Initiation de fissure dans les milieux fragiles - Prise en compte des contraintes résiduelles

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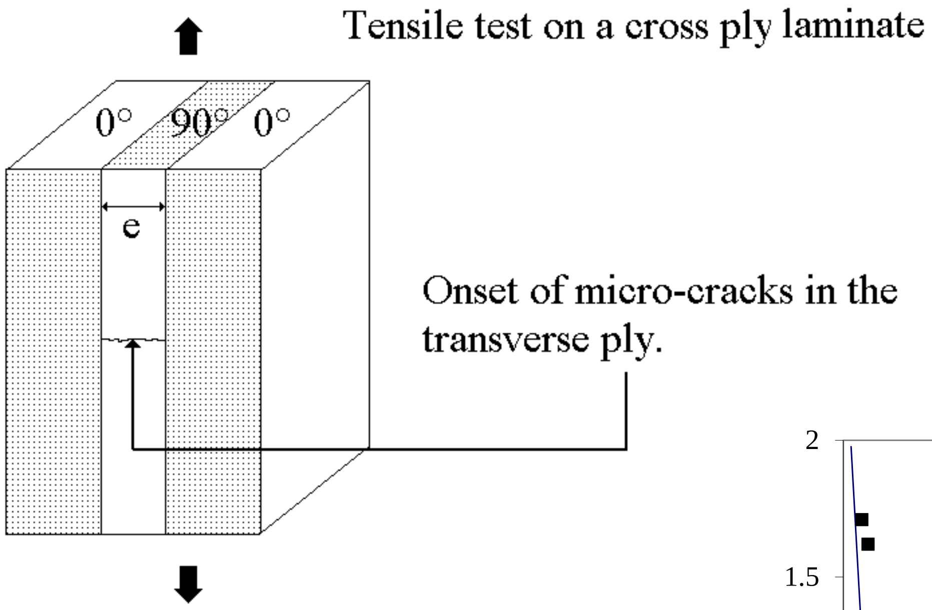
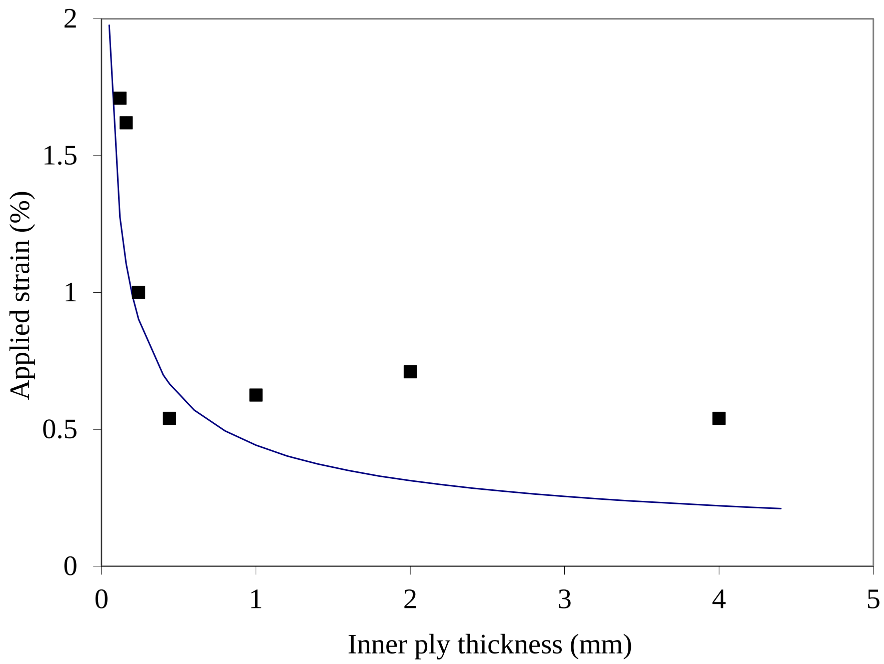
Parvizi, Garrett and Bailey experiments (1978)

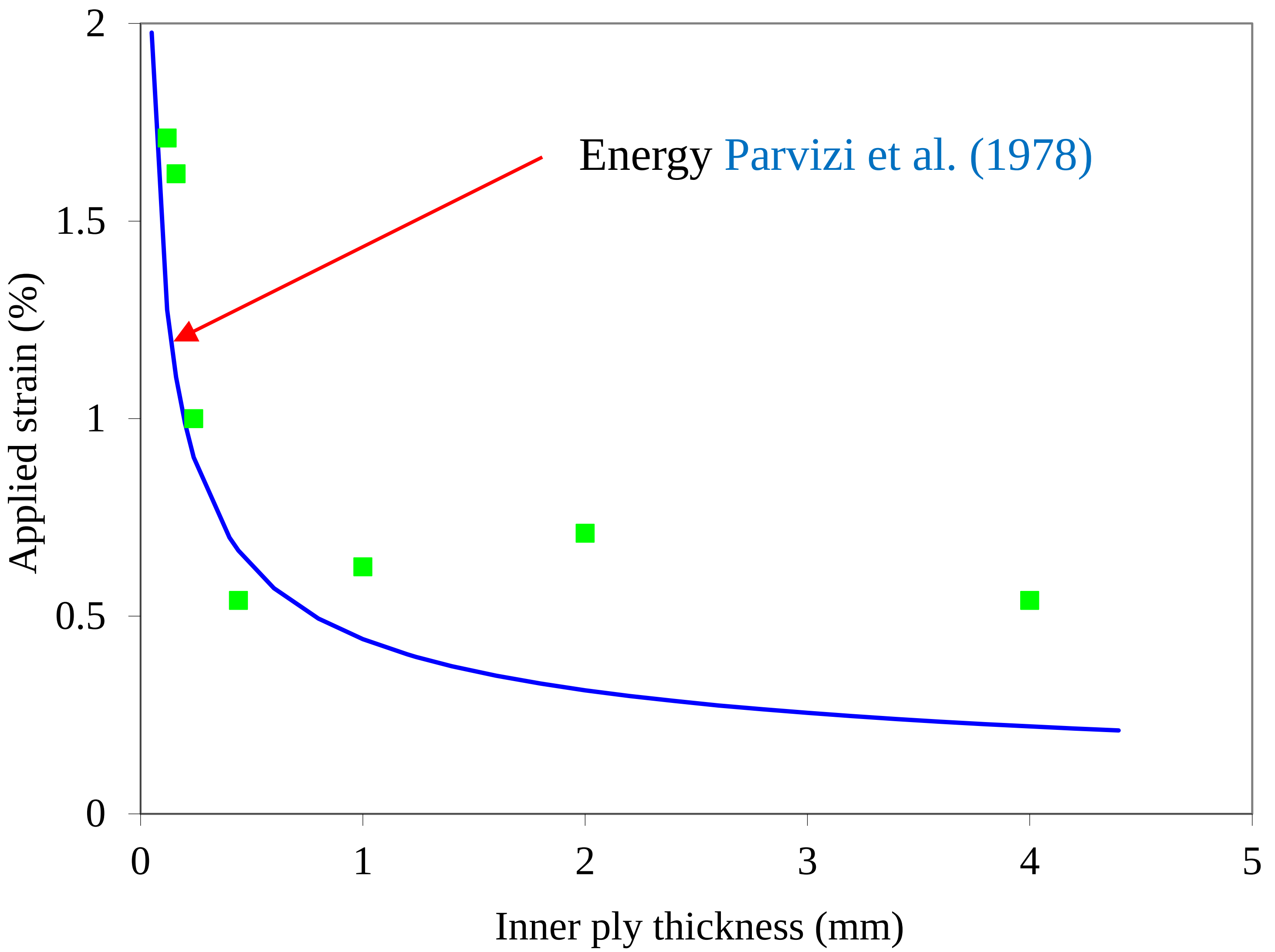




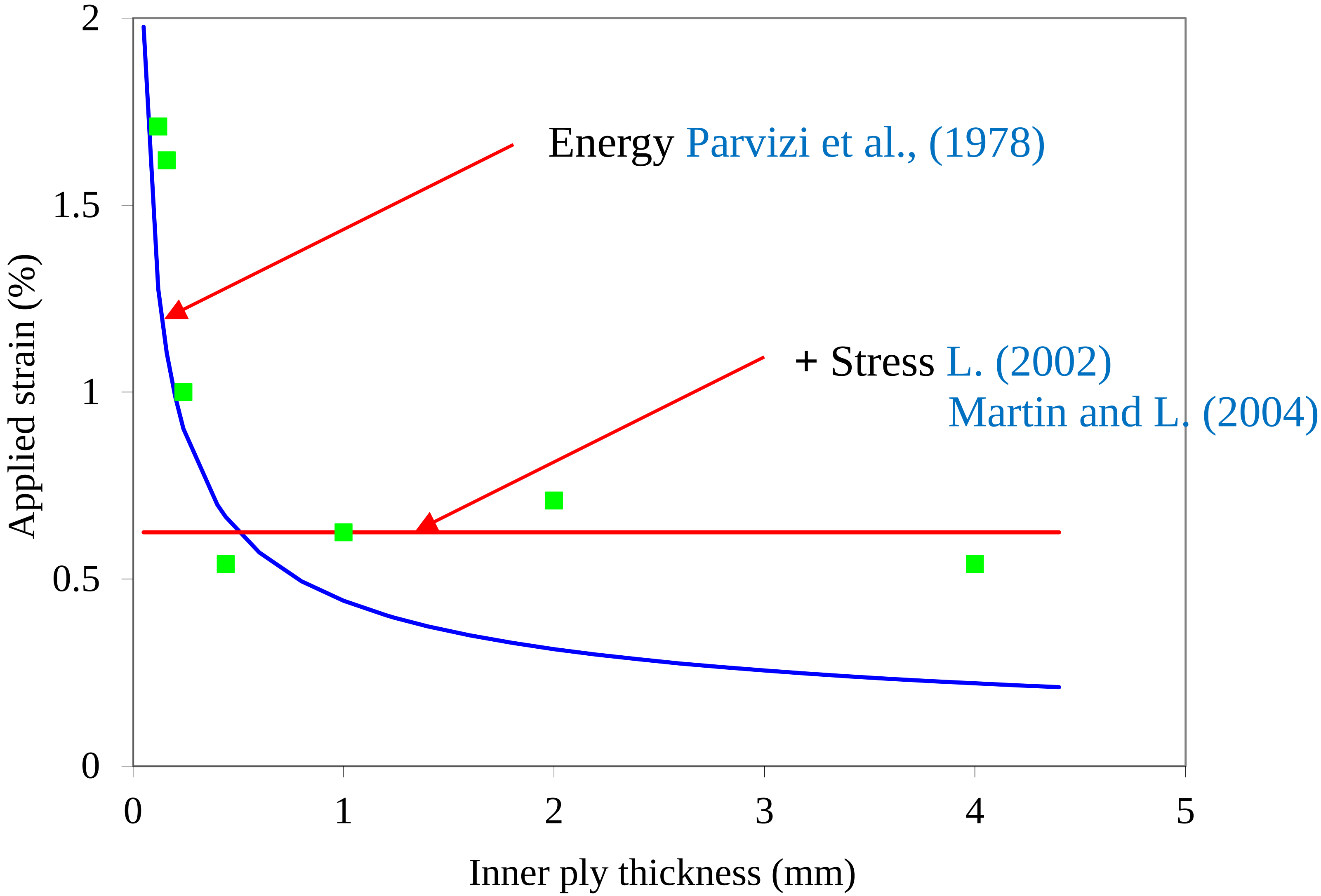
Is it a Weibull effect?

Controversy: Kelly (1988), L. et al. (2015)

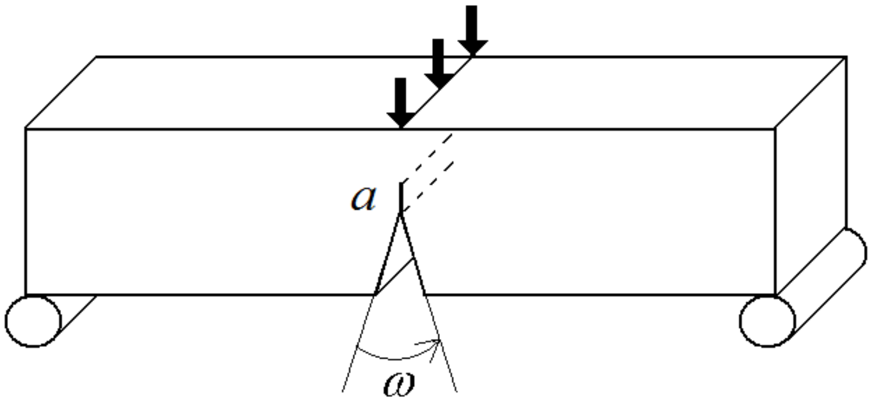
Parvizi, Garrett and Bailey experiments (1978)



Parvizi, Garrett and Bailey experiments (1978)



3-point bending of V-notched specimens



Stress criterion → onset whatever the applied load for $\omega < \pi$.

Energy criterion (Griffith) → no onset whatever the applied load if $\omega > 0$.

These conclusions are contradictory and do not match with the experiments, obviously onset occurs but for a finite applied load.

How to state a criterion for the prediction of the crack onset? The starting point is a very simple energy balance

$$\delta W^{\mathrm{P}} + \delta W^{\mathrm{K}} + G_{\mathrm{c}} a = 0$$

And the observation that in quasi-static tests $\delta W^{\mathrm{K}} \geq 0$

The crack nucleation occurs abruptly. The crack jumps from 0 to a length a without any equilibrium state in between (except if it occurs at the tip of a pre-existing crack)

Two conditions must be fulfilled simultaneously (L., 2002):

• there is enough available energy to create a crack of length a

• the tensile stress is greater than the tensile strength all along the presupposed crack path (i.e. between 0 and a)

$$\begin{cases} -\delta W^{\mathrm{P}}(a) \geq G_{\mathrm{c}}\, a \\ \sigma(z) \geq \sigma_{\mathrm{c}} \quad \text{for} \quad 0 \leq z \leq a \end{cases}$$

G_{c} is the material toughness

σ_{c} is the tensile strength

$\delta W^{\mathrm{P}}(a)$: change in potential energy, z: distance to the notch root

$\sigma(z)$ is a decreasing function of z then

$$\frac{G^{\mathrm{inc}}(a)}{G_{\mathrm{c}}} \geq 1 \quad \text{and} \quad \frac{\sigma(a)}{\sigma_{\mathrm{c}}} \geq 1$$ with $G^{\mathrm{inc}}(a) = -\delta W^{\mathrm{P}}(a)/a$ incremental energy release rate

a is unknown (“incremental” vs. “differential” considering the limit $a \to 0$)

$$G^{\text{inc}}(a) = \frac{W^{\text{P}}(0) - W^{\text{P}}(a)}{a} \quad ; \quad G(a) \simeq \frac{W^{\text{P}}(a) - W^{\text{P}}(a+\delta a)}{\delta a}$$ (Martin and L., 2004)

G^{inc}: incremental energy release rate
G: energy release rate (Griffith, 1920)

- Crack nucleation $\dfrac{G^{\text{inc}}(a)}{G_{\text{c}}} \geq 1$ and $\dfrac{\sigma(a)}{\sigma_{\text{c}}} \geq 1$

- Crack growth $\dfrac{G(a)}{G_{\text{c}}} \geq 1$

The coupled criterion – matched asymptotic expansions

Under some assumptions, the previous expressions can be explicitly derived using matched asymptotic expansions (L., 1989)

$$\begin{cases} G^{\text{inc}}(a) = Ak^2 a^{2\lambda-1} + \ldots \\ G(a) = 2\lambda Ak^2 a^{2\lambda-1} + \ldots \\ \sigma(a) = ka^{\lambda-1} + \ldots \end{cases}$$

A is a scaling coefficient
k is the generalized stress intensity factor of the singularity at the corner
λ is the exponent of the singularity
Recalling that the solution of a linear problem of elasticity can develop in the vicinity of the corner as follows (Williams, 1958)

$$\underline{U}(r,\theta) = \underline{C} + k\, r^{\lambda} \underline{u}(\theta) + \ldots$$

r and θ are polar coordinates with origin at the V-notch root.

