Rappel

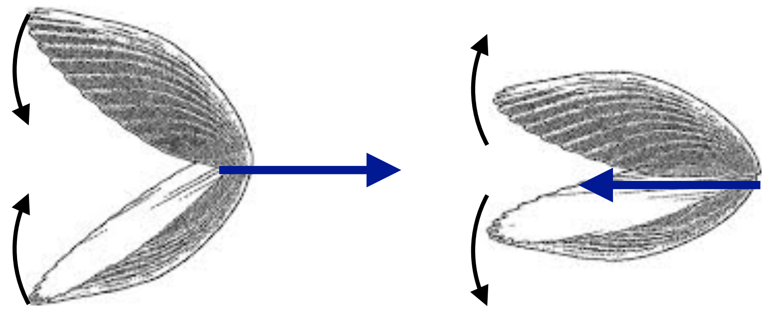
Équation de Navier & Stokes:

$$\cancel{\rho\frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\nabla)}\mathbf{u} = -\nabla p + \eta\Delta\mathbf{u} + \rho\mathbf{f}$$

$$Re \ll 1$$

Équation linéaire ⇒ solutions additives

Équation réversible dans le temps



"Scallop theorem"

Qui et al., Nature Communications (2014)

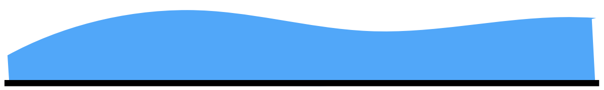
Les écoulements à petit nombre de Reynolds

Pas d'inertie

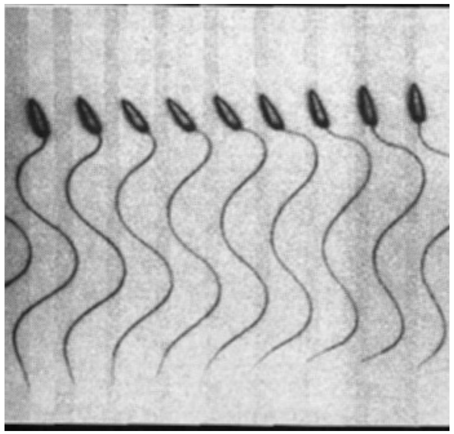
Navier-Stokes $\longrightarrow$ Équation de Stokes

$$\eta \Delta \mathbf{u} = \nabla p$$

Les écoulements en couche mince et l'approximation de lubrification



Les forces sur les objets solides, la réversibilité cinématique et la propulsion à petit Reynolds



