Dans un fluide newtonien, avec écoulement macroscopique

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\mathbf{u}\right)=\rho\mathbf{f}+\nabla.\sigma$$

$$\nabla.\sigma=-\nabla p+\eta\Delta\mathbf{u}$$

Équation de Navier-Stokes

$$\rho\frac{\partial \mathbf{u}}{\partial t}+\rho(\mathbf{u}.\nabla)\mathbf{u}=-\nabla p+\eta\Delta\mathbf{u}+\rho\mathbf{f}$$

$$\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}.\nabla)\mathbf{u}=-\frac{1}{\rho}\nabla p+\nu\Delta\mathbf{u}+\mathbf{f}$$

viscosité cinématique $\nu=\eta/\rho$

L'équation de Navier-Stokes dans toute sa splendeur (horreur ?)

$$\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2} \right) + f_x$$

$$\frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2} \right) + f_y$$

$$\frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2} \right) + f_z$$

Comment la simplifier ?

Une analyse dimensionnelle de l'équation de Navier-Stokes

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\nabla)\mathbf{u} = -\nabla p + \eta \Delta \mathbf{u} + \rho \mathbf{f}$$

$$\mathbf{u} \sim U$$

$$\nabla \sim 1/L \qquad \Delta \sim 1/L^2$$

Échelle de longueur pertinente qui caractérise la variation spatiale de vitesse

$$\rho\ \mathbf{u}.\nabla\mathbf{u} \sim \rho\ \frac{U^2}{L}$$

Terme inertiel

$$\eta\ \Delta\mathbf{u} \sim \eta\ \frac{U}{L^2}$$

Terme visqueux

$$\frac{\rho\mathbf{u}.\nabla\mathbf{u}}{\eta\Delta\mathbf{u}} \sim \frac{\rho U L}{\eta} = \frac{UL}{\nu}$$

Nombre de Reynolds

Quelles conditions aux limites pour le champ de vitesse ?

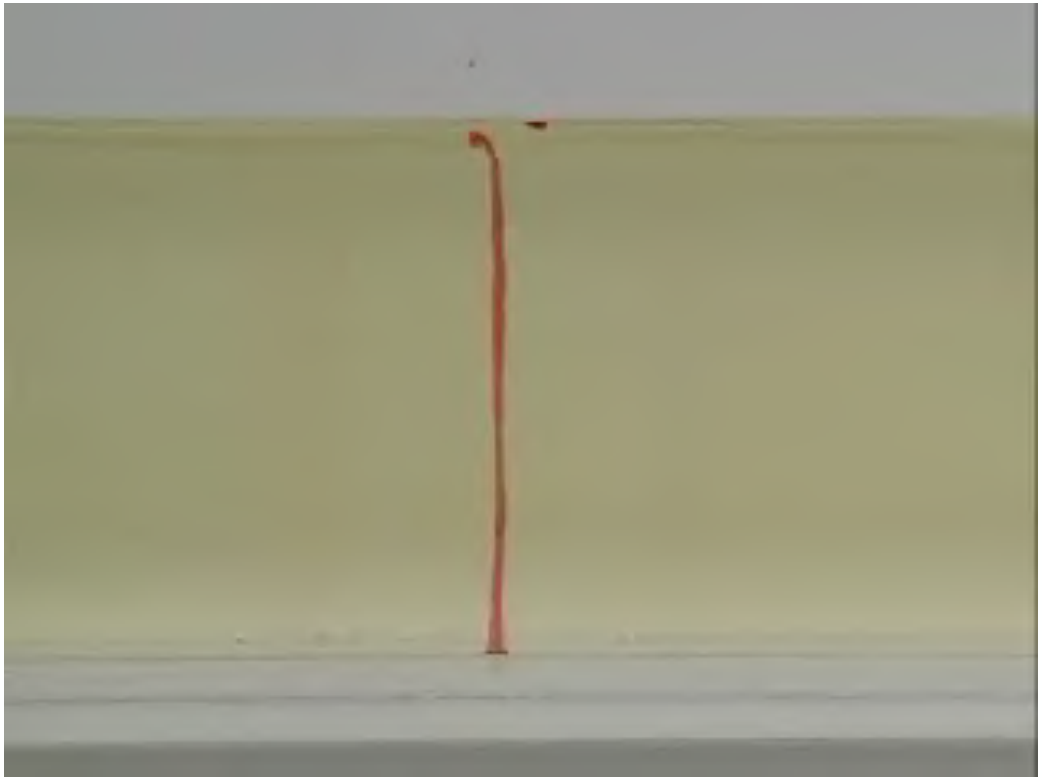
Interface fluide/solide :

