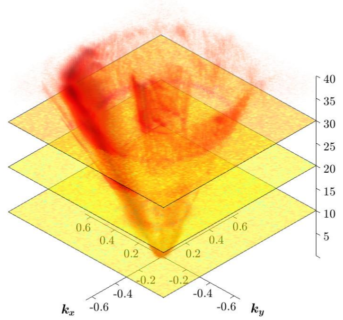


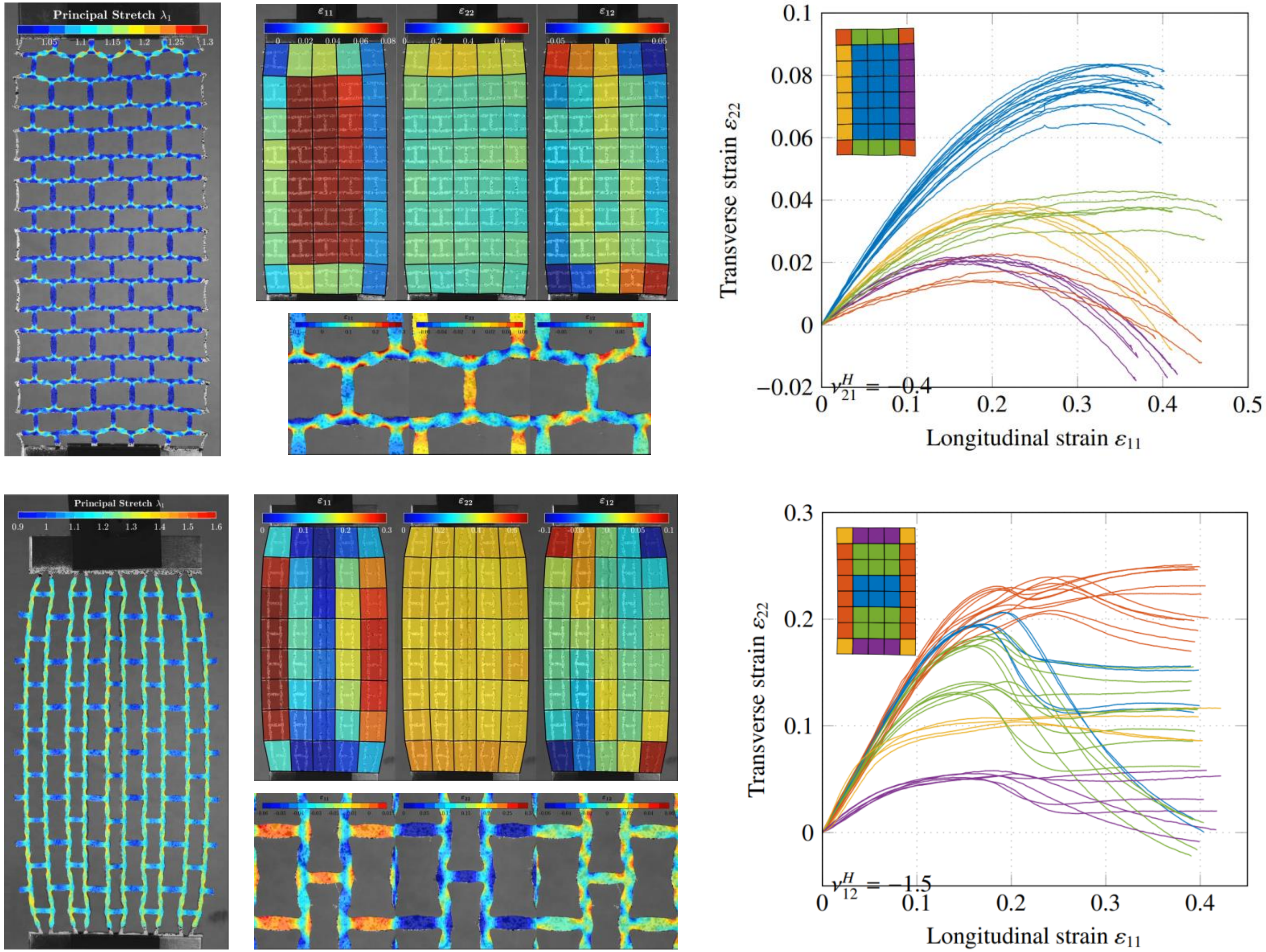
Experimental extraction of dispersion surfaces : application to wave propagation in periodic media

GDR MéPhy, 15 juin 2022

Pierre Margerit

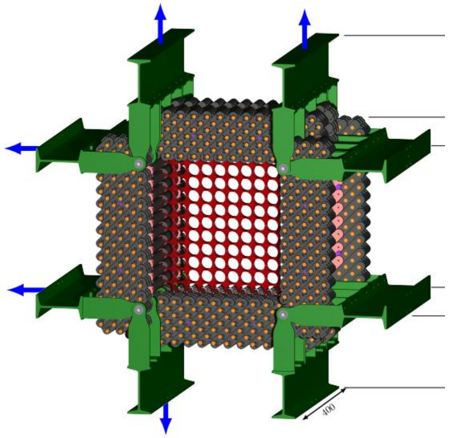
Experimental characterization of multiscale media

What could be an unitary test for a multiscale structure ? [Agnelli 2021]



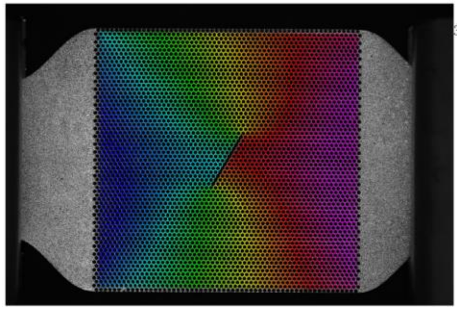
How to deal with the **heterogeneity** of **macro**scopic fields ?

...minimize



Experimental application of KUBC [Dassonville 2020]

...maximize



Experimental identification of a second gradient model [Réthoré 2015]

Full-field measurements: from statics to dynamics

Principle of Virtual Powers:

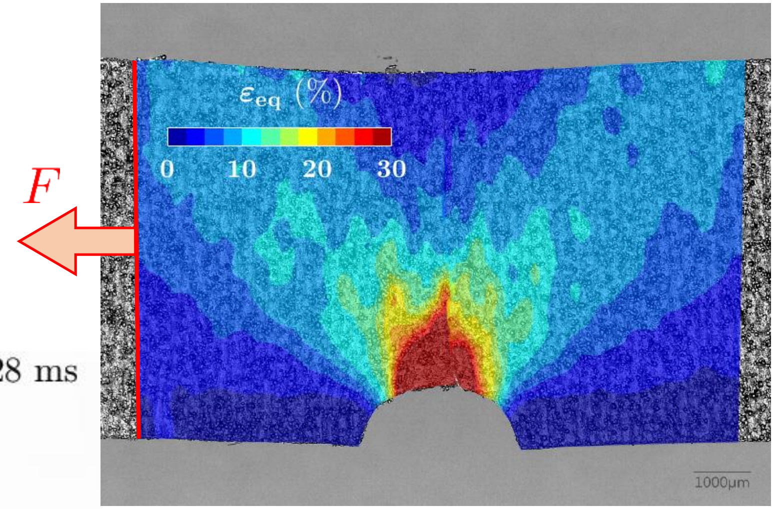
$$\forall \underline{v}^* \,,\; \mathcal{P}_\mathrm{i}(\underline{v}^*) + \mathcal{P}_\mathrm{e}(\underline{v}^*) = \mathcal{P}_\mathrm{a}(\underline{v}^*)$$

