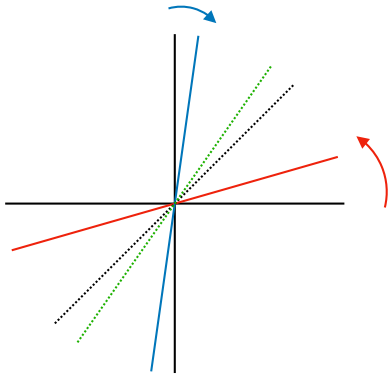


- Tourbillons
- Portance
- Trainée

Tourbillons

vorticité

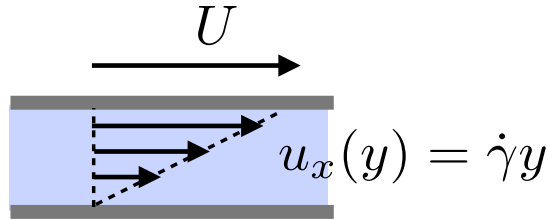
$$\omega = \nabla \wedge \mathbf{u}$$



Vitesse angulaire

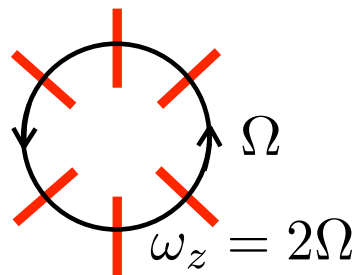
$$\frac{1}{2} \left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} \right) = \frac{\omega_z}{2}$$

Cisaillement simple



$$\omega_z = \frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y} = -\dot{\gamma}$$

Rotation solide



$$u_\theta = \Omega r$$

$$\omega_z = \frac{1}{r} \frac{\partial}{\partial r} (r u_\theta) = 2\Omega$$

Un tourbillon sans vorticité ?

Ecoulement irrotationnel

$$\mathbf{u} = \nabla \phi$$

Φ potentiel de vitesse

Fluide incompressible

$$\nabla \cdot \mathbf{u} = \Delta \phi = 0$$

Pas de composante radiale

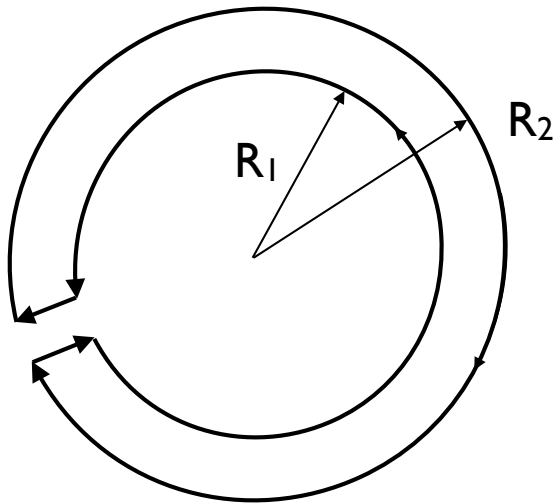
$$u_r = \frac{\partial \phi}{\partial r} = 0$$

$$\frac{\partial^2 \phi}{\partial \theta^2} = 0$$

$$\phi = A\theta$$

$$u_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = \frac{A}{r}$$

Un tourbillon sans vorticit   ?



$$\oint \mathbf{u} \cdot d\mathbf{l} = 2\pi R_1 u_\theta(R_1) - 2\pi R_2 u_\theta(R_2) = \int \omega dS = 0$$

$$\frac{u_\theta(R_1)}{u_\theta(R_2)} = \frac{R_2}{R_1}$$

$$u_\theta(r) = \frac{\Gamma}{2\pi r}$$

Γ Circulation du tourbillon

Un tourbillon sans vorticit  ?

Ecoulement irrotationnel, sans viscosit  : Bernoulli $p_{\infty} = p + \frac{1}{2}\rho u^2$

$$p = p_{\infty} - \frac{\rho \Gamma^2}{8\pi^2 r^2}$$

Forte tornade, $U \sim 100 \text{ m/s}$, $\Delta p \sim 10^4 \text{ Pa}$



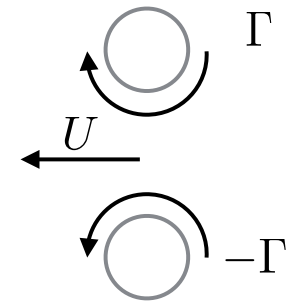
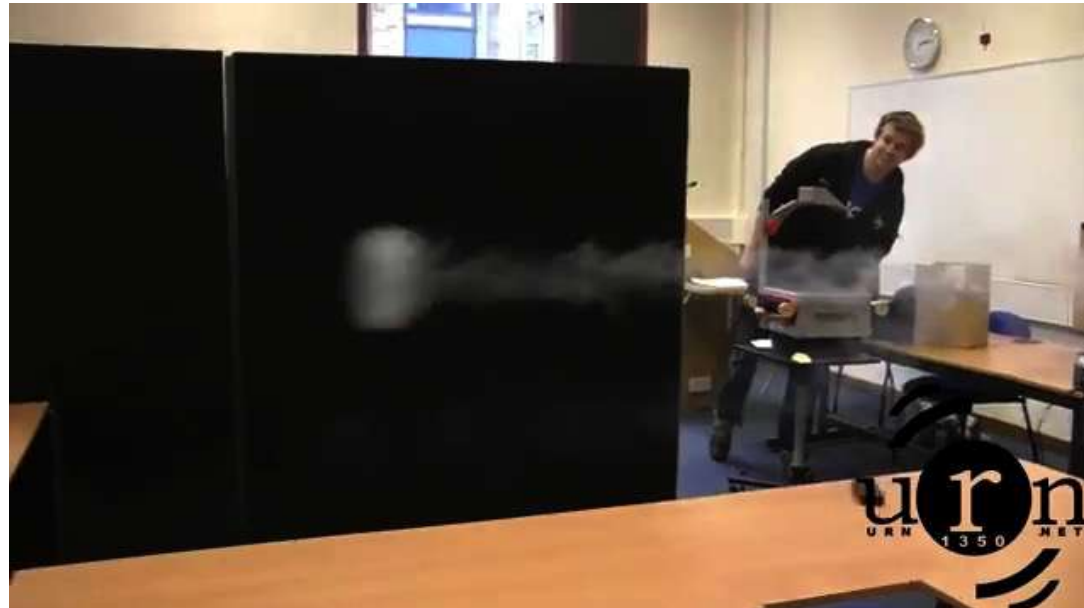
Image NOAA



Canon à vortex

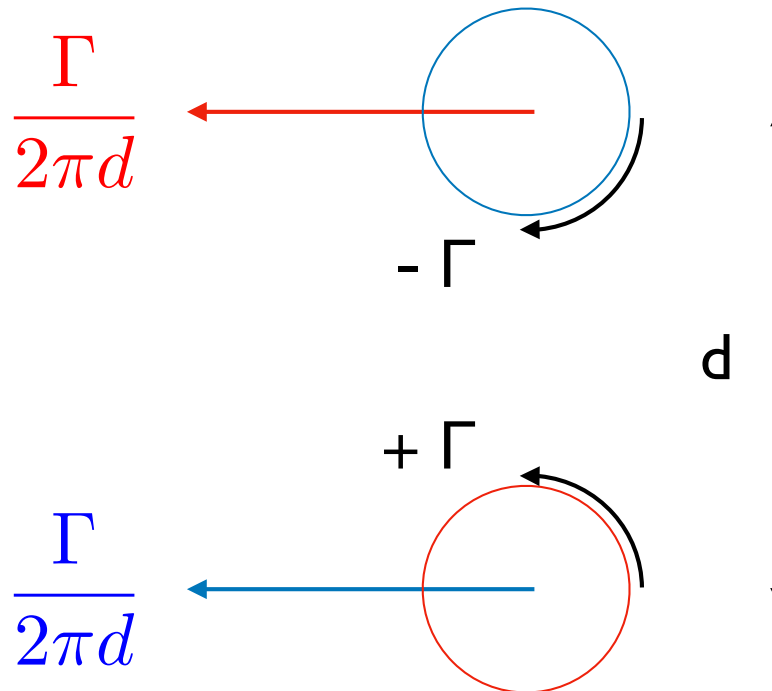
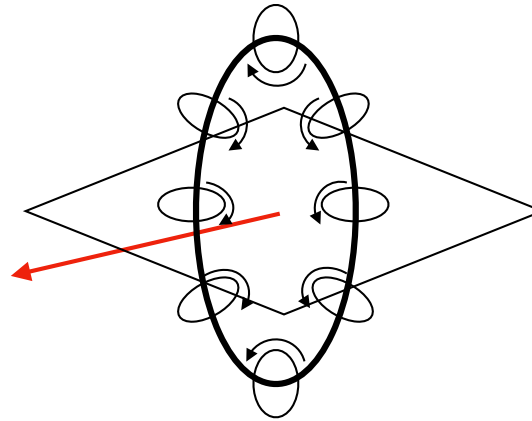