condition de non glissement à la paroi solide

Interface entre deux fluides non miscibles :

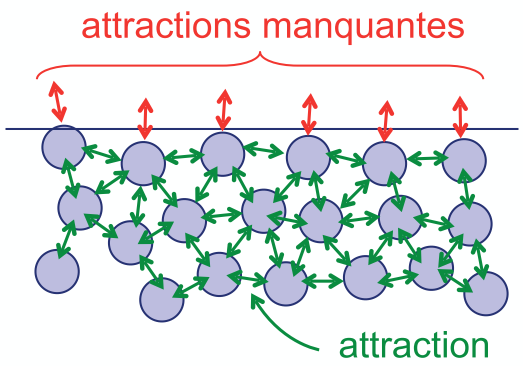
continuité des vitesses

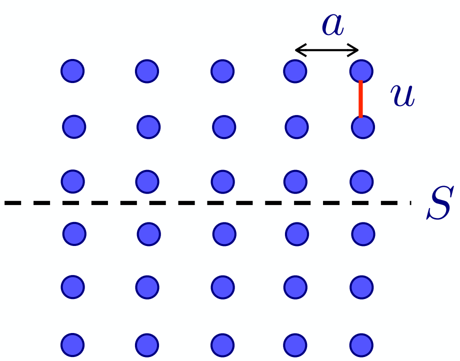
continuité des contraintes



La tension de surface et la pression capillaire

Interfaces





u énergie d'interaction entre deux molécules

Coût énergétique pour créer S :

$$U_s \sim \frac{S}{a^2} u = \gamma S$$

γ mesure ce défaut d'énergie par unité de surface

$$\gamma \sim \frac{u}{a^2} \sim \frac{kT}{a^2}$$

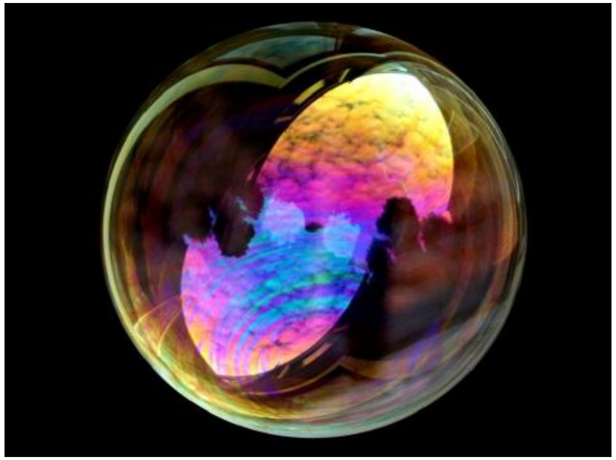
$$\left. \begin{array}{c} u \sim 2 - 3\,kT \sim 10^{-20}\,\mathrm{J} \\ a \sim 0.3\,\mathrm{nm} \end{array} \right\} \gamma \sim 100\,\mathrm{mJ/m^2} \quad \mathrm{ou\ mN/m}$$

$$\gamma_{eau} = 72\ \mathrm{mN/m}$$

$$\gamma_{huile} \simeq 20\ \mathrm{mN/m}$$

$$U_S = \gamma S$$

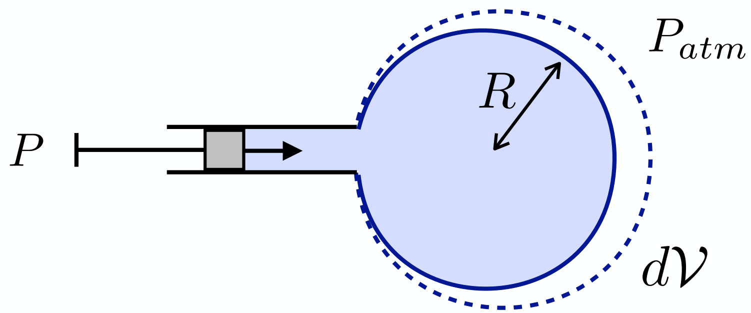
La sphère est l'objet qui minimise
sa surface à volume fixé