Un monde de très petites forces dans l'eau et l'air

$$Re = \frac{UL}{\nu} \leq 1$$

$$0 = -\nabla p + \eta \Delta \mathbf{u}$$

$$0 \sim \frac{p}{L} + \eta \frac{U}{L^2}$$

$$p \sim \eta \frac{U}{L}$$

$$\boxed{F \sim pL^2 \sim \eta \; U \;\; L}$$

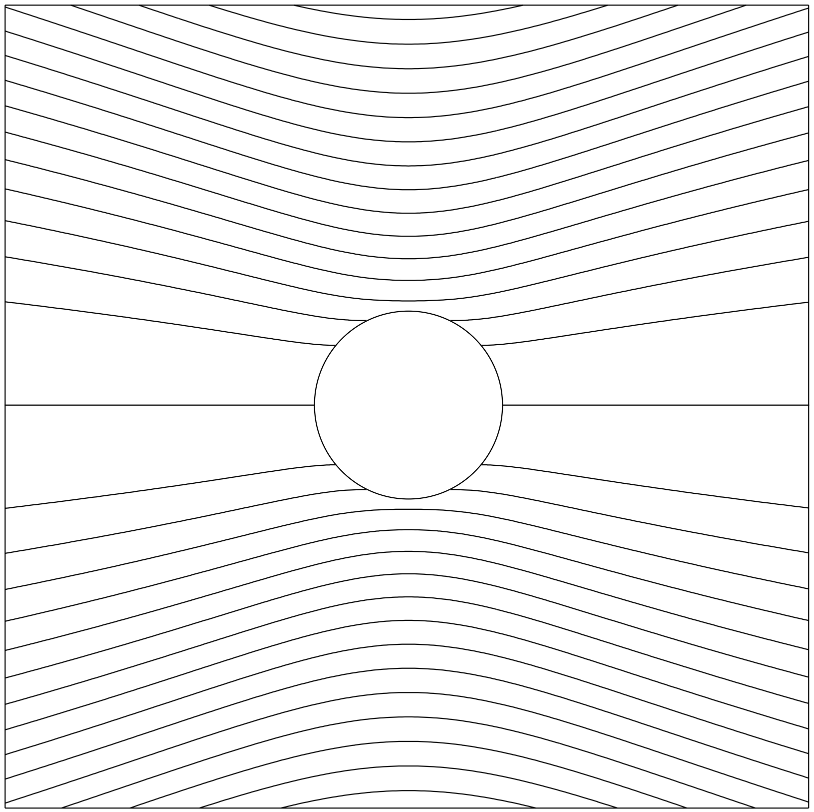
$$F \leq \eta \; \nu$$

0,000000001 N
1 nN dans l'eau

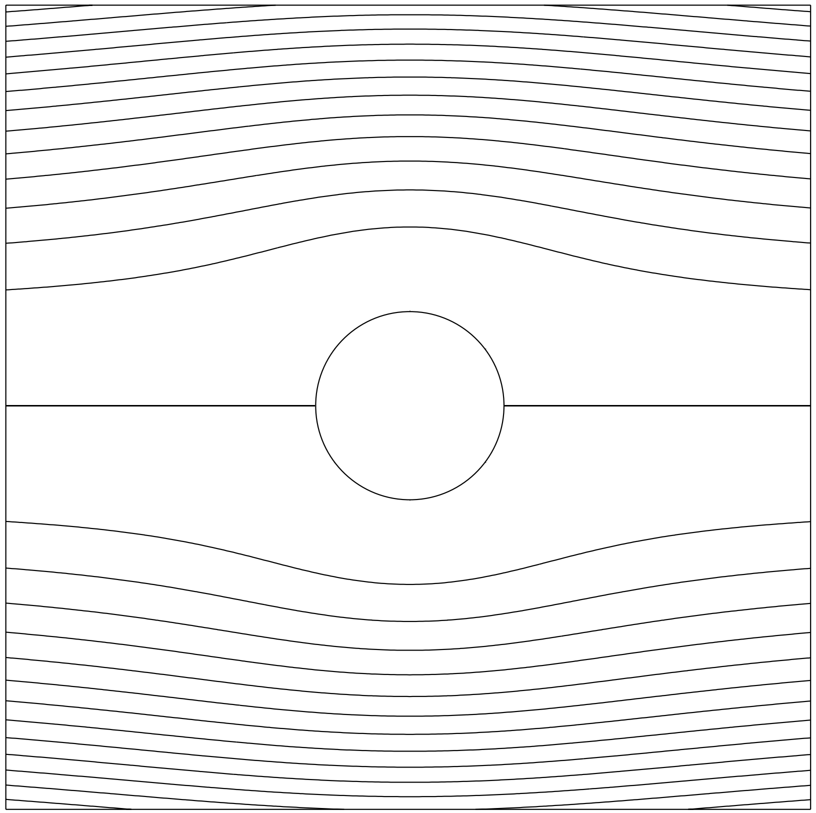
0,0000000002 N
0,2 nN dans l'air

Les forces sur un objet en mouvement : la sphère

Deux sphères de rapport de diamètre égal à 2



Fluide immobile à l'infini



Sphère immobile

Vitesse de chute d'une sphère

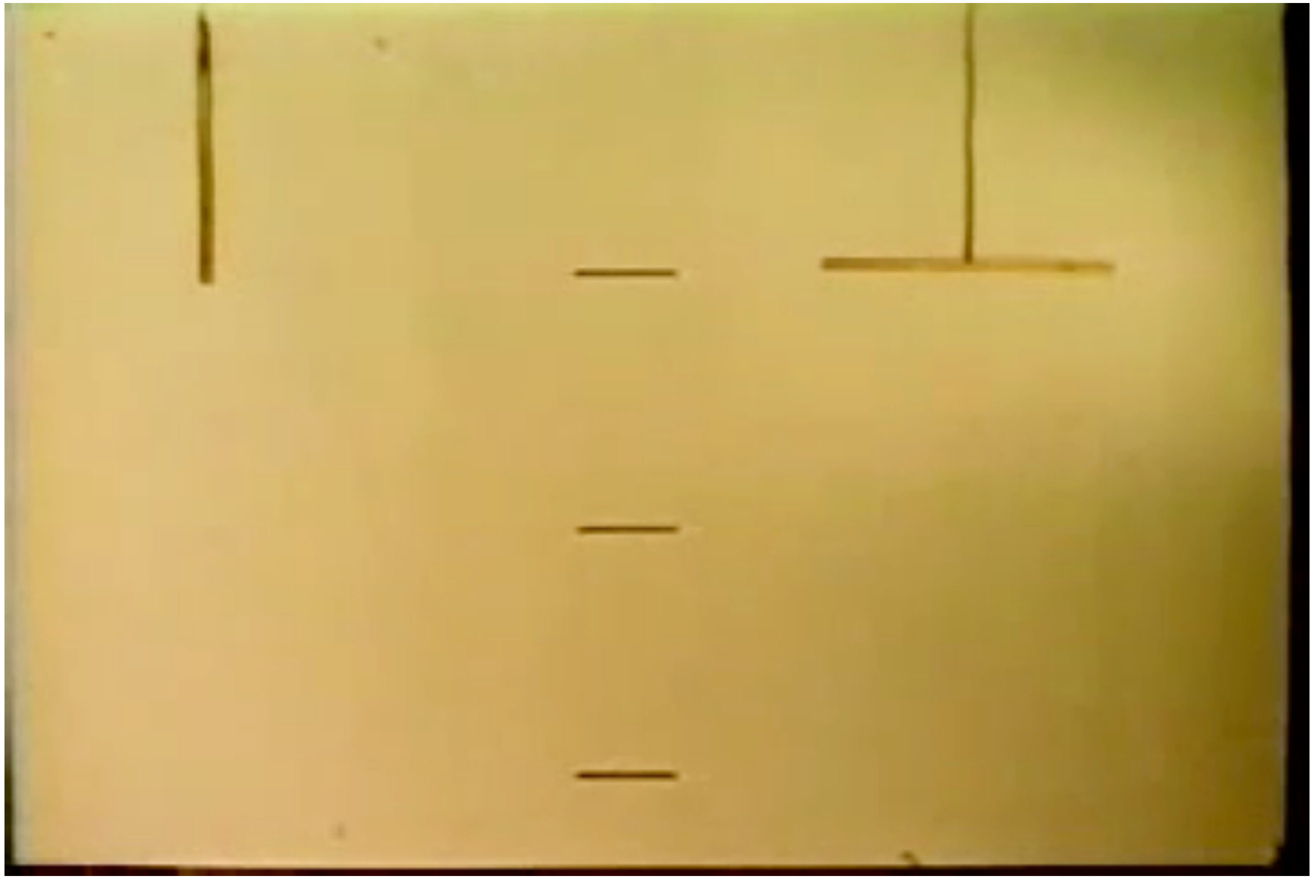
$$F \sim \eta \, U \; R$$

$$F = 6\pi\eta \, U \; R$$

$$P_{app} = \delta\rho \; g \; \frac{4}{3}\pi R^3$$

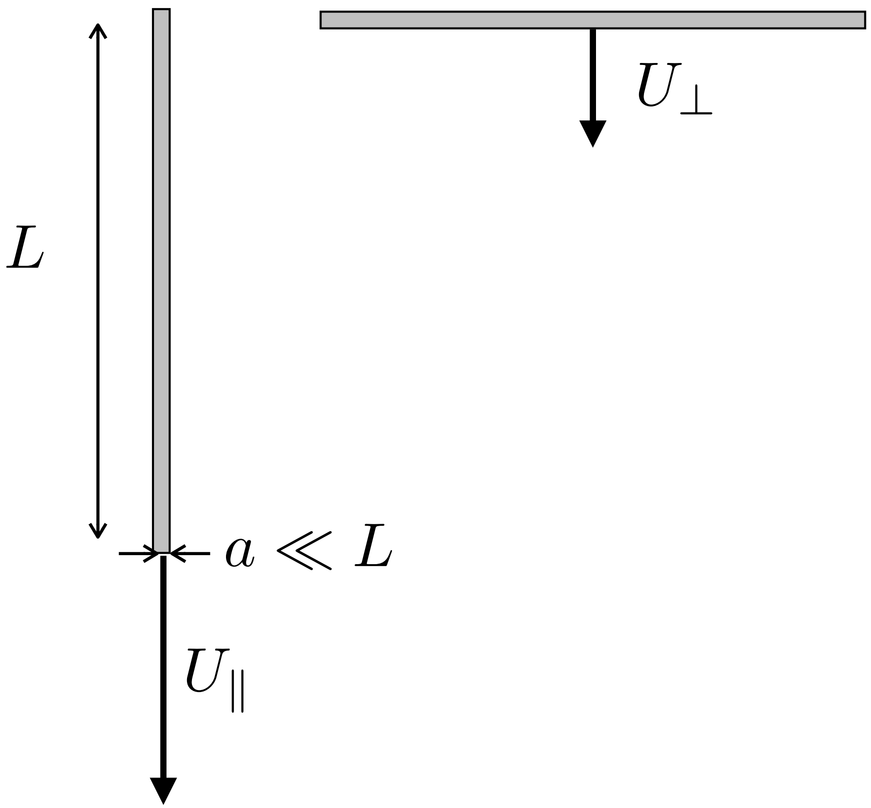
$$U_{sed} = \frac{P_{app}}{6\pi\eta R} = \frac{2}{9}\frac{\delta\rho \; g \; R^2}{\eta}$$

