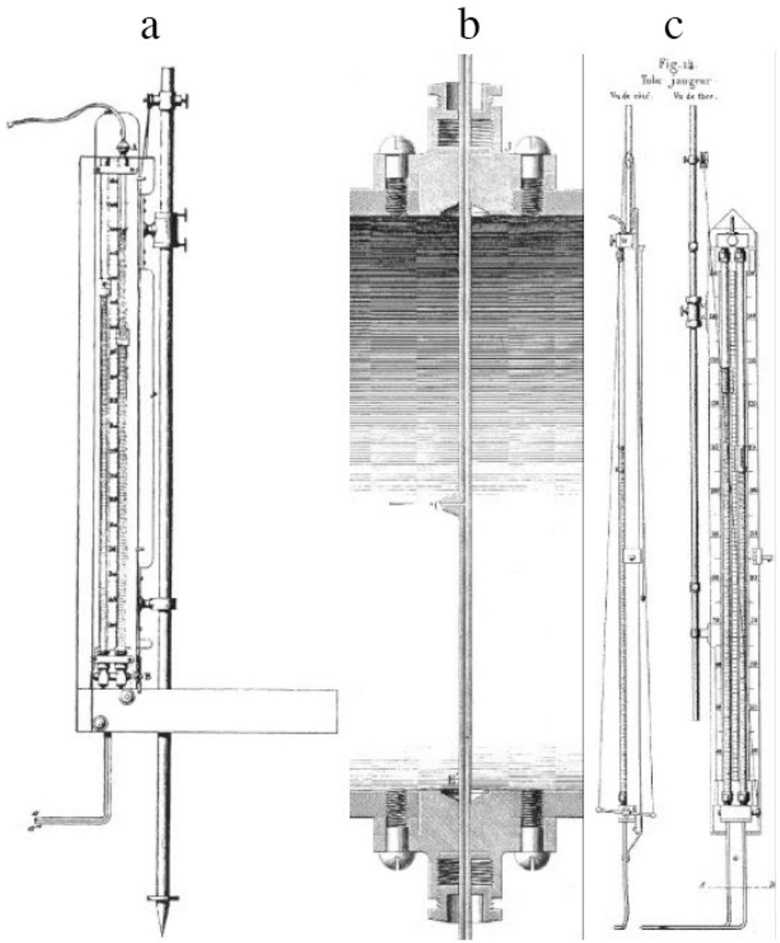
Loi de Bernoulli instationnaire

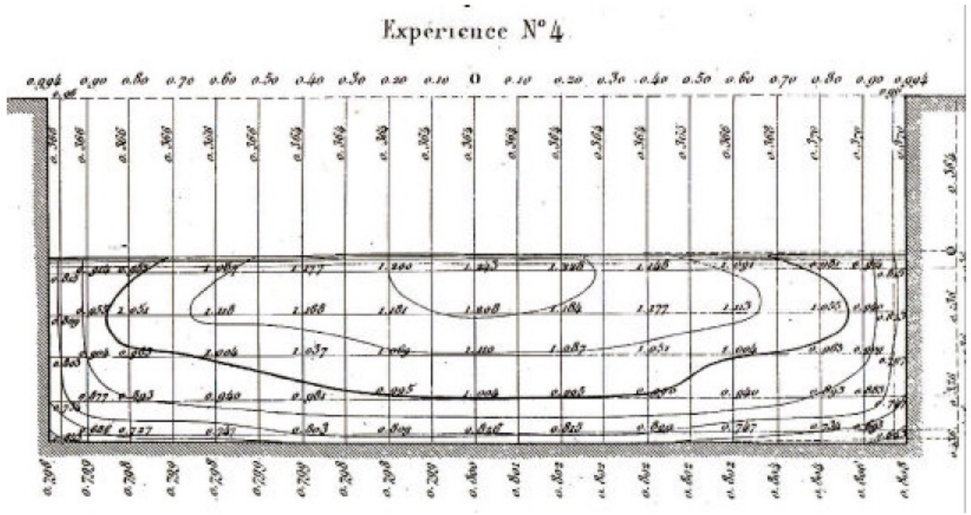
Loi de Bernoulli :

$$\boxed{p + \frac{1}{2}\rho u^2 + \rho g z = C}$$

Loi de Bernoulli en écoulement stationnaire

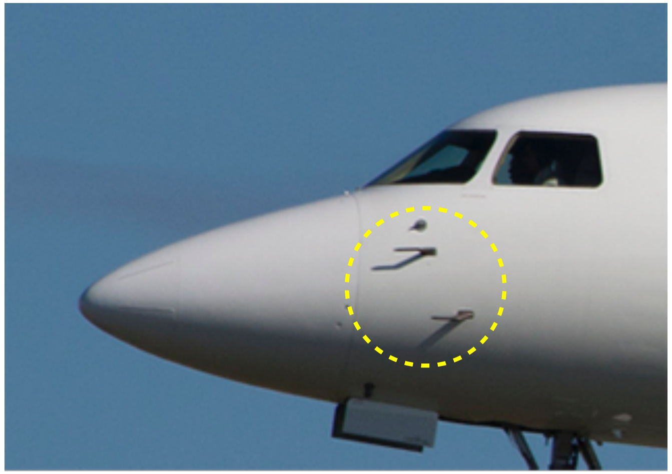
Tube de Pitot (1732) -Darcy(~1855)

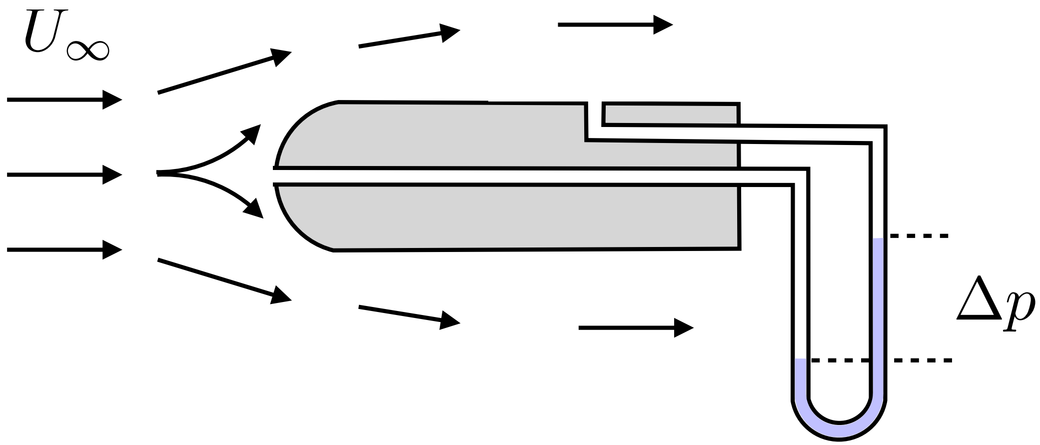




Tube de Pitot

Comment ça marche ?