Hergé "On a marché sur la lune"

Scott Kelly www.nasa.gov/station

Une goutte gonflée



On applique une surpression P par rapport à P_{atm}

$$dW_{piston} = P\, d\mathcal{V}$$

$$dW_{atm} = -P_{atm}\, d\mathcal{V}$$

Travail reçu par la goutte :

$$dW = dW_{piston} + dW_{atm} = (P - P_{atm})\, d\mathcal{V}$$

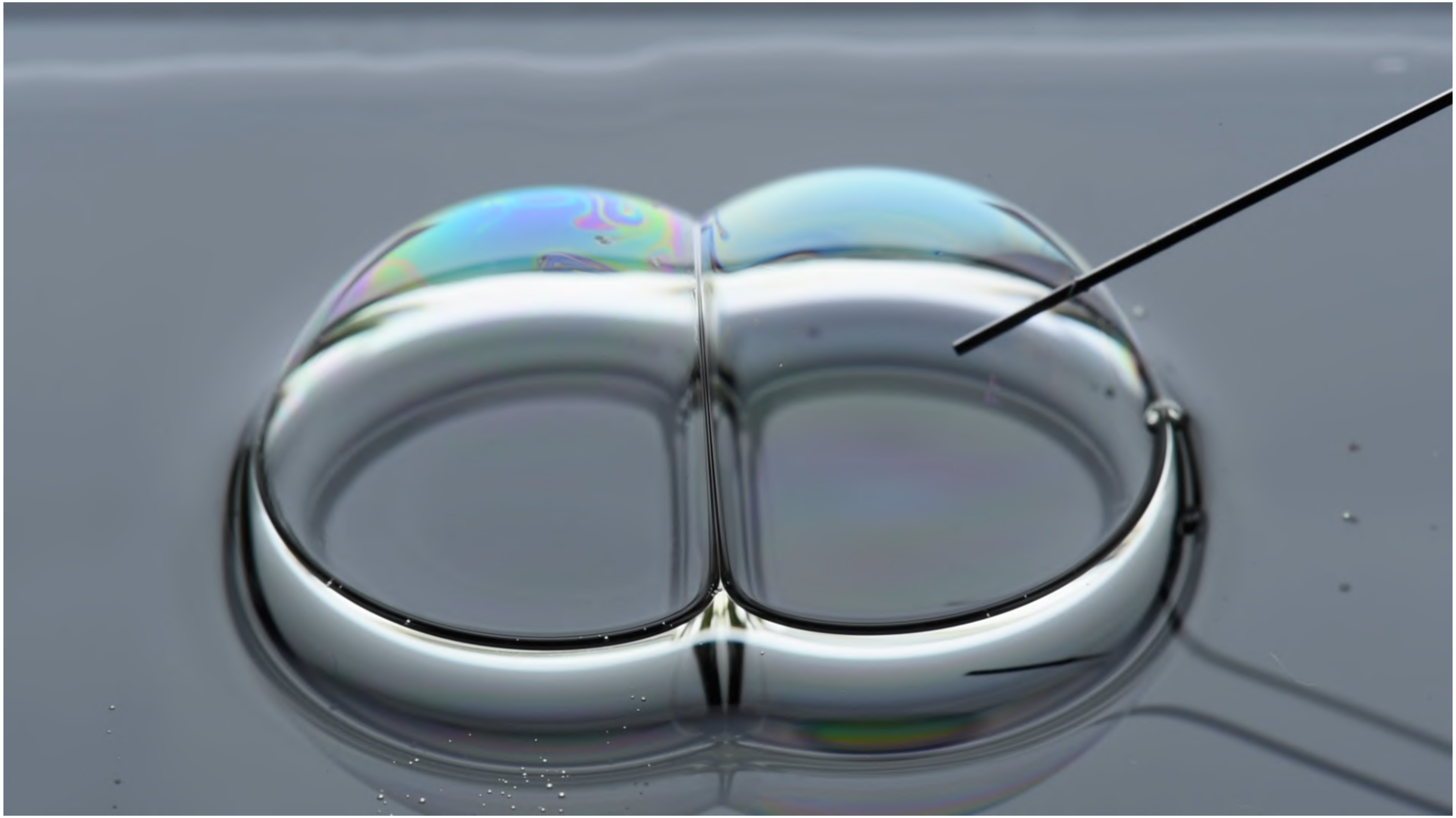
$$d\mathcal{V} = d(\frac{4}{3}\pi R^3) = 4\pi R^2\, dR$$