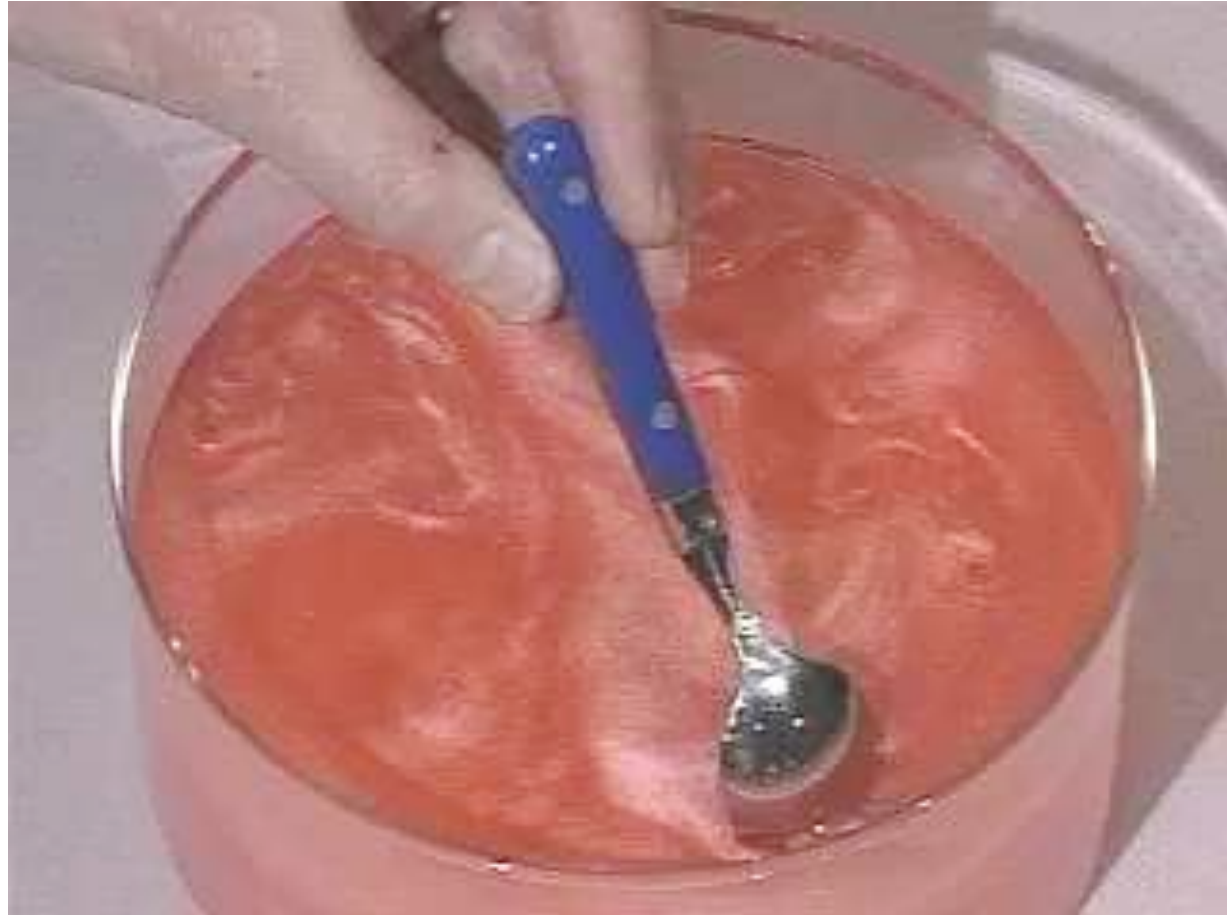
Canon à vortex



Canon à vortex

Vortex annulaire

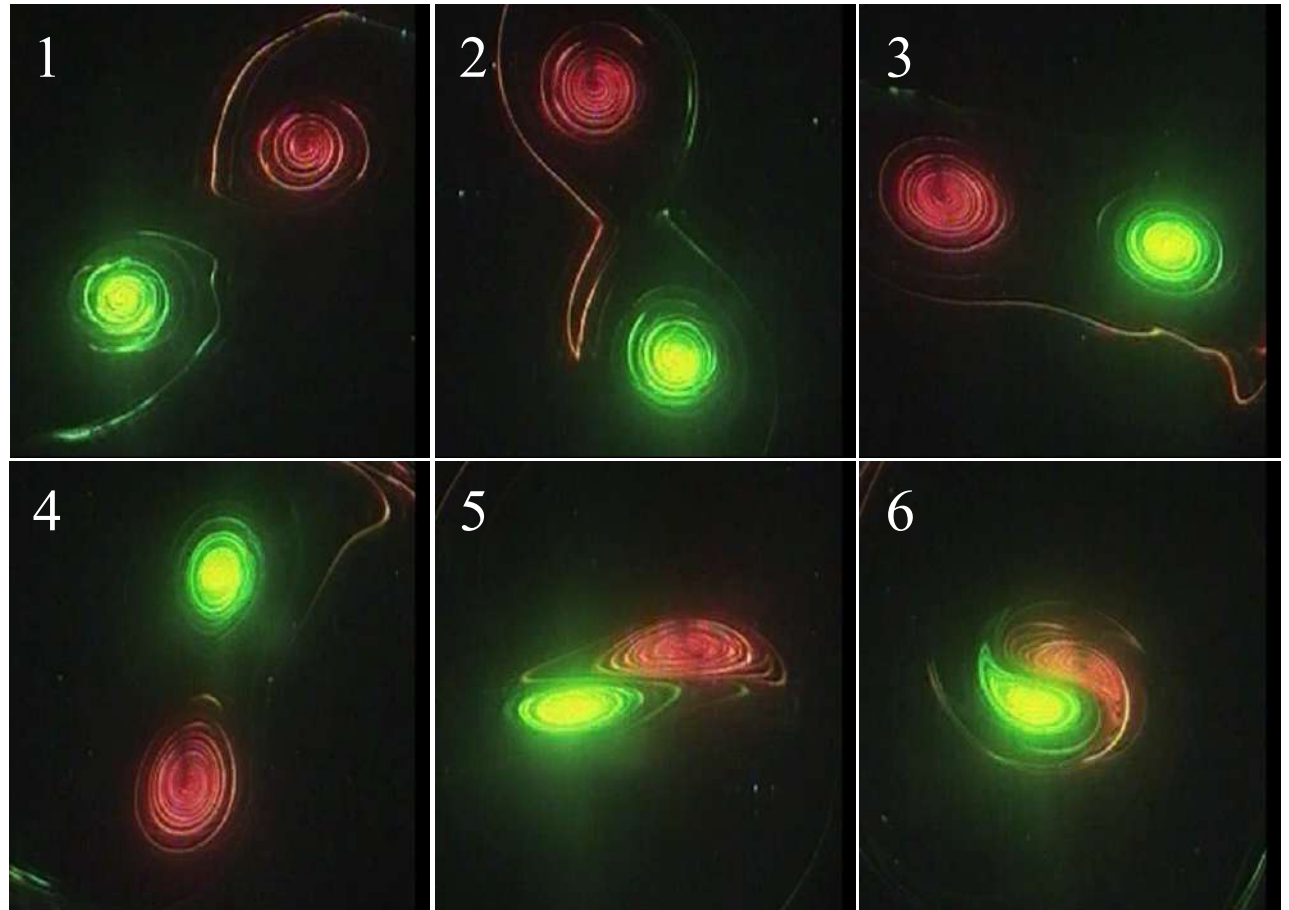
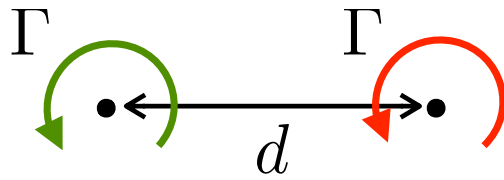
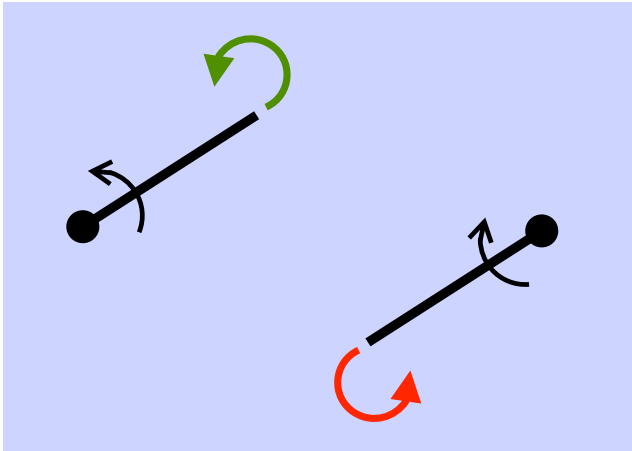