Les forces sur un objet en mouvement : un bâtonnet



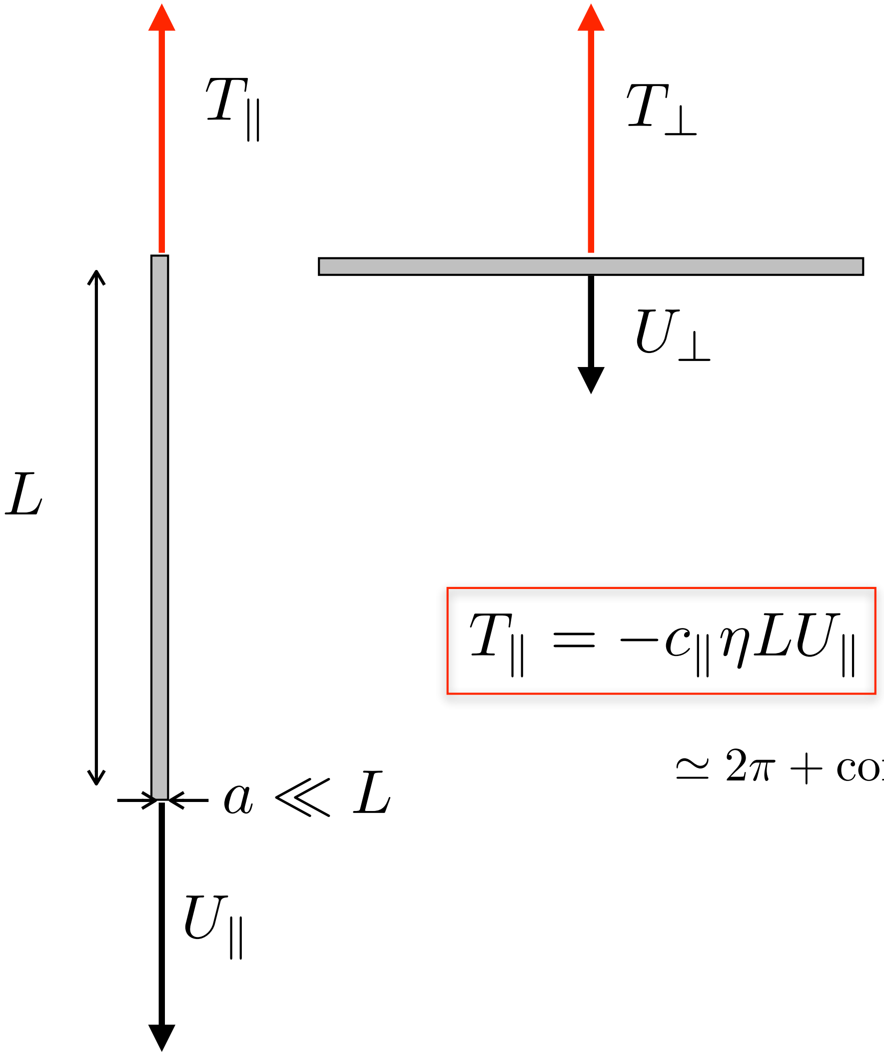
Deux bâtonnets identiques orientés perpendiculairement l'un de l'autre,
puis un bâtonnet incliné

Chute d'une tige



$$U_\parallel \simeq 2U_\perp$$

Chute d’une tige



$$U_{\|} \simeq 2U_{\perp}$$

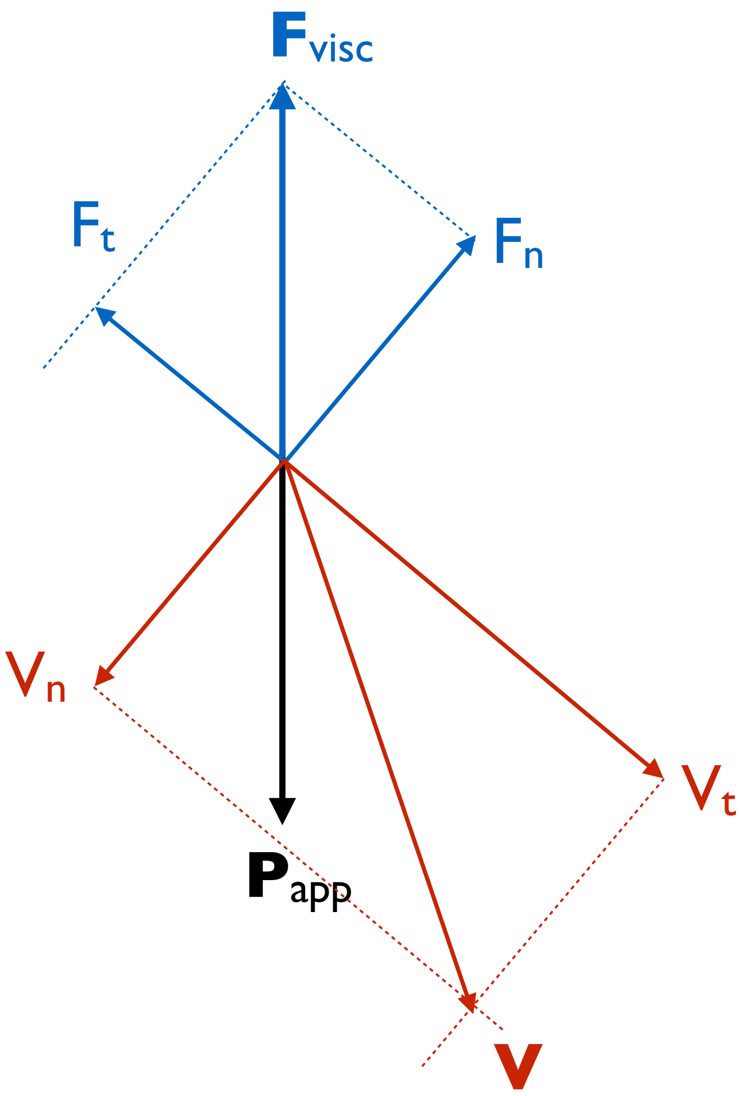
$$\boxed{T_{\|} = -c_{\|}\eta L U_{\|}}$$

$$\simeq 2\pi + \text{correction}(a/L)$$

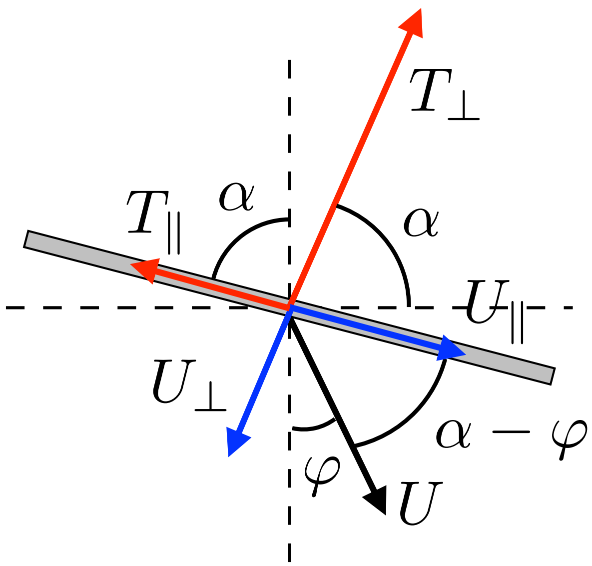
$$\boxed{T_{\perp} = -c_{\perp}\eta L U_{\perp}}$$

$$\boxed{c_{\perp} \simeq 2c_{\|}}$$

Forces sur un bâtonnet incliné par rapport au mouvement



Chute d'une tige inclinée (calcul en bonus)