Tube de Pitot

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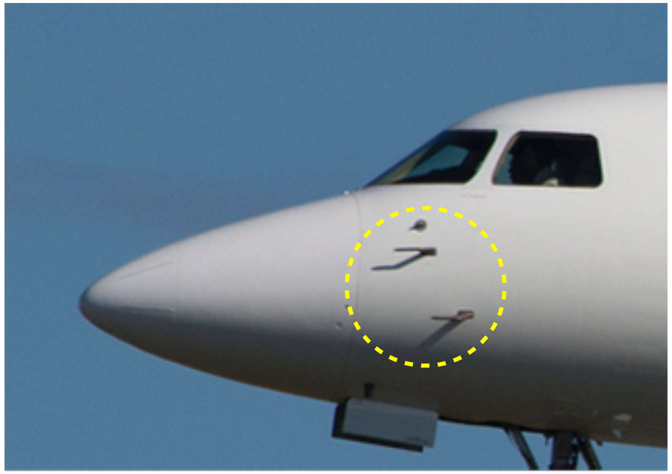
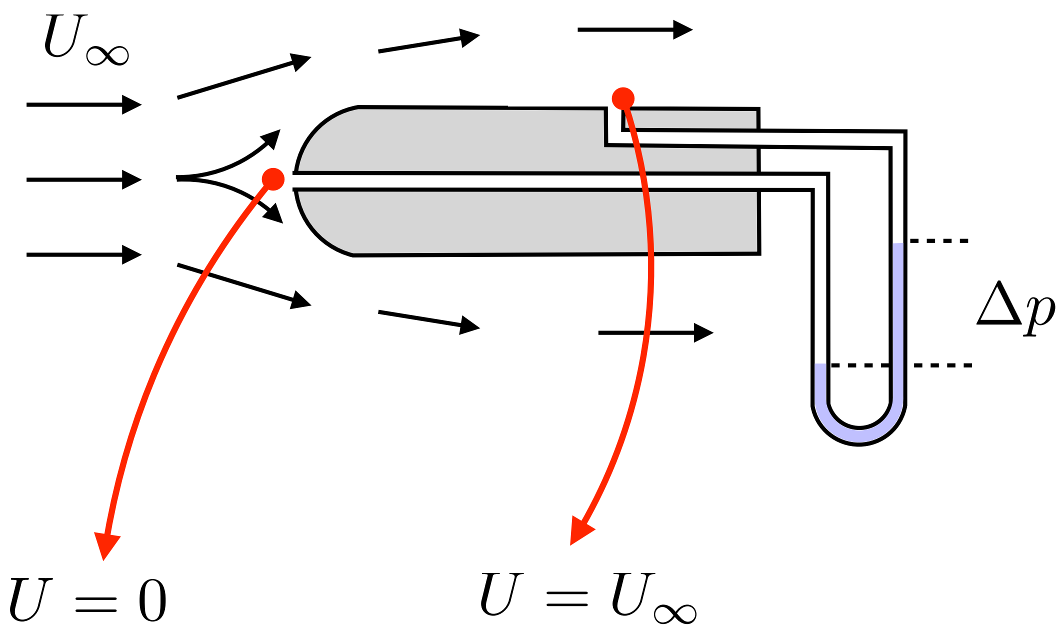
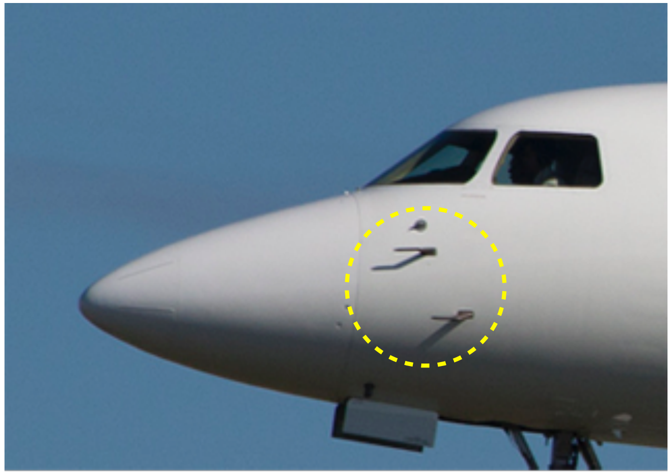


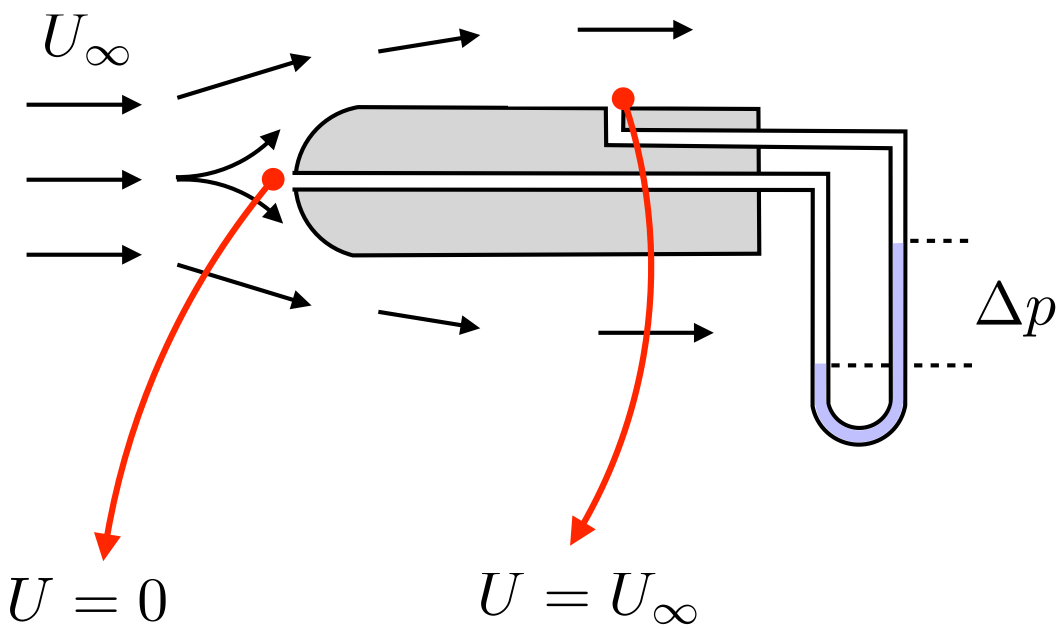


Explication classique

Tube de Pitot

Comment ça marche ?