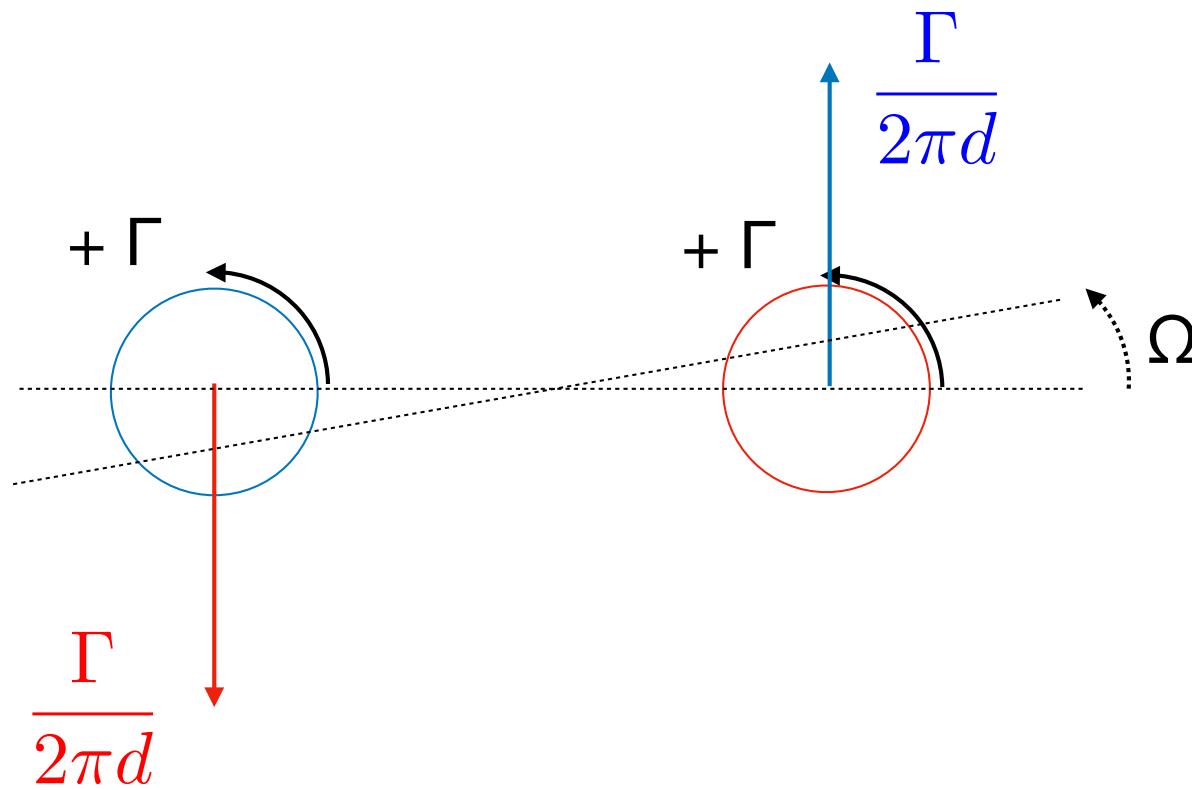
Valse de tourbillons



Valse de tourbillons



Valse de tourbillons



$$\Omega \frac{d}{2} = \frac{\Gamma}{2\pi d}$$

$$\Omega = \frac{\Gamma}{\pi d^2}$$

Dissipation des tourbillons



Tourbillon stationnaire

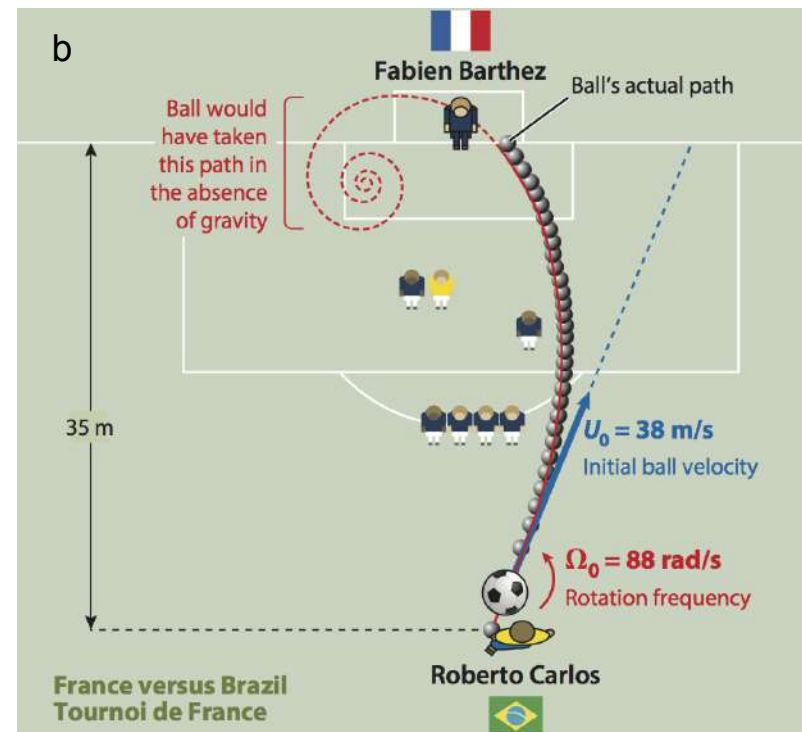
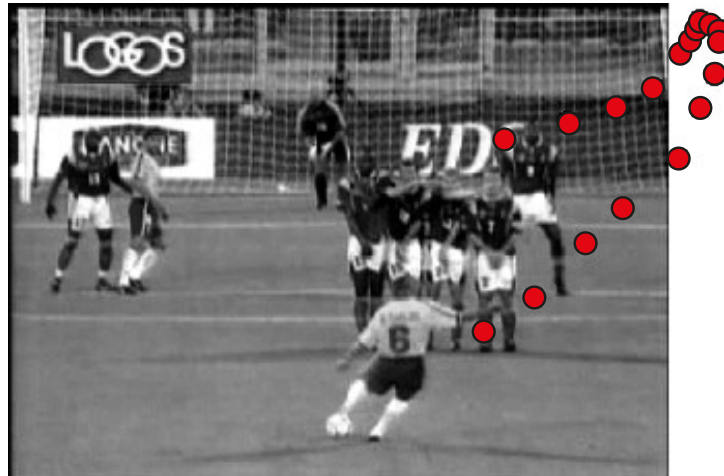


Portance

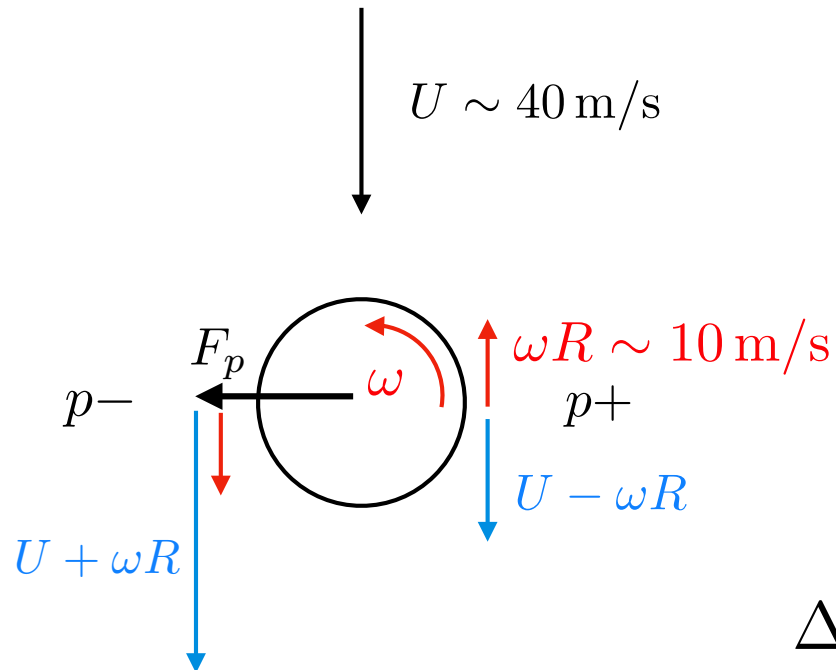
Le but de Roberto Carlos



a



Le but de Roberto Carlos : force latérale sur le ballon



$$\Delta p = \frac{1}{2} \rho [(U + \omega R)^2 - (U - \omega R)^2]$$

$$\Delta p = 2 \rho U \omega R$$

$$F_p \sim \Delta p \pi R^2 = 2 \pi \rho U \omega R^3$$

$$\text{A.N. : } F_p \simeq 6.1.40.100.10^{-3} \simeq 24 \text{ N}$$

Portance sur un cylindre tournant

A 2D, un écoulement potentiel incompressible peut être décrit :

par une fonction potentielle ϕ telle que : $\mathbf{u} = \nabla \phi$

et par une fonction de courant ψ telle que : $\mathbf{u} = \nabla \wedge (\psi \mathbf{e}_z)$

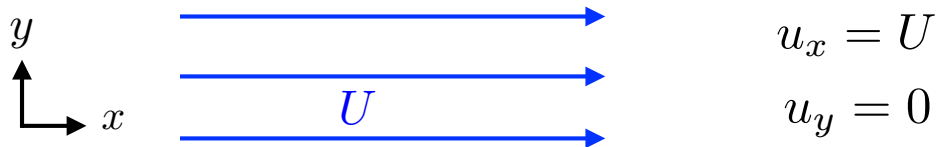
Rappel : en coordonnées cartésiennes à 2D

$$\nabla u = \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \quad \left| \quad \begin{array}{l} u_x = \frac{\partial \psi}{\partial y} \\ u_y = -\frac{\partial \psi}{\partial x} \end{array} \right.$$

$$\Delta \phi = \Delta \psi = 0$$

Une fonction potentielle ϕ telle que : $\mathbf{u} = \nabla \phi$
 Une fonction de courant ψ telle que : $\mathbf{u} = \nabla \wedge (\psi \mathbf{e}_z)$

