Initiation de fissure dans les milieux fragiles - Prise en compte des contraintes résiduelles

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Is it a Weibull effect?

Parvizi, Garrett and Bailey experiments (1978)

Applied strain (%) vs INNER ply thickness (mm)

Energy Parvizi et al. (1978)
Parvizi, Garrett and Bailey experiments (1978)

Applied strain (%) vs. Inner ply thickness (mm)

- Energy Parvizi et al., (1978)
These conclusions are contradictory and do not match with the experiments, obviously onset occurs but for a finite applied load.

How to state a criterion for the prediction of the crack onset? The starting point is a very simple energy balance

\[ \delta W^p + \delta W^k + G_c a = 0 \]

And the observation that in quasi-static tests \( \delta W^k \geq 0 \).
The crack nucleation occurs abruptly. The crack jumps from 0 to a length $a$ without any equilibrium state in between (except if it occurs at the tip of a pre-existing crack).

Two conditions must be fulfilled simultaneously (L., 2002):
- there is enough available energy to create a crack of length $a$
- the tensile stress is greater than the tensile strength all along the presupposed crack path (i.e. between 0 and $a$)

\[
\begin{align*}
-\delta W^p(a) & \geq G_c a \\
\sigma(z) & \geq \sigma_c \quad \text{for } 0 \leq z \leq a
\end{align*}
\]

$G_c$ is the material toughness

$\sigma_c$ is the tensile strength

$\delta W^p(a)$: change in potential energy, $z$: distance to the notch root

$\sigma(z)$ is a decreasing function of $z$ then

\[
\frac{G^{\text{inc}}(a)}{G_c} \geq 1 \quad \text{and} \quad \frac{\sigma(a)}{\sigma_c} \geq 1 \quad \text{with} \quad G^{\text{inc}}(a) = -\delta W^p(a) / a \quad \text{incremental energy release rate}
\]

$a$ is unknown (“incremental” vs. “differential” considering the limit $a \to 0$)
The coupled criterion

\[ G^{\text{inc}}(a) = \frac{W^p(0) - W^p(a)}{a} ; \quad G(a) \approx \frac{W^p(a) - W^p(a + \delta a)}{\delta a} \]  

(Martin and L., 2004)

\( G^{\text{inc}} \): incremental energy release rate
\( G \): energy release rate (Griffith, 1920)

- Crack nucleation
  \[ \frac{G^{\text{inc}}(a)}{G_e} \geq 1 \quad \text{and} \quad \frac{\sigma(a)}{\sigma_e} \geq 1 \]

- Crack growth
  \[ \frac{G(a)}{G_e} \geq 1 \]
The coupled criterion – matched asymptotic expansions

Under some assumptions, the previous expressions can be explicitly derived using matched asymptotic expansions (L., 1989)

\[
\begin{aligned}
G^{\text{inc}}(a) &= Ak^2 a^{2\lambda - 1} + ... \\
G(a) &= 2\lambda Ak^2 a^{2\lambda - 1} + ... \\
\sigma(a) &= ka^{2\lambda - 1} + ...
\end{aligned}
\]

\(A\) is a scaling coefficient  
\(k\) is the generalized stress intensity factor of the singularity at the corner  
\(\lambda\) is the exponent of the singularity  
Recalling that the solution of a linear problem of elasticity can develop in the vicinity of the corner as follows (Williams, 1958)

\[
U(r, \theta) = C + k r^2 u(\theta) + ...
\]

\(r\) and \(\theta\) are polar coordinates with origin at the V-notch root.
Then the coupled criterion takes the special form (Irwin-like form) (L., 2002)

\[
\begin{align*}
    a &= \frac{G_c}{A\sigma_c^2} \\
    k \geq k_c &= \left( \frac{G_c}{A} \right)^{1-\lambda} \sigma_c^{2\lambda-1}
\end{align*}
\]

Remind

- Crack nucleation
  \[ \frac{G^{\text{inc}}(a)}{G_c} \geq 1 \quad \text{and} \quad \frac{\sigma(a)}{\sigma_c} \geq 1 \]

- Crack growth
  \[ \frac{G(a)}{G_c} \geq 1 \]
Case 1: monotonic case (energy)

3-point bending on V-notched homogeneous specimen (L., 2002), 4-point bending on V-notched laminated ceramics (Bermejo et al., 2006)
Case 1: monotonic case (energy)

\[ F = 0.8 F_{\text{rupt}} \]
Case 1: monotonic case (energy)

\[ F = 0.97 \, F_{\text{rupt}} \]
Case 1: monotonic case (energy)

\[ F = F_{\text{rupt}} \]
Case 1: monotonic case (energy)

\[ F = F_{\text{rupt}} \]
Case 2: energy governed non monotonic case

Double cleavage drilled specimen (L. et al., 2015), Fiber interface debonding in composite material (Martin et al., 2008) → High $G_c$, low $\sigma_c$
Case 2: energy governed non monotonic case

\[ F = 0.9 F_{\text{rupt}} \]

\[ G^{\text{inc}} \]
Case 2: energy governed non monotonic case
Case 2: energy governed non monotonic case
Case 2: energy governed non monotonic case