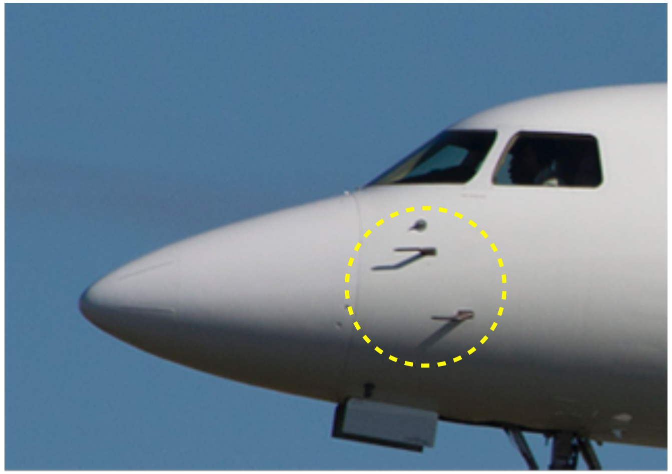


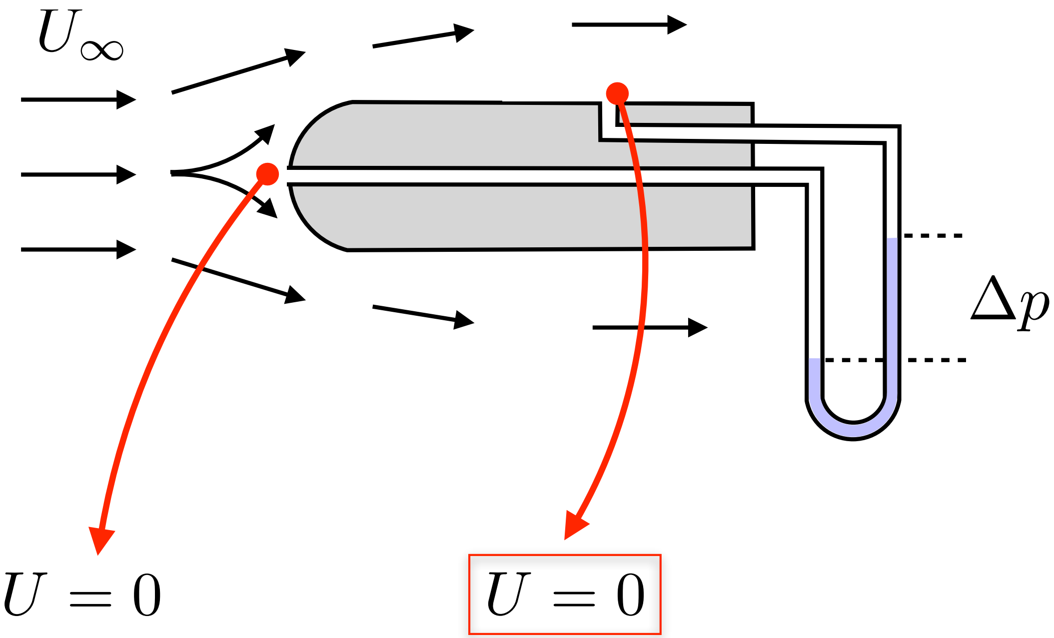
Explication classique

$$\boxed{\Delta p = \frac{1}{2}\rho U_\infty^2}$$

Tube de Pitot

Comment ça marche ?

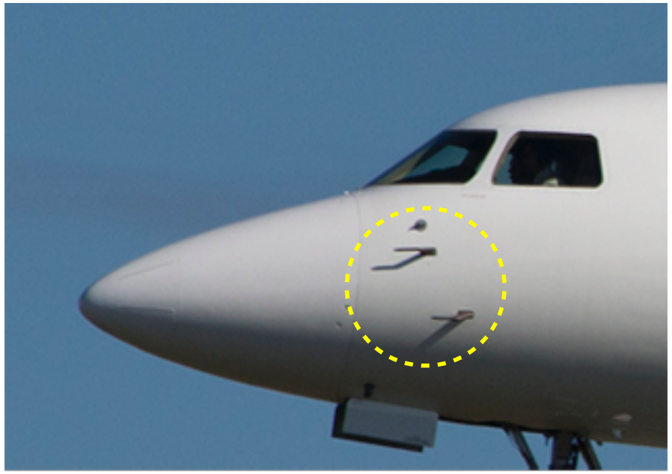


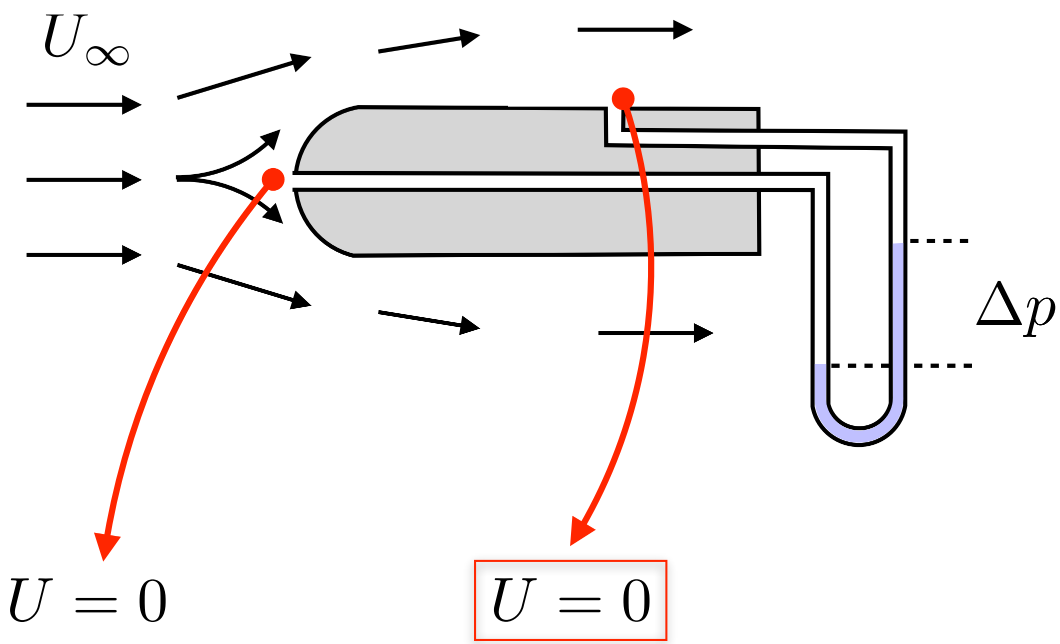


mais fluide réel: non glissement au niveau des parois

Tube de Pitot

Comment ça marche ?