Then the coupled criterion takes the special form (Irwin-like form) (L., 2002)

$$\begin{cases} a = \dfrac{G_c}{A\sigma_c^2} \\ k \geq k_c = \left(\dfrac{G_c}{A}\right)^{1-\lambda} \sigma_c^{2\lambda-1} \end{cases}$$

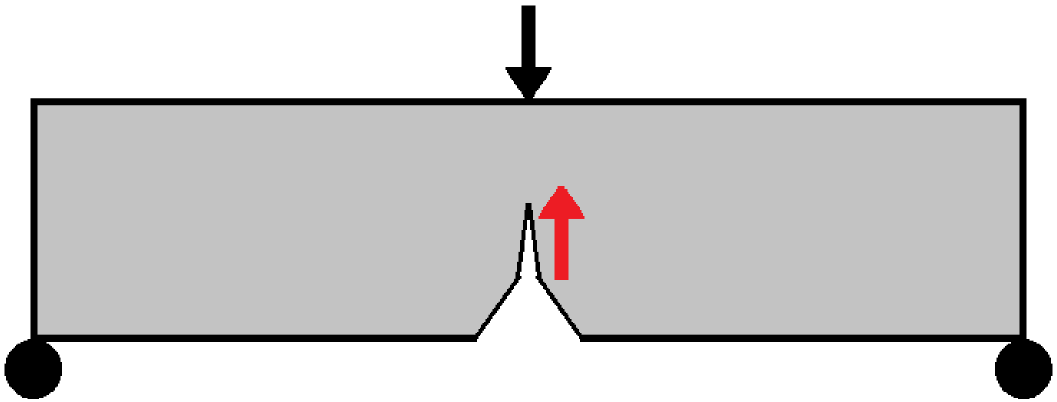
Remind

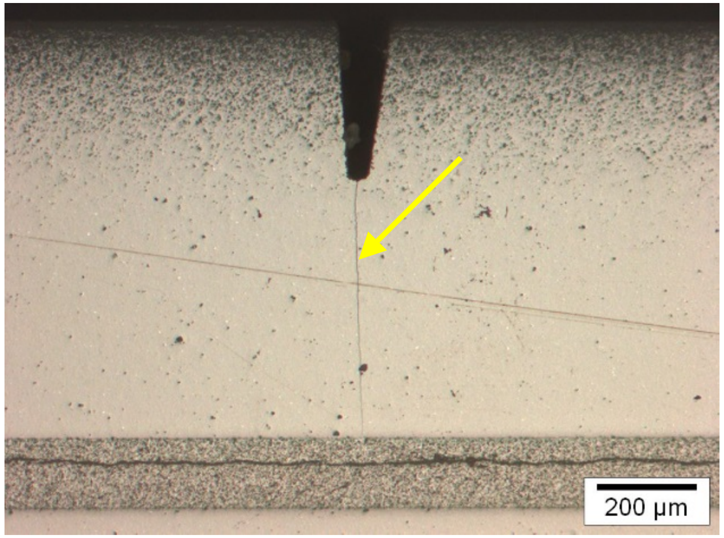
- Crack nucleation $\dfrac{G^{\text{inc}}(a)}{G_c} \geq 1$ and $\dfrac{\sigma(a)}{\sigma_c} \geq 1$

- Crack growth $\dfrac{G(a)}{G_c} \geq 1$

Case 1: monotonic case (energy)

3-point bending on V-notched homogeneous specimen (L., 2002), 4-point bending on V-notched laminated ceramics (Bermejo et al., 2006)





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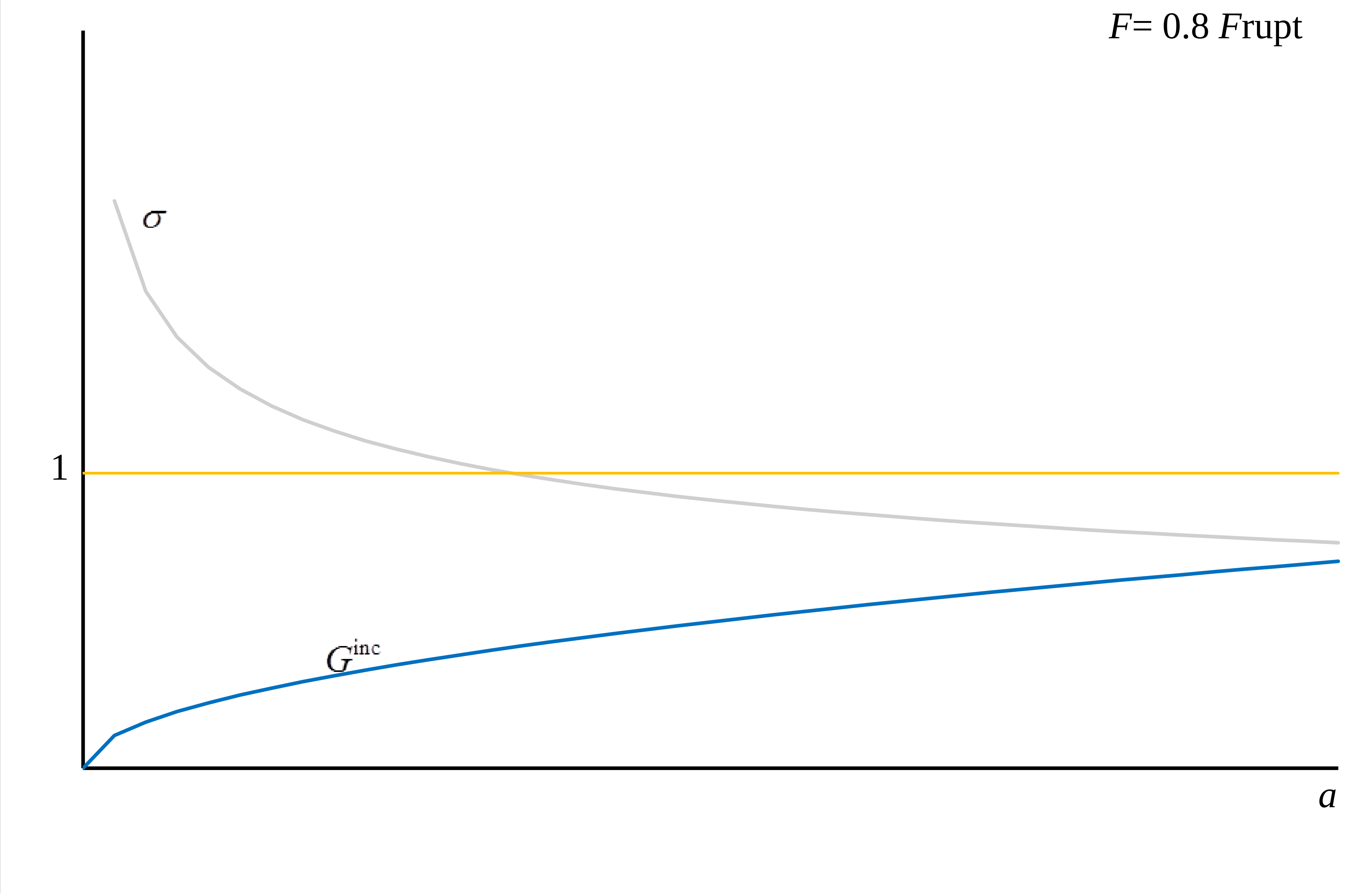

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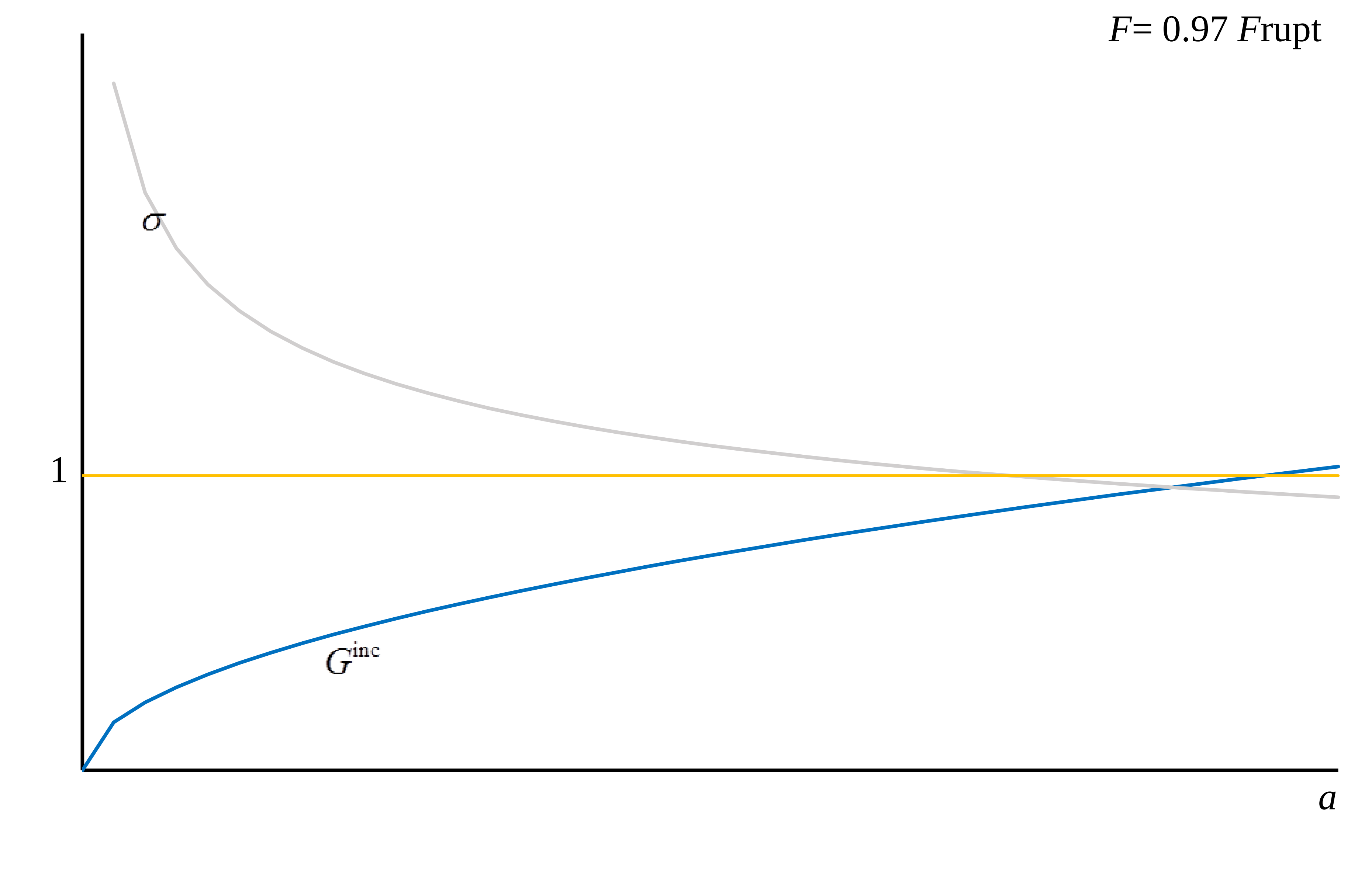

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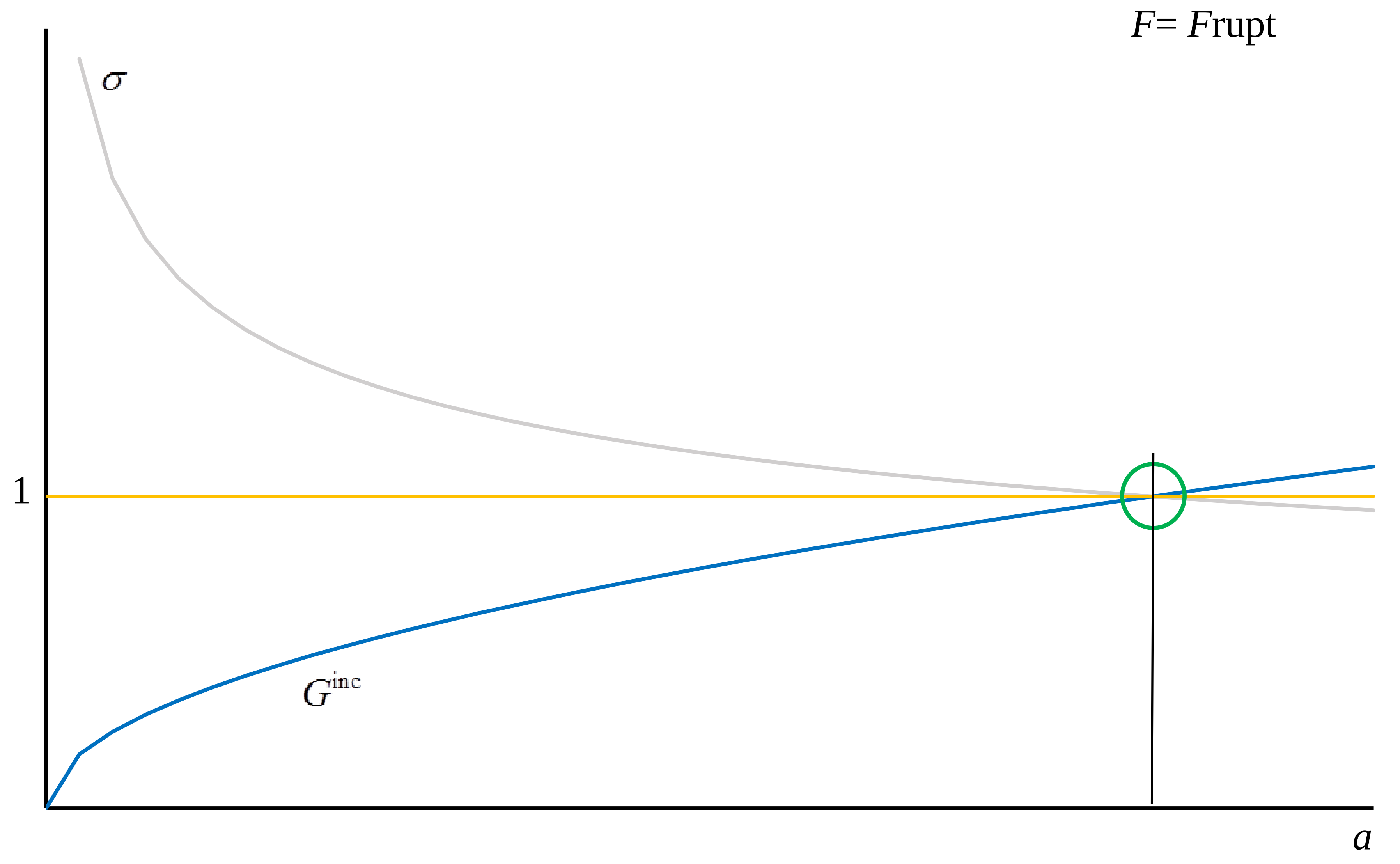