équilibre forces horizontales

$$T_\parallel \sin\alpha = T_\perp \cos\alpha$$

$$\tan\alpha = \frac{T_\perp}{T_\parallel} = \frac{c_\perp \eta U_\perp L}{c_\parallel \eta U_\parallel L} = 2\frac{U_\perp}{U_\parallel}$$

$$\frac{U_\perp}{U_\parallel} = \tan(\alpha - \varphi)$$

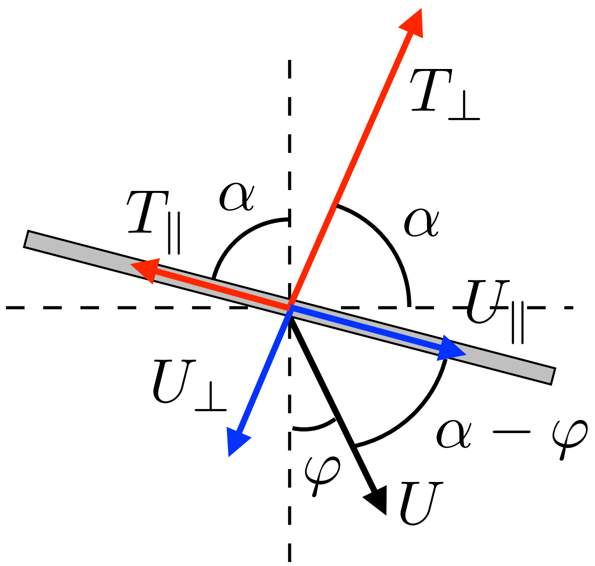
$$\tan(\alpha) = 2\tan(\alpha - \varphi) = 2\frac{\tan(\alpha) - \tan(\varphi)}{1 + \tan(\alpha)\tan(\varphi)}$$

$$\boxed{\tan(\varphi) = \frac{\tan\alpha}{2 + \tan^2\alpha}}$$

$\varphi_{max} = 19.5°$ pour $\alpha_m = 54.7°$

Chute d'une tige inclinée (calcul en bonus)

Vitesse de chute ?



équilibre forces verticales

$$T_\parallel \cos\alpha + T_\perp \sin\alpha = \pi a^2 \Delta\rho L g$$

$$T_\parallel \cos\alpha \left(1 + \frac{T_\perp}{T_\parallel}\tan\alpha\right) = \pi a^2 \Delta\rho L g$$

$$\tan\alpha$$

$$T_\parallel \frac{1}{\cos\alpha} = \pi a^2 \Delta\rho L g$$

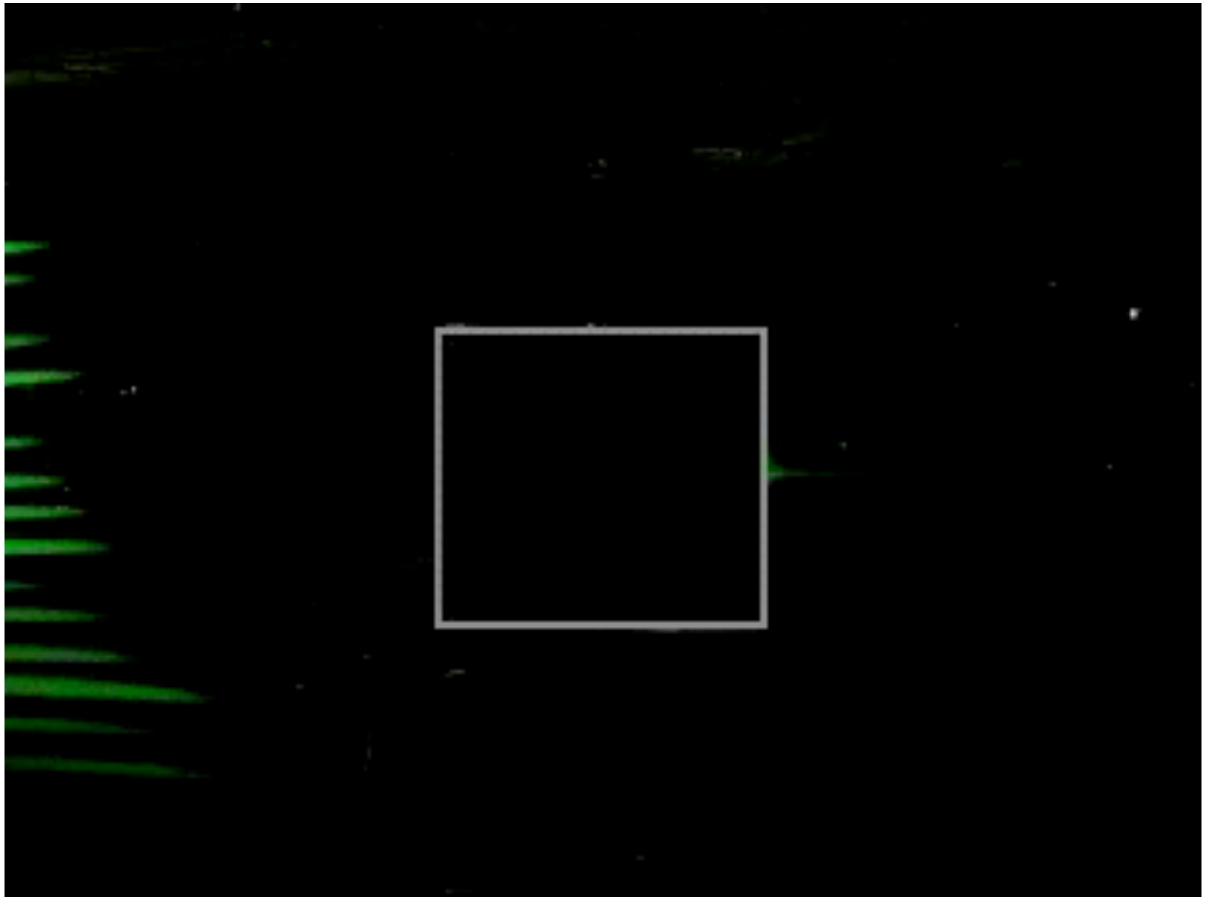
$$c_\parallel \eta L U_\parallel = \pi a^2 \Delta\rho L g \cos\alpha$$

$$U_\parallel = \frac{\pi a^2 \Delta\rho g \cos\alpha}{c_\parallel \eta}$$

$$U_\perp = \tan(\alpha - \varphi) U_\parallel = \frac{\tan\alpha}{2} U_\parallel$$