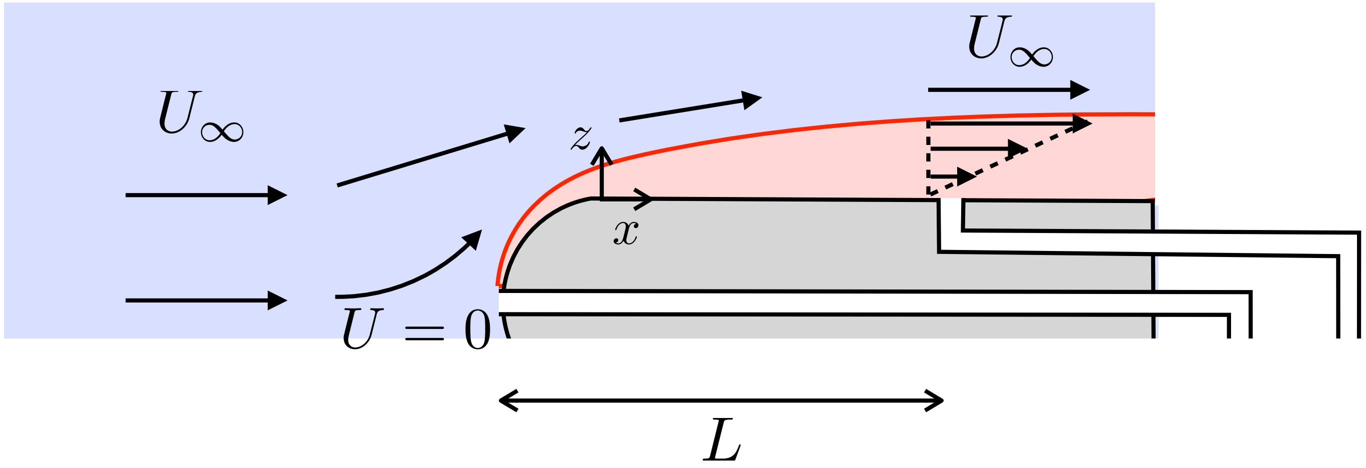


mais fluide réel: non glissement au niveau des parois

pourquoi la formule classique fonctionne-t-elle?

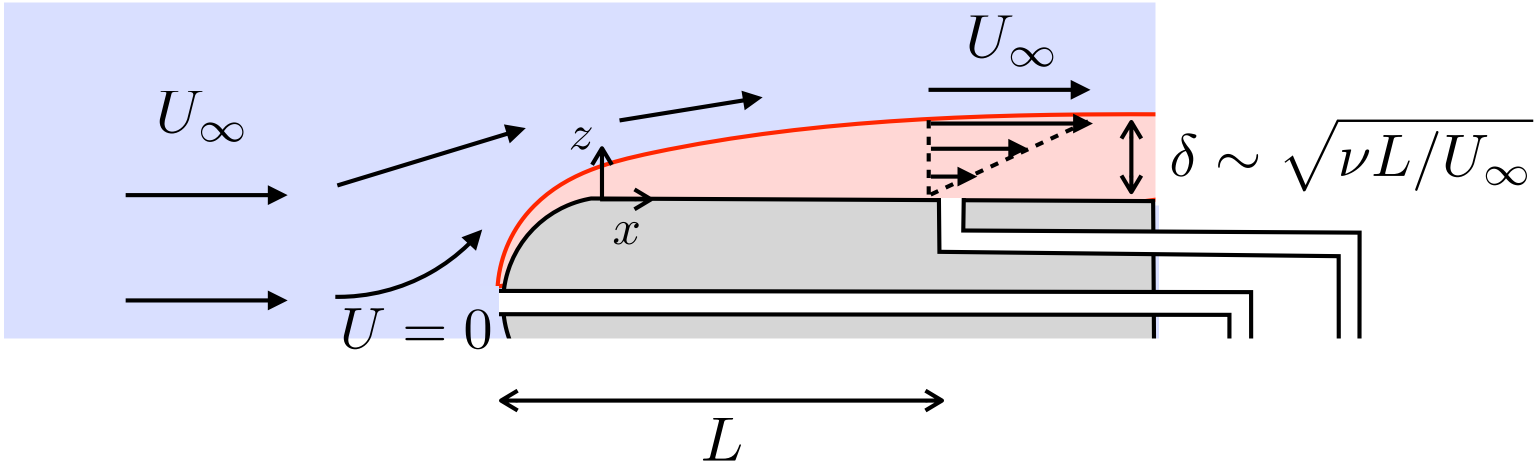
Tube de Pitot

Sauvé par les couches limites !



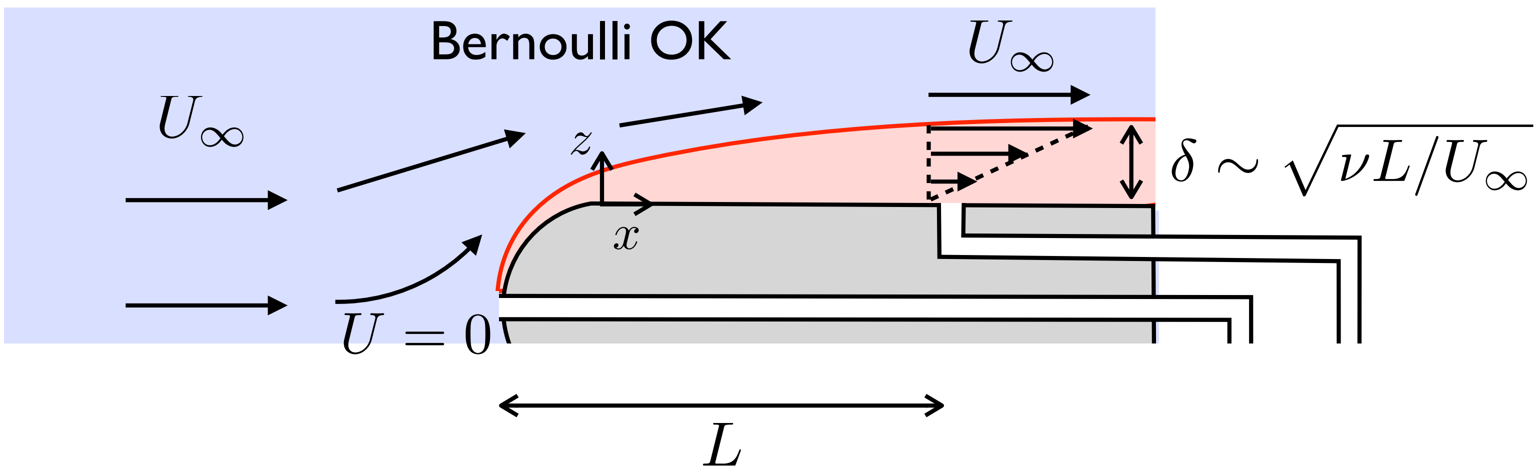
Tube de Pitot

Sauvé par les couches limites !