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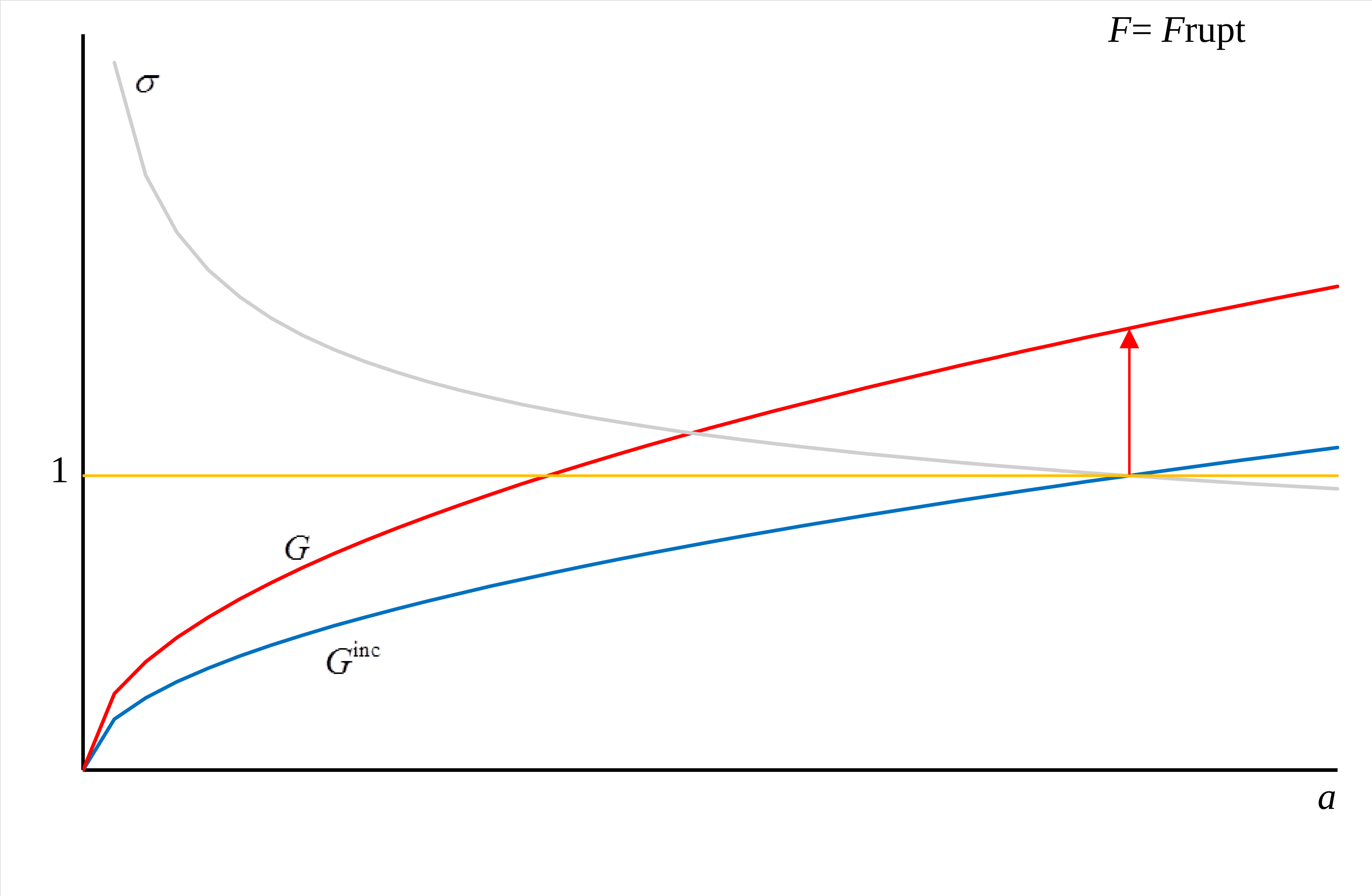
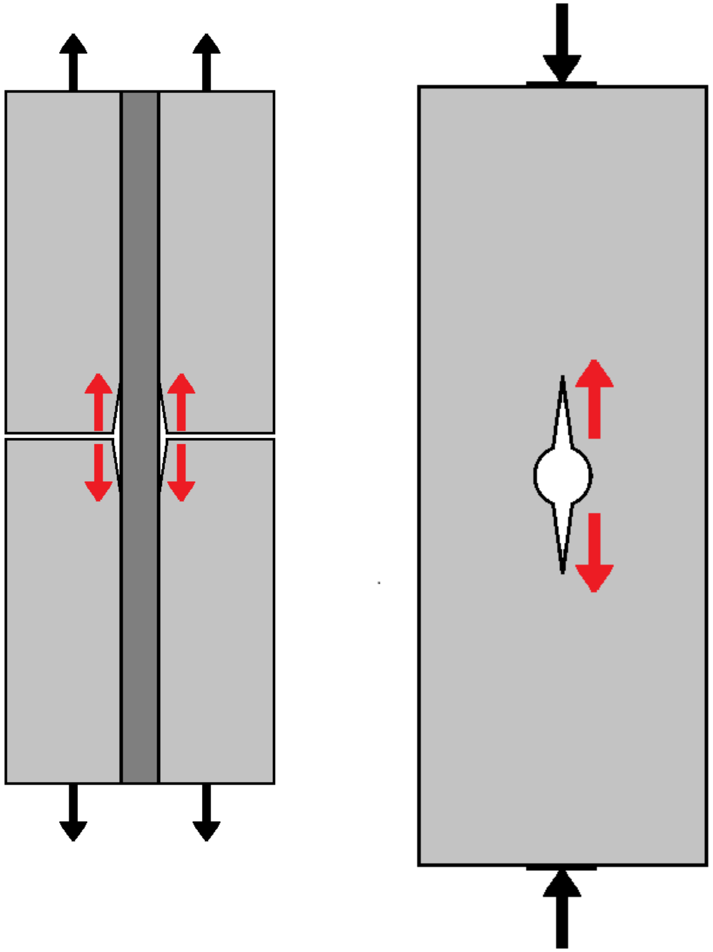


Case 2: energy governed non monotonic case

Double cleavage drilled specimen (L. et al., 2015), Fiber interface debonding in composite material (Martin et al., 2008) ➔ High G_c, low σ_c



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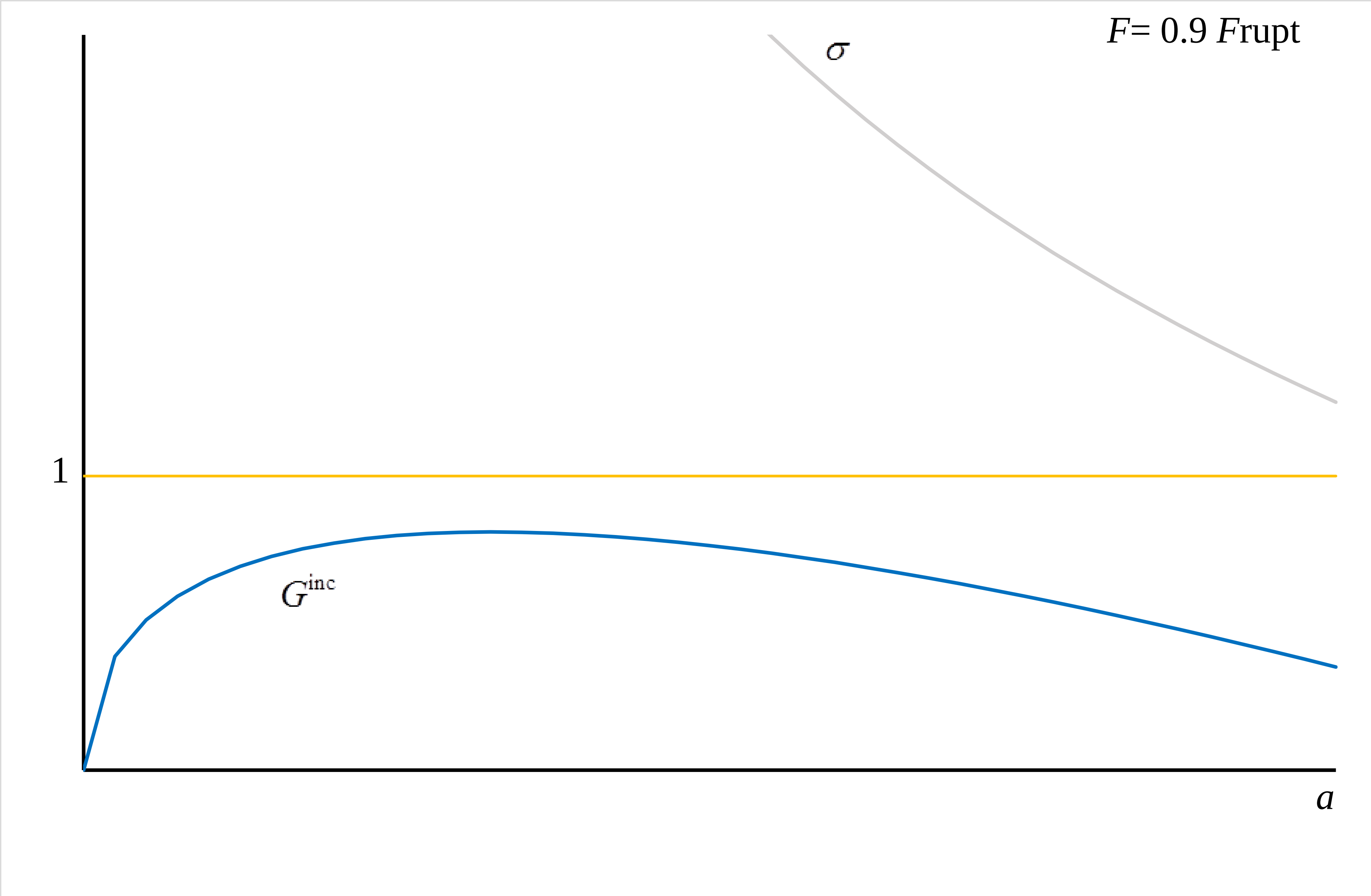

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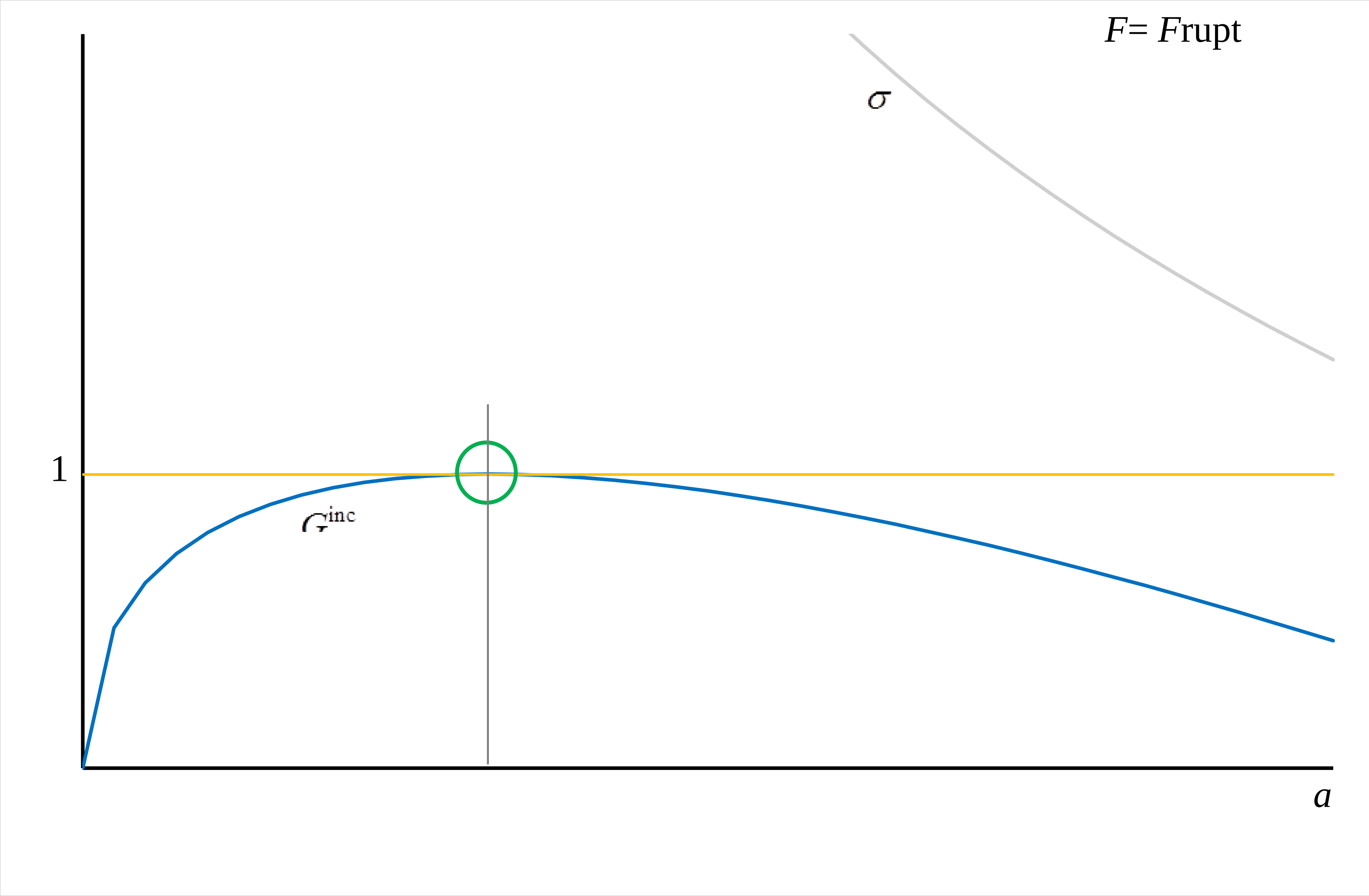