$\underline{\underline{\sigma}}(\underline{\underline{\varepsilon}})$ — **comportement**

$\underline{T}$ — **efforts**

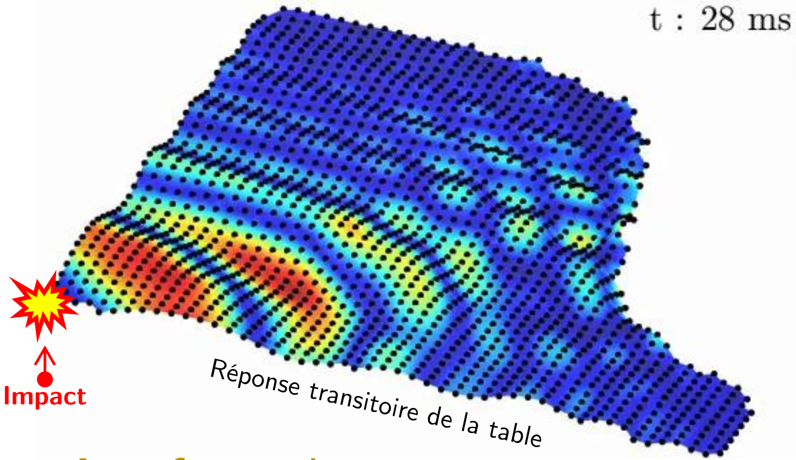
$\underline{\ddot{u}}$ — **accélération**

Fracture tests on additively manufactured notched samples[1]

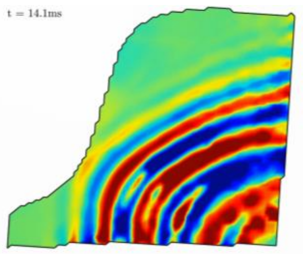


Multiscale response: characteristic **wavelength**

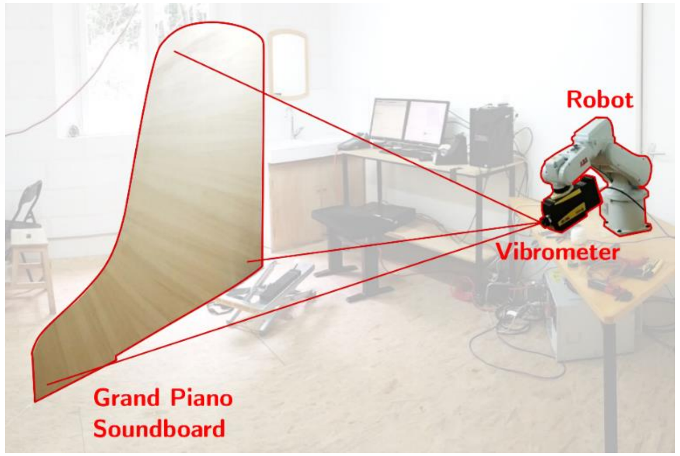
$$\lambda(\omega) = \frac{2\pi}{k(\omega)}$$



How to extract the **meaningful information** from the **rich data** available ?



Robotized measurement system [2]



[1] **Margerit, Pierre**, et al. "Tensile and ductile fracture properties of as-printed 316L stainless steel thin walls obtained by directed energy deposition." Additive Manufacturing 37 (2021)

[2] **Margerit, Pierre**, et al. "The robotized laser doppler vibrometer: On the use of an industrial robot arm to perform 3D full-field velocity measurements." Optics and Lasers in Engineering 137 (2021)

Experimental characterization of multiscale media

Is there a way to make use of the separation of scales ?

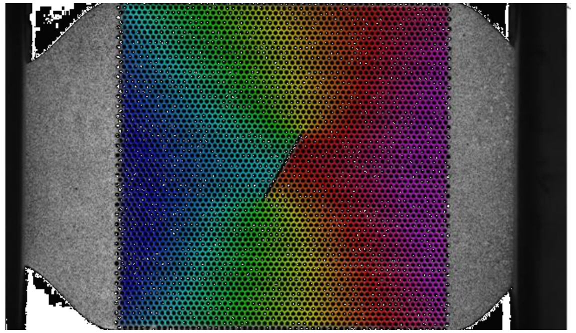
$$f(x) = f(y, Y)$$

microscopic coordinate (y)

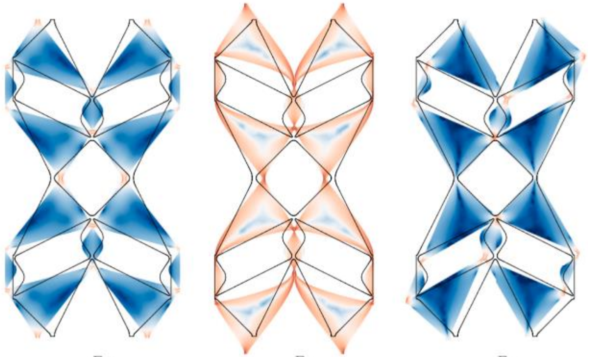
macroscopic coordinate (Y)

Motivations:

- As an **experimentalist**, formulate data/problem **reduction** methods



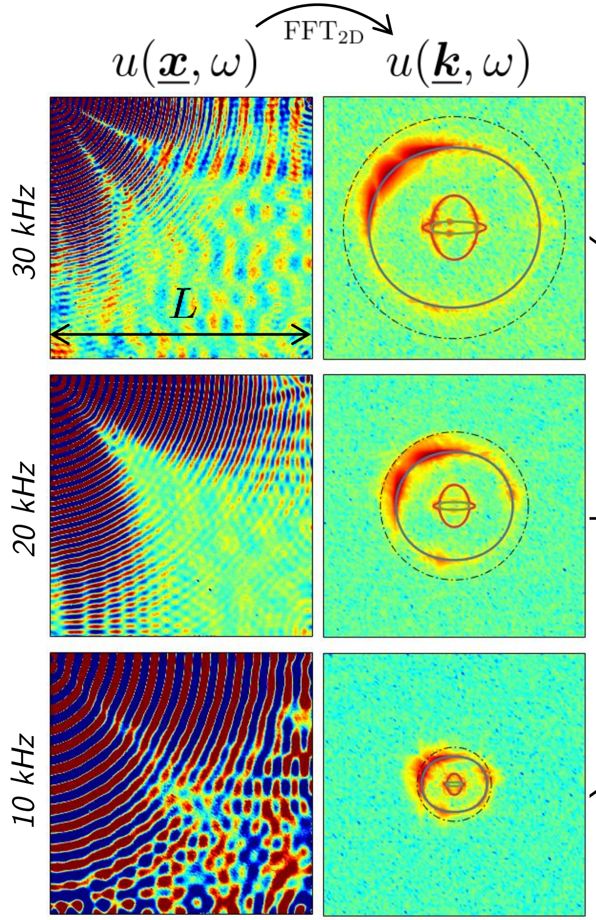
- Working with **theoricians**, provide **experimental twins** *[Durand 2022]*