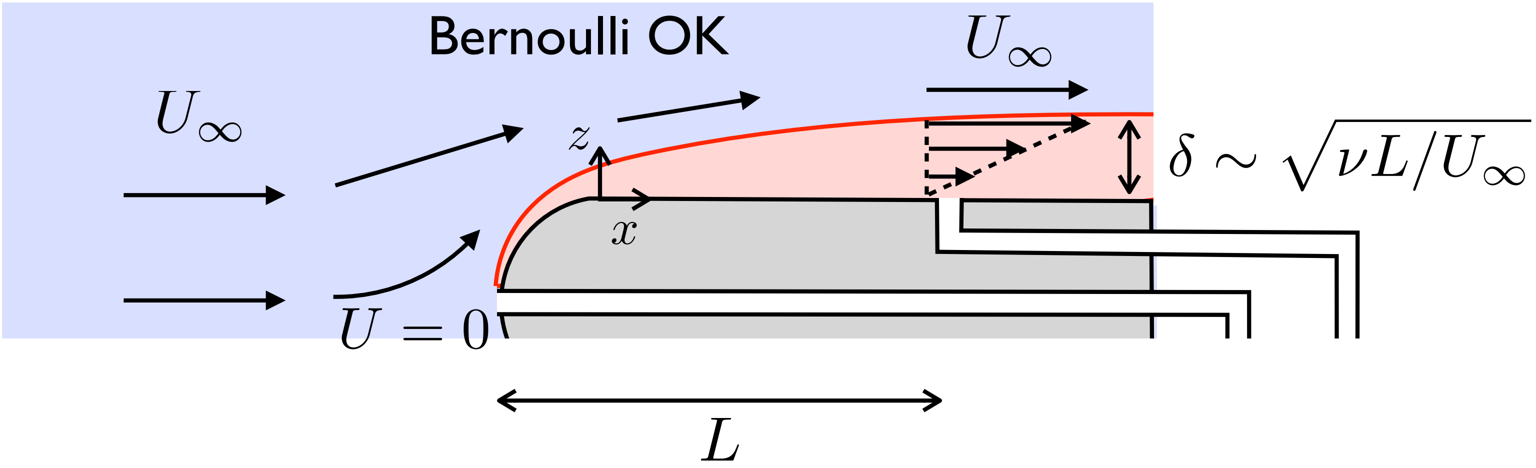
Tube de Pitot

Sauvé par les couches limites !



Tube de Pitot

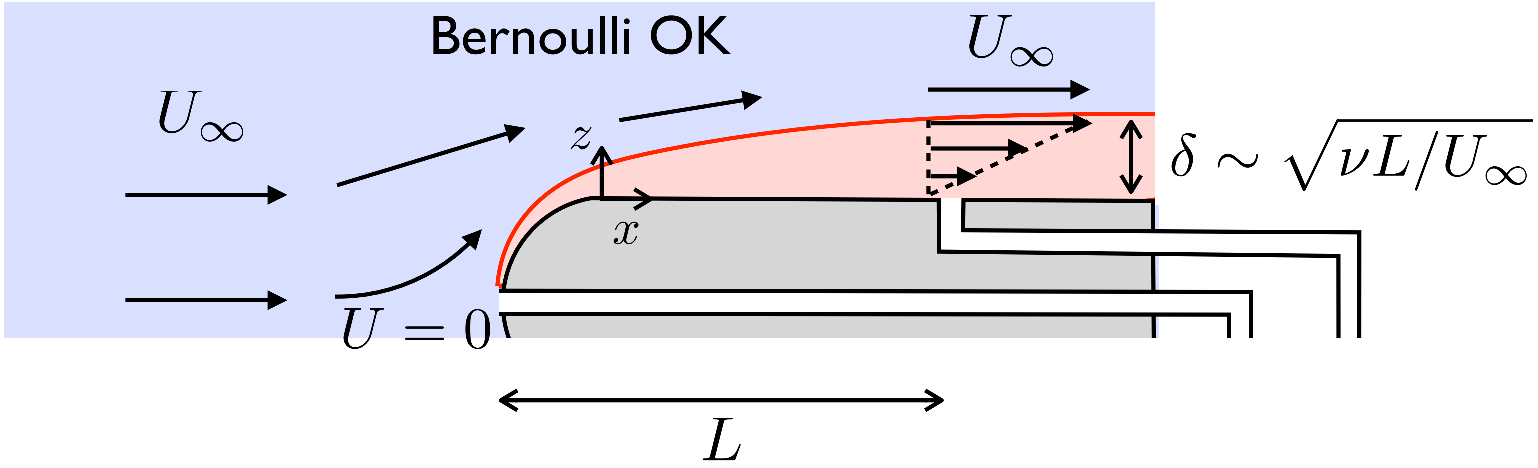
Sauvé par les couches limites !



écoulement selon u_x ?

Tube de Pitot

Sauvé par les couches limites !



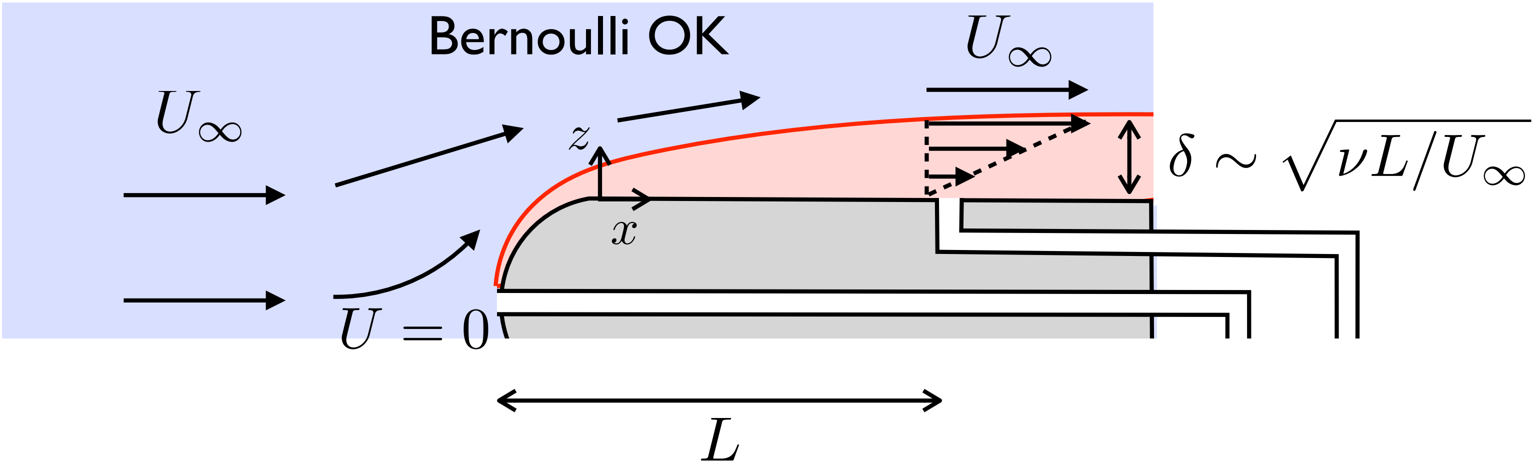
écoulement selon u_x ?