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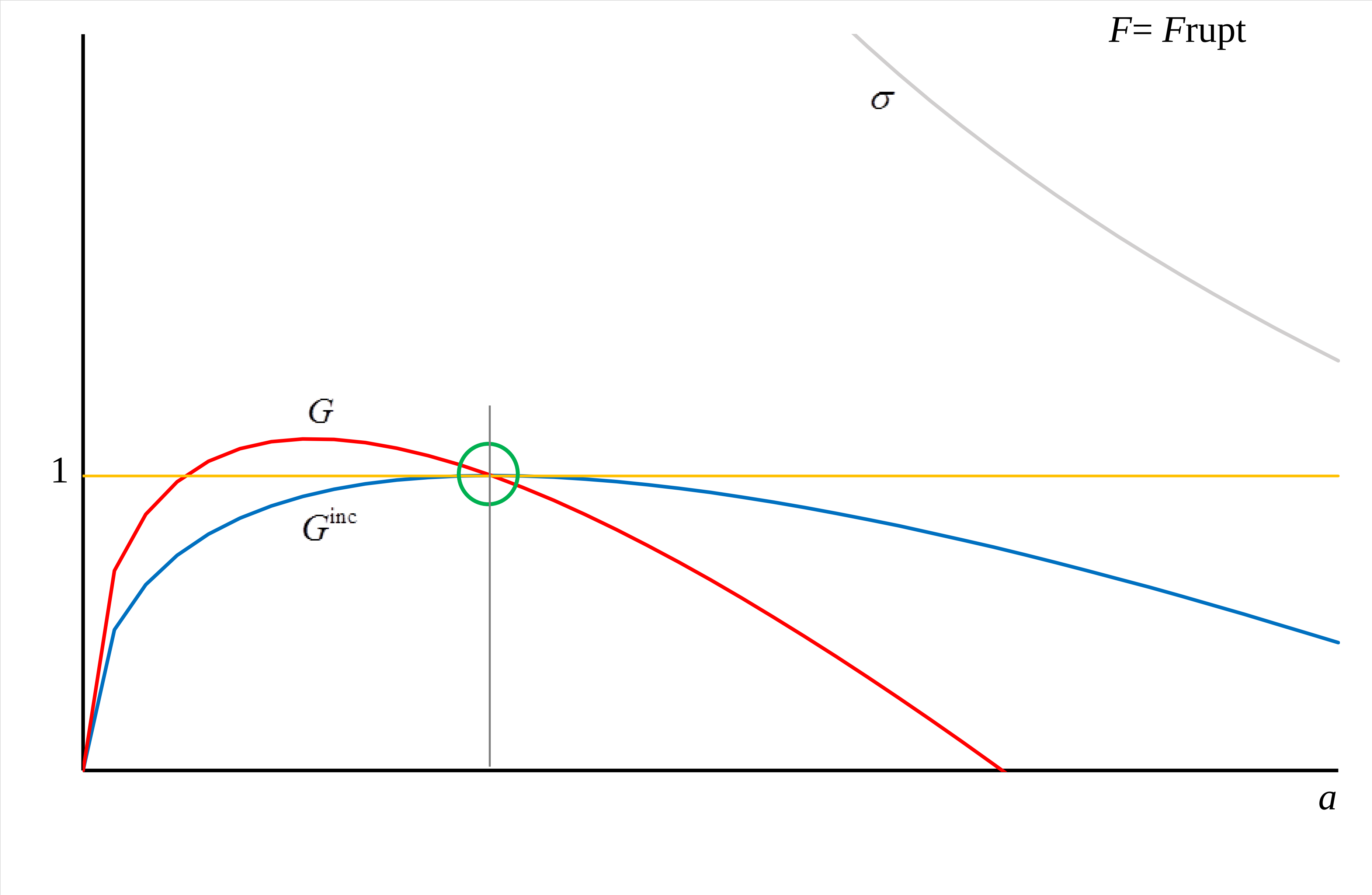

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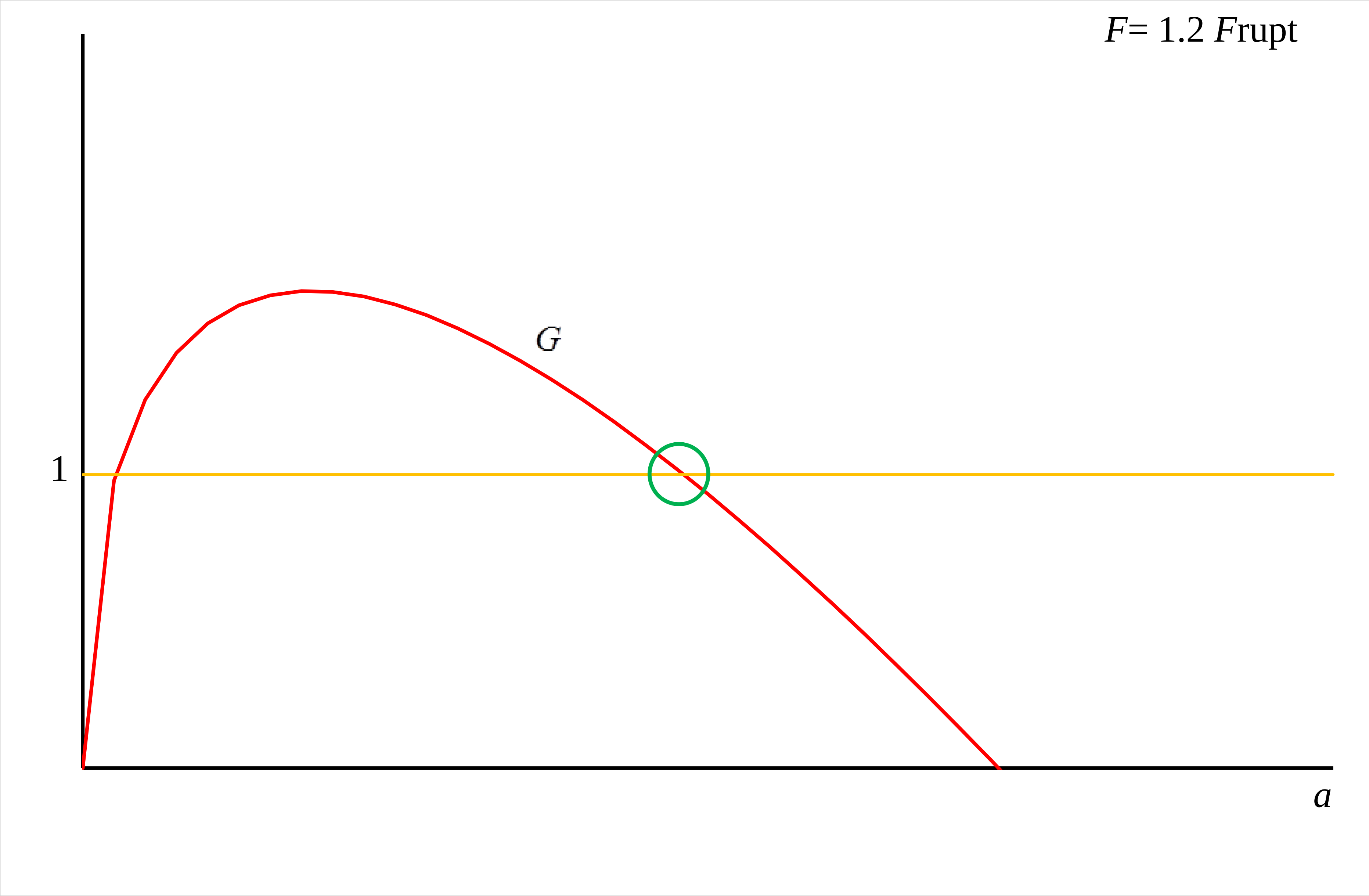
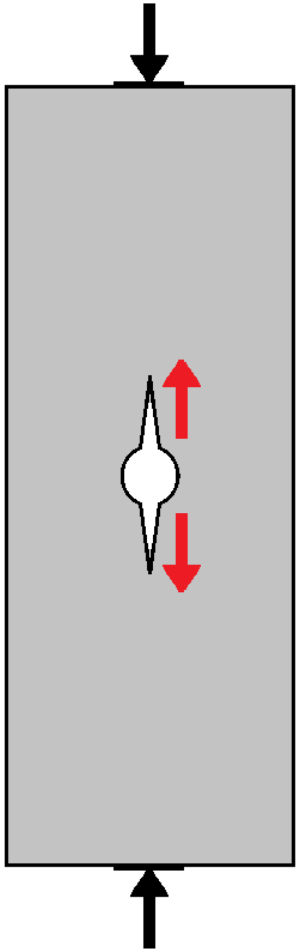
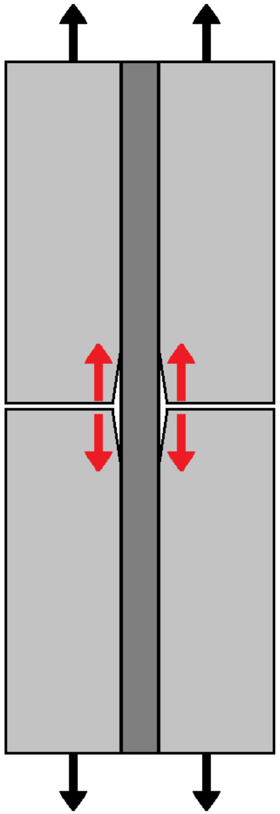


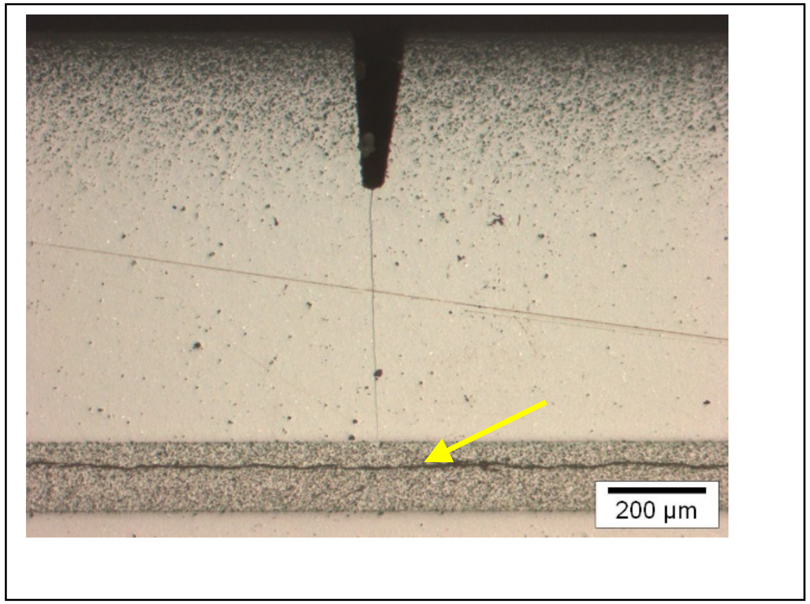
Case 3: stress governed non monotonic case

Double cleavage drilled specimen (L. et al., 2015), interface debonding in composite material (Martin et al., 2008), edge cracking (Chen et al., 2010; L. et al., 2015)