- Ecoulement parallèle uniforme

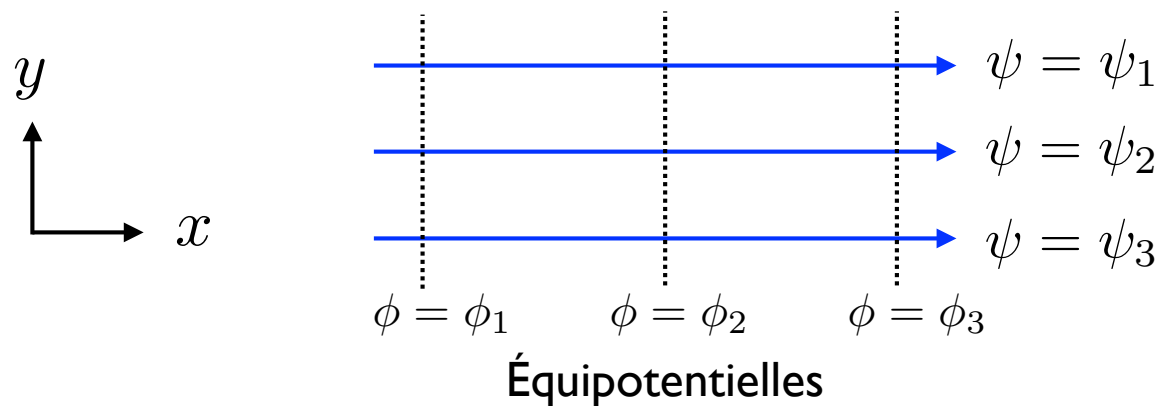


$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} = u_x = U$$

$$\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x} = u_y = 0$$

$$\phi = Ux$$

$$\psi = Uy$$



Lignes de courant

Une fonction potentielle ϕ telle que : $\mathbf{u} = \nabla \phi$
 Une fonction de courant ψ telle que : $\mathbf{u} = \nabla \wedge (\psi \mathbf{e}_z)$

- Tourbillon autour de l'axe Oz

$$u_r = 0$$

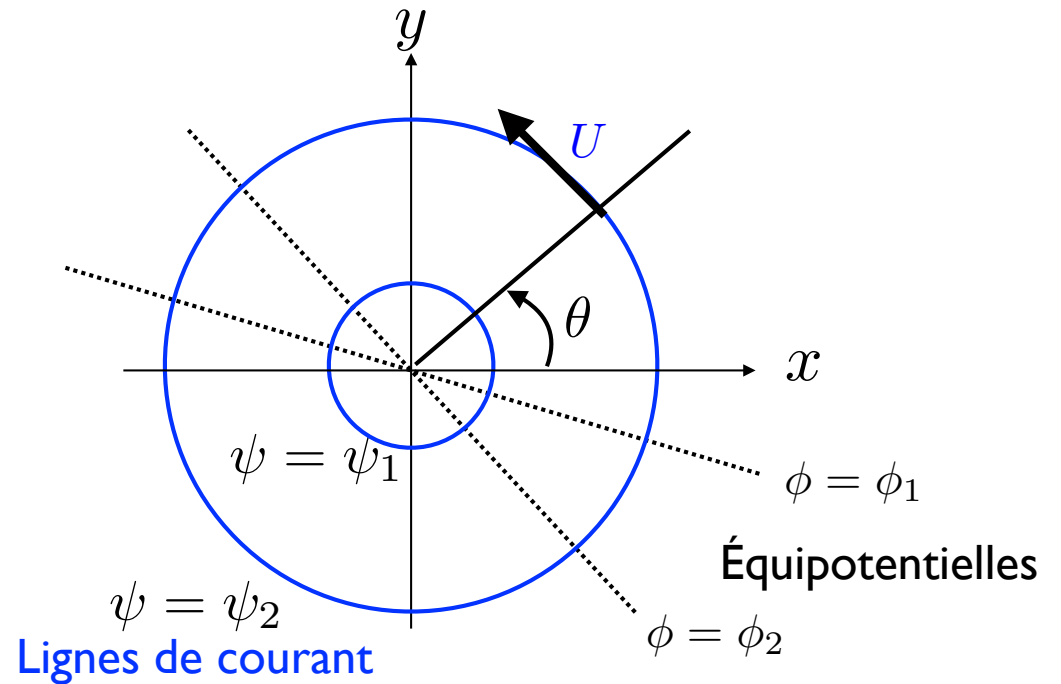
$$u_\theta = \frac{\Gamma}{2\pi r}$$

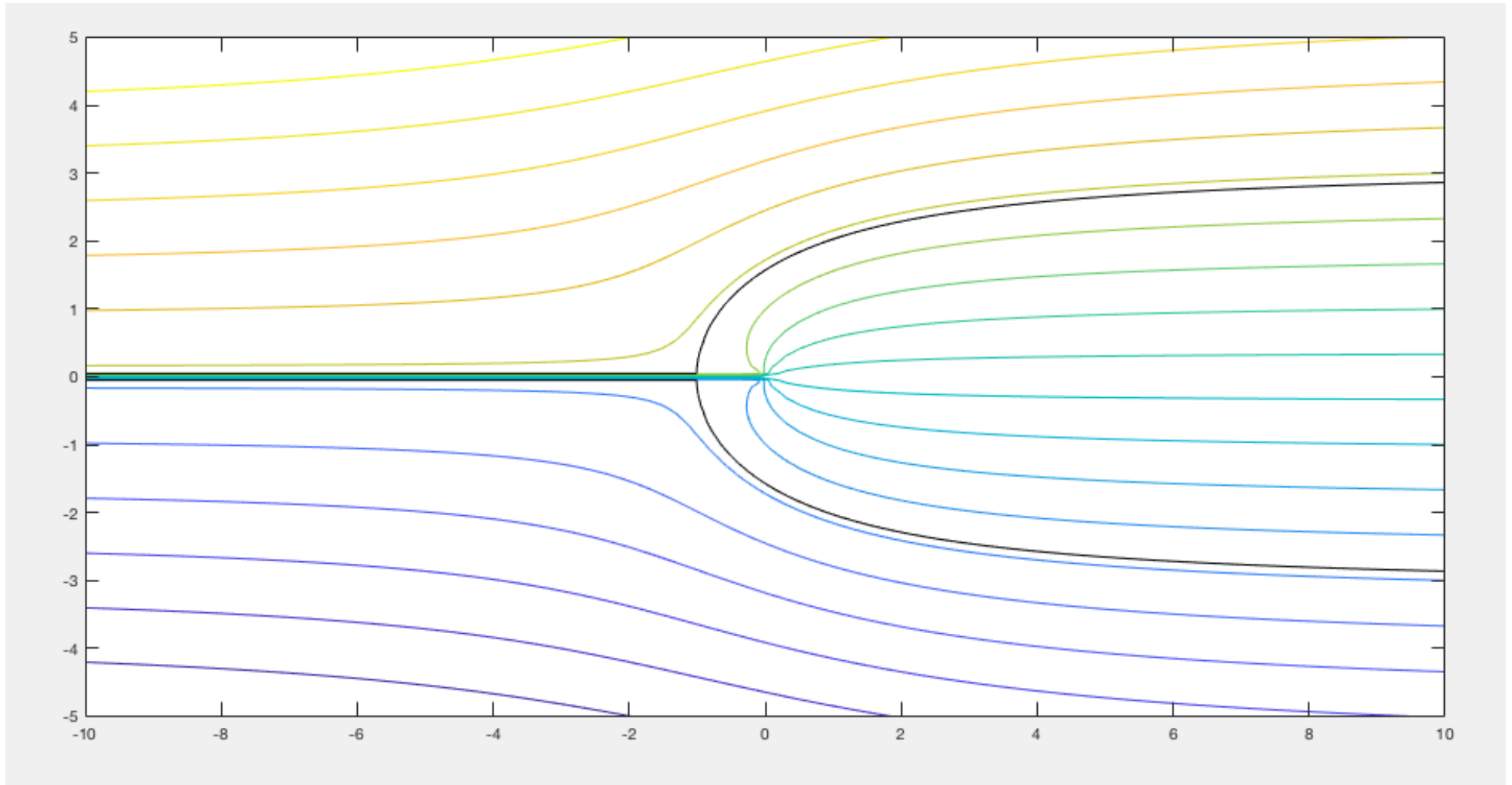
$$\frac{\partial \phi}{\partial r} = \frac{1}{r} \frac{\partial \psi}{\partial \theta} = u_r = 0$$

$$\frac{1}{r} \frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r} = u_\theta = \frac{\Gamma}{2\pi r}$$

