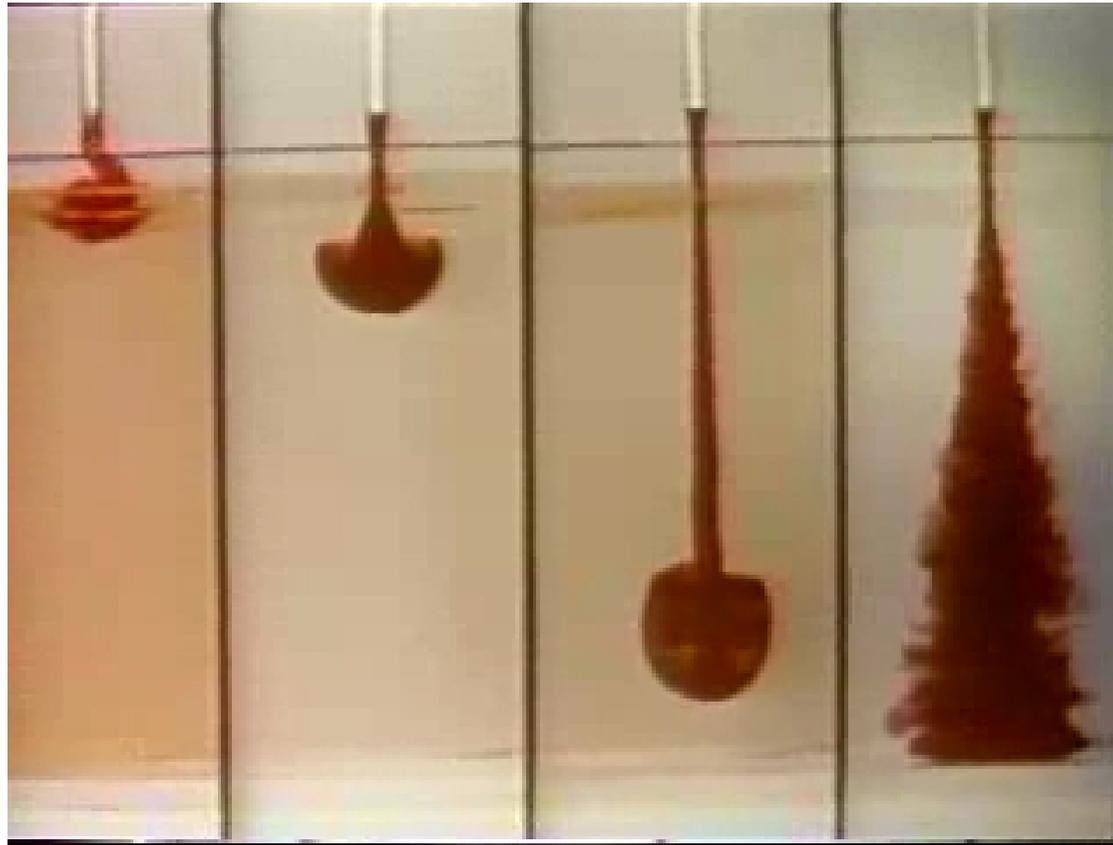


# Inertie contre Viscosité



sirop glycérine eau +  
glycérine eau

# Un monde sans inertie



# Les écoulements à petit nombre de Reynolds

Pas d'inertie

Navier-Stokes



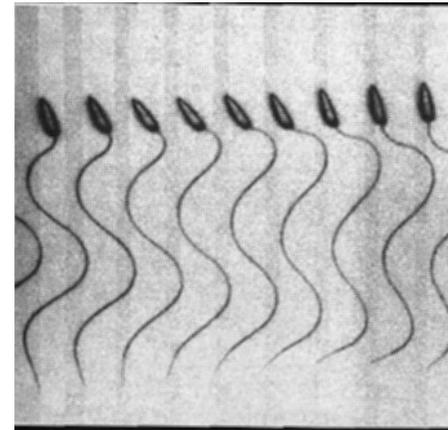
Équation de Stokes

$$\eta \Delta \mathbf{u} = \nabla p$$