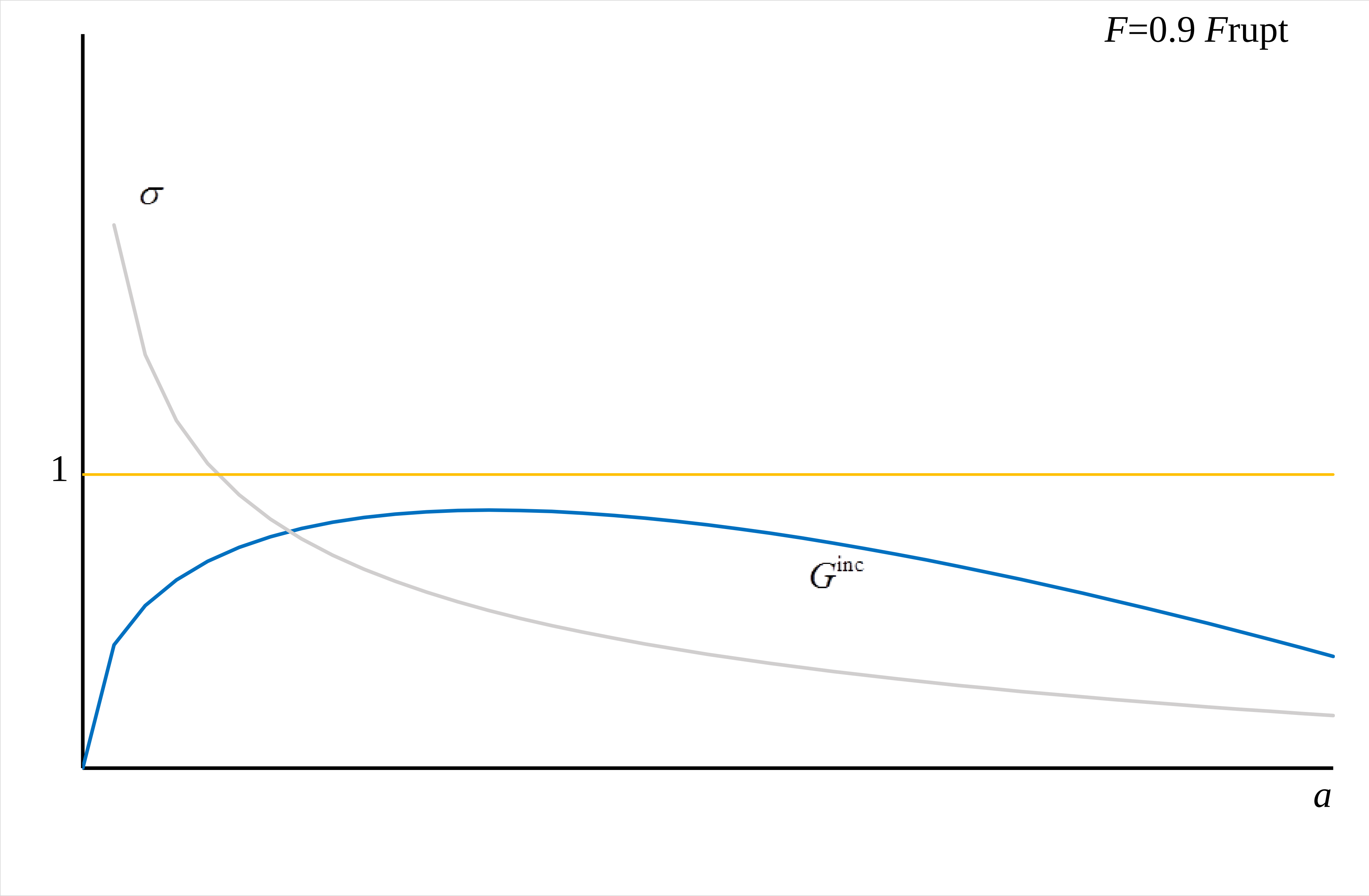
➔ Low G_c, high σ_c







Case 3: stress governed non monotonic case



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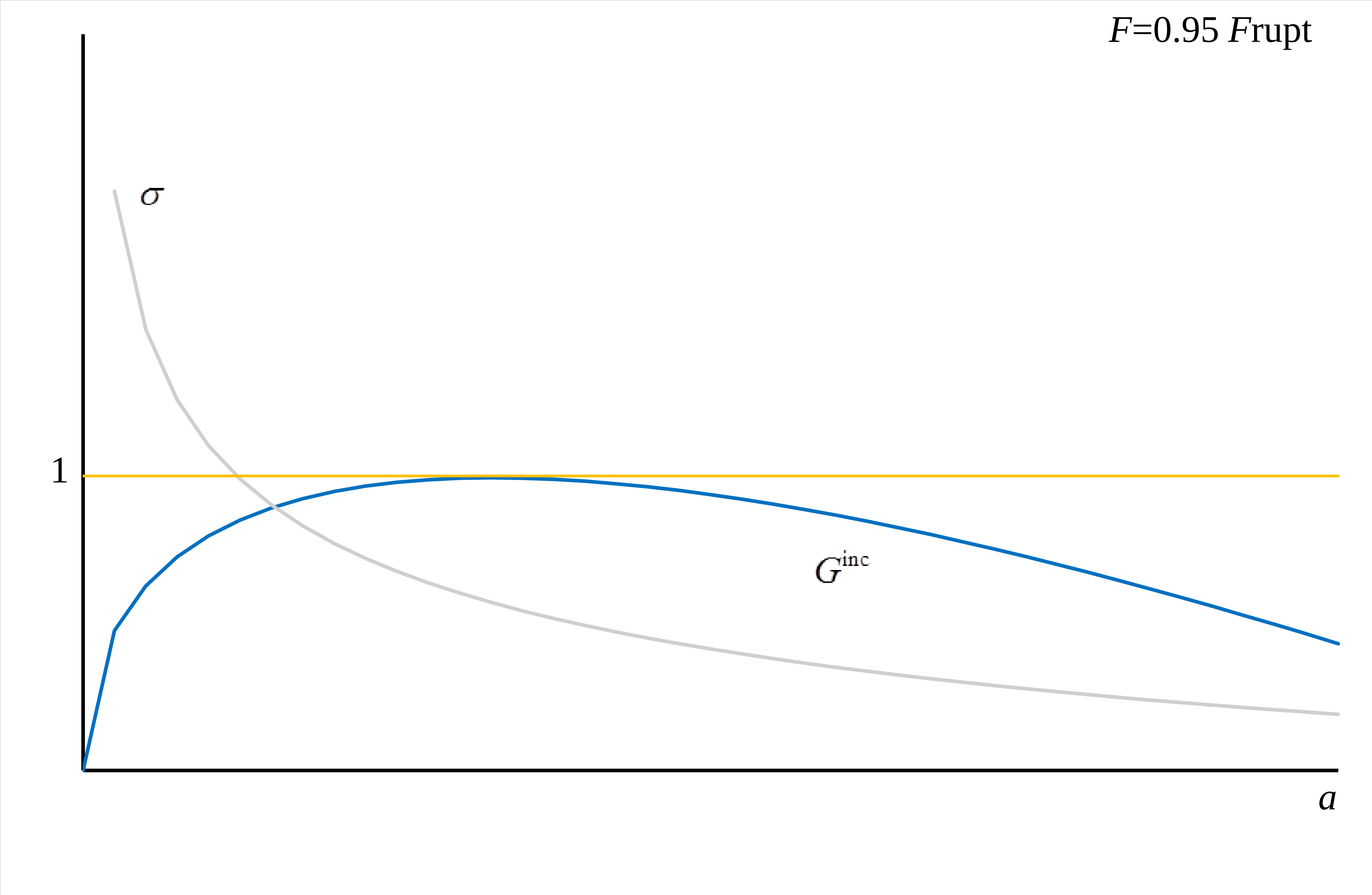

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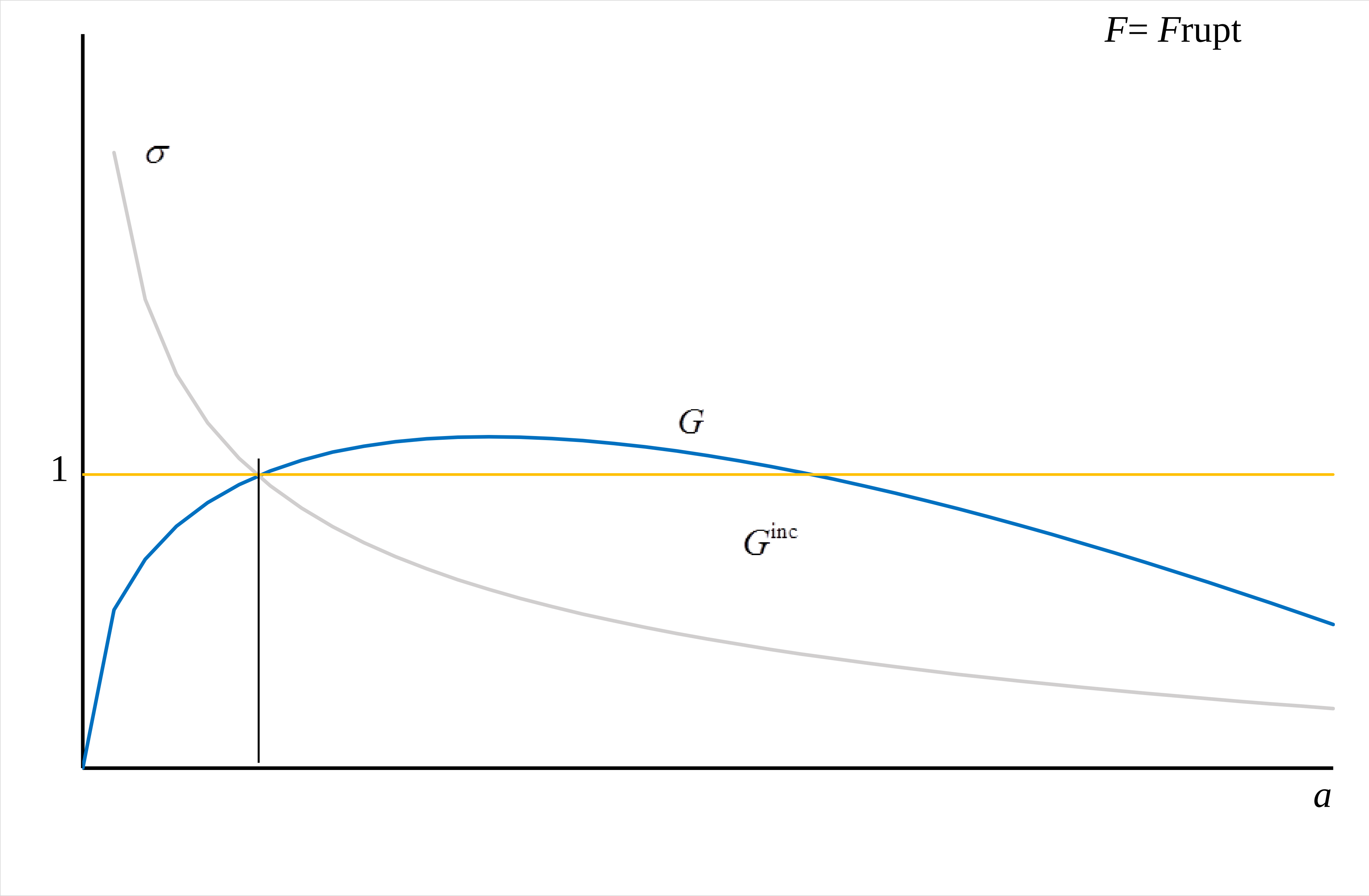

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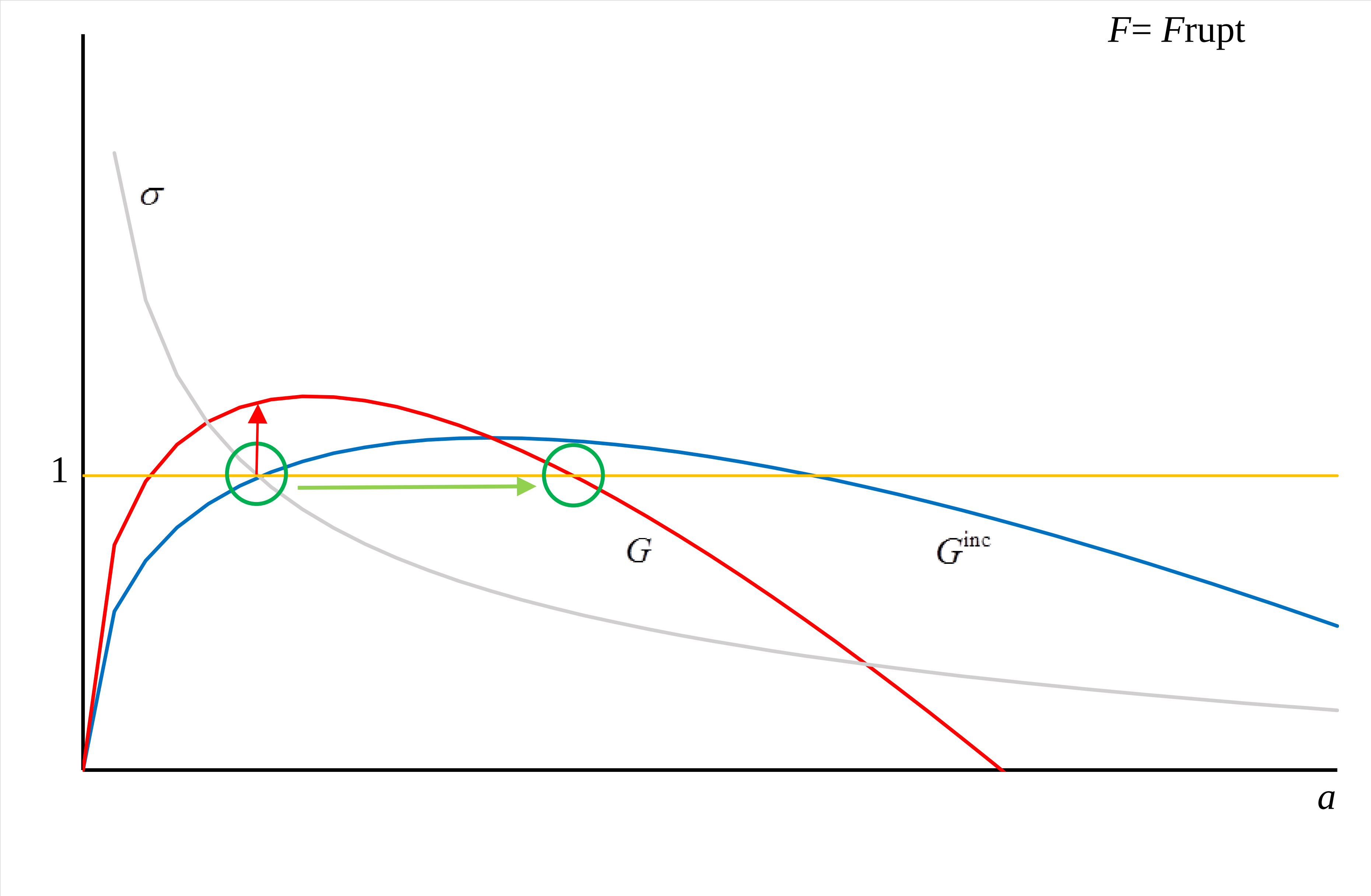
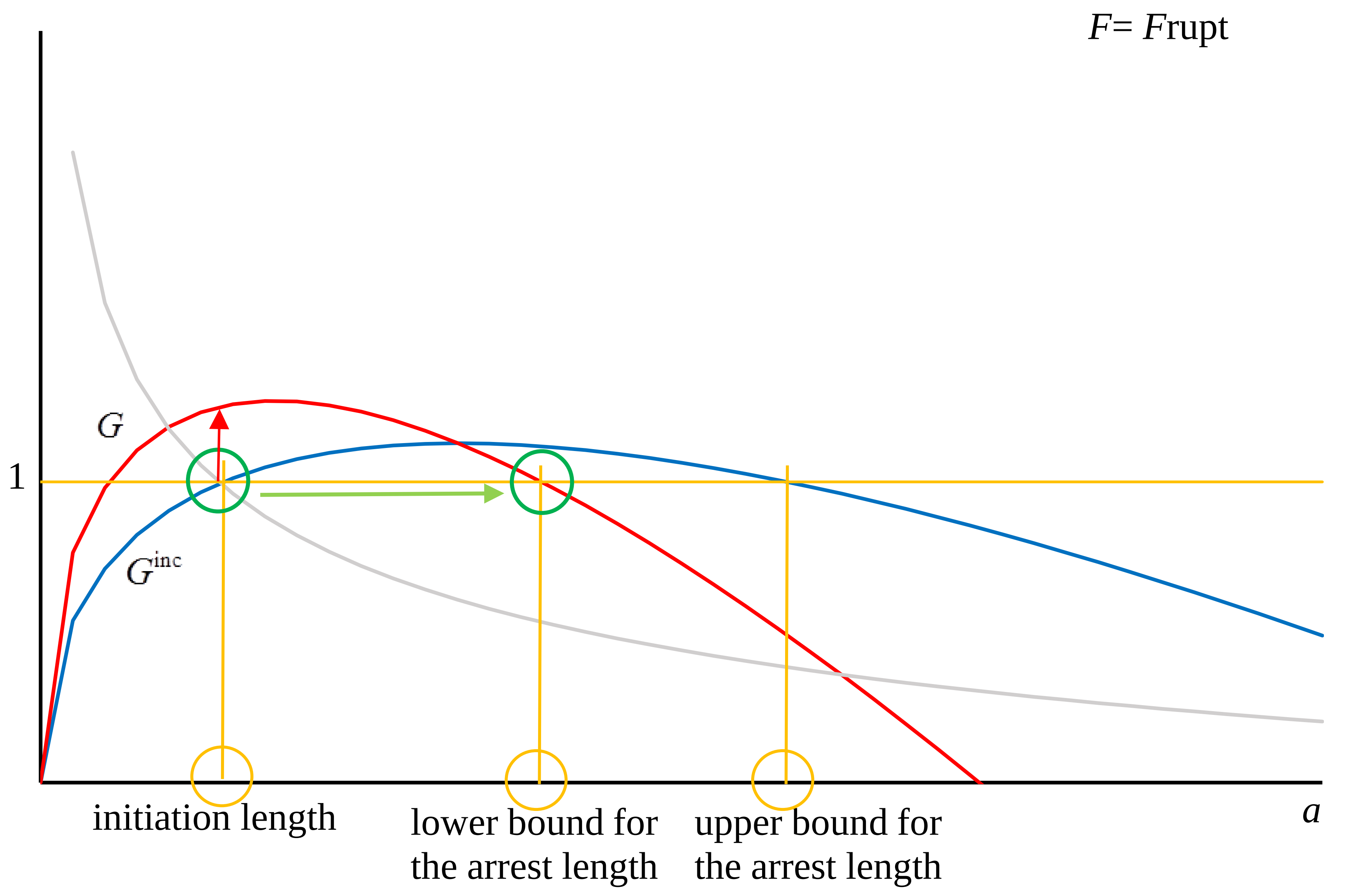
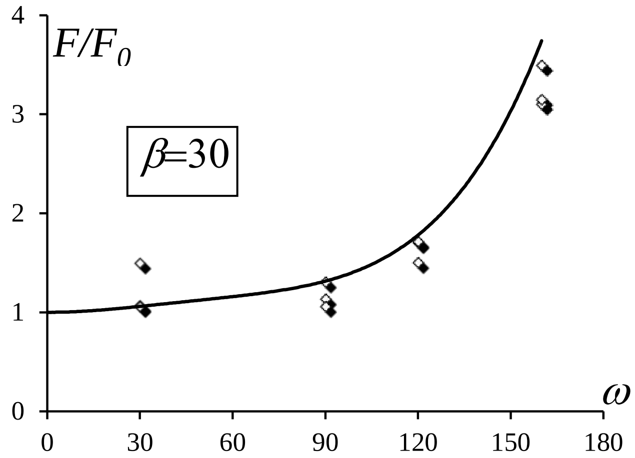
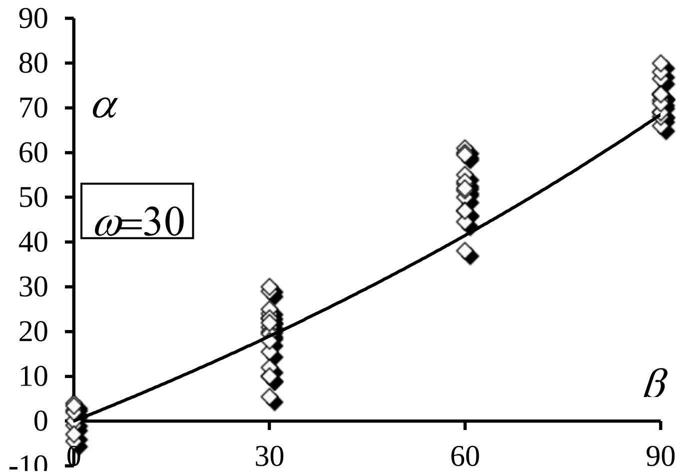


