TP Champ de contraintes en tête de fissure - *sur place*

Quelles sont les questions scientifiques et techniques ?
Champ de contraintes au voisinage d'une pointe de fissure : singularité universelle, facteur d'intensité de contraintes, fonctions angulaires. Zone de process plastique. Effets de bord.

Par quelles expériences y répondre ?
Technique de la photoélasticimétrie appliquée à une éprouvette CT (Compact Tension) en polycarbonate, entaillée, pré-fissurée et chargée en mode I (ouvrant).

Quelles techniques expérimentales ?
Observation d'une éprouvette chargée entre deux polariseurs croisés. Mesure des isoclines pour obtenir les directions propres du champ de contraintes. Mesure des isochromes pour obtenir l'intensité du champ de contrainte (déviateur). Analyse d'images à partir de photos.

Introduction

Les fissures font partie intégrante de notre quotidien. Toutes les pièces mécaniques en contiennent, et ce à diverses échelles. Les fissures les plus grandes sont observables à l'œil nu. Par exemple, chaque conducteur est conscient du danger de rouler avec un impact sur son pare-brise. Malgré les précautions prises dans la réalisation des pare-brises (multi-couches), un simple impact de quelques millimètres (une fissure de petite taille en fait) peut se propager très rapidement sous charge, jusqu'à couvrir toute l'étendue du pare-brise.

Figure 1: Impact sur un pare-brise, fissuration de la banquise et rupture des ponts et des coques des bateaux 'liberty ships' (source: google images).

Pour analyser les structures et dans le but de prévenir leur rupture, plusieurs techniques peuvent être utilisées (par exemple, méthodes de Moiré, interférométrie, caustiques, corrélation d'images, photoélasticimétrie). **La photoélasticimétrie** est une technique industrielle de prévision des contraintes qui vient compléter des méthodes numériques telles que la méthode des éléments finis. Elle est largement utilisée dans des secteurs de technologie avancée comme l'aéronautique. Sa capacité à simuler des structures complexes a conduit à élaborer des démarches hybrides calcul-photoélasticimétrie. Plus qu'une complémentarité, il s'agit d'une véritable intégration de ces deux moyens prévisionnels. Certains bureaux d'études, possédant de gros moyens de calculs, ont recours systématiquement à la photoélasticité afin d'élaborer par exemple des hypothèses plausibles sur les conditions aux limites. Cette technique est fondée sur le phénomène de **biréfringence accidentelle ou effet photoélastique**: tout matériau solide transparent acquiert une biréfringence lorsqu'il est soumis à des sollicitations mécaniques extérieures. L'ensemble des lois physiques décrivant ce phénomène constitue la photoélasticité.

Montage expérimental

Le modèle consiste en une plaque photoélastique (un matériau photoélastique est un matériau dont l'indice de réfraction dépend de la contrainte) $60 \times 60mm$, en polycarbonate ou en autres matériaux, comportant une entaille d'environ $25mm$ de long, terminée par une amorce de fissure réalisée au moyen d'une lame de rasoir. Un système vis-écrou permet d'appliquer une force provoquant l'ouverture de la fissure (figure 2). Le banc de photoélasticité est composé d'une lampe à vapeur de mercure (utilisé pour étudier les isoclines, figure 3), d'une lampe à vapeur de sodium (utilisé pour étudier les isochromes, figure 3) et du modèle placé entre polariseurs croisés et lentilles (figure 2).