Les écoulements en  
couche mince et  
l'approximation de  
lubrification

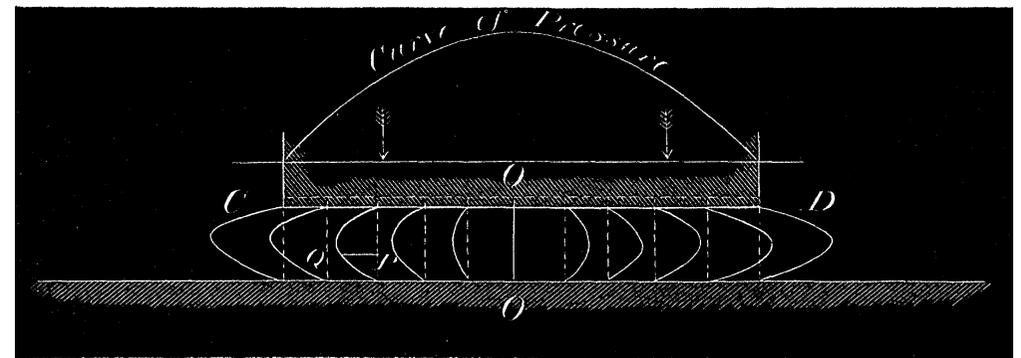
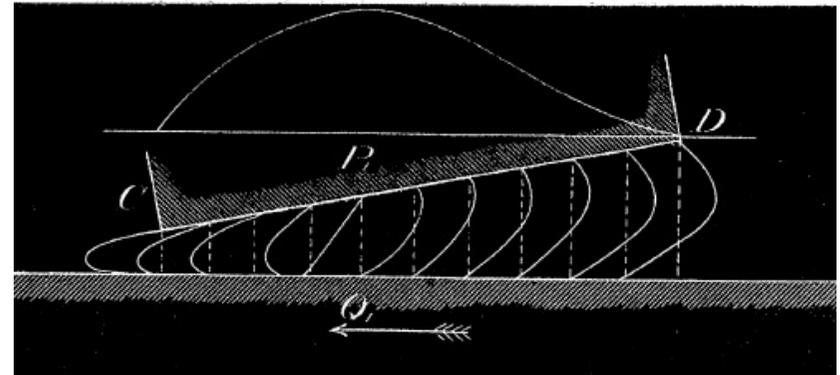
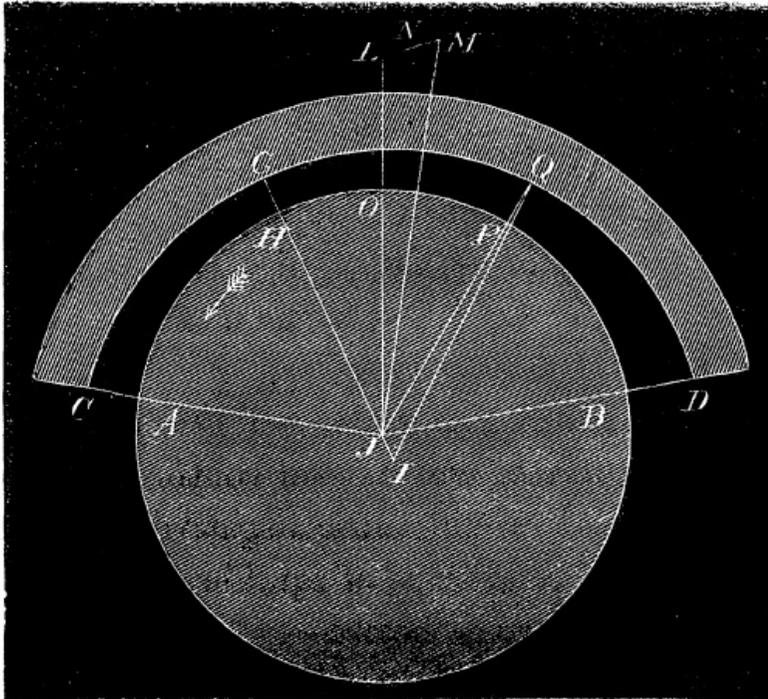


Les forces sur les objets  
solides, la réversibilité  
cinématique et la  
propulsion à petit  
Reynolds