$$dU_s = d(4\pi R^2 \gamma) = 8\pi R\gamma\, dR$$

$$dU = dU_s = dW$$

$$(P - P_{atm})\, 4\pi R^2\, dR = 8\pi R\gamma\, dR$$

$$P - P_{atm} = \frac{2\gamma}{R}$$



$$P_{in} - P_{atm} = \frac{4\gamma}{R}$$

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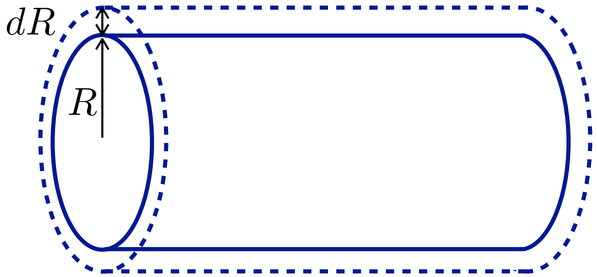


$$U_s = \gamma\, 2\pi H\, dR$$

$$d\mathcal{V} = 2\pi R H\, dR$$

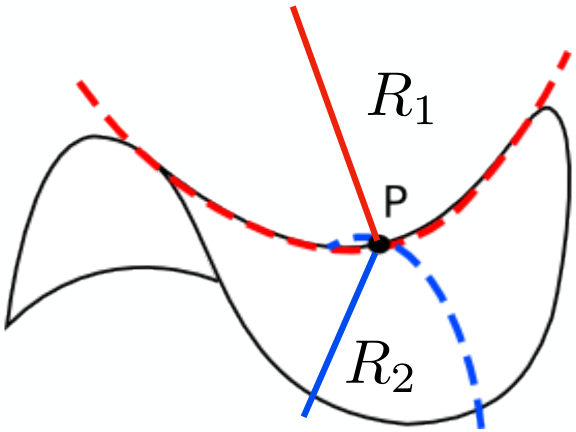
$$(P - P_{atm})\, 2\pi R H\, dR = 2\pi H \gamma\, dR$$

$$\boxed{P - P_{atm} = \frac{\gamma}{R}}$$

Dans le cas général :

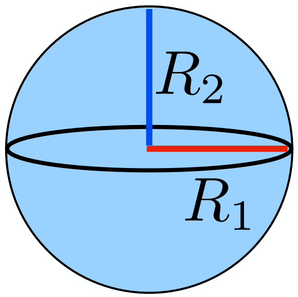
$$\Delta P = \gamma\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \gamma \mathcal{C}$$