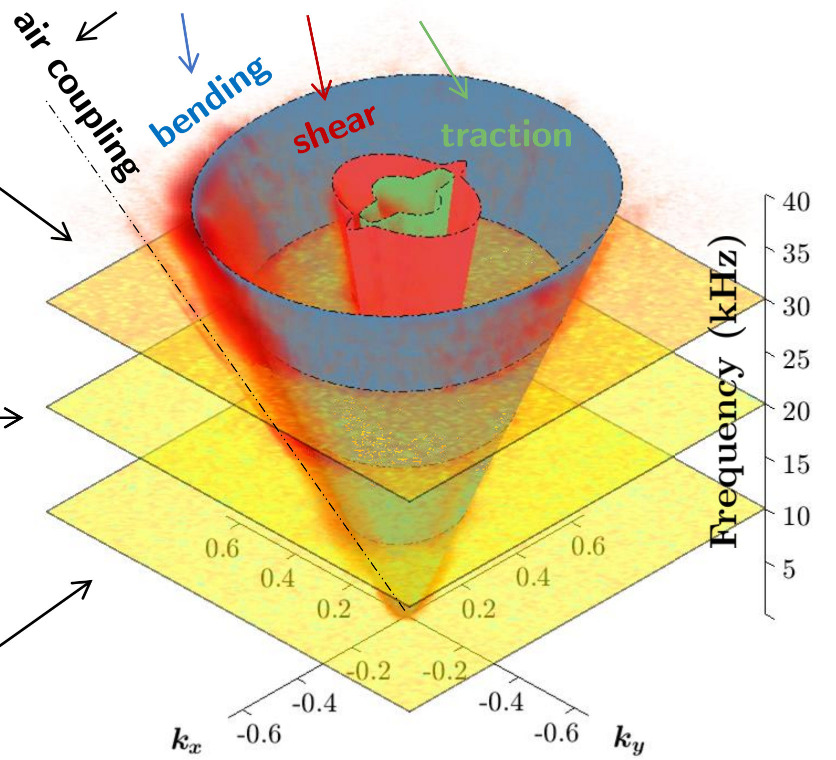
Approaches:

- Principal Component Analysis: $f(y, Y) = \sum_r^R \sigma_r \psi_r(y) \Phi_r(Y)$
- Asymptotic Expansion: $f(y, Y) = \Phi(Y) + \sum_r^R \psi_r(y) \dfrac{\partial^{(r)} \Phi(Y)}{\partial Y^r}$
- Wave Decomposition: $f(y, Y) = \sum_r^R \psi_r(y) \exp(\boldsymbol{i}\, k Y)$

Dispersion surfaces



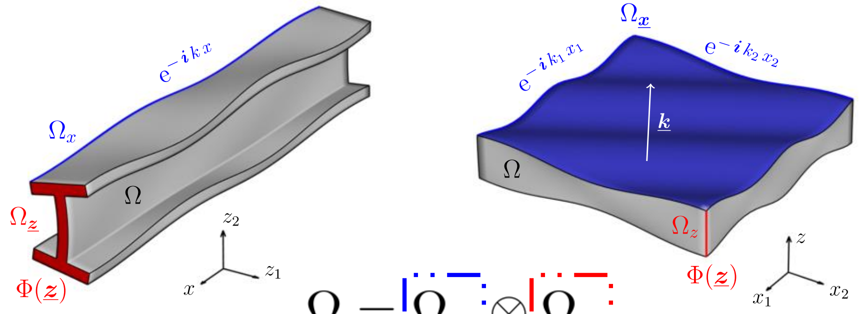
Separation of mechanisms of motion



Independent to experimental boundary conditions: **in-situ**

Uncertainty principle : $\Delta k \geqslant \dfrac{2\pi}{L}$

- **Waveguides**, variable **separation**



$$\Omega = \Omega_{\underline{x}} \otimes \Omega_{\underline{z}}$$

geometry — characteristic section

- **Plane** wave **decomposition**

$$\underline{\tilde{u}}(\underline{z}, \underline{x}, t) = \underline{\Phi}(\underline{z})\, e^{i(\omega t - \underline{k}\cdot\underline{x})}$$

mode — frequency — wavevector

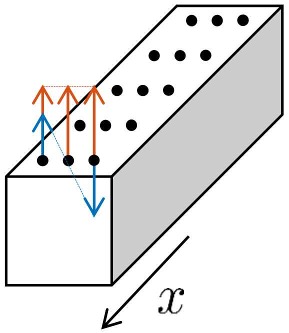
- **Dispersion** equations

$$\underline{\underline{\mathcal{D}}}(\underline{k}, \omega) \cdot \underline{\Phi}(\underline{z}) = \underline{0}$$

Tangent behavior : **propagation** of **perturbations** w.r.t. current configuration