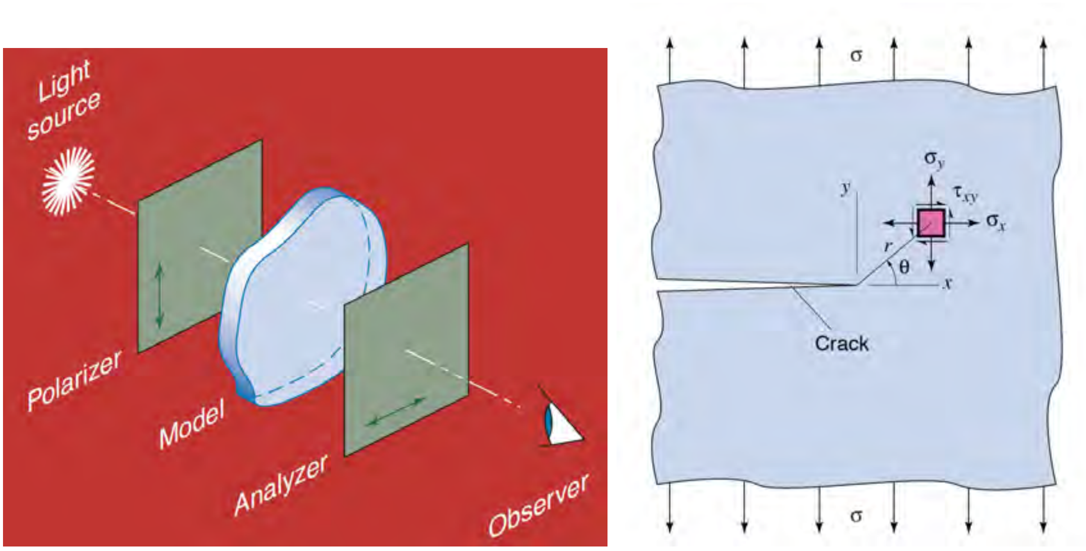

Figure 2: (gauche) Montage expérimental. (droit) Une plaque photoélastique avec le élément de contrainte au voisinage de la pointe de fissure. (source: Experimental Stress Analysis, James W. Phillips).

Figure 3: Une lampe à vapeur de mercure et une lampe à vapeur de sodium (source: google images).

Photoelastometry

In a photoelastic material the stress in some region of material influences the propagation of light through that region. In particular, there is a birefringence effect: light polarized along the axis of maximal principle stress, and the light polarized orhogonally (i.e., along axis of minimal principle stress) are delayed in their propagation by different amounts. Based on Maxwell's equations for birefringence, the *angular phase difference* accumulated between the two orthogonal light waves is

$$\Delta = 2\pi hc\frac{\sigma_1 - \sigma_2}{\lambda},$$

where $\sigma_{1,2}$ are the two principal stresses (which are for us determined by the crack), the λ is the wavelength of light (which is set by the lamp used), while the two parameters intrinsic to the chosen piece of material are h, its thickness, and c, the relative stress-optic coefficient, which is independent of λ.

In our setup with an orthogonal polarizer and analyzer which sandwich the material (figure 2), given the phase shift between two orthogonal components of light, it is straightforward to find the intensity of observed light:

$$I = a^2 \sin(2\alpha)^2 \sin\left(\frac{\Delta}{2}\right)^2,$$

where α is the angle between the axis of maximal principal stress (σ_1) and the axis of the analyzer, the Δ is the phase shift described above, and a is the amplitude of light leaving the polarizer.

The application of photoelastometry in this TP is to extract information about the crack-induced stress in the material, namely the principle stresses $\sigma_{1,2}$ and the orientation of their axes at various positions in the material, from observing the light intensity as you rotate the polarizer-analyzer pair with respect to the material. The light intensity you will observe has two main features, **isoclines** and **isochromes**, that will enable you to extract, respectively, the principle axes and the principal stresses.

Isoclines

To start with, you will study the evolution of the isoclines emanating from the crack tip. Each point in the material $M(r, \theta)$ is defined using polar coordinates r and θ with respect to the coordinate system of the crack (figure 4). The θ is taken positive in the counterclockwise direction and ranges from $-\pi$ to π as you circle from just below the crack to just above it.