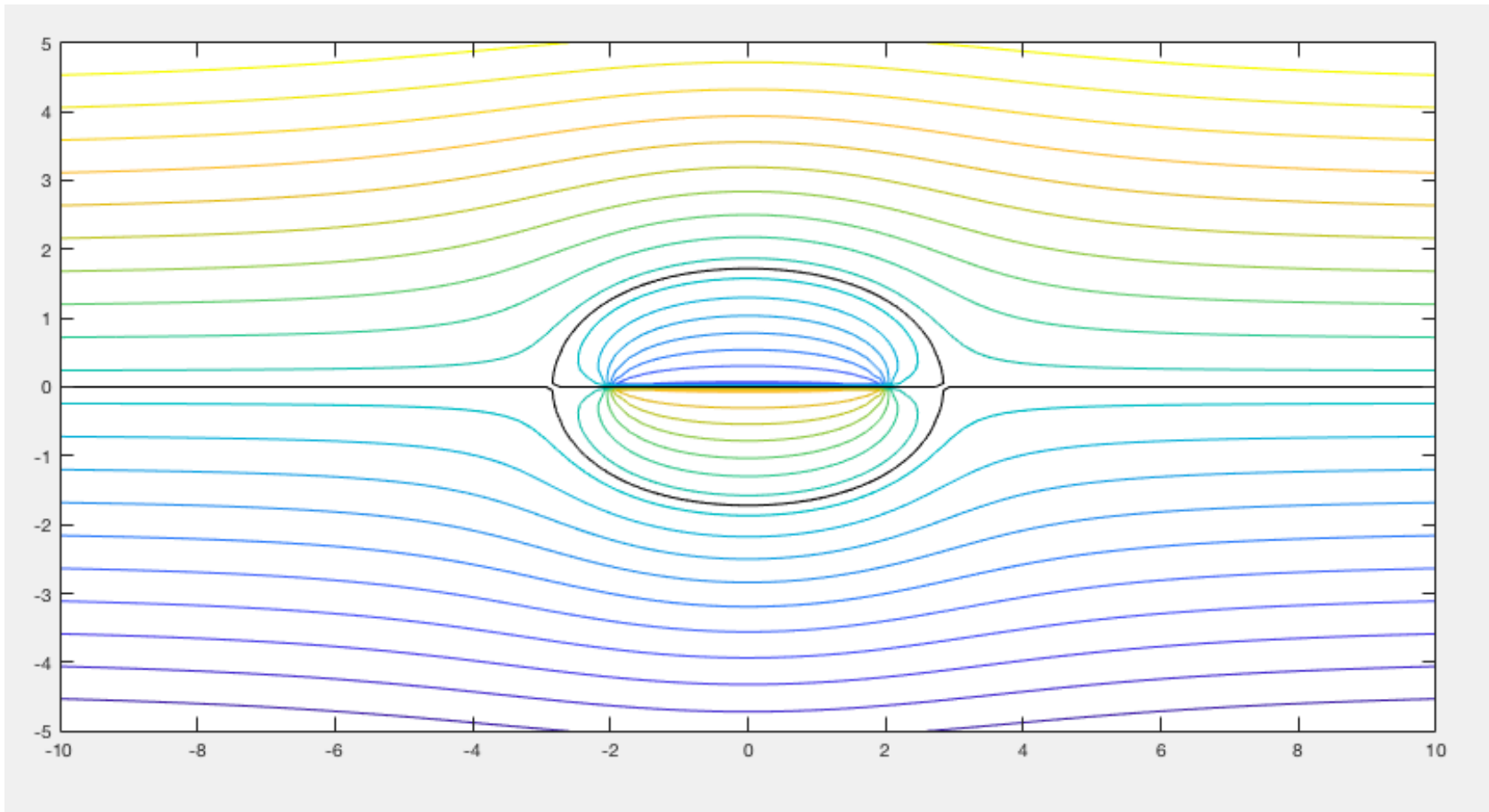
$$\phi = \frac{\Gamma \theta}{2\pi}$$

$$\psi = -\frac{\Gamma}{2\pi} \ln \frac{r}{r_0}$$

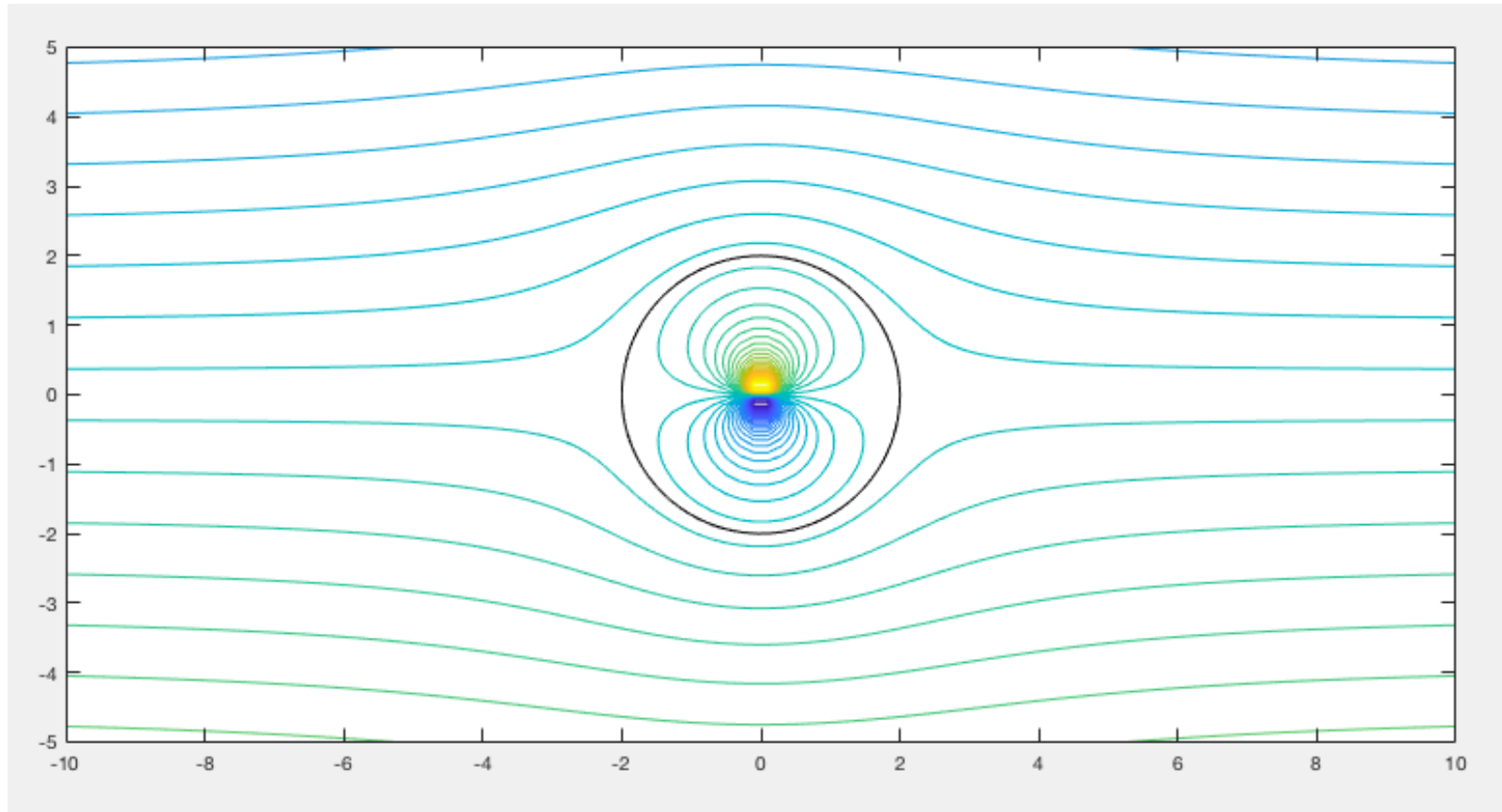