Saut de pression de Laplace

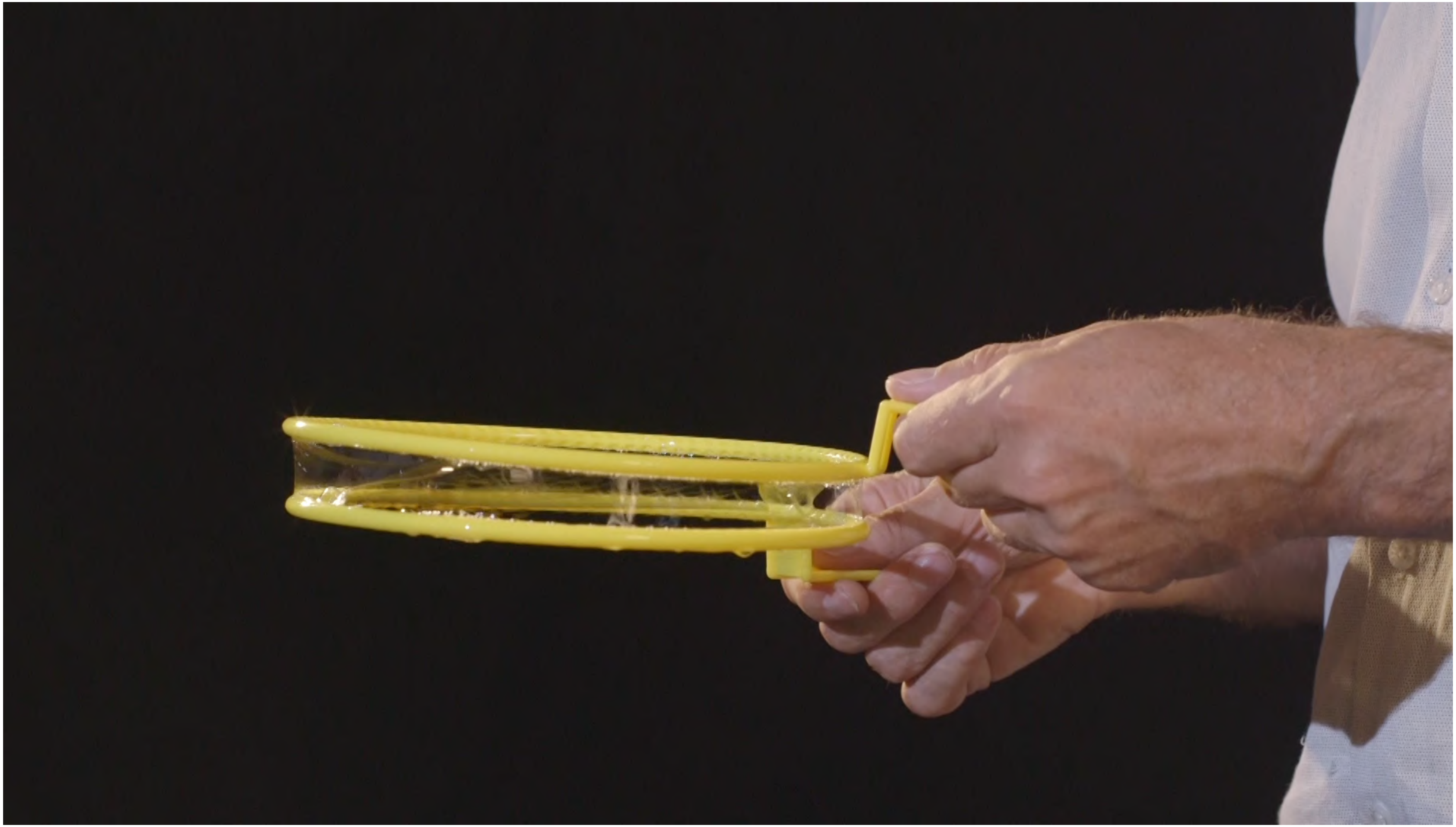


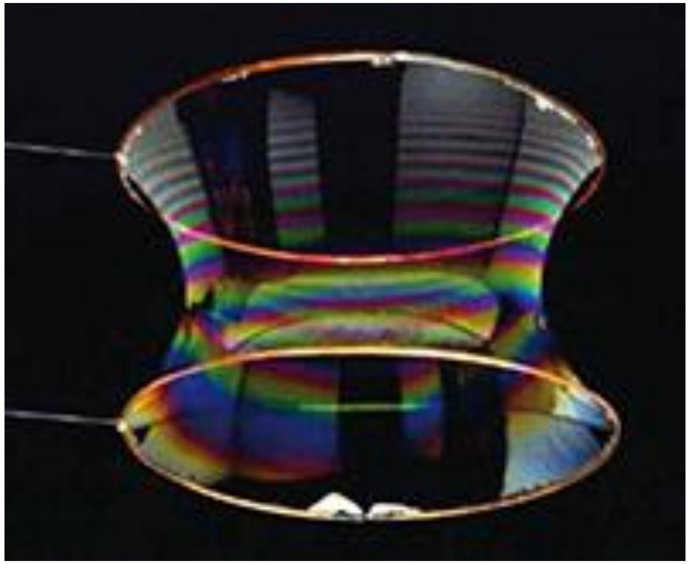
$R_1 > 0$

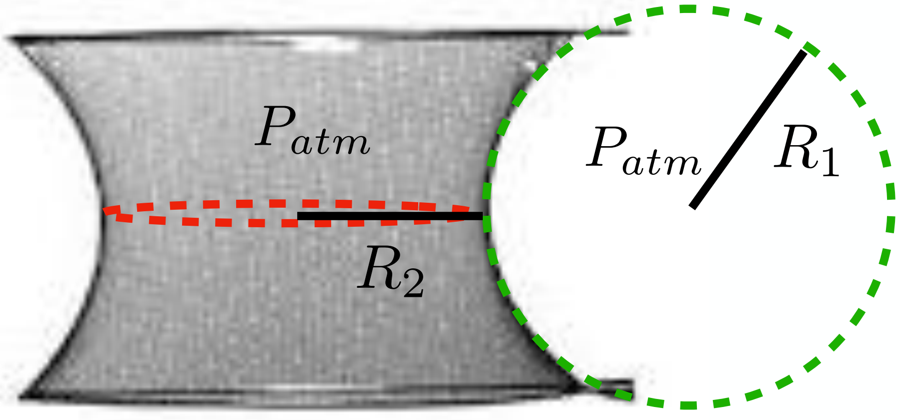
$R_2 < 0$



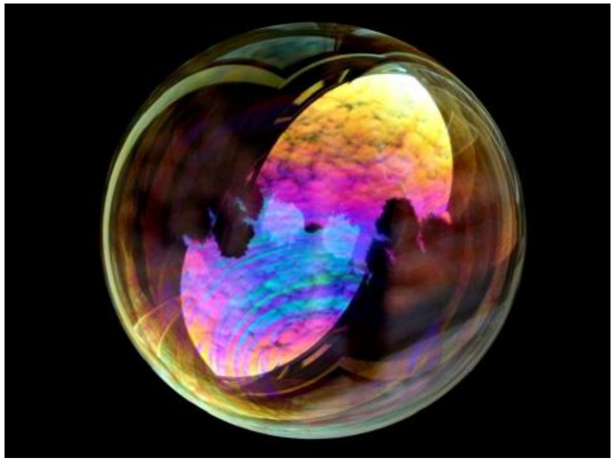
$R_1 = R_2 = R$

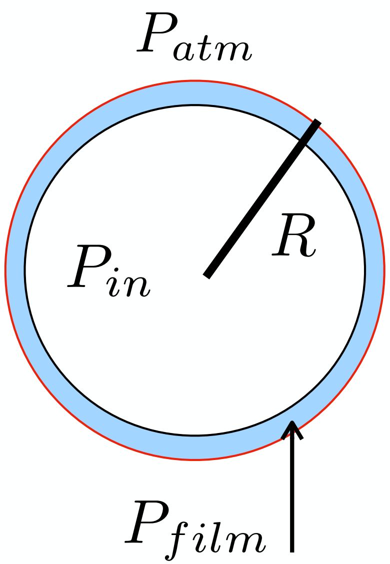






$$R_1 = -R_2$$



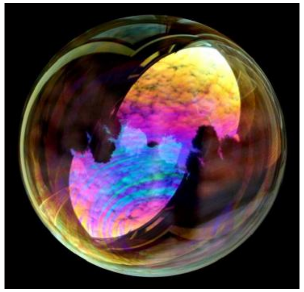


$$P_{film} - P_{atm} = \frac{2\gamma}{R}$$

$$P_{in} - P_{film} = \frac{2\gamma}{R}$$

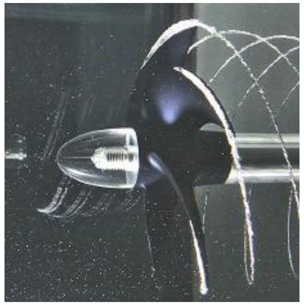
$$\boxed{P_{in} - P_{atm} = \frac{4\gamma}{R}}$$