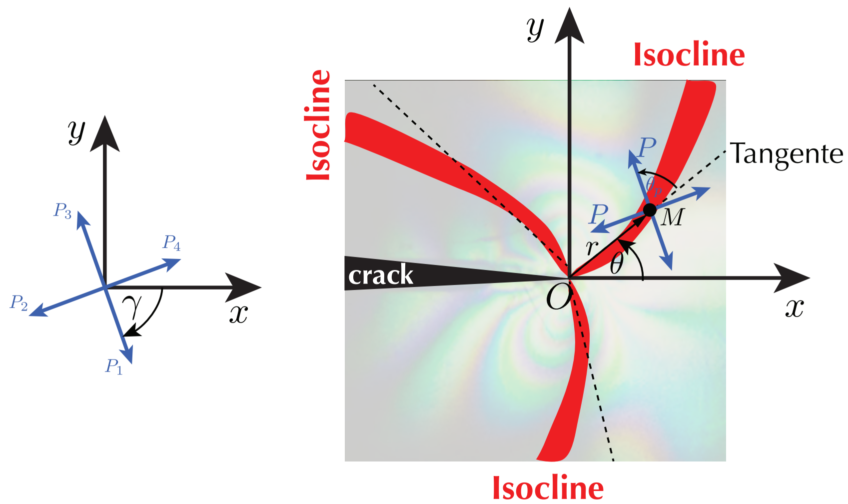

Figure 4: (left) γ is the angle the polarizer axis P_1 makes with the $x-axis$ measured in the clockwise direction. (right) θ is the angle between the $x-axis$ and the "tangent" passing through the isocline measured in the counterclockwise direction. θ_P is the angle between the isocline and the closest polarizer axis P_i measured in the counterclockwise direction. By definition θ_p cannot be larger than 90^o

Isoclines are lines at which light completely vanishes, $I = 0$, no matter the color of light. They are therefore given by the condition $\sin(2\alpha)^2 = 0$, which is equivalent to $\alpha = m\frac{\pi}{2}$, where $m = 0, \pm1, \pm2, \ldots$. In other words, at each point in the material $M(r, \theta)$ where the isocline is located, the local principle stress axes are aligned with the axes defined by the polarizer and analyzer (figure 4).

We first define the angle γ as the angle between one direction P_1 on the polarizer axis and the x-axis of the crack (figure 4). You will change this angle in the range 0 to 2π by rotating the polarizer dial (the analyzer should always remain perpendicular to the polarizer). In an infinitely large photoelastic plate the isoclines would be straight lines, so you will approximate each isocline with a "tangent" (like in figure 4), and you will assume that the isocline passes through all points $M(r, \theta)$, i.e., through all r along the tangent with a certain value of θ. For any point $M(\theta)$ on an isocline you will record the two angles θ and γ (as explained we take this independent of r). According to the photoelastometry model of isoclines, we can only deduce that the principle stress axes coincide with the cross defined by $P_{1,2,3,4}$ in figure 4. Knowing which of the arms of the cross is the P_1 (tracked by $\gamma \in [0, 2\pi)$) does not add information about principle stress axes. For this reason, and also for usage in later part of the TP, we introduce a new angle variable θ_p which ranges only from 0 to $\pi/2$. This range is enough to define the orientation of the cross and therefore the cross of principle stress axes. The angle $\theta_p \in [0, \pi/2)$ for an isocline point $M(\theta)$ is defined as the counterclockwise rotation angle needed to align the radial direction at $M(\theta)$ with a P_i axis closest to it (figure 4). In other words, at a certain point $M(\theta)$ the angle θ_p measures the counterclockwise misalignment of the cross defined by principle stress axes and the cross defined by radial coordinates. Automatically, you may find the four directions P_i as the set $P_n \equiv \{\gamma + n\frac{\pi}{2}\}$, for $n = 0, 1, 2, 3$, and then choose the minimum distance $\theta_p = \min_n\{(-P_n - \theta) \bmod 2\pi\}$ (note: (1) the minus sign in front of P_n is due to opposite sense of rotation of γ and θ; (2) the minimization should find the single distance that is in the range 0 to $\pi/2$). Collect data from all useful isoclines as you increase the polarizer angle γ in increments of 15^o.