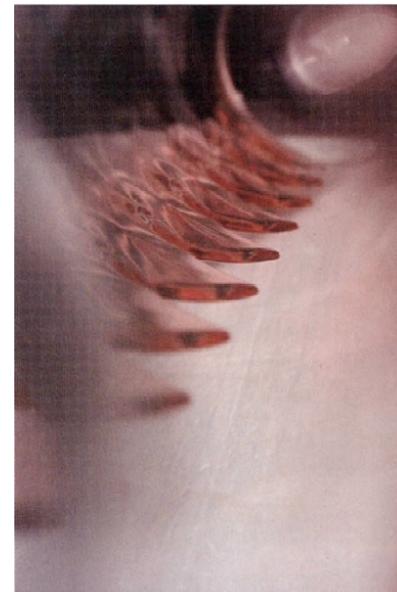
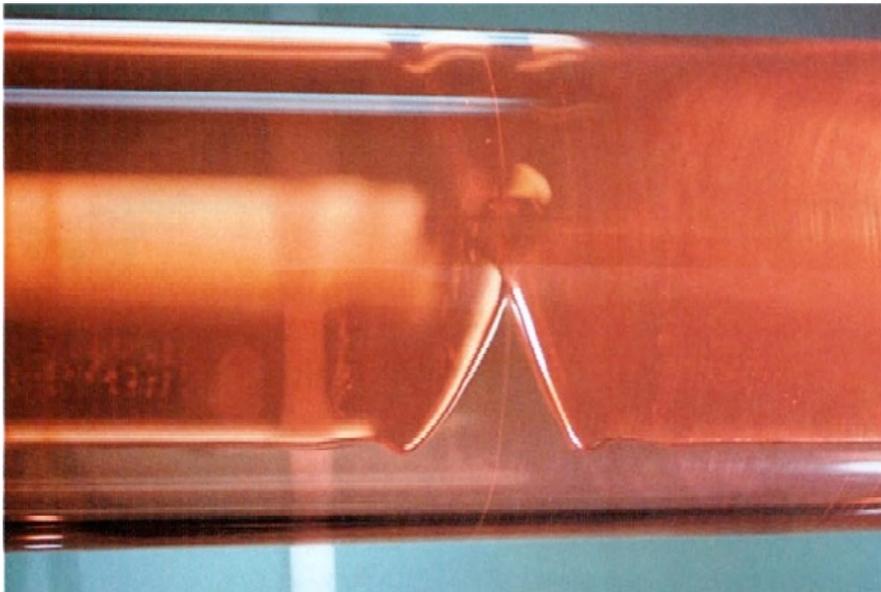


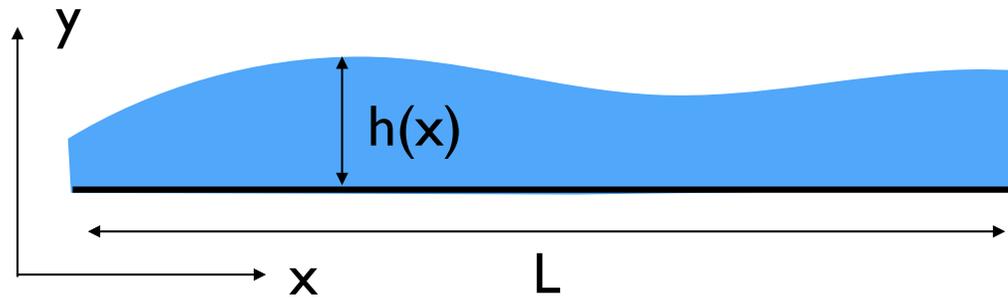
# Les écoulements visqueux en couches minces

## Lubrification (Reynolds 1882)



# Écoulements en couche mince à surface libre

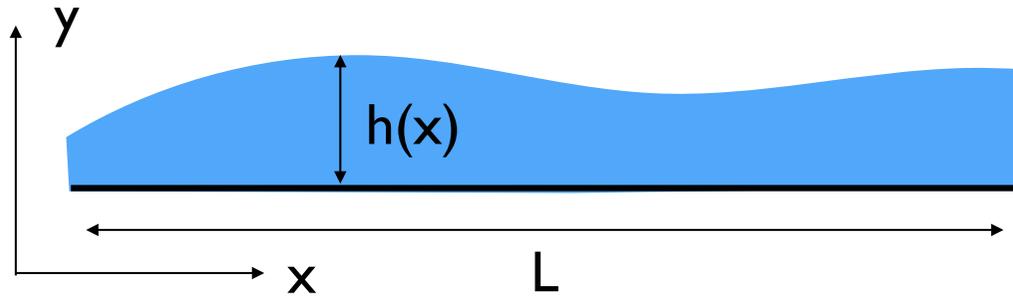




$$h \ll L$$

$$\nabla \cdot \mathbf{u} = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} \sim \frac{u_x}{L} + \frac{u_y}{h}$$