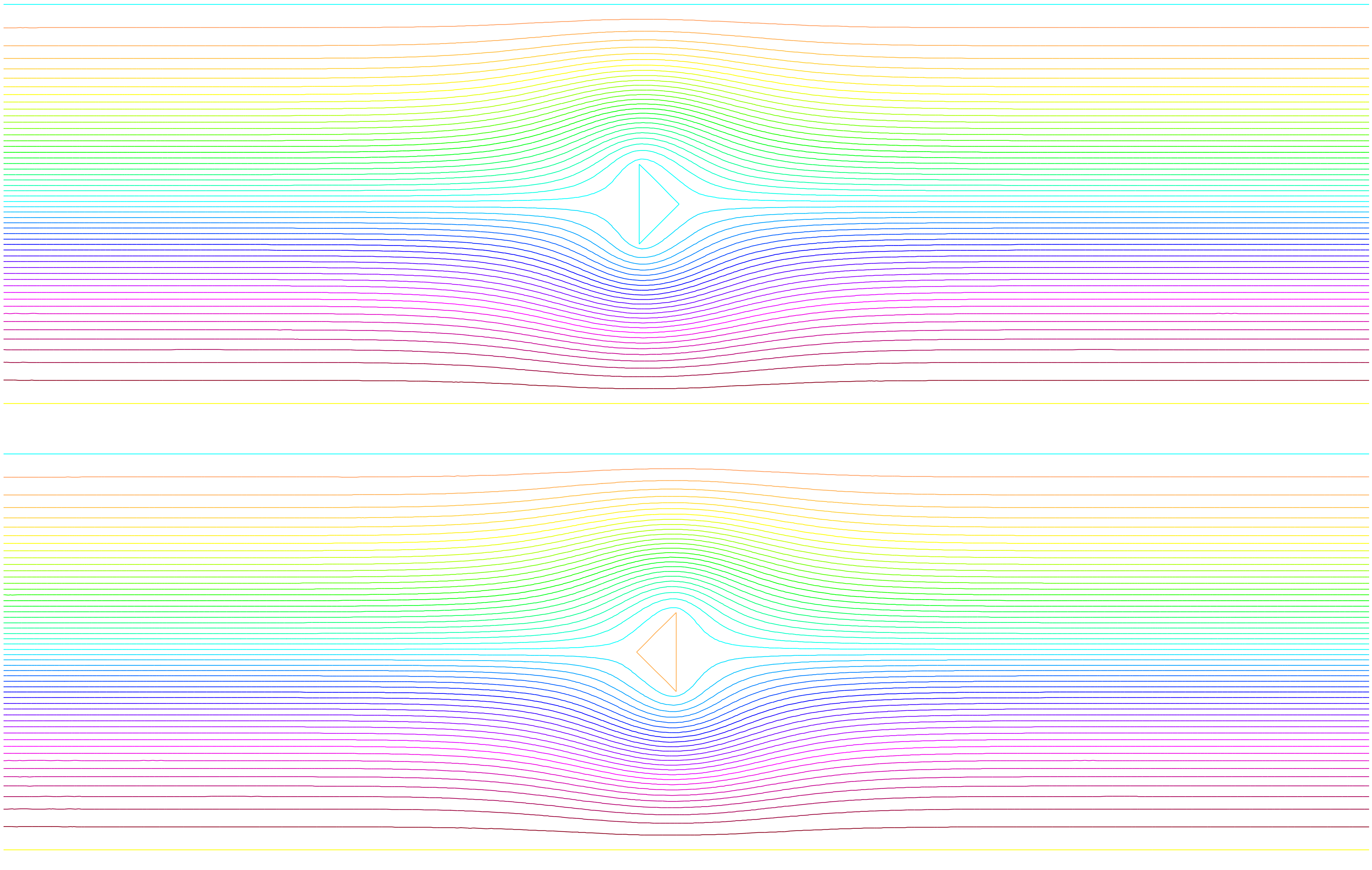
$$\boxed{U = \left(1 + \frac{\tan^2\alpha}{4}\right)^{1/2} \frac{\pi a^2 \Delta\rho g \cos\alpha}{c_\parallel \eta}}$$

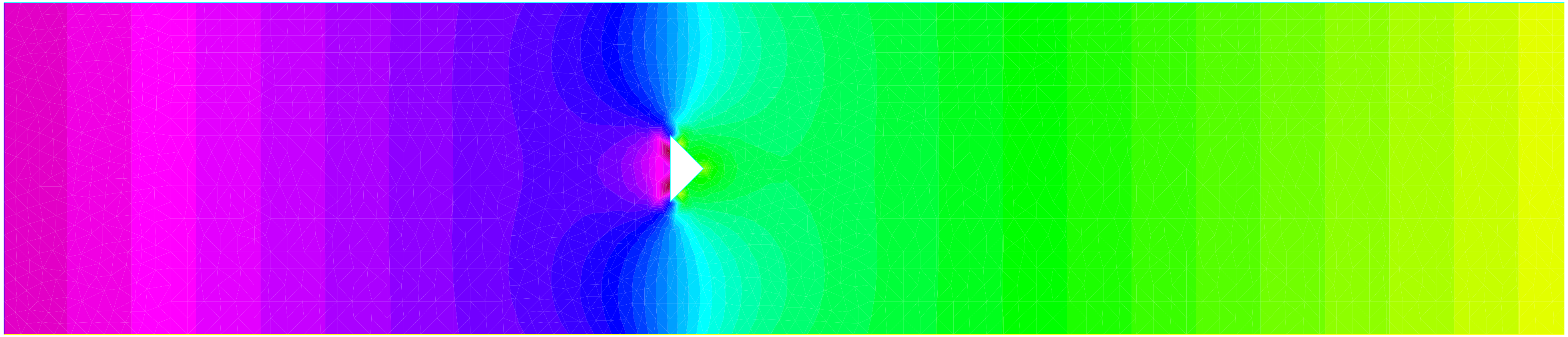
Propriétés de symétrie des écoulements de Stokes