Beam under bending & torsion

- **Homogeneous** beam

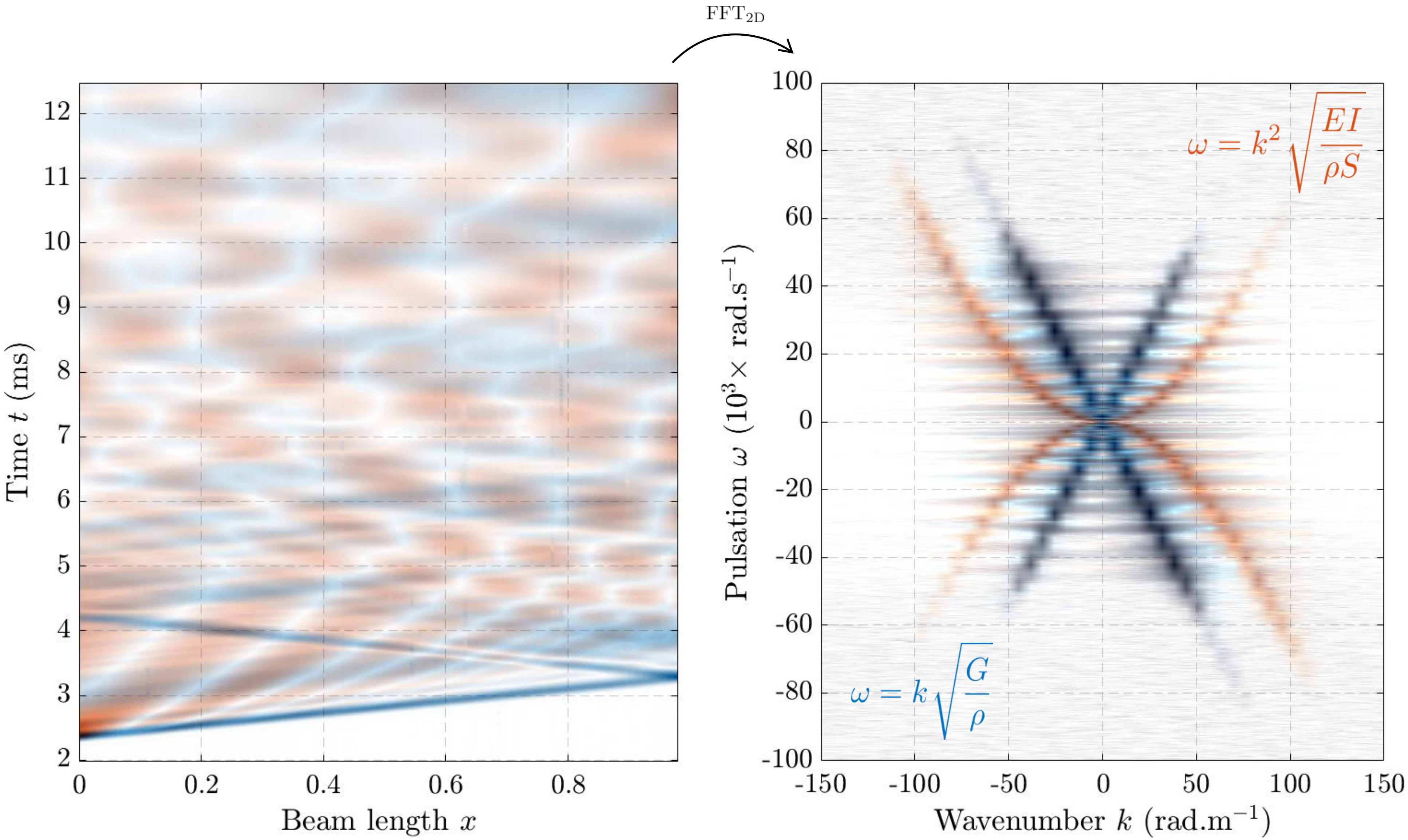


- **Bending**

$$EI\frac{\partial^4 u}{\partial x^4} + \rho S\frac{\partial^2 u}{\partial t^2} = 0$$

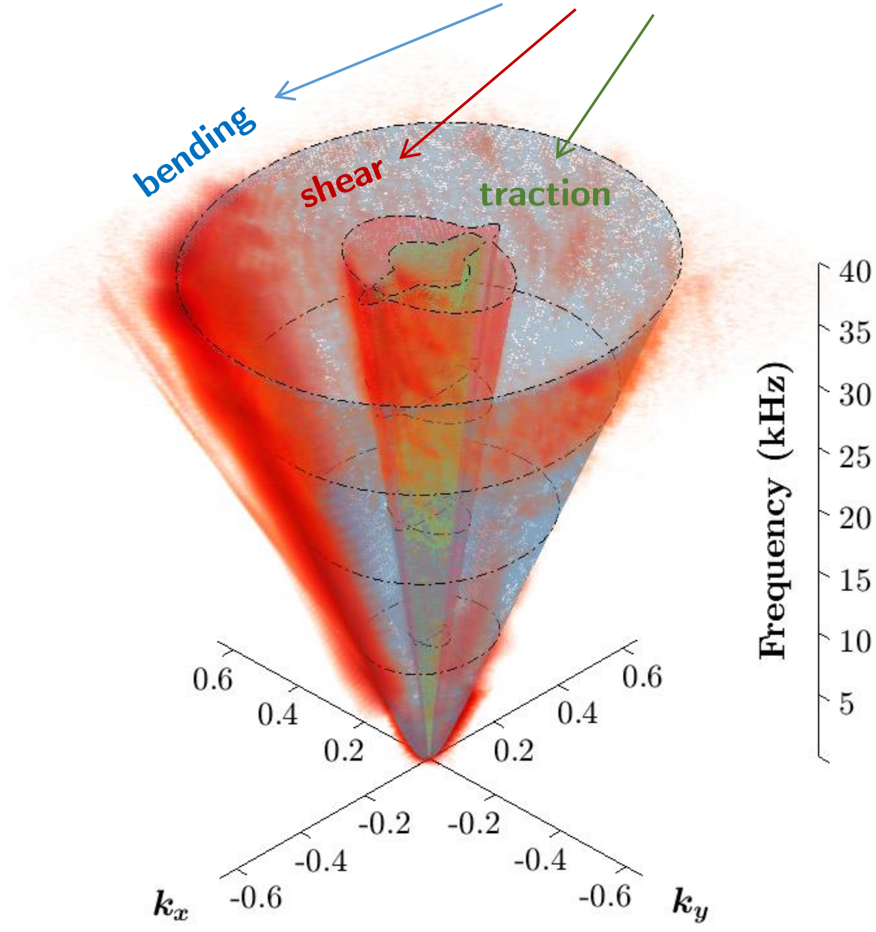
- **Torsion**

$$G\frac{\partial^2 \theta}{\partial x^2} - \rho\frac{\partial^2 \theta}{\partial t^2} = 0$$

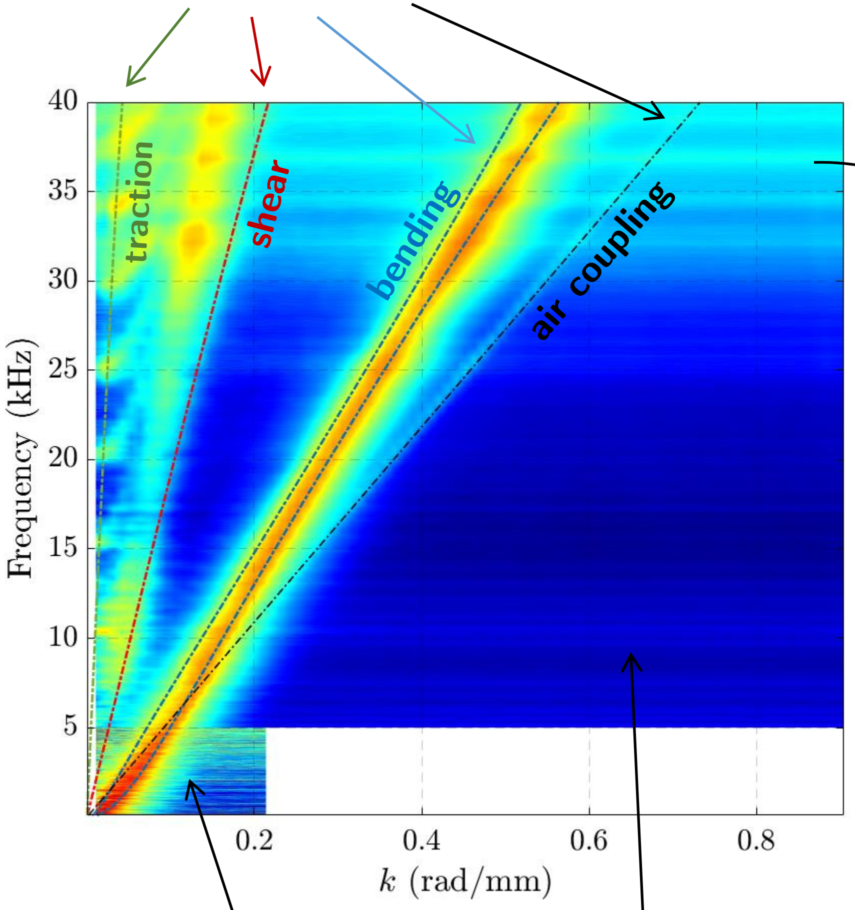


Sandwich plate with multiple wave modes

1) Correlation with **theoretical** dispersion surfaces



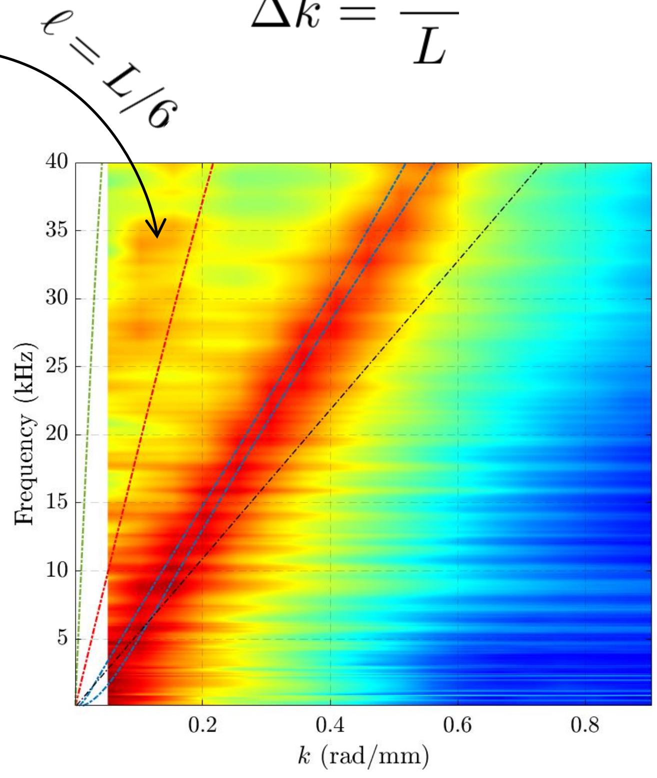
2) Natural **separation** of the **mechanisms** of motion