écoulement incompressible: $\nabla \cdot \mathbf{u} = 0$

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = 0$$

Tube de Pitot

Sauvé par les couches limites !



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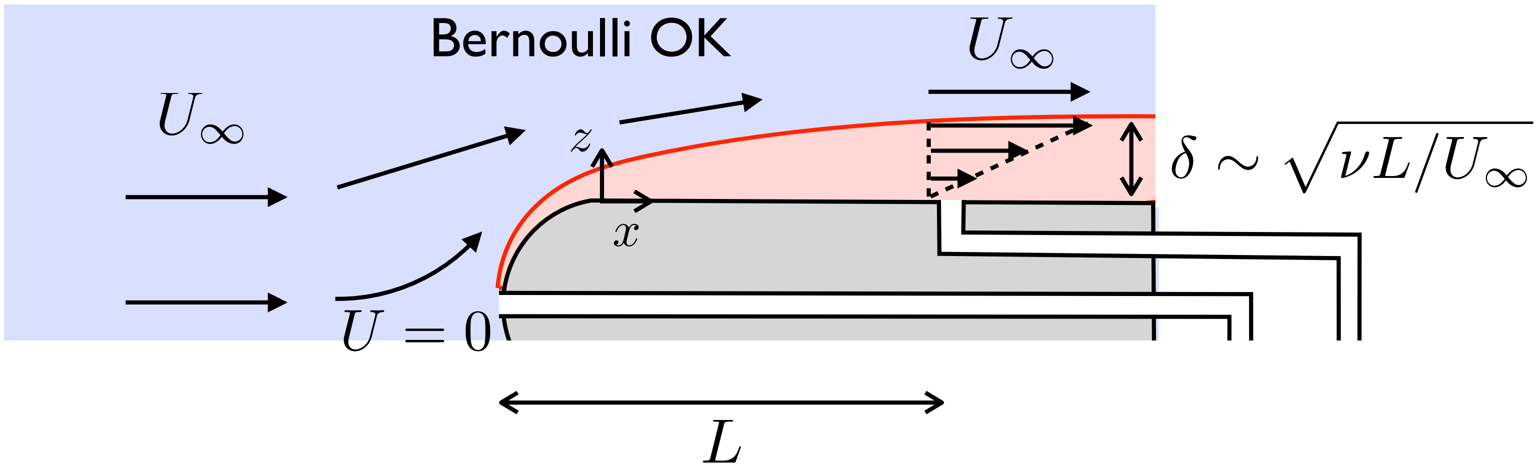
$$\frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} = 0 \quad \Rightarrow \frac{U_\infty}{L} \sim \frac{u_z}{\delta} \Rightarrow u_z \sim U_\infty \frac{\delta}{L}$$

Tube de Pitot

Sauvé par les couches limites !



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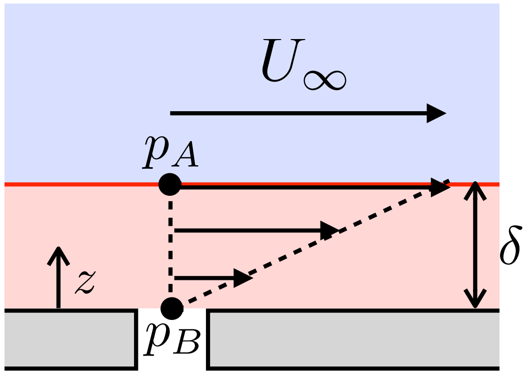
u_z négligeable si $\delta \ll L$

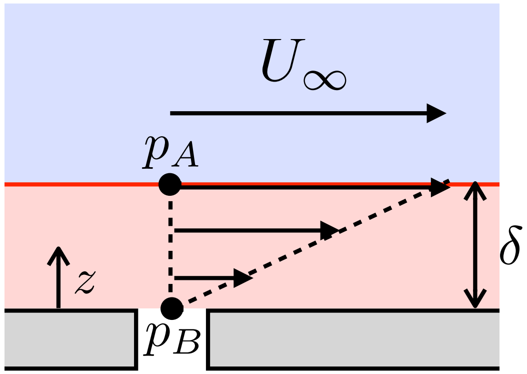
Tube de Pitot



~ fluide parfait (Bernoulli)

écoulement visqueux

Tube de Pitot



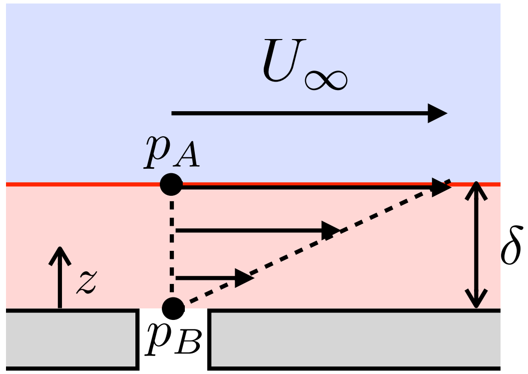
~ fluide parfait (Bernoulli)

écoulement visqueux

équation de Stokes sur z

$$\frac{\partial p}{\partial z} = -\rho g \qquad p_B = p_A + \rho g \delta$$

Tube de Pitot



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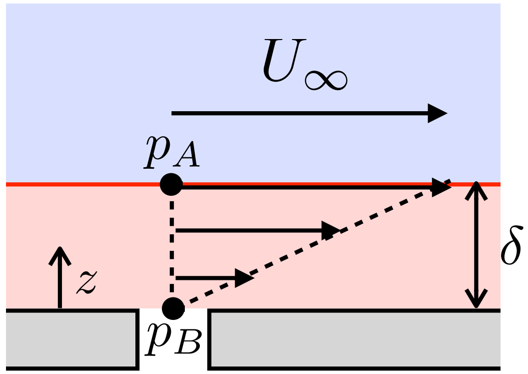
Tube de Pitot



~ fluide parfait (Bernoulli)