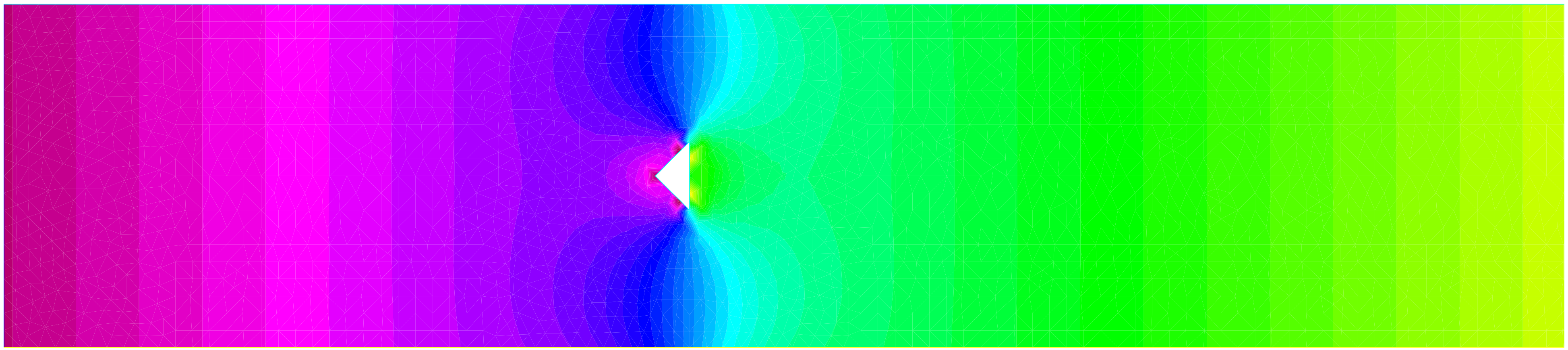


$$\eta \Delta \mathbf{u} = \nabla p$$

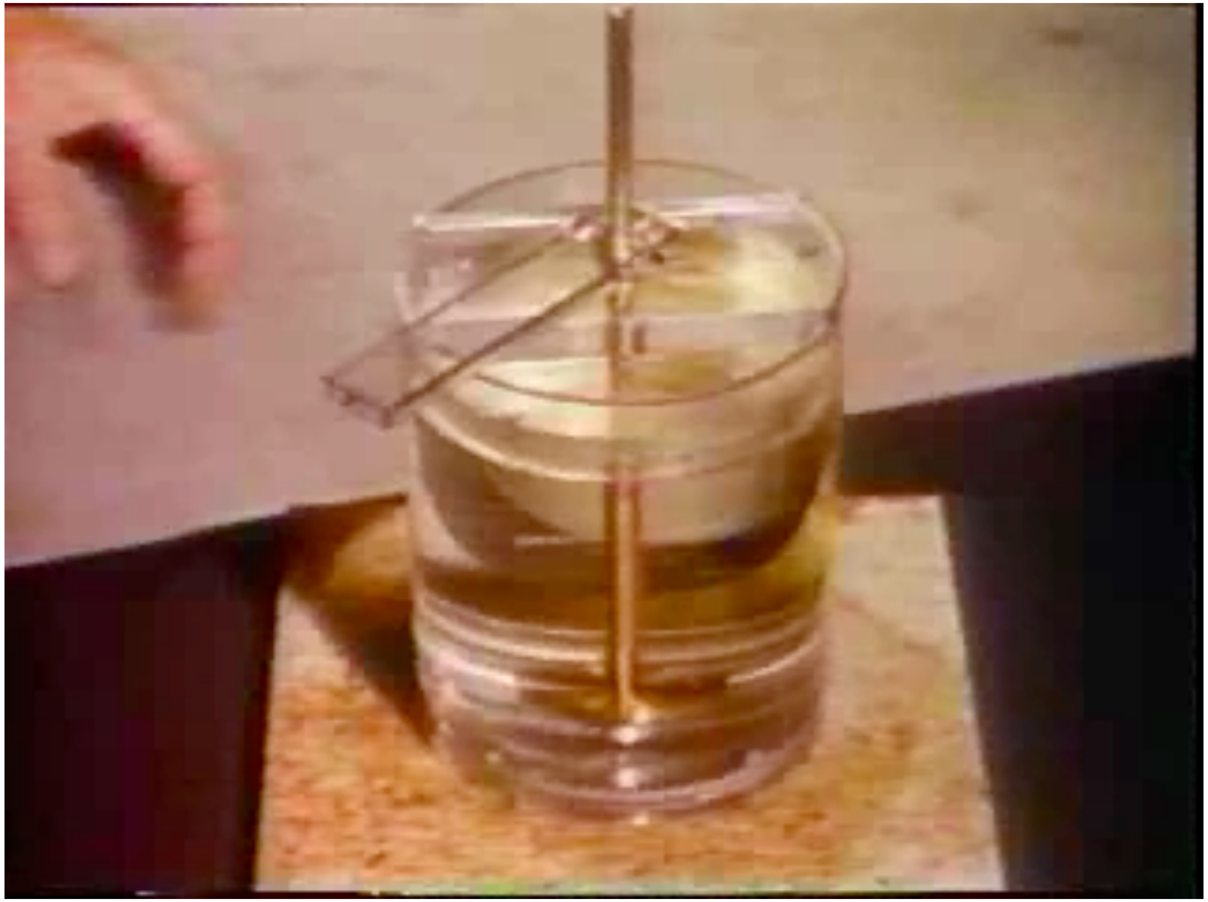
$$\nabla p \rightarrow -\nabla p \qquad \qquad \mathbf{u} \rightarrow -\mathbf{u}$$





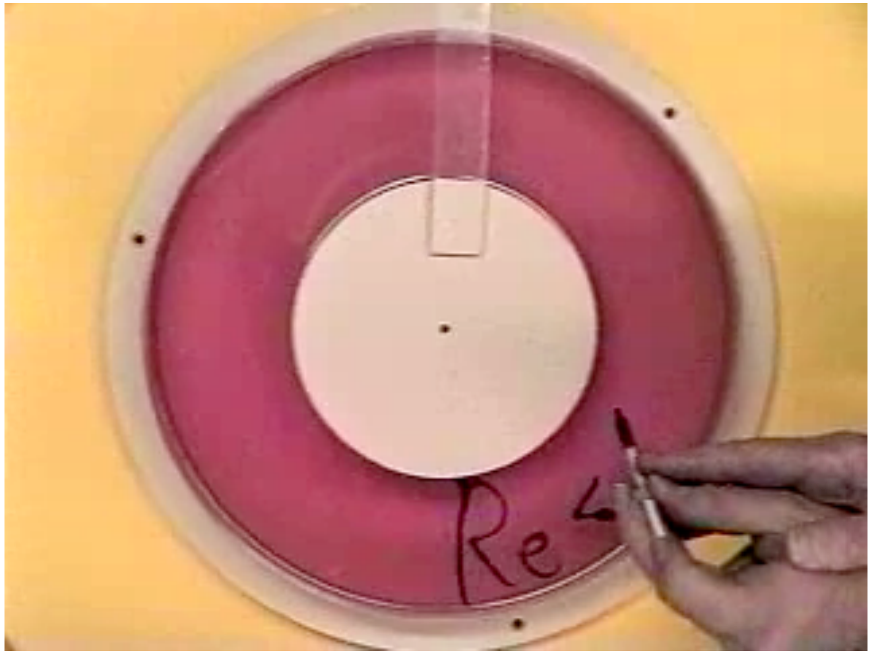


Réversibilité cinématique



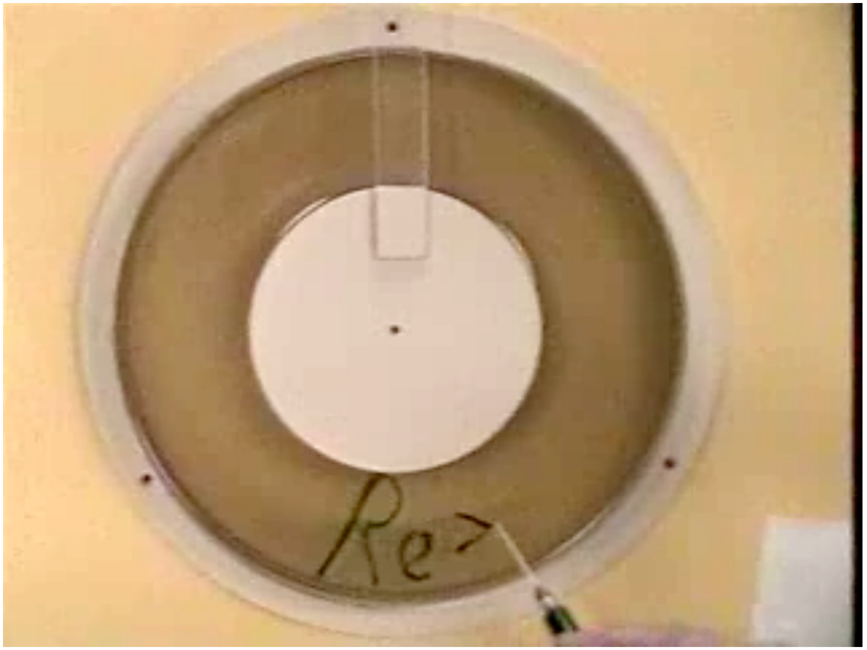
Issu de notre « MOOC » favori par G.I. Taylor:

https://www.youtube.com/watch?v=51-6QCJTAjU



$$Re \ll 1$$

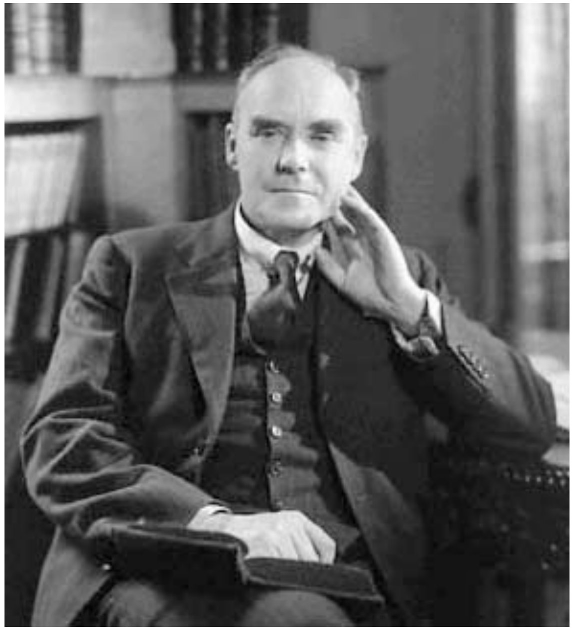
Réversibilité cinématique



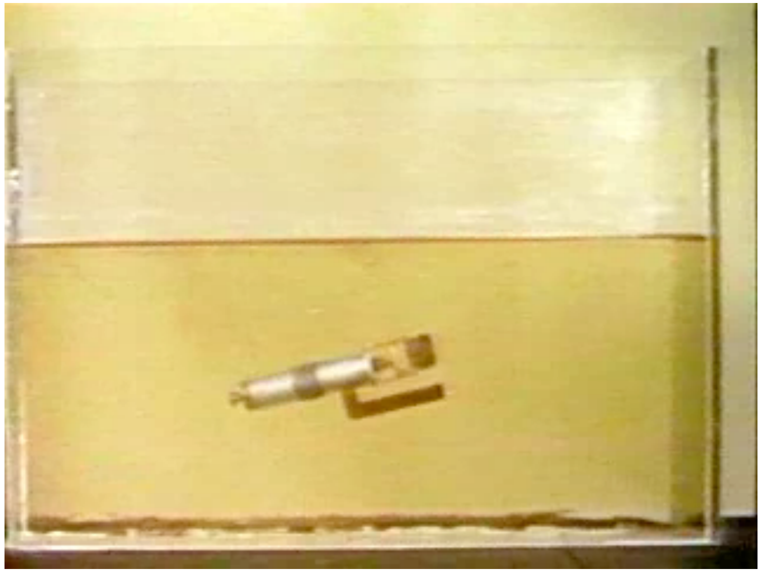
$$Re \gg 1$$

Irréversibilité cinématique

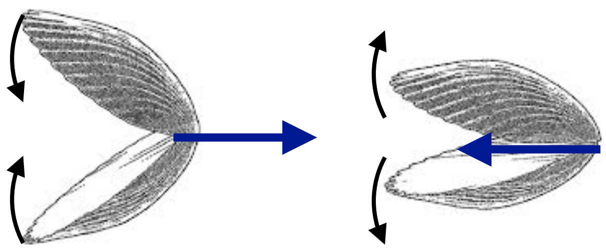
Réversibilité cinématique et propulsion



G. I. Taylor



Le théorème de la coquille St Jacques



“Scallop theorem”

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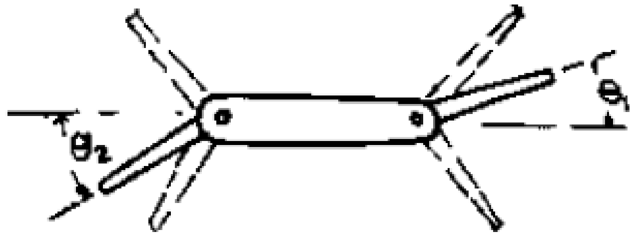
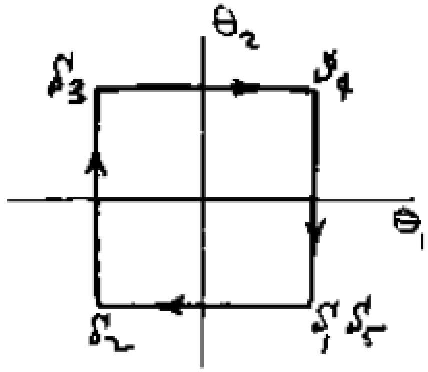
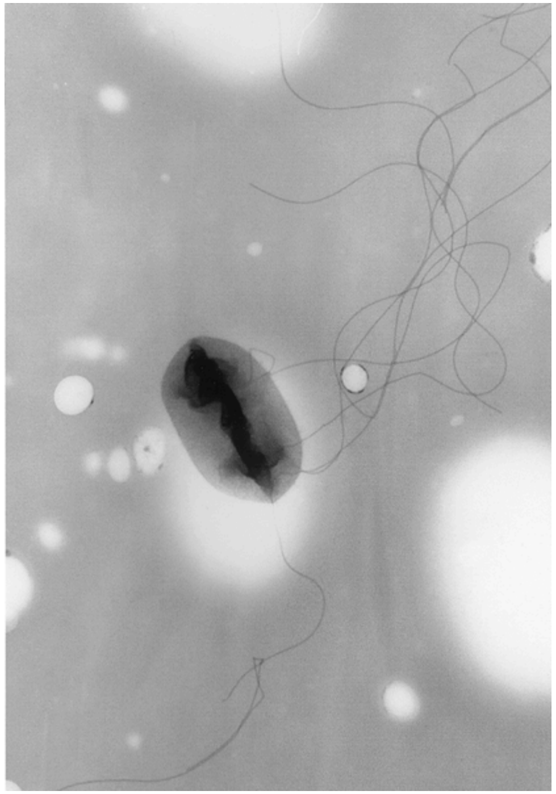


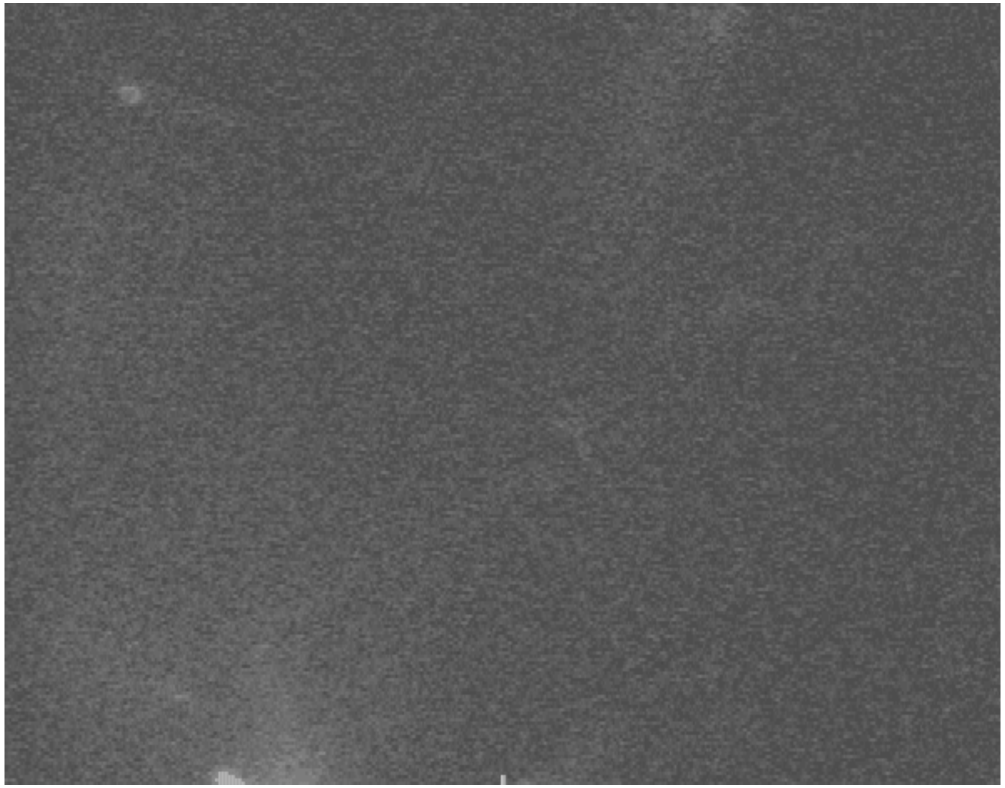
1 seul degré de liberté :
réversibilité imposée