Initiation and arrest lengths



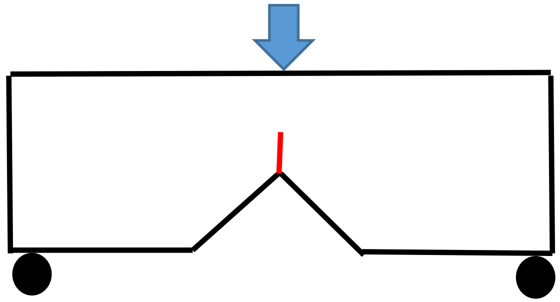
The first generalization is to consider a mode mix loading: both the critical load and the crack direction must be predicted.



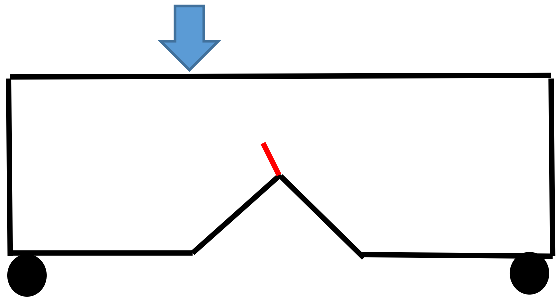


(Yosibash et al., 2006 ; L. et al., 2009)

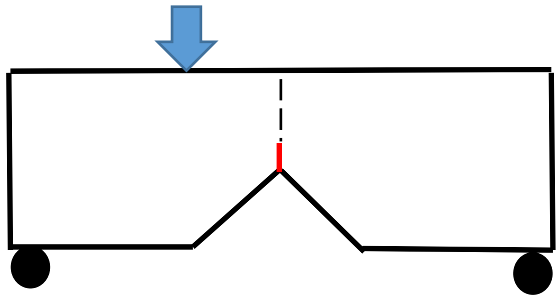
Increasing complexity



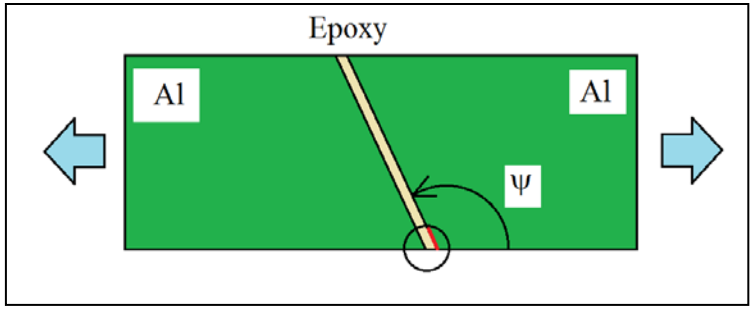
Symmetric loading, opening mode
Data: G_{Ic}, σ_c (Leguillon, 2002)
1 unknown (critical load)



Complex loading, opening mode
Data: G_{Ic}, σ_c (Yosibash et al., 2006)
2 unknowns (critical load + crack direction)



Complex loading, mixed mode (guided crack)
Data: G_{Ic}, G_{IIc}, σ_c, τ_c (Tran et al., 2012)
1 unknown (critical load)

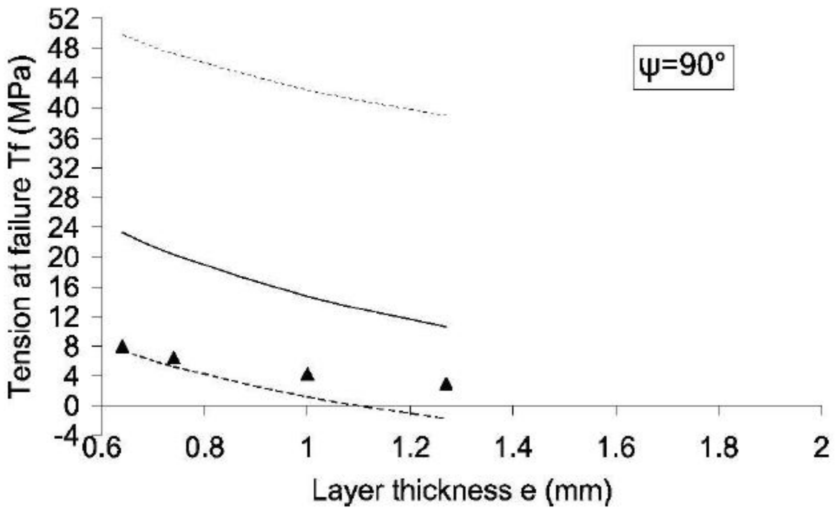


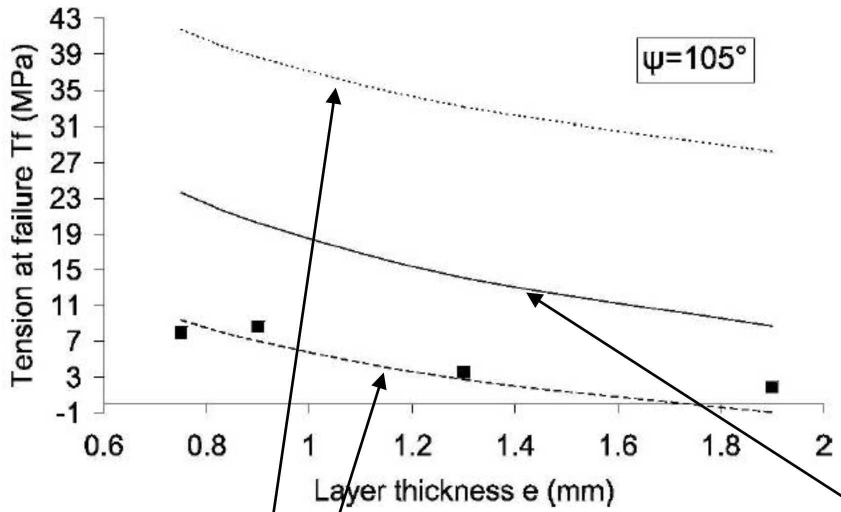
Residual stresses in an adhesive layer (epoxy).
Matched asymptotic expansions

$$k = k^{\mathrm{m}} + k^{\mathrm{t}} \geq k_{\mathrm{c}} = \left(\frac{G_{\mathrm{c}}}{A}\right)^{1-\lambda} \left(\sigma_{\mathrm{c}} - \Delta\Theta\bar{\sigma}\right)$$

(Henninger and Leguillon, 2008)

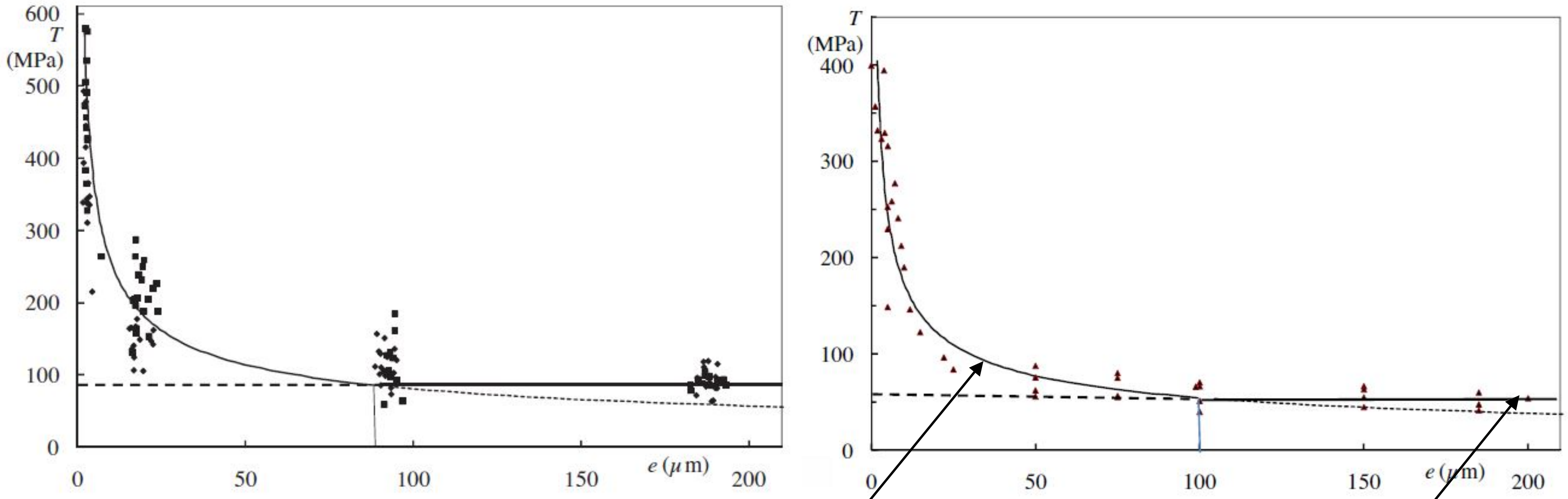
Comparison with experiments (Kian and Akisanya, 1998)





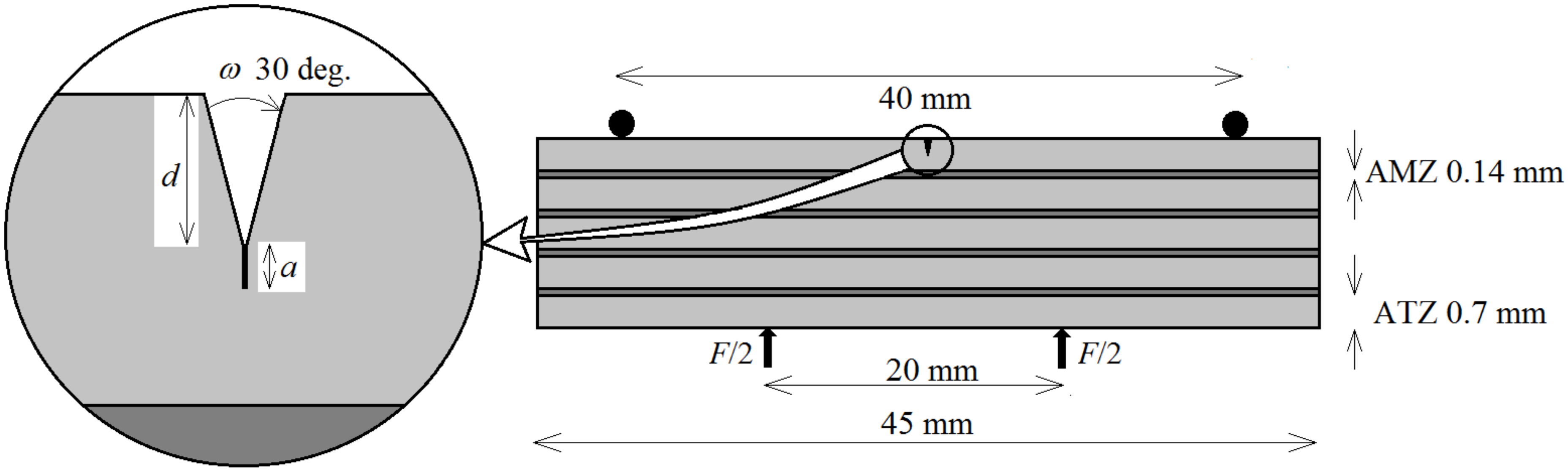
$\Delta\Theta = -140$ deg. (i) $(\sigma_{\mathrm{c}}, G_{\mathrm{c}}) = (45\ \text{MPa},\ 46\ \text{Jm}^{-2})$ neglecting the residual stresses; with residual stresses; (ii) $(\sigma_{\mathrm{c}}, G_{\mathrm{c}}) = (45\ \text{MPa},\ 19\ \text{Jm}^{-2})$ with residual stresses.