\[ F = 1.2 \ F_{\text{rupt}} \]
Double cleavage drilled specimen (L. et al., 2015), interface debonding in composite material (Martin et al., 2008), edge cracking (Chen et al., 2010; L. et al., 2015) ➞ Low $G_c$, high $\sigma_c$
Case 3: stress governed non monotonic case

\[ F = 0.9 \ F_{\text{rupt}} \]
Case 3: stress governed non monotonic case

\[ F = 0.95 \cdot F_{rupt} \]
Case 3: stress governed non monotonic case

\[ F = F_{\text{rupt}} \]
Case 3: stress governed non monotonic case

\[ F = F_{\text{rupt}} \]
Initiation and arrest lengths

$F = F_{\text{rupt}}$

Variables:
- $G$
- $G^{\text{inc}}$

Initiation length: $1$
Lower bound for the arrest length
Upper bound for the arrest length
The first generalization is to consider a mode mix loading: both the critical load and the crack direction must be predicted.

(Yosibash et al., 2006; L. et al., 2009)
Increasing complexity

Symmetric loading, opening mode
Data: $G_{lc}, \sigma_c$ (Leguillon, 2002)
1 unknown (critical load)

Complex loading, opening mode
Data: $G_{lc}, \sigma_c$ (Yosibash et al., 2006)
2 unknowns (critical load + crack direction)

Complex loading, mixed mode (guided crack)
Data: $G_{lc}, G_{llc}, \sigma_c, \tau_c$ (Tran et al., 2012)
1 unknown (critical load)
Residual stresses in an adhesive layer (epoxy).

Matched asymptotic expansions

\[ k = k^m + k^t \geq k_e = \left( \frac{G_c}{A} \right)^{1-\lambda} (\sigma_c - \Delta \Theta \bar{\sigma}) \]

(Henninger and Leguillon, 2008)

Comparison with experiments (Kian and Akisanya, 1998)

\[ \Delta \Theta = -140 \text{ deg.} \]

(i) \( (\sigma_c, G_c) = (45 \text{ MPa}, 46 \text{ Jm}^{-2}) \) neglecting the residual stresses; with residual stresses; (ii) \( (\sigma_c, G_c) = (45 \text{ MPa}, 19 \text{ Jm}^{-2}) \) with residual stresses.
A similar result can be obtained in brazed assemblies of SiC structures (Nguyen et al., 2012).

Comparison with experiments carried out in CEA Grenoble: load at failure vs. solder thickness. The transition between the energy driven criterion and the stress driven one is visible.
The crack initiates at the notch root and then grows unstably and penetrates the next layer and stop as $k_1 = 0$ not $k_1 = k_{lc}$ (i.e. the cracks is closed due to the compressive residual stresses). Then after reloading, it kinks (Leguillon et al., 2015). Only pure FE calculations are allowed because the layers thickness is similar to the crack length.
Laminated ceramics: before kinking

$F = 0.9 \, F_{rupt}$

$\sigma$

$G^{inc}$

$a$
Laminated ceramics: kinking

\[ F = F_{\text{rupt}} \]

\[ \sigma \]

\[ G^{\text{inc}} \]
Laminated ceramics: kinking

$F = F_{\text{rupt}}$

$\sigma$

$G$

$G^{\text{inc}}$

$a$

1
Laminated ceramics: stable growth

\[ F = 1.2 F_{\text{rupt}} \]
Laminated ceramics: instability

\[ F = 1.4 F_{\text{rupt}} \]
Laminated ceramics: a complete scenario

Crack kinking followed by a stable growth of the deflected crack

The pop-in occurs at $F=52$ N in the experiments and is predicted at $F=46$ N in the model.

Instability

![Graph showing load vs. displacement with points labeled 1, 2, and 3, and a pop-in at 52 N in experiments and 46 N in the model.](image)
Residual stresses in a polymer due to oxidation

- Oxidized layer 600 h
- Sound material
- Pristine polymer or oxidized 200 h
- Oxidized polymer 400 h or 600 h
- Abrupt failure
- Multi-cracking
The challenge here is that in the oxydized layer, the Young modulus, the tensile strength and the toughness vary. Again, only pure FE calculations are allowed. A comparison with experiments (Pannier et al., 2014; Leguillon et al., 2016) shows a good agreement, both in terms of applied load at initiation of crack(s) and in terms of cracks density.
Grinding a notch in zirconia produces big stresses at the notch which induces a phase change and generates high compressive residual stresses (around 1800 MPa) in a thin layer of a few micrometers (< 10 µm). They are released by a heat treatment.
Experiments by Andersons et al. (2008)
Thermal residual stresses in sub-micron films

Coating intrinsic strain (%) vs. Film thickness (µm)

Hashin (energy based criterion)
Coating intrinsic strain (%) vs. Film thickness (µm)

Thermal residual stresses in sub-micron films

Graph showing the relationship between coating intrinsic strain and film thickness.
Scaled crack density vs. Scaled applied strain (µm$^{1/2}$)

Theory

- 0.043 µm
- 0.067 µm
- 0.090 µm
- 0.120 µm
- 0.320 µm