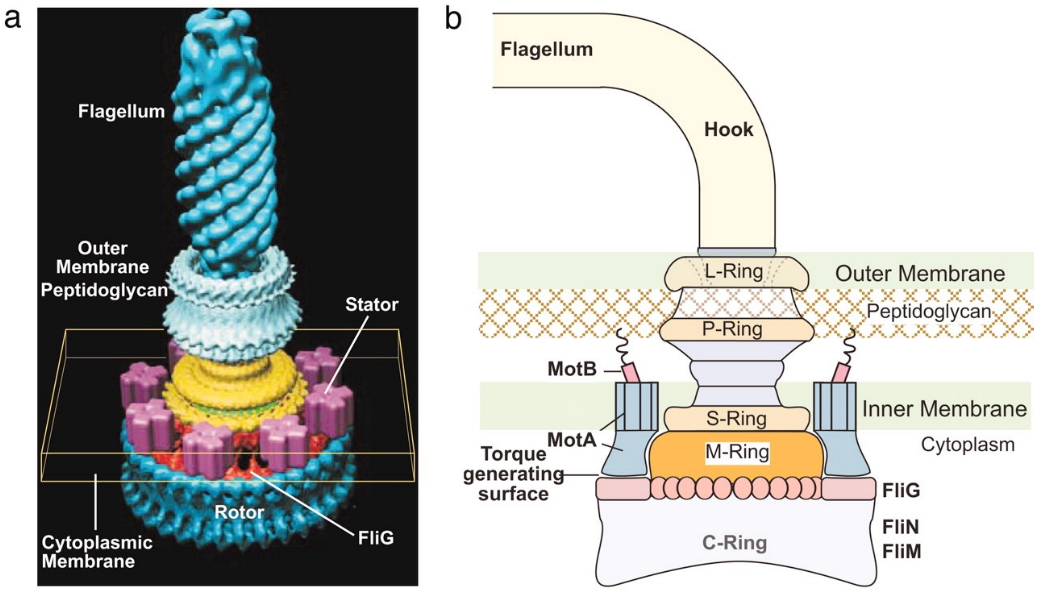
2 degrés de liberté :
possibilité de briser la
symétrie +t/-t



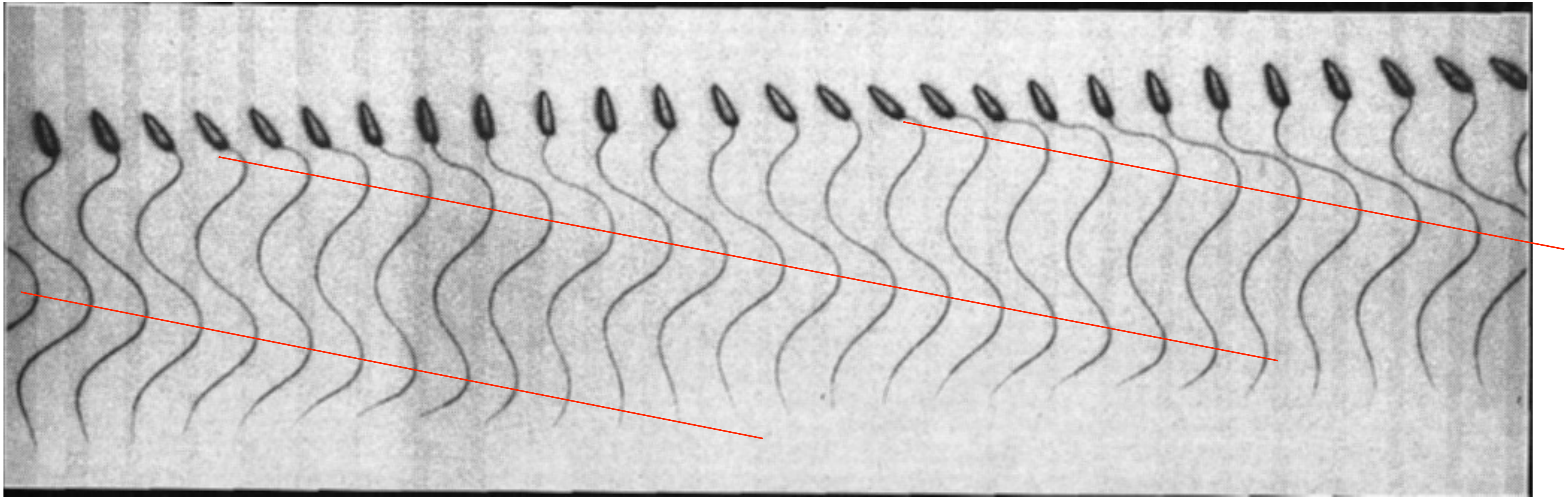
E. Purcell, life at low
Reynolds number

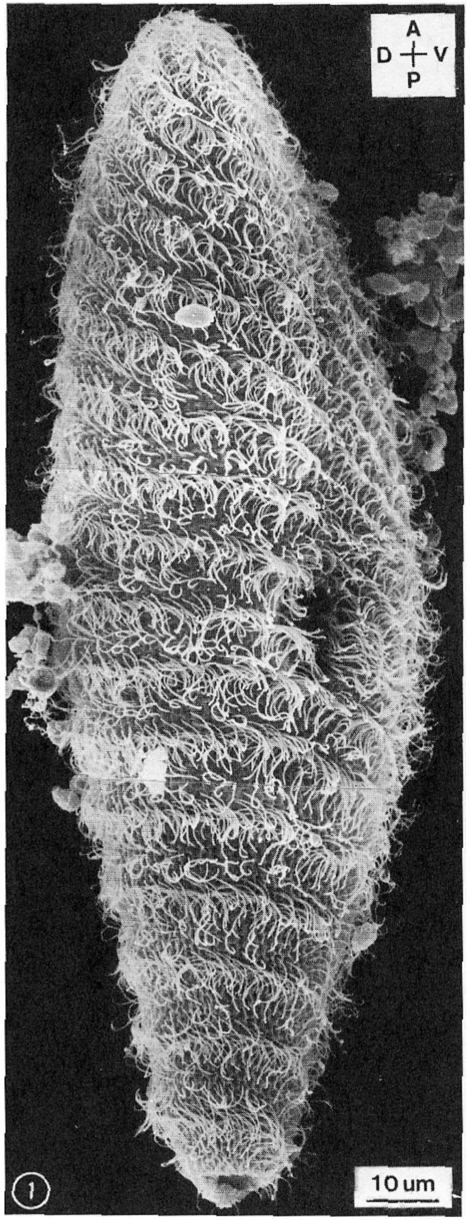






Escherischia Coli





Mouvement collectif des cils