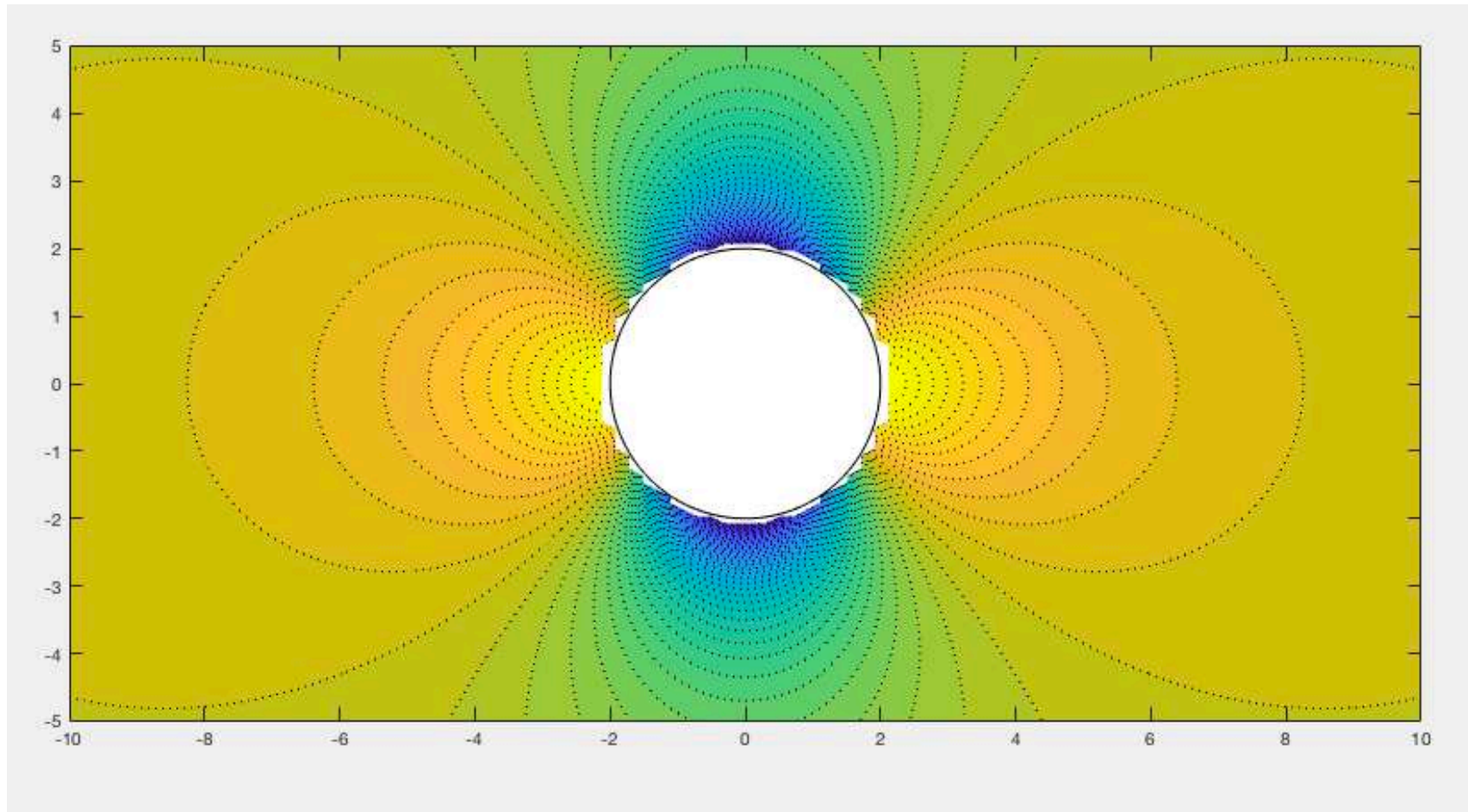
Uniforme + source : lignes de courant



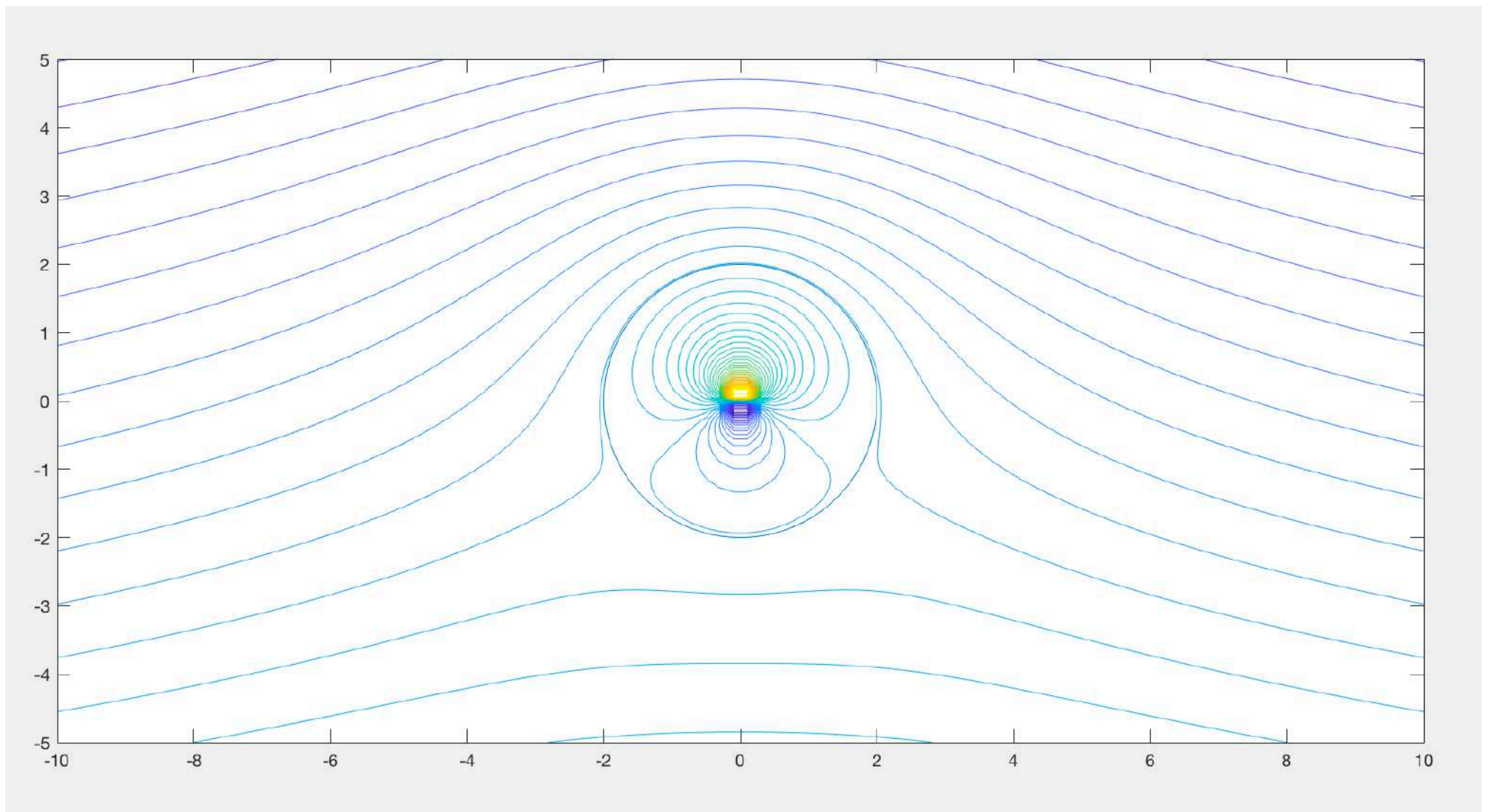
Uniforme + source+ puits : lignes de courant