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équation de Stokes sur z

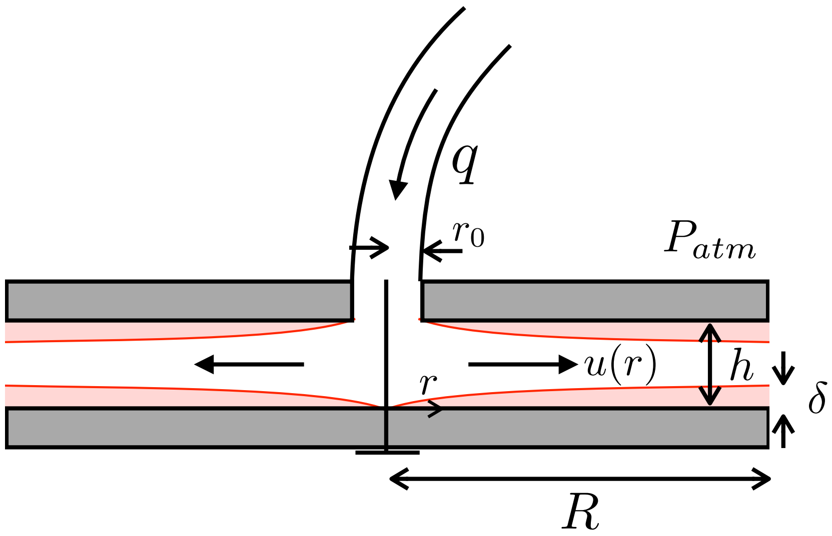
$$\frac{\partial p}{\partial z} = -\rho g \qquad\qquad p_B = p_A + \cancel{\rho g \delta}$$

$$p_A \;\leftrightarrow\; \text{Bernoulli}$$

Relation classique OK (à la condition $\delta \ll L$)

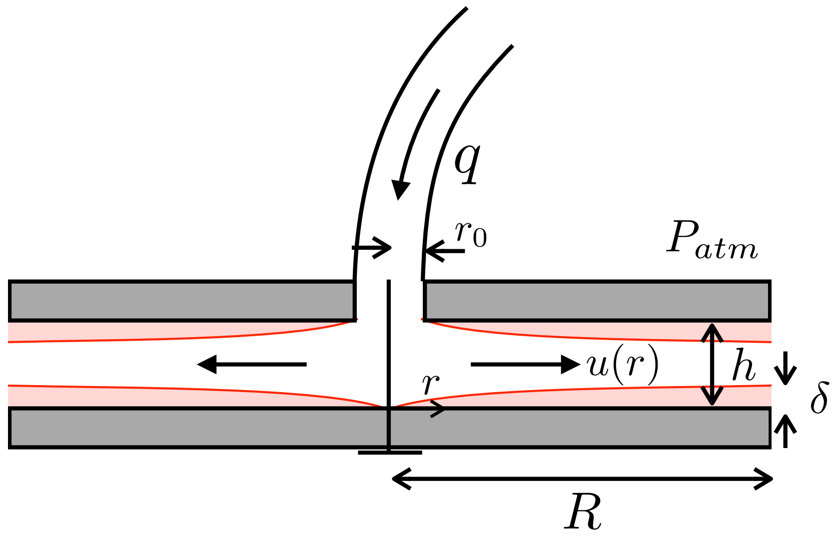
2. Clap

L'explication à la Bernoulli



2. Clap

L'explication à la Bernoulli

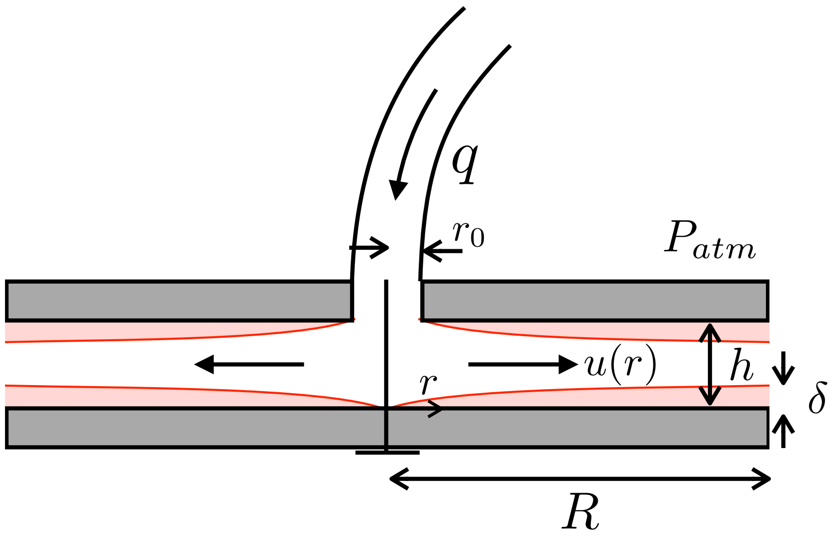


couche limite: $\delta \sim \sqrt{\nu R/U}$

si $h \gg \delta$ couche limite négligeable

2. Clap

L'explication à la Bernoulli



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conservation du débit: $u(r) = \dfrac{q}{2\pi r h}$

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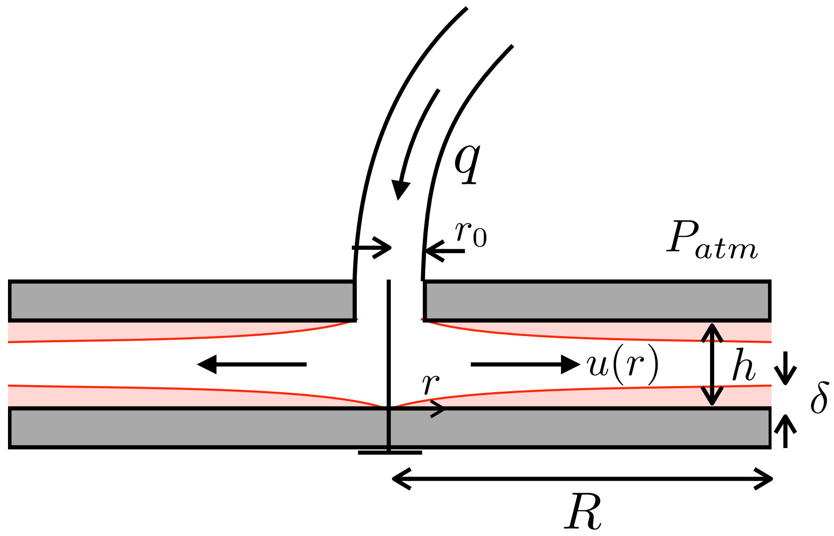