Your task is therefore to:

- Deduce the dependence $\theta_p(\theta)$ of the angle defining the principle stress axes on the position in material θ. The fitting function should be $\theta_p = m\theta + \theta_0$. For all the measurements use ImageJ and for fitting use Python or Matlab.

NB:
(1) Due to effects not included in our modeling, you will may see either three or four isoclines.
(2) Isoclines in vicinity of the x-axis appear quite distorted, so avoid using them.

Isochromes

Isochromes are lines of vanishing light intensity, $I = 0$, obtained from the second condiction, namely, $\sin\left(\frac{\Delta}{2}\right)^2 = 0$, which is equivalent to $\Delta = 2\pi n$, or

$$\sigma_1 - \sigma_2 = n\frac{\lambda}{ch}, \tag{1}$$

where $n = 0, \pm1, \pm2, \ldots$. As we consider a variation of color of light (λ), this condition will be satisfied at varying points in the material, therefore non-monochromatic light produces a rainbow-like pattern of colors. For this part of the exercise we therefore switch lamps and work with a monochromatic source. Note that the isoclines are still present, but we instead focus on the isochromes which are dark fringes identified by the red lobes in figure 5.

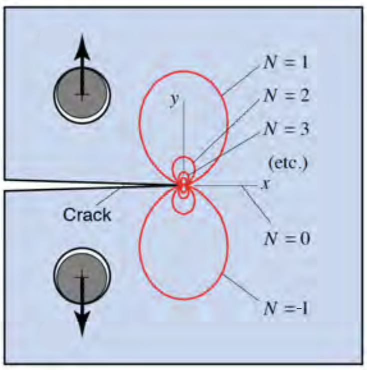

Figure 5: Isochromes - theoretical fringes for a K_I (mode I) dominant field (source: Experimental Stress Analysis, James W. Phillips).

You will analyze the stress tensor in the material using the isochromes. In the simplest theoretical model for the crack, the stress tensor has the form:

$$\sigma_{ij} = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta), \tag{2}$$

so that the magnitude of any stress, e.g., the principal stresses, depends on the distance from the crack tip through a factor $1/r^{0.5}$.

Analysis on line $\theta = const$: finding $f_1(r)$

Following the line at $\theta = 90^o$ which passes through the crack tip, find the intersections of this line and the isochrome fringes. Measure each distance $r(N)$ from the crack point to an intersection, which is labeled by the isochrome fringe order N. To obtain enough data points, you will have to load the plate to see around $8 - 10$ isochromes.

Your tasks are to:

- Fit the datapoints $(r, S) \equiv (r(N), N)$ to the curve $S = \frac{a}{r^\alpha}$ to find the exponent α.
- Find the domain of validity of eq. 2.
- Record the fit function $S = f_1(r)$ you obtained.

NB:
(1) Be careful when loading the sample, there is a risk of propagating the crack, or even breaking the plate.
(2) Follow attentively the order N of each isochrome as you load the plate.

Analysis on line $r = const$: finding $f_2(\theta)$

Consider a circle of radius r centered on the crack tip (use ImageJ to draw the circle), and measure the angular positions $\theta(N)$ of successive isochromes N; choose the radius r such that you stay within the domain of validity of eq. 2 studied above.

Your tasks are:

- In the measured datapoints $(\theta(N), N)$ if the $\theta(N)$ is negative, also set its N to be negative (see figure 5).
- Fit the datapoints $(\theta, S) \equiv (\theta(N), N)$ by attempting simple trigonometric curves and record the best function $S = f_2(\theta)$. to find the exponent α. Record the fit function $S = f_1(r)$ you obtained. Comment on the domain of validity of eq. 2.
- Note that the free edges of the crack deform the stress field in the plate; in the ideal case, the field would be perfectly symmetric across the crack line. Consider the symmetry which the candidate functions for f_2 should obey in this idealized case.