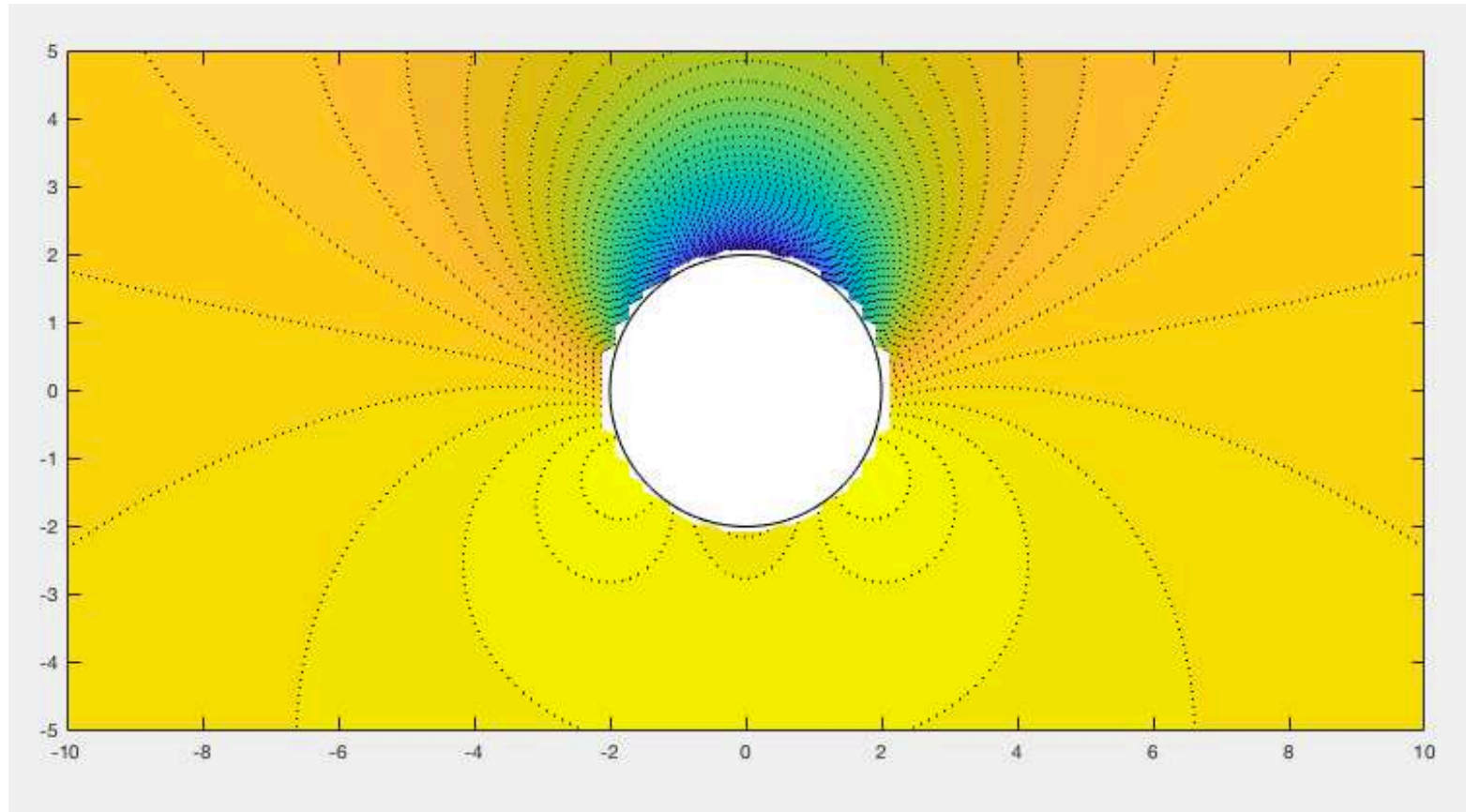
Uniforme + dipôle : lignes de courant



Uniforme + dipôle : champ de pression

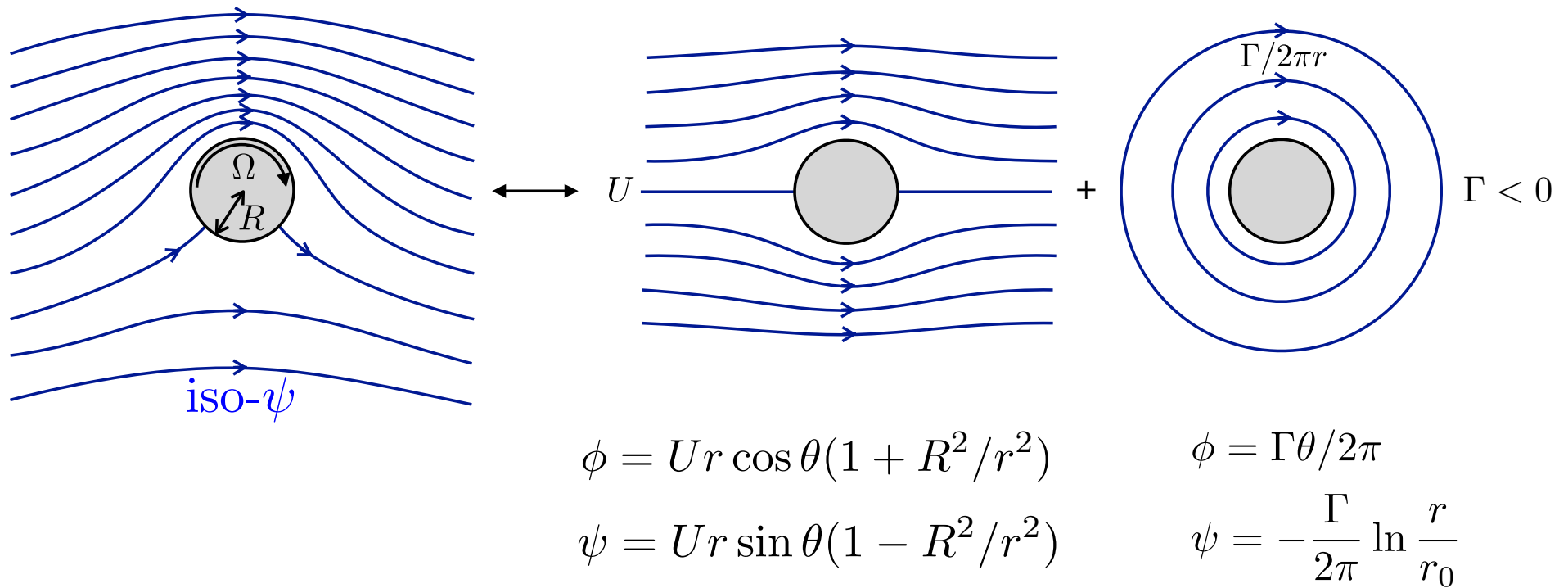


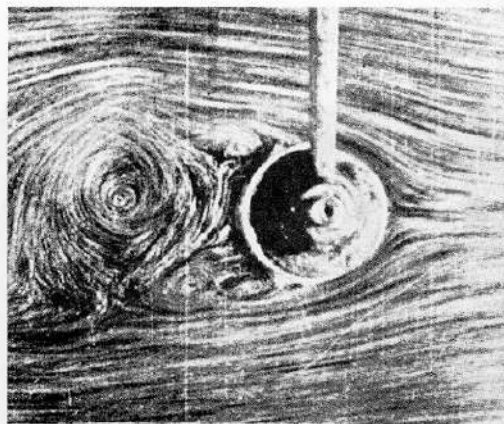
Uniforme + dipôle + vortex



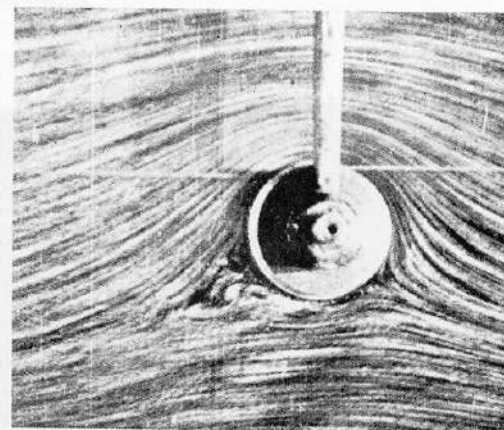
Uniforme + dipôle + vortex : champ de pression

Portance sur un cylindre tournant

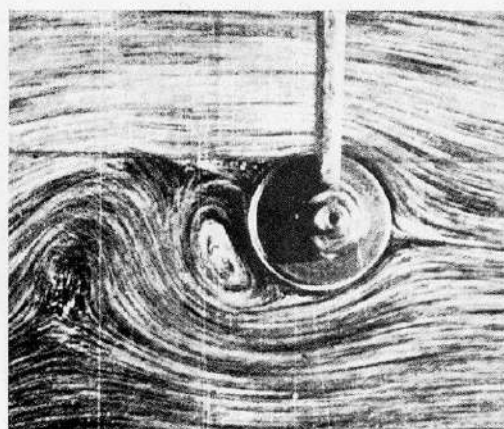




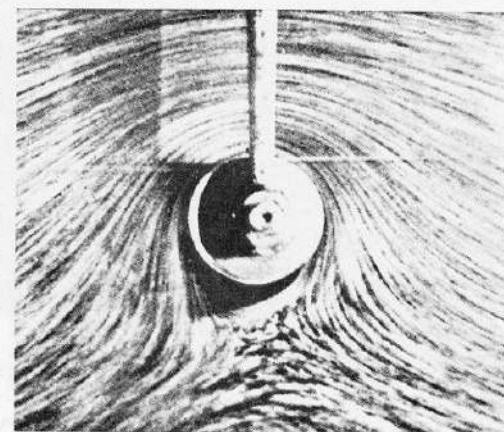
$$a\Omega/U = 0$$



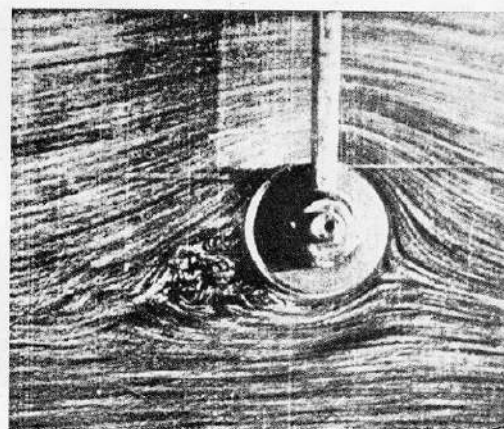
$$a\Omega/U = 3$$



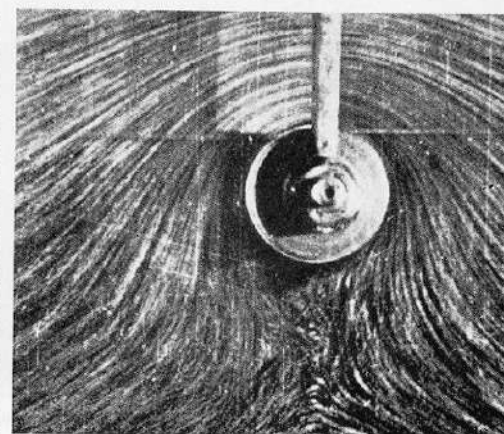
$$a\Omega/U = 1$$



$$a\Omega/U = 4$$



$$a\Omega/U = 2$$



$$a\Omega/U = 6$$

La pression sur le cylindre est donnée par l'équation de Bernoulli :

$$p + \frac{1}{2}\rho u^2(R) = p_\infty + \frac{1}{2}\rho U^2 \quad \left(\begin{array}{l} u_\theta(R) = -2U \sin \theta + \frac{\Gamma}{2\pi R} \\ u_r(R) = 0 \end{array} \right)$$

$$p = p_\infty + \frac{1}{2}\rho U^2 (1 - 4 \sin^2 \theta - \gamma^2 + 4\gamma \sin \theta) \quad \text{avec} \quad \gamma = \frac{\Gamma}{2\pi R U}$$

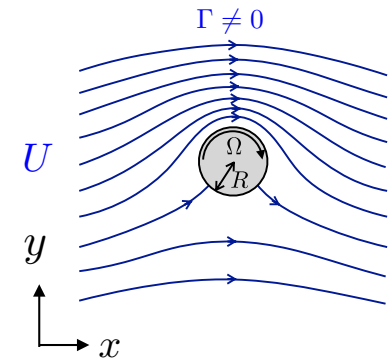
la circulation réduite

La **trainée par unité de longueur de cylindre** est donnée par :