Pression de Laplace : quelques ordres de grandeur…

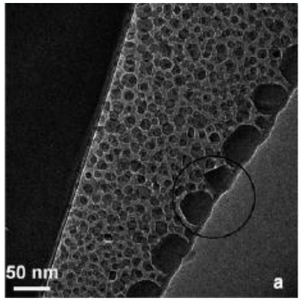


Bulles de savon : $R \sim 5$ cm, $\gamma \sim 35$ mN/m
$\Delta P \sim 3$ Pa $\sim 3.10^{-5}$ atm

Bulles de champagne : $R \sim 0.1$ mm, $\gamma \sim 50$ mN/m
$\Delta P \sim 1000$ Pa $\sim 10^{-2}$ atm



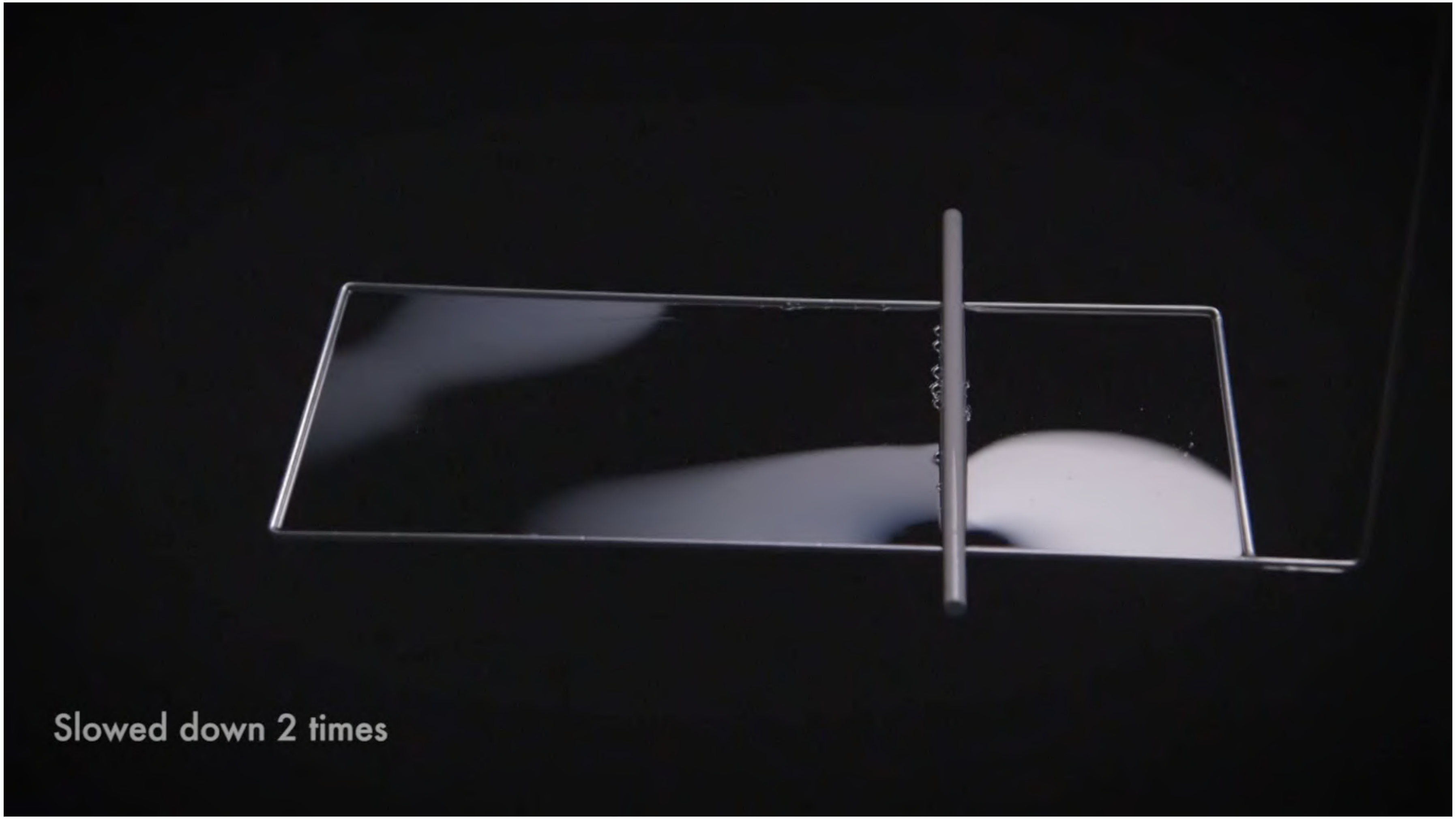
Bulles de cavitation : $R \sim 0.5$ µm, $\gamma \sim 50$ mN/m
$\Delta P \sim 10^{5}$ Pa ~ 1 atm