3) Independent from experimental **boundary conditions**

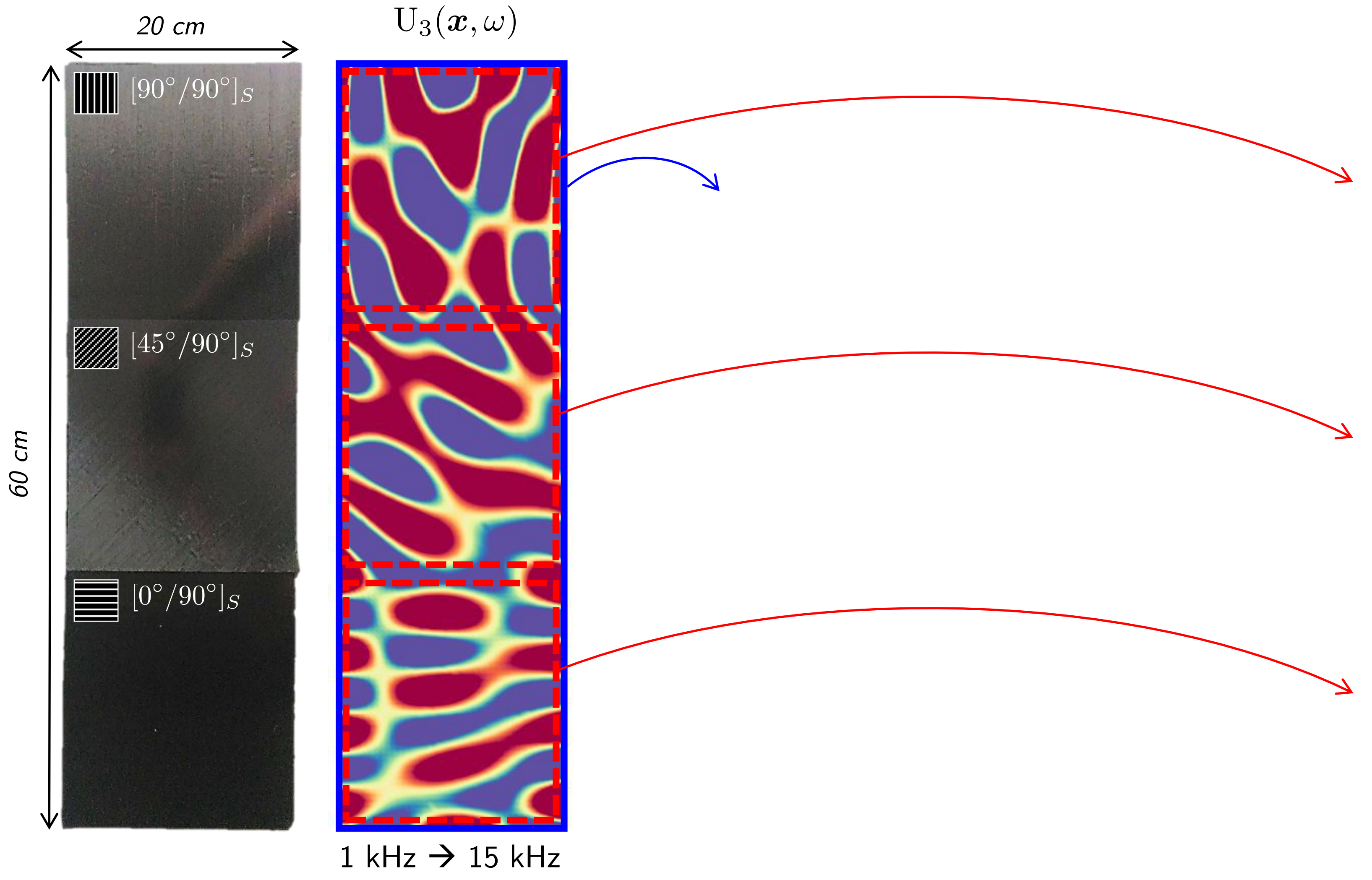
Uncertainty principle is limiting

$$\Delta k = \frac{2\pi}{L}$$



Limited Resolution !

Heterogeneous plate

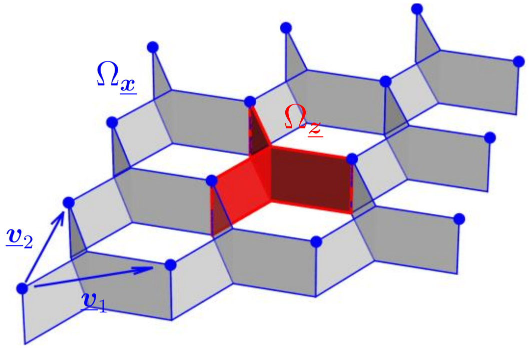


Dispersion in periodic structures

- ❑ **Scale** separation

$$\Omega = \Omega_{\boldsymbol{Y}} \otimes \Omega_{\boldsymbol{y}}$$

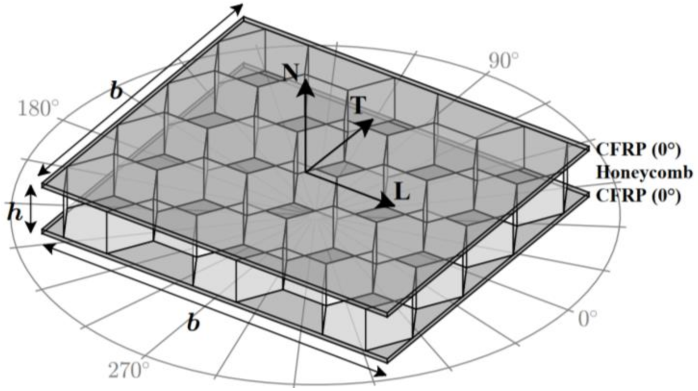
geometry   unit cell



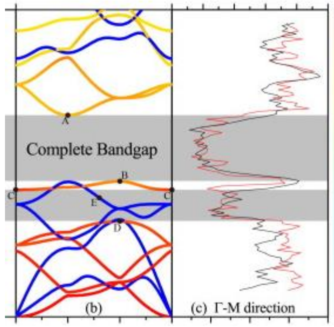
- ❑ **Bloch** waves

$$\boldsymbol{u}(\boldsymbol{y}, \boldsymbol{Y}) = \widetilde{\boldsymbol{u}}(\boldsymbol{y})\, \mathrm{e}^{i \boldsymbol{k} \cdot \boldsymbol{Y}}$$

periodic spectrum !