$$F_t = F_x = - \int_0^{2\pi} p \cos \theta \, R d\theta$$

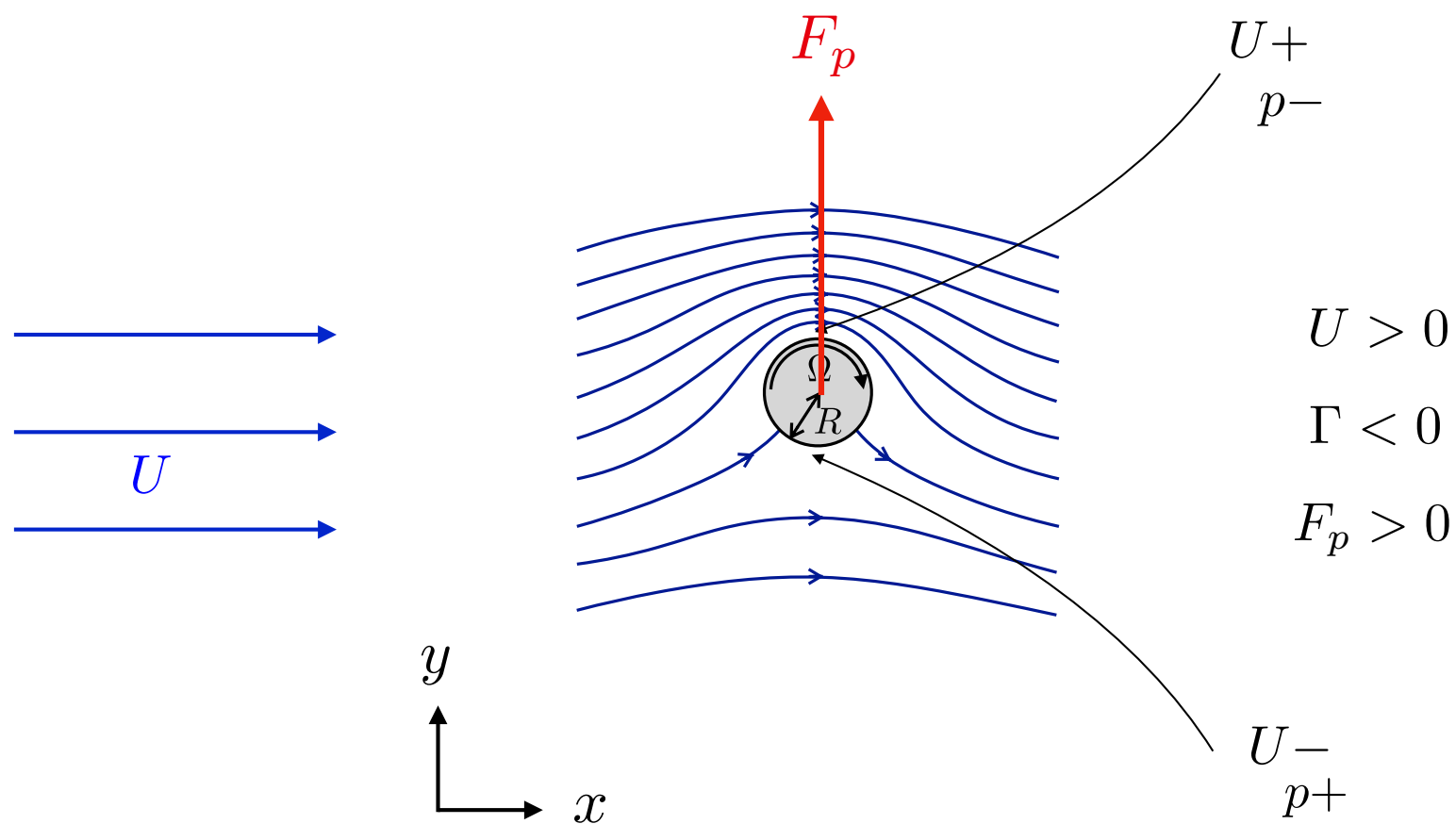
La **portance par unité de longueur de cylindre** est donnée par :

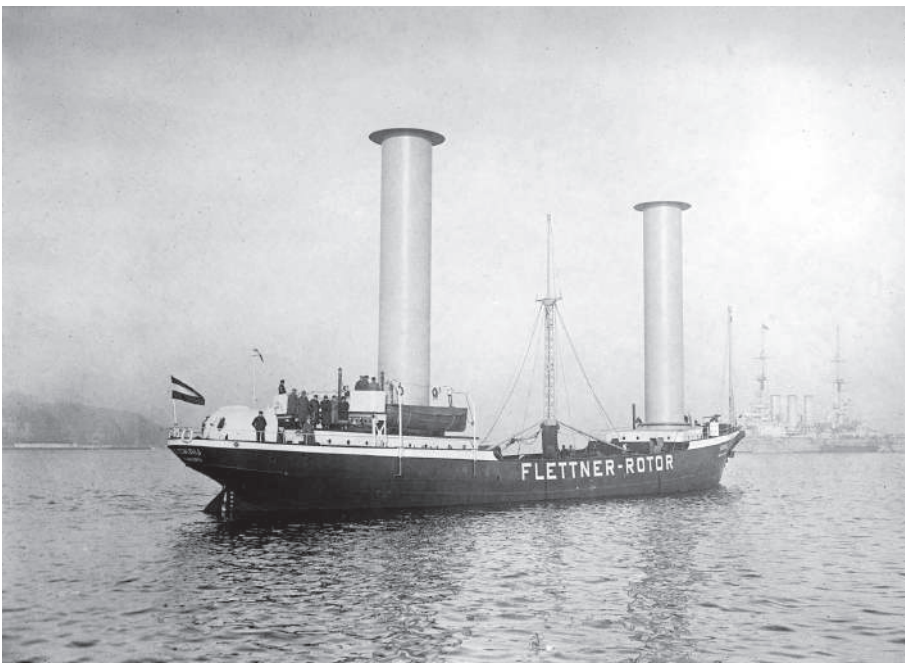
$$F_p = F_y = - \int_0^{2\pi} p \sin \theta \, R d\theta$$



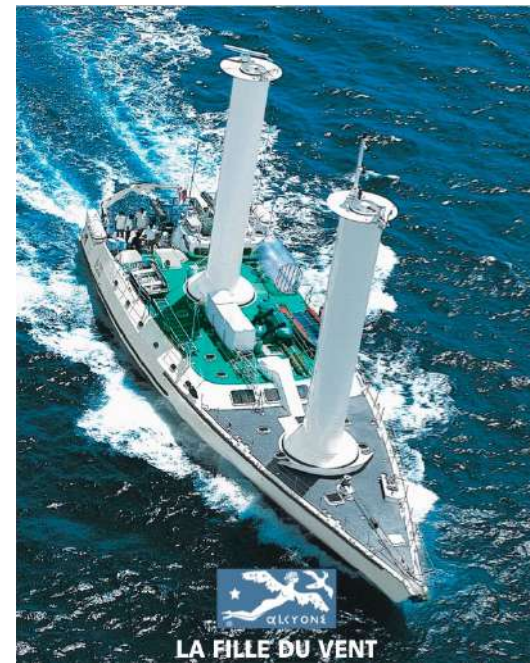
$$F_p = -\rho U \Gamma \quad \text{Formule générale de Joukovski qui lie la portance à la circulation}$$

$$F_p = -\rho U \Gamma$$





Rotorship Flettner ca 1930

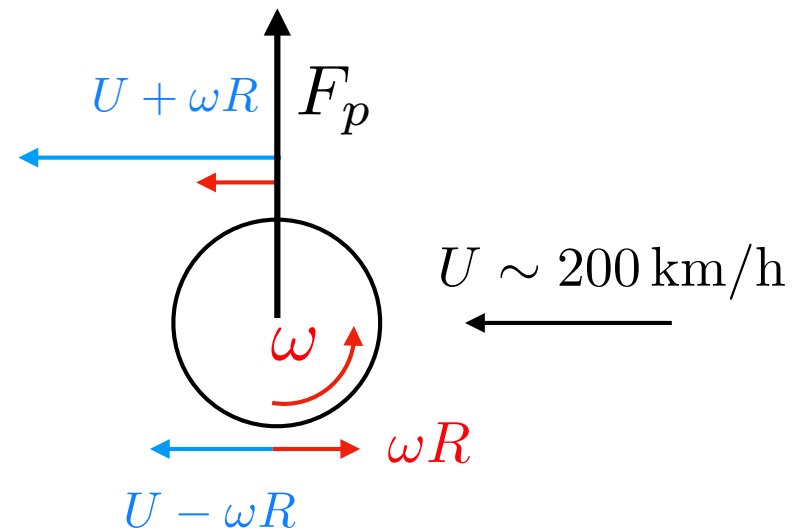
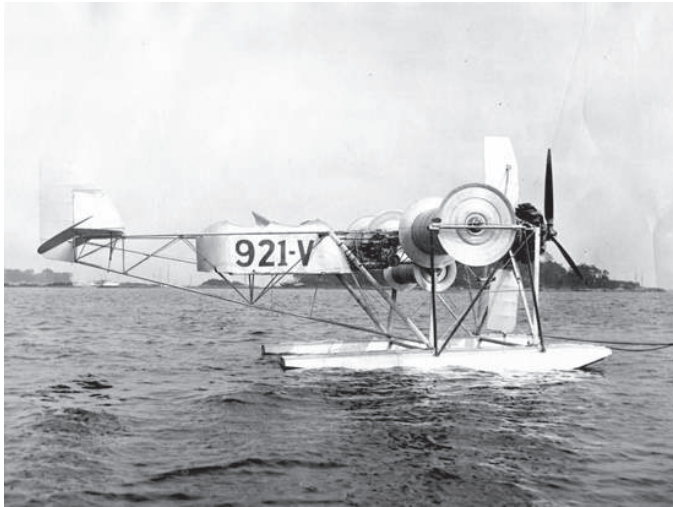


Turbovoile Malavard-Cousteau



E Ship I 2010

Un avion à ailes tournantes ?



$$F_p \sim \rho U \Gamma W \quad \text{et} \quad \Gamma = 2\pi R(R\omega)$$

$$F_p \sim \rho U (2\pi R^2 \omega) W$$

$$\omega \sim \frac{Mg}{2\pi \rho R^2 U W}$$

$$M \simeq 500 \text{ kg}, U \simeq 50 \text{ m/s}, W = 4 \text{ m}, R = 0.3 \text{ m}$$

$$\omega \simeq 40 \text{ rad/s} \simeq 6,5 \text{ tours/s}$$