$$\frac{u_x}{u_y} \sim \frac{L}{h} \gg 1$$



$$\frac{u_x}{u_y} \sim \frac{L}{h} \gg 1$$

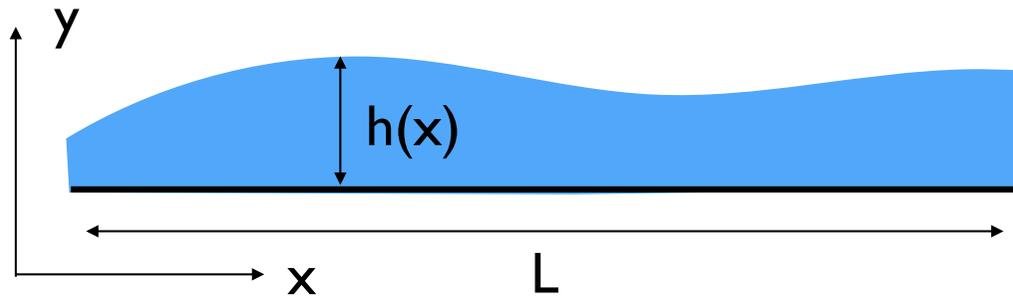
$$0 = -\frac{\partial p}{\partial x} + \eta \left( \cancel{\frac{\partial^2 u_x}{\partial x^2}} + \frac{\partial^2 u_x}{\partial y^2} \right)$$

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u_x}{\partial y^2}$$

$$\frac{U}{L^2} \quad \frac{U}{h^2}$$

$$0 = -\frac{\partial p}{\partial y} + \eta \left( \cancel{\frac{\partial^2 u_y}{\partial x^2}} + \cancel{\frac{\partial^2 u_y}{\partial y^2}} \right)$$

$$\frac{\partial p}{\partial y} \sim 0$$



$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u_x}{\partial y^2}$$

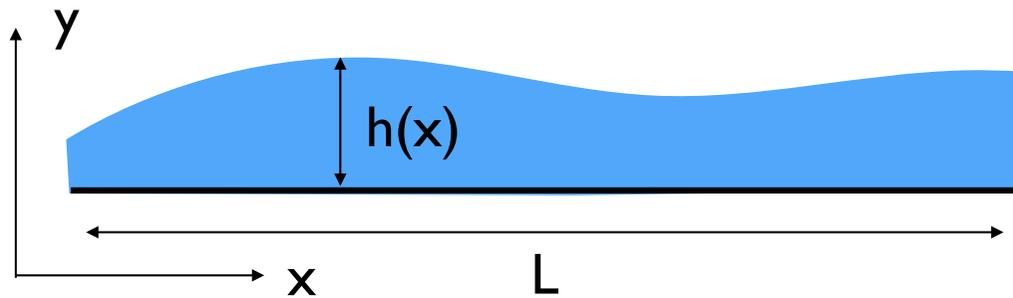
$$\frac{p}{L} \sim \eta \frac{U}{h^2}$$

$$F_y \sim pL \sim \eta U \frac{L^2}{h^2}$$

$$\sigma_{xy} = \eta \frac{\partial u_x}{\partial y} \sim \eta \frac{U}{h}$$

$$F_x \sim \sigma_{xy}L \sim \eta U \frac{L}{h}$$

$$\frac{F_y}{F_x} \sim \frac{L}{h} \gg 1$$



$$\frac{dh}{dx} \ll 1$$

$$\frac{\partial p}{\partial x} = \eta \frac{\partial^2 u_x}{\partial y^2}$$

Intégration suivant y à h fixé  
avec conditions aux limites en  $y = 0$  et  $y = h$

Distribution de pression suivant y

Intégration suivant x



Forces globales