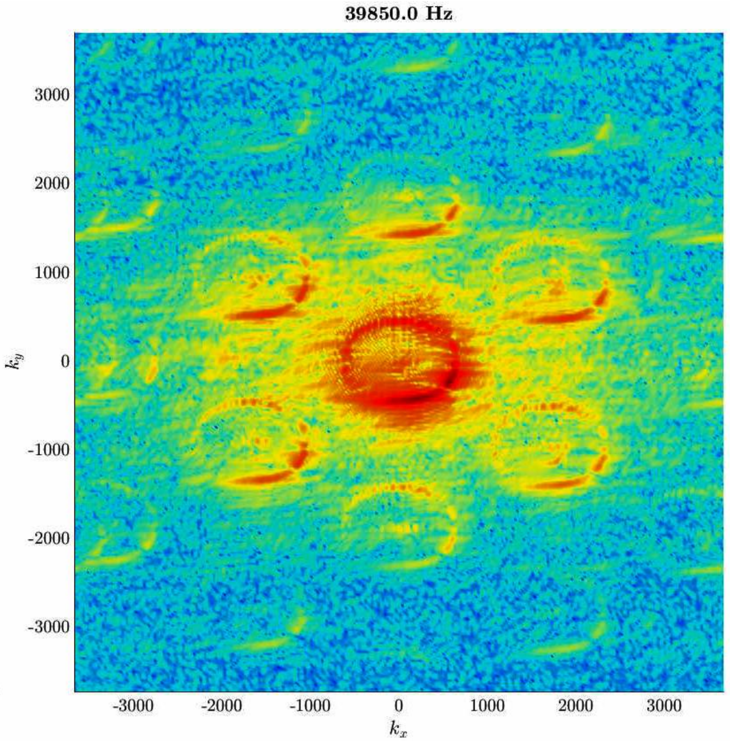


Need for **rich data** to characterize the **high** dynamical **behavior complexity** [Hussein 2014]



[Yang 2017]

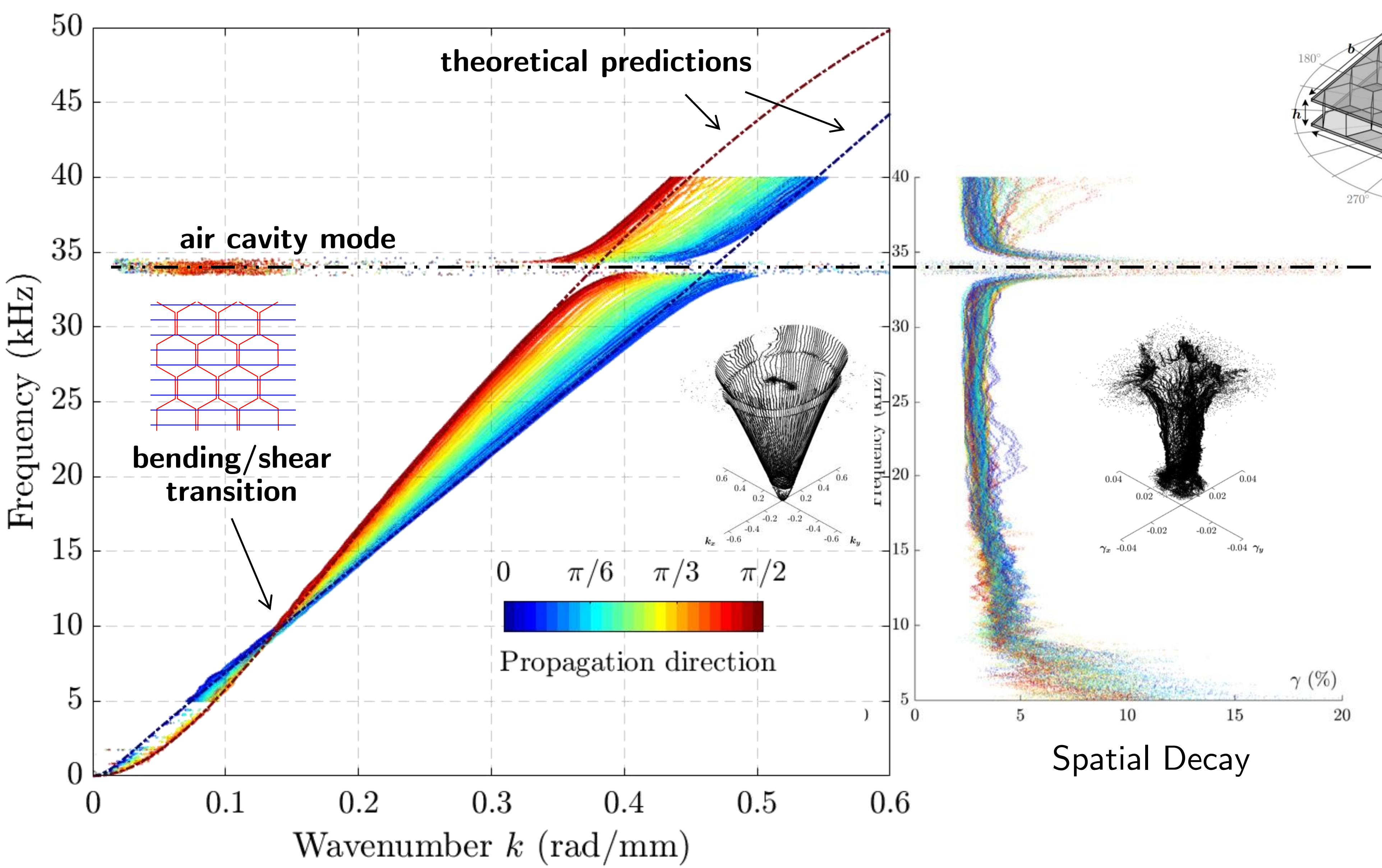
Dispersion Surfaces are a the ideal **experimental twin**



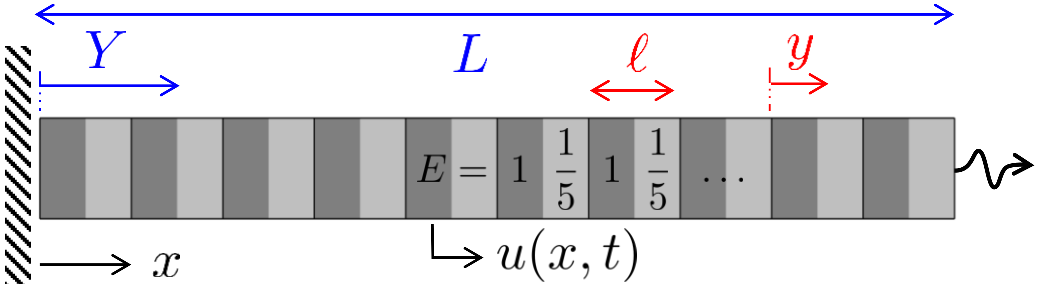
- ❑ **Characterization**
- ❑ **Validation**
- ❑ **Identification**