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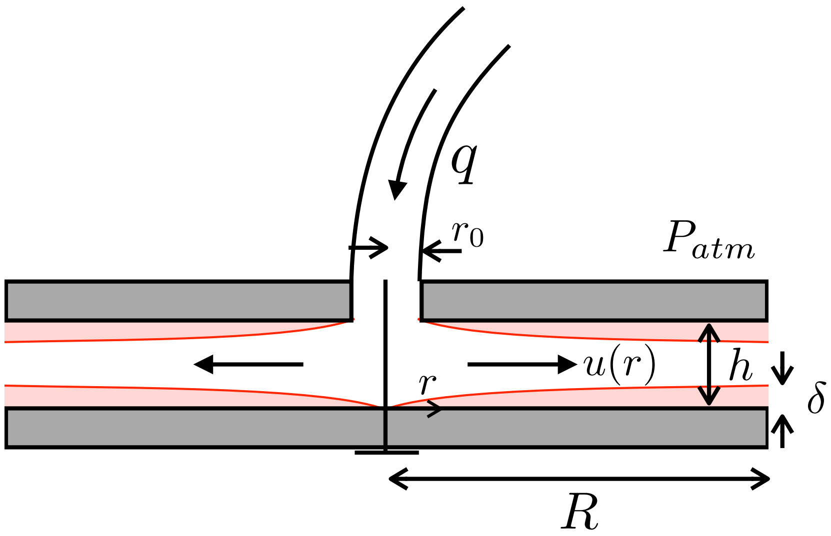
Bernoulli: $P(r) + \dfrac{1}{2}\rho u^2(r) = P_{atm}$

$$P(r) = P_{atm} - \frac{\rho}{8\pi^2}\left(\frac{q}{hr}\right)^2$$

2. Clap

L'explication à la Bernoulli



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conservation du débit: $u(r) = \dfrac{q}{2\pi r h}$

Bernoulli: $P(r) + \dfrac{1}{2}\rho u^2(r) = P_{atm}$

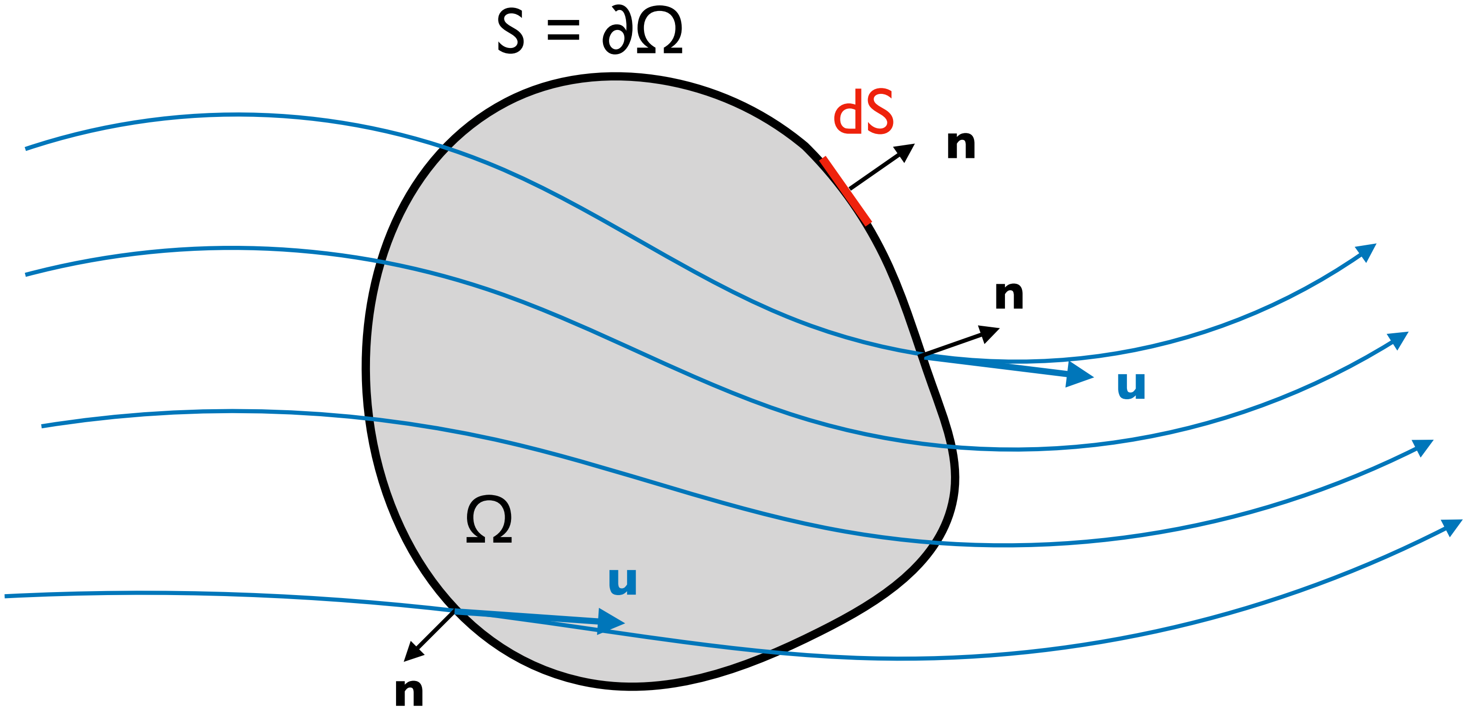
$$P(r) = P_{atm} - \frac{\rho}{8\pi^2}\left(\frac{q}{hr}\right)^2$$

Force:

$$F = \int_{r_0}^{R} 2\pi r (P(r) - P_{atm}) dr = -\frac{\rho q^2}{4\pi h^2}\int_{r_0}^{R}\frac{dr}{r} = -\frac{\rho q^2}{4\pi h^2}\ln(R/r_0)$$

$\Rightarrow$ attraction

La conservation de la quantité de mouvement en « fluide parfait »



$$\int_\Omega \frac{\partial \rho \mathbf{u}}{\partial t} dV = \int_\Omega \rho \mathbf{f}\ dV - \int_{\partial\Omega} \rho \mathbf{u}\ \mathbf{u}.\mathbf{n}\ dS\ + \int_{\partial\Omega} \underline{\underline{\sigma}}.\mathbf{n}\ dS$$

vecteur scalaire contraintes

En négligeant la viscosité :

$$\sigma = \begin{pmatrix} -p & 0 & 0 \\ 0 & -p & 0 \\ 0 & 0 & -p \end{pmatrix}$$

$$\int_\Omega \frac{\partial \rho \mathbf{u}}{\partial t} dV = \int_\Omega \rho \mathbf{f} \ dV - \int_{\partial\Omega} \rho \mathbf{u} \ \mathbf{u}.\mathbf{n} \ dS - \int_{\partial\Omega} p\mathbf{n} \ dS$$

En écoulement stationnaire :

$$\int_\Omega \rho \mathbf{f} \ dV - \int_{\partial\Omega} \rho \mathbf{u} \ \mathbf{u}.\mathbf{n} \ dS - \int_{\partial\Omega} p\mathbf{n} \ dS = 0$$