Bulles d'hélium (métal irradié): $R \sim 10$ nm, $\gamma \sim 10^{3}$ mN/m
$\Delta P \sim 2.10^{8}$ Pa ~ 2000 atm

Glam et al., J. Nuclear Mat. (2009)

1.2 Surfaces sous tension



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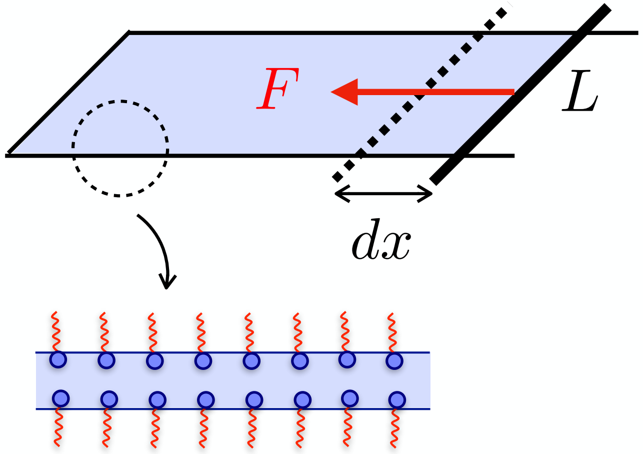
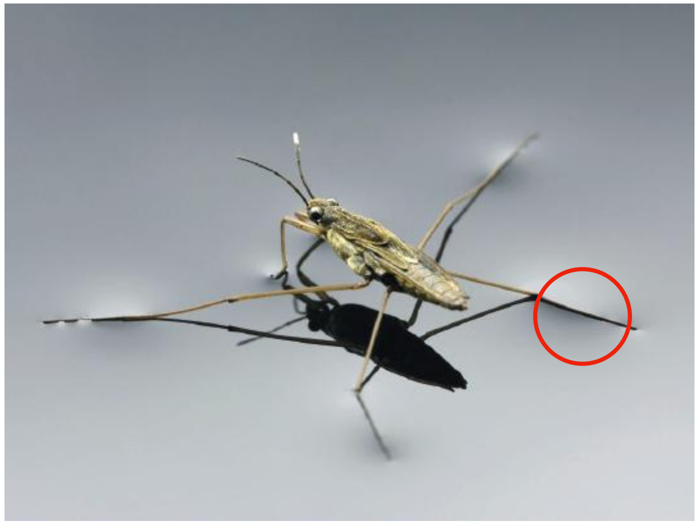


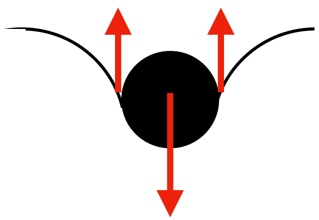
$$dU_s = 2\gamma\, L\ dx \quad = dW = F dx$$

$$\boxed{F = 2\gamma L}$$

γ est bien une force par unité de longueur!

$F = \gamma\ p$ $\quad$ p le périmètre





$$M_{max}\ g \sim 8\ L\ \gamma$$