Lecture 1 – Physics of miniaturization
Scaling laws relate the properties of a system to its dimensions.
Example: diffusion law

\[ \tau \sim \frac{l^2}{D} \]
Example: capillary forces

\[ \Delta P = \frac{\gamma}{r} \]
Example: capillary forces
To establish scaling laws, one estimates terms implied in the equations of the system.

The same approach applies to partial-derivative equations.
0 = -\nabla P + \mu \Delta u
A list of scaling laws relevant to microfluidics

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Scaling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capillary force</td>
<td>$l$</td>
</tr>
<tr>
<td>Flow speed in a microfluidic system</td>
<td>$l$</td>
</tr>
<tr>
<td>Thermal power transferred by conduction</td>
<td>$l$</td>
</tr>
<tr>
<td>Viscous force in a microfluidic system</td>
<td>$l^2$</td>
</tr>
<tr>
<td>Electrostatic force</td>
<td>$l^2$</td>
</tr>
<tr>
<td>Diffusion time</td>
<td>$l^2$</td>
</tr>
<tr>
<td>Gravity force</td>
<td>$l^3$</td>
</tr>
<tr>
<td>Magnetic force with an external field</td>
<td>$l^3$</td>
</tr>
<tr>
<td>Magnetic force without external field</td>
<td>$l^3$</td>
</tr>
</tbody>
</table>
Scaling laws in nature – Kleibler law

Slope 0.75

Graph showing the relationship between metabolic rate (W) and body weight (kg) for various animals.
How to measure the metabolic rate?

Warburg respirometer

CO₂ absorbant
The consequences of Kleibler law

\( W \sim l^{9/4} \)

Blood volume \( \sim l^3 \)

Heart size \( \sim l \)

Lung volume \( \sim l^3 \)

Heart pulsation \( \sim l^{3/4} \)

Longevity \( \sim l^{3/4} \)
The consequences of Kleibler law

\[ T = 10^M^{1/4} \]
Muscular force varies as $l^2$.

Volume musculaire $\sim l^3$

Energie musculaire $E \sim l^3$

$F_{max} \sim l^2$
Maximum speed of the animals

Running develops a work equal to $l^3$

Kinetic energy is in $V^2 l^3$

Conclusion: speed independant of size
At what speed Tyrannosaurus rex runs?
Speeds of two-legs animals

Ostrich (144 pounds, 65.3 kg)—34.5 mph (15.4 m/s)
Velociraptor (44 pounds, 20 kg)—24.2 mph (10.8 m/s)
Dilophosaurus (948 pounds, 430 kg)—23.5 mph (10.5 m/s)
Tyrannosaurus (13,230 pounds, 6,000 kg)—17.9 mph (8 m/s)
Human (157 pounds, 71 kg)—17.7 mph (7.9 m/s)

From Manning, Sellers (Manchester Univ.)
Thermal losses $\Delta T \cdot l$ must be compensated by food $\sim N \cdot l^3$

This leads to:

$$l \sim (\Delta T)^{1/2} N^{-1/2}$$

$N$ cannot be increased indefinitely. There is therefore a minimum size for mammals.
Heat transfer in animals

Mouse: $\frac{1}{3}$ of its own weight per day

Elephant: $\frac{1}{100}$ of its own weight per day
Pygmy shrew is the smallest mammifer on earth

The terrible life of the pygmy shrew
No problem for the whale to keep its internal heat
No problem for the whale to keep its internal heat

Whale explodes on the streets of Taiwan?

Scaling of adhesion

Labonte D et al
Phil. Trans. R. Soc. B
Pros:

- Jump high ($H \sim 1^0$),
- Walk on water

Con: Being trapped in a soap bubble
Scientific errors in “Honey, I shrunk the kids”
Scientific errors in “Honey, I shrunk the kids”
Scientific errors in “Honey, I shrunk the kids”
A useful tool for microfluidics: π theorem

\[ a = f(a_1, a_2, \ldots, a_n) \text{ and } k \text{ dimensions.} \]

\( f \) can be rewritten into a simpler function \( g \) :

\[ \Pi = g(\Pi_1, \Pi_2, \Pi_3, \ldots, \Pi_{n-k}) \]

Where \( \Pi_n \) are independant products formed on combination sof \( a, a_1, a_2, \ldots, a_n \)

From \( n+1 \) variables we move to \( n+1-k \) variables
Exemple 1:

Demonstration of Pythagore theorem
Exemple 2 : Dynamical similarity theorem

\[ u = U g(x/l, Re) \]

Reynolds number = \( Ul/\nu \)
Démonstration : \( u(x) = f(x,U,\rho,\mu,l) \)

\[ n = 5 \quad k = 3 \]

One can define \( 6-3 = 3 \) dimensionless numbers

\[ \frac{u}{U} = g\left( \frac{x}{l}, \text{Re} \right) \]

With \( \text{Re} = Ul/\nu \), the Reynolds number
Analysis of a microjet

Ordinary jet

Miniaturized jet

[Image of children playing with a water ball, spraying water]

[Diagram showing break-off of droplets with times in microseconds]
3 dimensionless numbers:

\[ \text{Re} = \frac{Ua}{\nu} \approx 10 \quad ; \quad Ca = \frac{\mu U}{\gamma} \approx 10^{-2} \quad ; \quad Bo = \frac{\rho a^2 g}{\gamma} \approx 10^{-3} \]

**Conclusion**: jet is laminar, droplets are spherical and gravity deformation of the shape is negligible
Some vocabulary....

<table>
<thead>
<tr>
<th>nom</th>
<th>puissance</th>
</tr>
</thead>
<tbody>
<tr>
<td>milli</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>micro</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>nano</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>pico</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>femto</td>
<td>$10^{-15}$</td>
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<tr>
<td>atto</td>
<td>$10^{-18}$</td>
</tr>
<tr>
<td>zepto</td>
<td>$10^{-21}$</td>
</tr>
<tr>
<td>yocto</td>
<td>$10^{-24}$</td>
</tr>
</tbody>
</table>
Vocabulary again ....

$10^9 \text{ Giga}$

$10^{12} \text{ Tera}$

$10^{15} \text{ Peta}$

$10^{18} \text{ Exa}$
Vocabulary again....

$10^{21}$ Zetta

$10^{24}$ Yotta
3) Examples in the MEMS domain
Miniaturisation of electrostatic systems

Attraction forces of the planes

\[ F = QE \sim CVE \sim \varepsilon_0 E^2 l^2 \]
The micromirror
The electrokinetic micromotor

Small torque, small power

**Rotating electric field**

\[ E \]

**Torque** \( C \sim Fl \sim l^3 \)

\[ P = C\Omega \sim \Omega l^3 \]
The MIT Microturbine
The MIT Microturbine

-Diameter/height : 12 mm/3mm
- Air flow rate: 0.15 g/s
- Température at the outlet: 1600 K
- Rotation speed: 2.4 $10^6$ tr/mn
- Power: 16W
- Weight: 1g
- Consumption of fuel: 7g/h
Miniaturisation of electromagnetic systems

\[ F = IBl \sim jBl^3 \]

\[ B \sim \mu ll^{-1} \sim \mu jl \]

\[ F \sim \mu j^2 l^4 \]
\[ F_e \sim \varepsilon_0 E^2 l^2 \]
\[ F_m \sim \mu_0 j^2 Nl^4 \]

Magnetic < Electrostatic
Scaling law for the vibration frequency of a Cantilever beam

\[ f \approx \frac{1.875}{2\pi} \sqrt{\frac{EI}{mL^4}} \sim l^{-1} \]

\[ I = \frac{wh^3}{12} \]
Miniaturisation of RF systems

![Diagram of miniaturised RF system](image)
Miniaturisation of heat exchangers

Fourier’s law

\[ q = -k \nabla T \]

For a conducting rod:

\[ Q = \frac{KS\Delta T}{L} \]

\[ Q \sim I/\Delta T \]
Source of heat $\sim l^3$
Conduction losses $\sim l \Delta T$

Increase of température $\Delta T \sim l^2$

Temperature control is feasible in microfluidics
4) Two limitations of microfluidics related to scaling laws
1st limitation: the small amount of material than can be analyzed

Number of molecules = Volume microsystème × Concentration

\[ N \sim l^3C \]

Order of magnitude

For 10^{-9} M/litre in a chamber of l \sim 10 \text{ mm}, N \sim 1 \text{ ymole}
2nd limitation: small throughput

Flow-rates = speed x area

\[ Q \sim l^3 \]

Order of magnitude

Microfluidics is able to produce ml/hour
Important to remember

- Table of scaling laws
- Examples in nature
- Examples in MEMS technology and microfluidics
End of Lecture 1