Analysis of stress field

- Using the results from this section, in view of eq. 1, write the principle stress difference $\sigma_1 - \sigma_2$ as function of position $M(r, \theta)$ in plate, i.e., if $\sigma_1 - \sigma_2 = J \cdot f_1(r) \cdot f_2(\theta)$, what are the values of $f_1(r)$ and $f_2(\theta)$.
- By convention the factor $J = \frac{K}{2\sqrt{2\pi}}$. What is the physical dimension of the constant J? To which physical quantity does it relate?

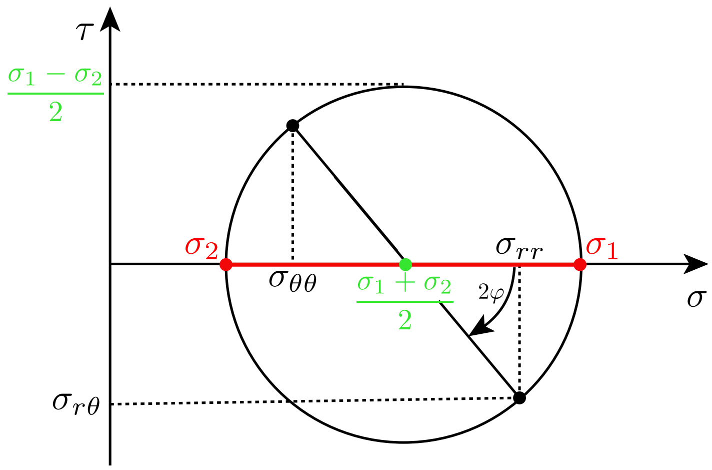

Figure 6: Mohr's circle for changing stress tensor under coordinate system rotations.

Mohr's circle and the stress field in polar coordinates

Mohr's circle enables us to simply and visually find the stress tensor in a rotated coordinate system (figure 6). Note that the stress tensor is of second rank so its components get rotated in a more complicated way than those of a vector. Nevertheless, Mohr's circle correctly encodes all the rules. You will use this tool together with a constraint equation on stresses to derive all components of stress tensor in polar coordinates, $\sigma_{ij}(r, \theta)$, $i, j \in \{r, \theta\}$, just by using two quantities: the rotation angle $\theta_p(\theta)$ (derived from isoclines) and $[\sigma_1 - \sigma_2](r, \theta)$ (derived from isochromes).

A constraint equation which relates normal and tangential stresses at each point of a material in the regime of linear elasticity and static equilibrium is:

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \tag{3}$$

By definition of θ_p, at each point $M(r, \theta)$ we need to rotate the principal stress axes by an angle $\varphi \equiv -\theta_p(\theta)$ to align them to the radial coordinate axes (r, θ). In Mohr's circle this always requires a rotation by 2φ, see figure 6. Importantly, the photoelastometry gives us only the value of the difference $\sigma_1 - \sigma_2$, which is seen by Mohr's circle to completely determine the maximal value of shear one can observe by rotating the coordinates. From the known rotation φ, the shear component $\sigma_{r\theta}$ follows straightforwardly.

Note that in the constraint equation you have: (1) the already found $\sigma_{r\theta}$, (2) the $\sigma_{rr} - \sigma_{\theta\theta}$ which depends only on the difference $\sigma_1 - \sigma_2$ (obvious in Mohr's circle), and (3) the derivative of the unknown field σ_{rr}. Integrate the partial differential equation and use the boundary condition that stress should vanish at infinity.

Your tasks are:

- Deduce $\sigma_{r\theta}(r, \theta)$ using Mohr's circle (note, $\sigma_{\theta r} \equiv \sigma_{r\theta}$).
- Derive both $\sigma_{rr}(r, \theta)$ and $\sigma_{\theta\theta}(r, \theta)$ by using the local constraint equation.