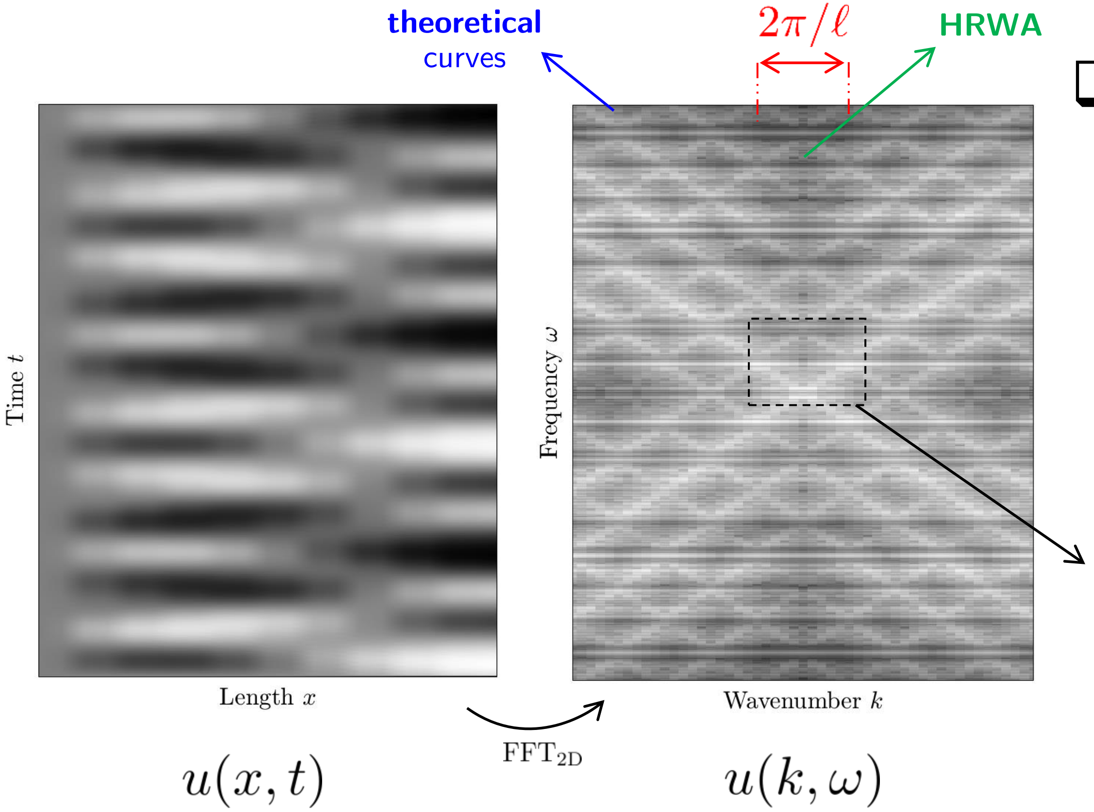
→ **Microstructure** effects
→ **Singularities** and **band gaps**
→ **Damping**

An experimental example



1D bilayered material





❑ Variable separation

$$u(x,t) = u(y,Y,t) = \int_\omega \int_k \tilde{u}(y,k,\omega)\, e^{i(\omega t - kY)}\, dk\, d\omega$$

microscopic wave **mode**

macroscopic **modulation**

wave**number**

❑ Low-order approximation

High-Resolution Wavenumber Analysis [Margerit 2019]

$$u(x,\omega) = u(y,Y,\omega) \approx \sum_r^R \tilde{u}_r(y,\omega) \exp(i\, k_r(\omega)\, Y)$$

discrete **dispersion curves**

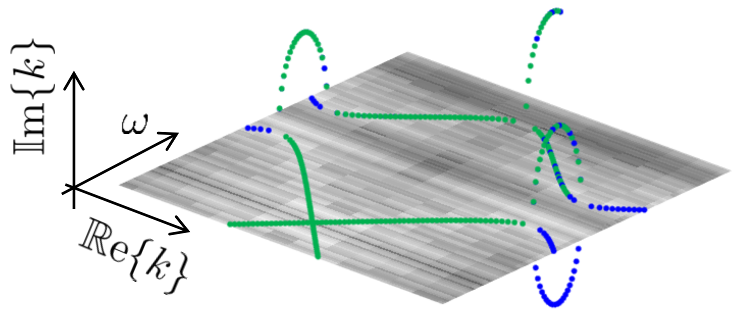
Rotational Invariance

cell shift

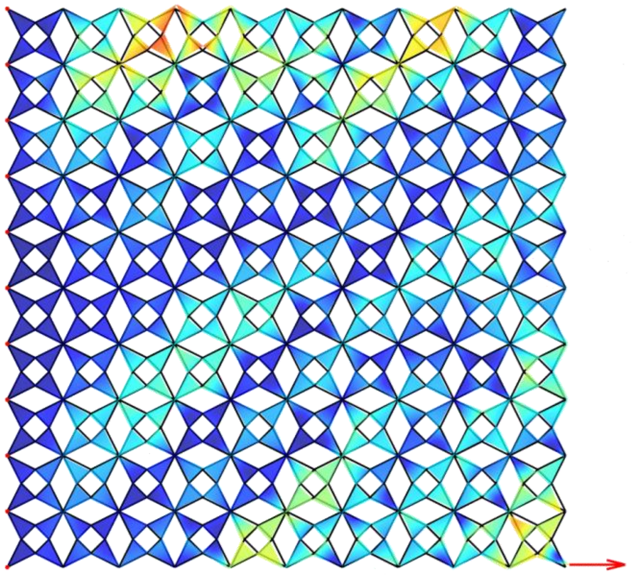
$$\exp(i\, k_r\, [Y + \ell]) = \exp(i\, k_r\, Y)\, \exp(i\, k_r\, \ell)$$

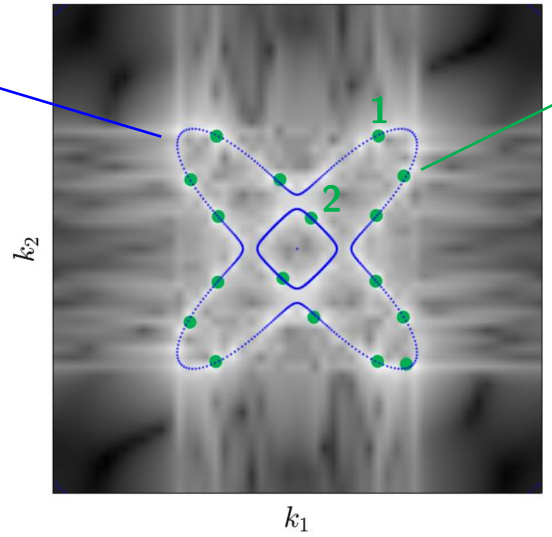
invariant

$k \in \mathbb{C}$



2D pantograph





- Variable separation

$$\underline{u}(\underline{x},t) = \underline{u}(\underline{y},\underline{Y},t) = \int_\omega \int_{\underline{k}} \underline{\widetilde{u}}(\underline{y},\underline{k},\omega)\, \mathrm{e}^{i(\omega t - \underline{k}\cdot\underline{Y})}\, \mathrm{d}\underline{k}\, \mathrm{d}\omega$$

microscopic wave **mode**; macroscopic **modulation**; wave**vector**

- Low-order approximation

High-Resolution Wavenumber Analysis [Margerit 2019]

$$\underline{u}(\underline{x},\omega) = \underline{u}(\underline{y},\underline{Y},\omega) \approx \sum_r^R \underline{\widetilde{u}}_r(\underline{y},\omega) \exp(i\, \underline{k}_r(\omega)\cdot \underline{Y})$$