A similar result can be obtained in brazed assemblies of SiC structures (Nguyen et al., 2012)

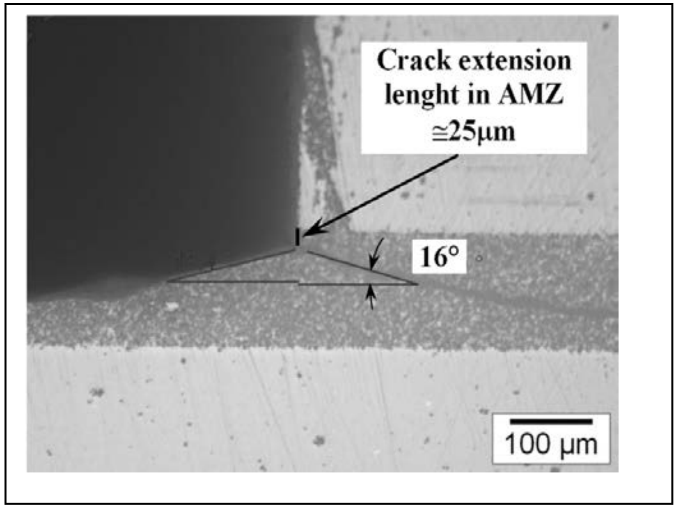


Comparison with experiments carried out in CEA Grenoble: load at failure vs. solder thickness. The transition between the energy driven criterion and the stress driven one is visible.

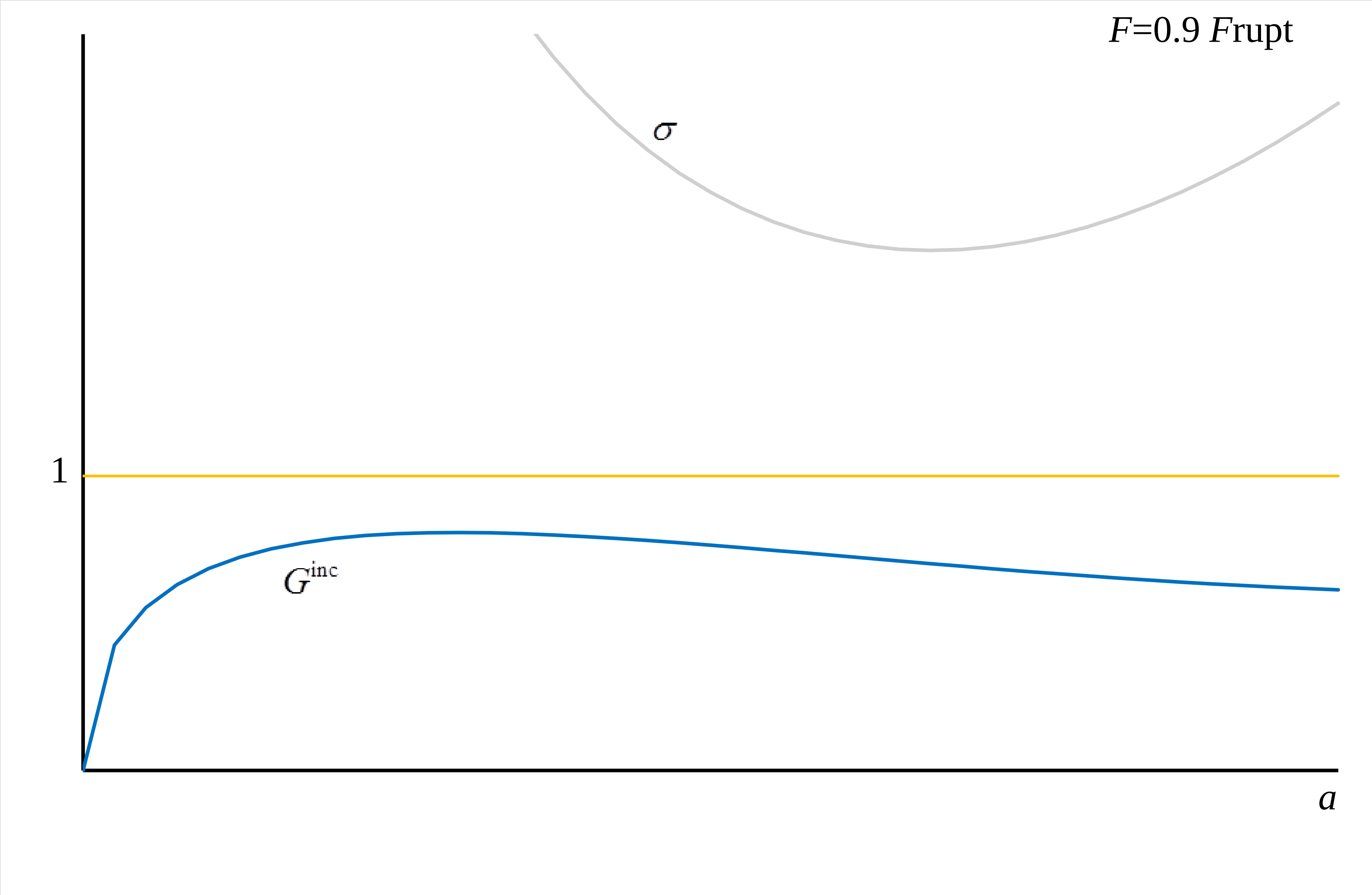
Thermal residual stresses in layered ceramics



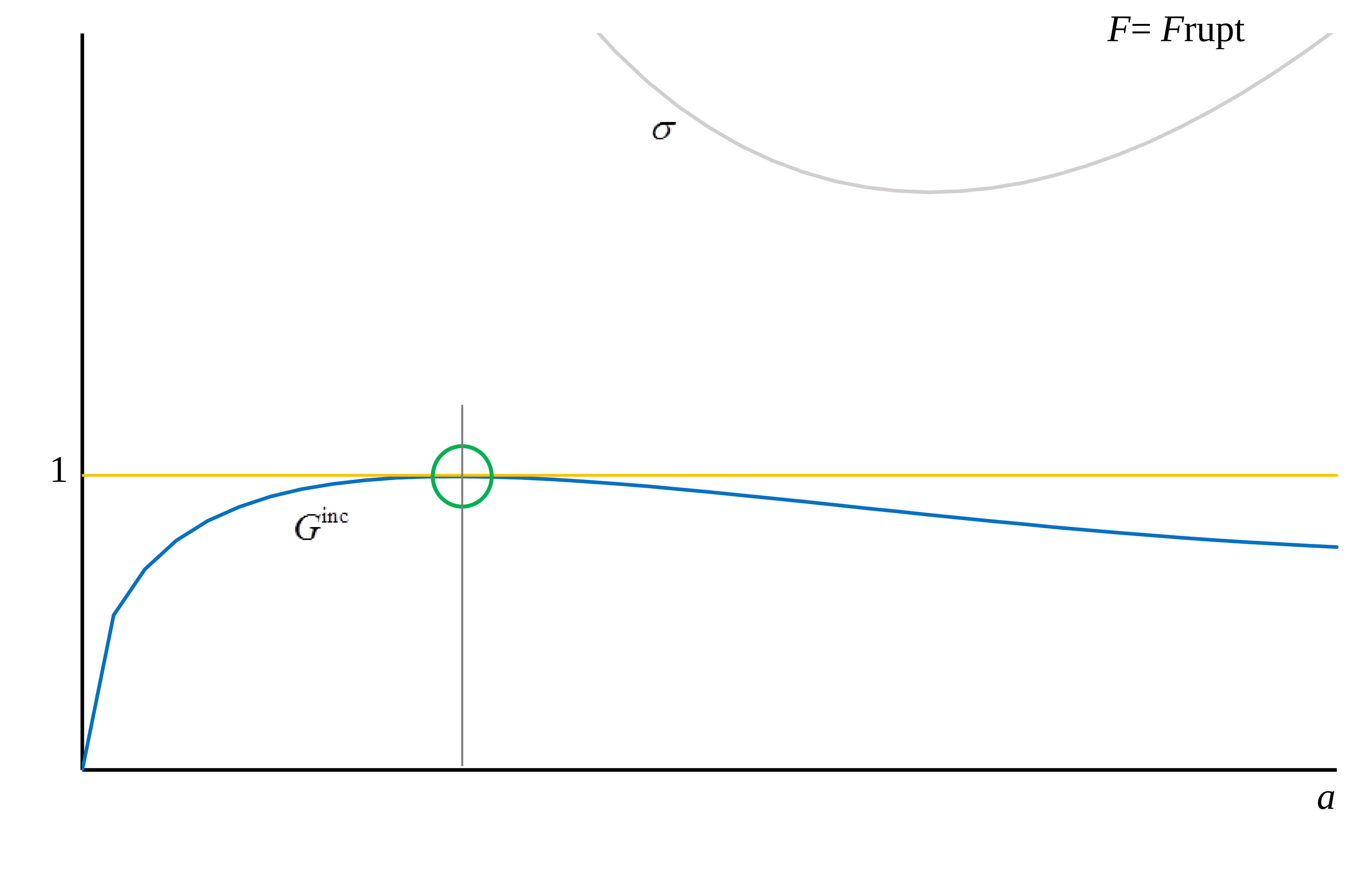
The crack initiates at the notch root and then grows unstably and penetrates the next layer and stop as $k_{\mathrm{I}} = 0$ not $k_{\mathrm{I}} = k_{\mathrm{Ic}}$ (i.e. the cracks is closed due to the compressive residual stresses). Then after reloading, it kinks (Leguillon et al., 2015).
Only pure FE calculations are allowed because the layers thickness is similar to the crack length.