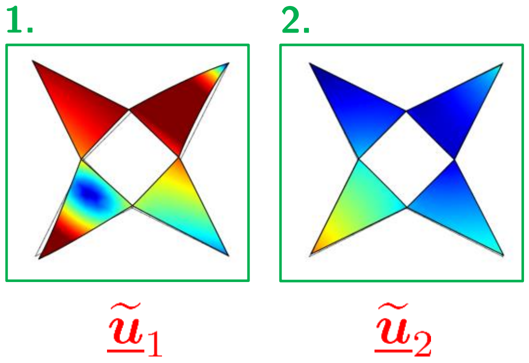
discrete **dispersion surfaces**

Rotational Invariance

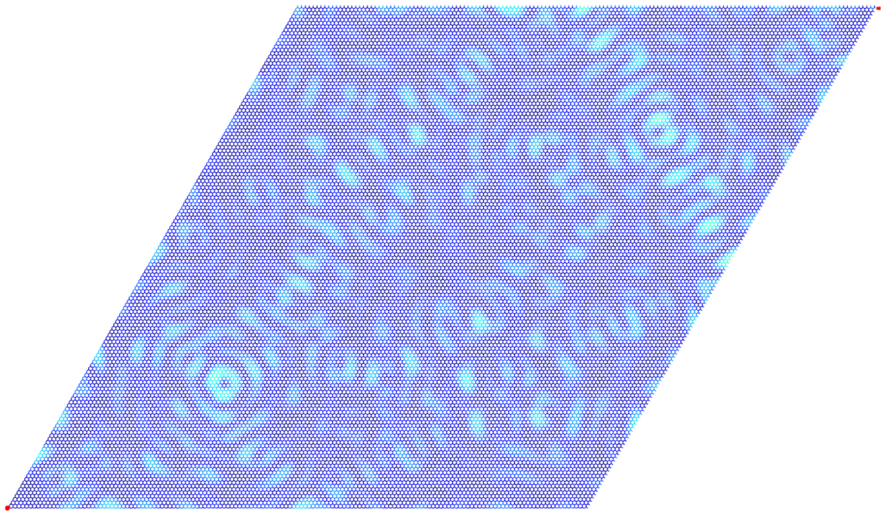
$$\exp(i\, \underline{k}_r \cdot [\underline{Y} + \underline{\ell}]) = \exp(i\, \underline{k}_r \cdot \underline{Y})\, \exp(i\, \underline{k}_r \cdot \underline{\ell})$$

cell shift; *invariant*

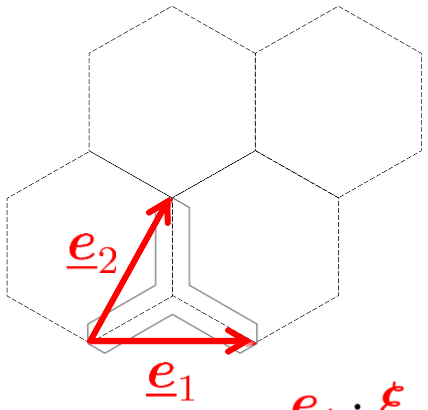
$$\underline{k} \in \mathbb{C}^2$$



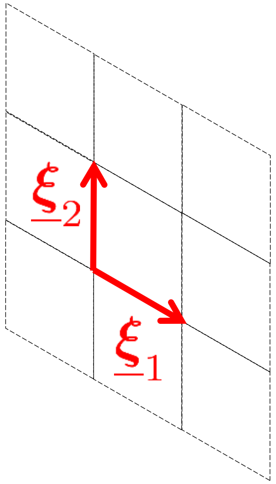
Honeycomb lattice



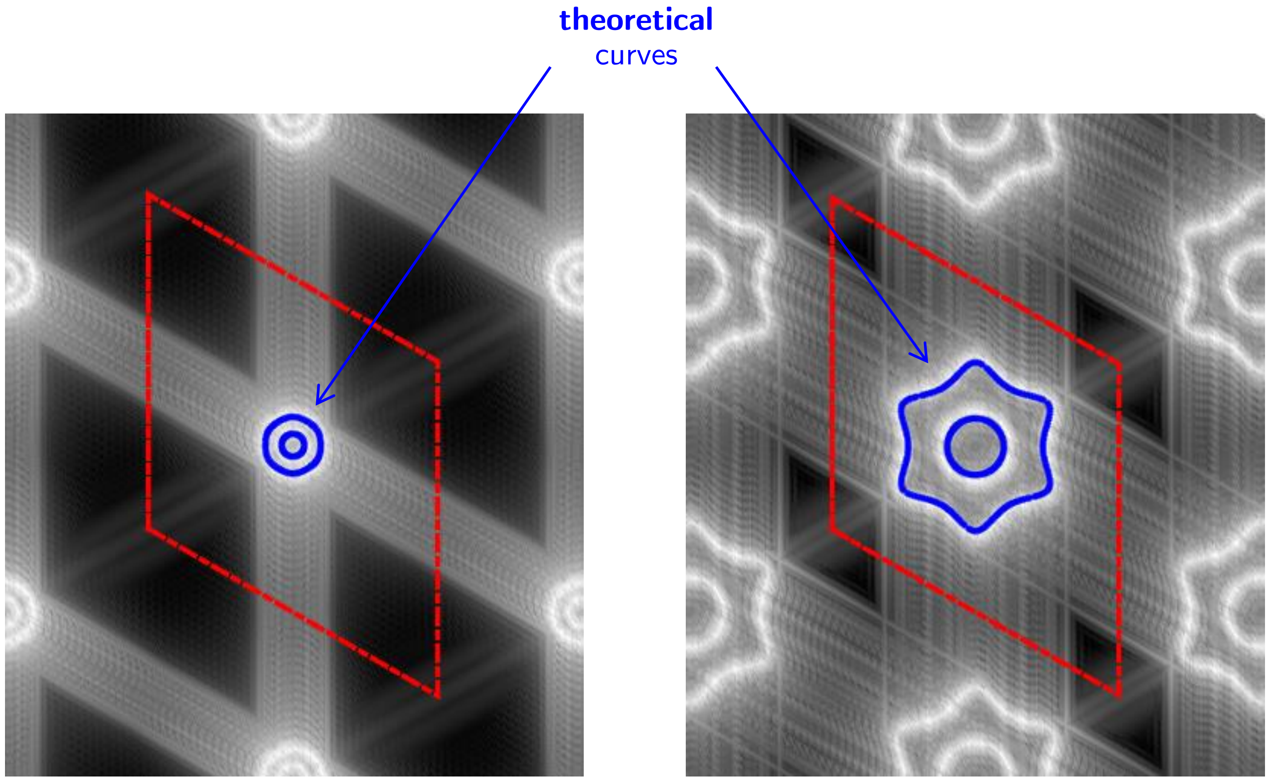
real space



reciprocal space



$$\underline{e}_i \cdot \underline{\xi}_j = 2\pi\delta_{ij}$$



$\omega = 0.1 \times \omega_0$

$\omega = 0.25 \times \omega_0$

Conclusions

Wave decomposition as an **experimental data reduction** & **identification**

- Includes **dynamical** effects while discarding **boundary conditions**
- The **wavelength** gives the **scale** of mechanical stimulation
- The wave **speed** provides information on the **stiffness**
- Wave **modes** can also be retrieved

- Limitations due to the **poor plane wave** approximation in (N>1)D.

- We need to apply these concepts to **real experimental data** !