Laminated ceramics: before kinking



Laminated ceramics: kinking



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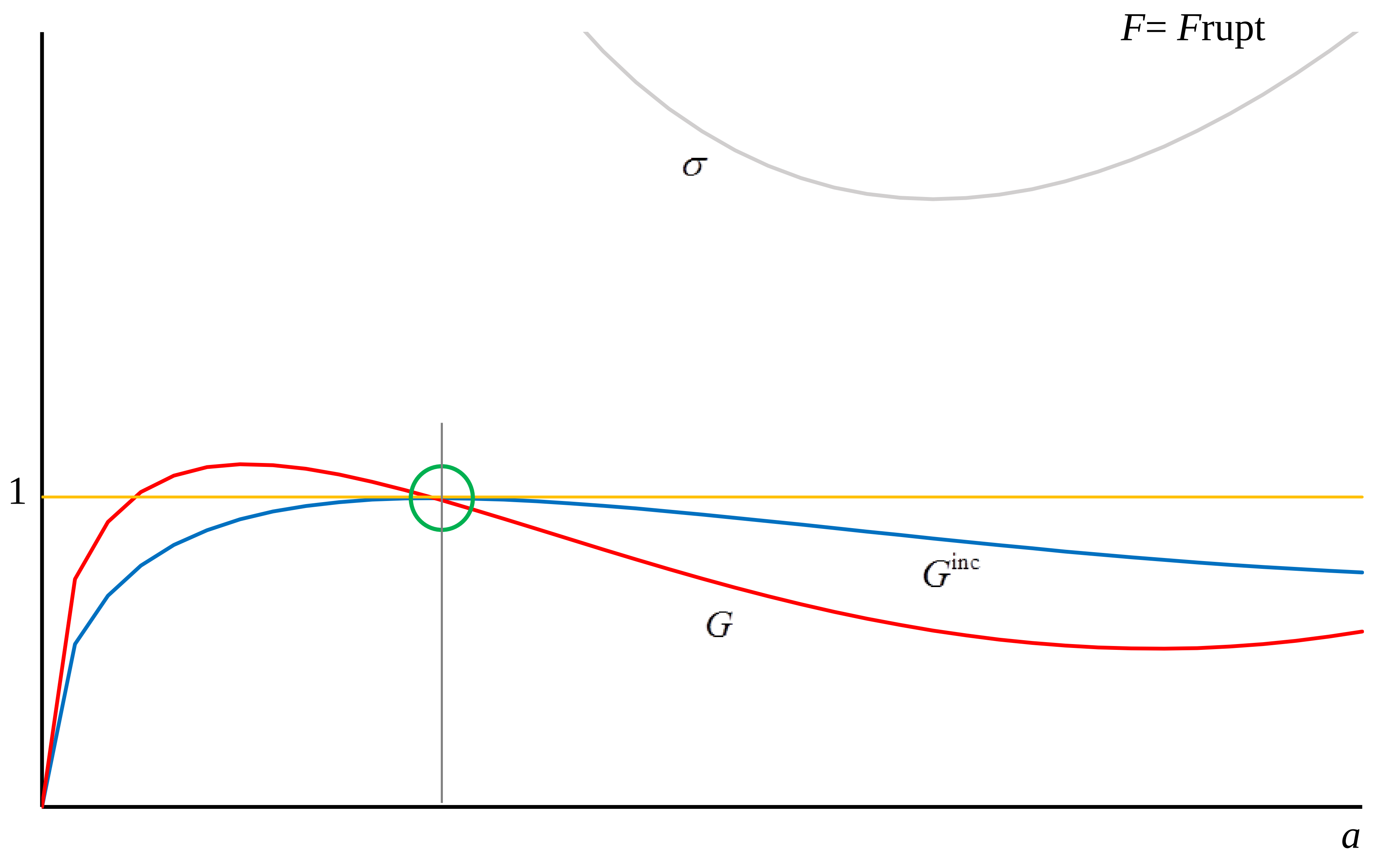
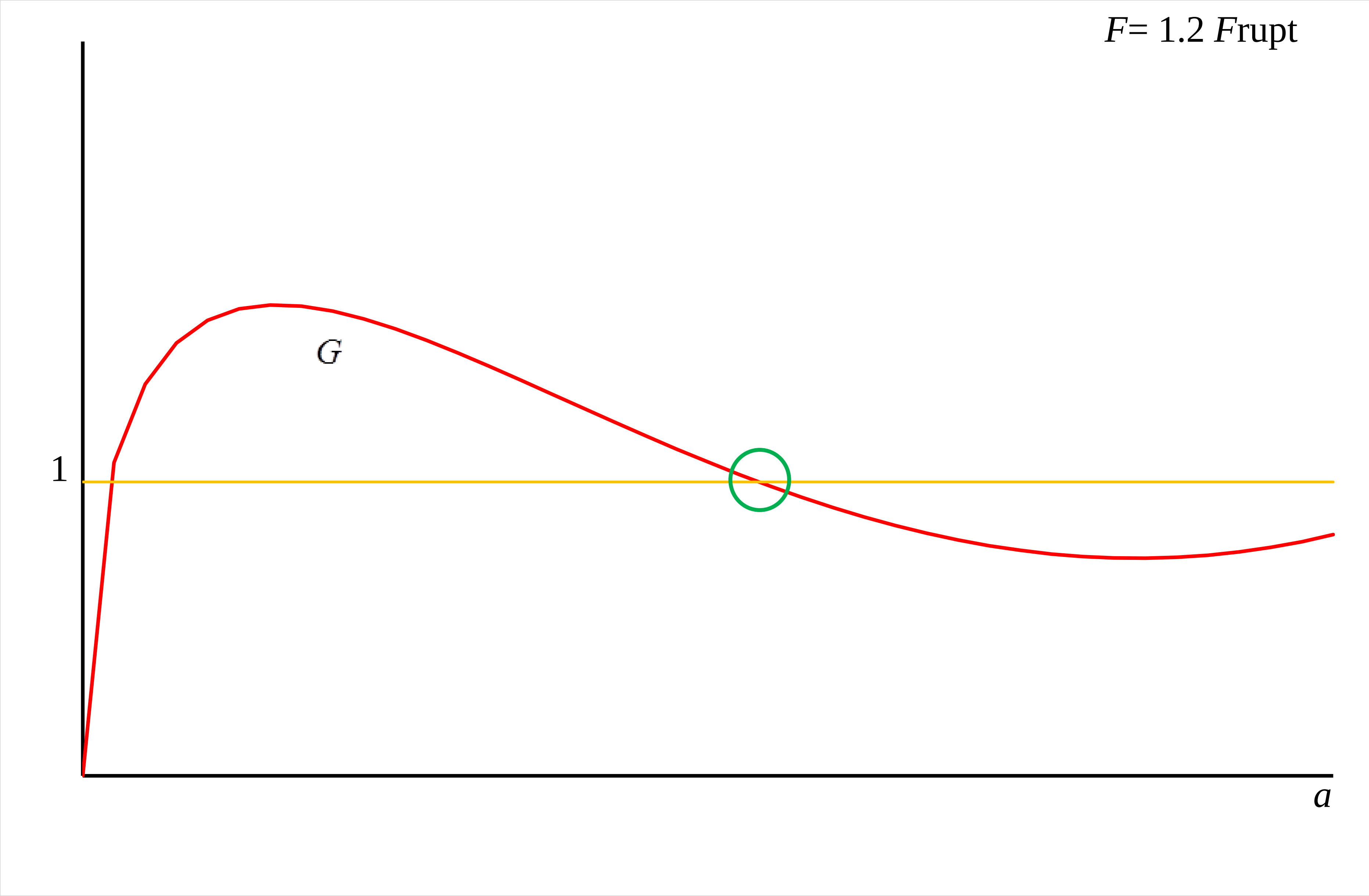
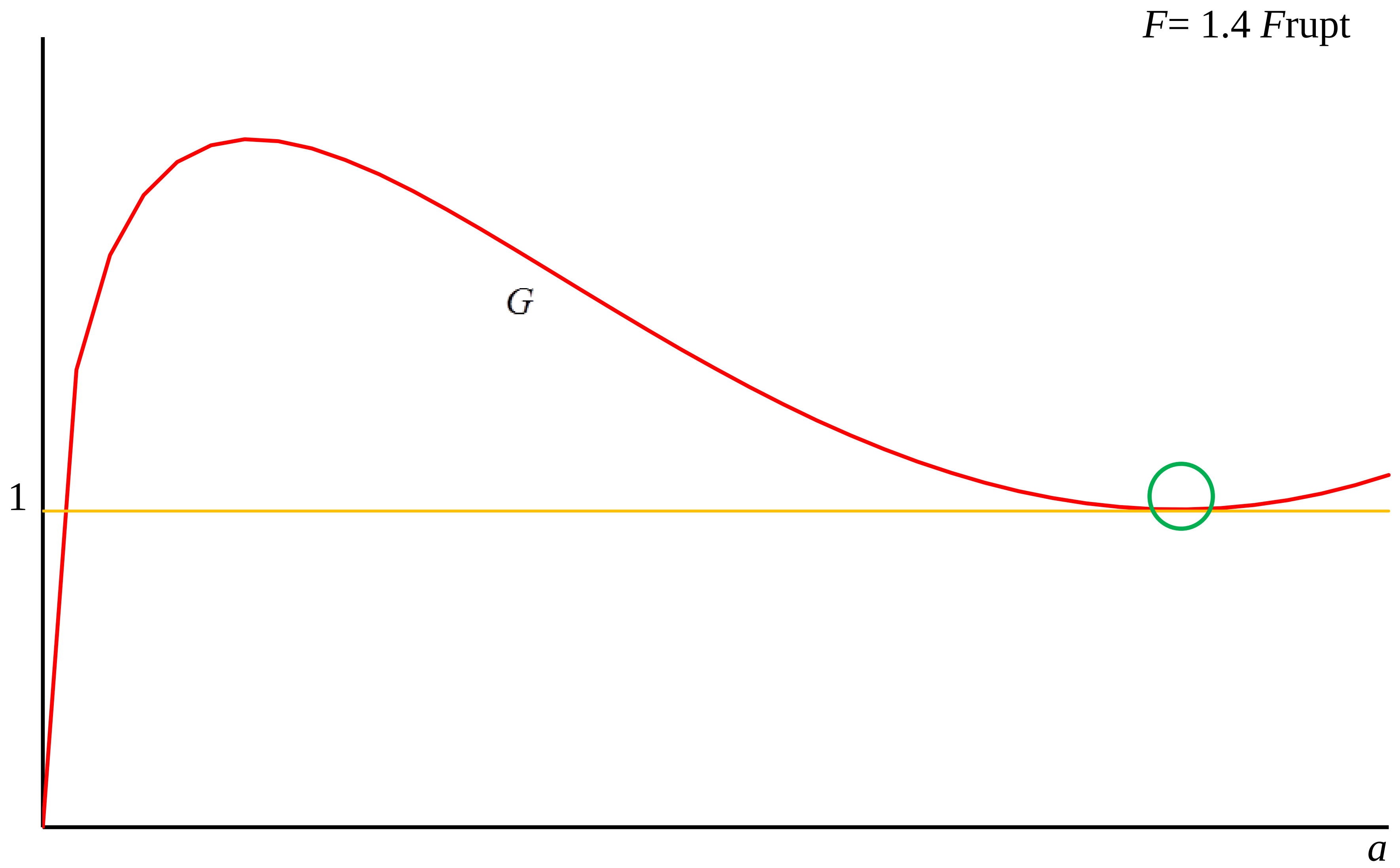


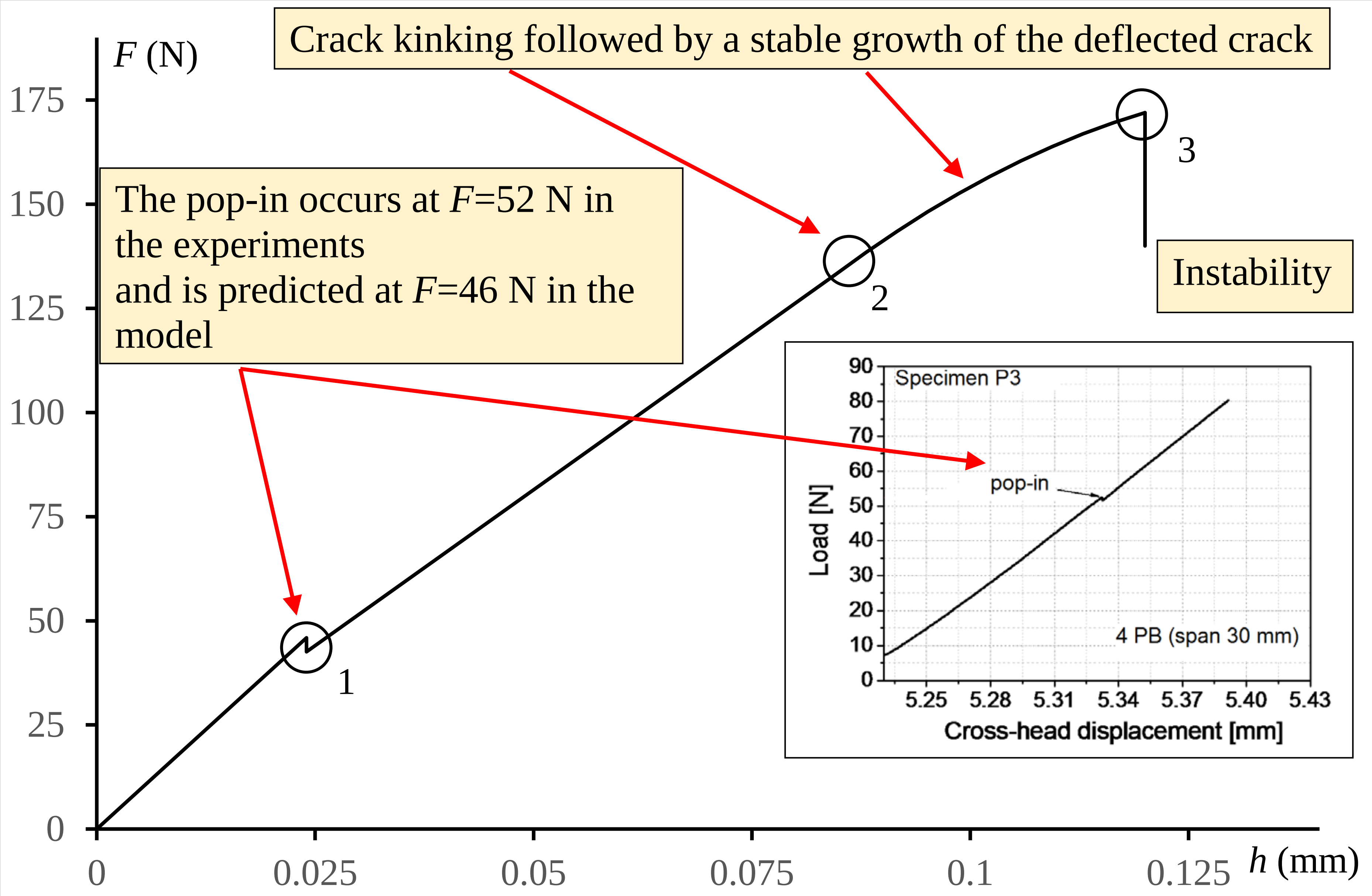
Laminated ceramics: stable growth



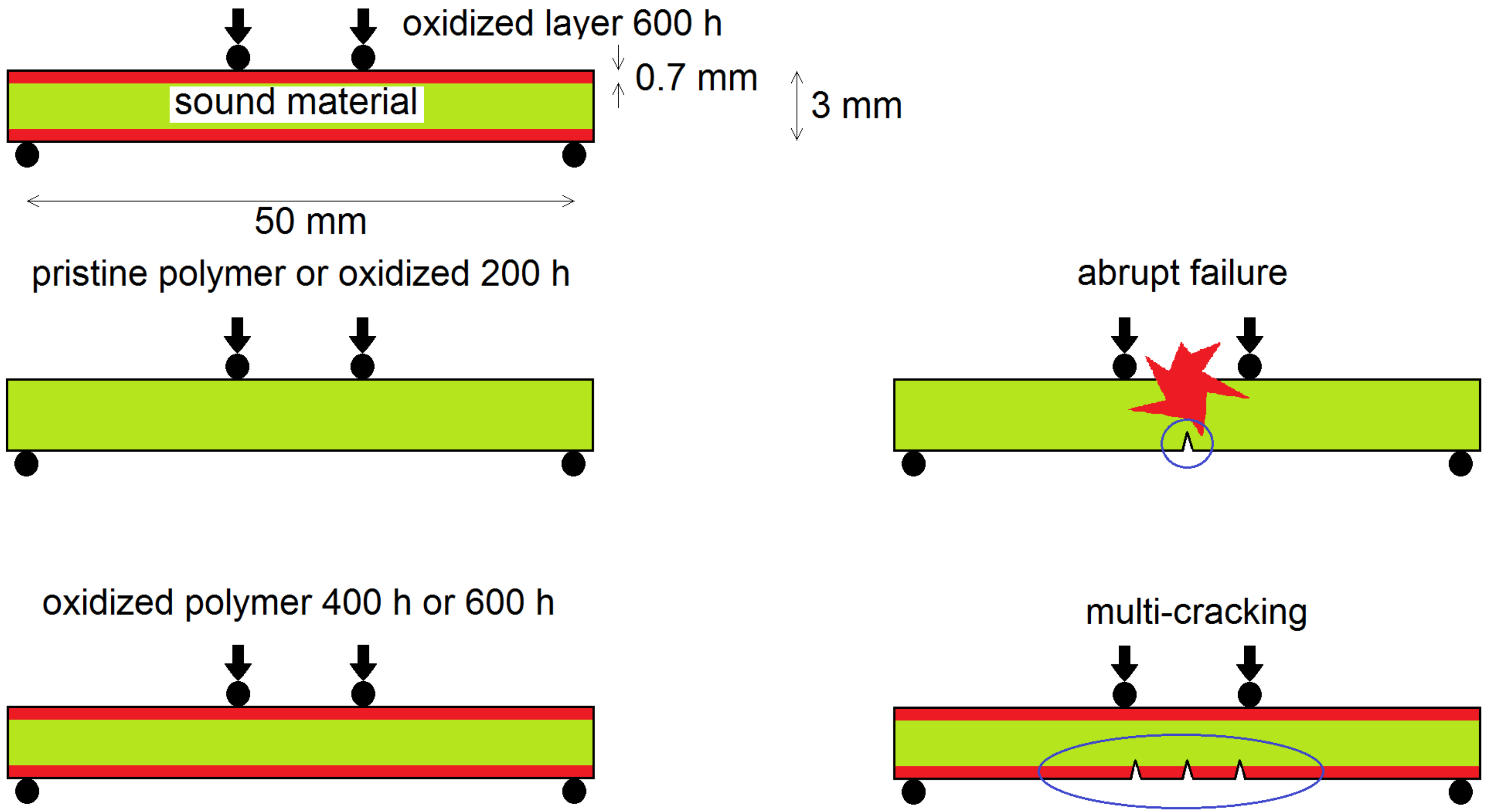
Laminated ceramics: instability



Laminated ceramics: a complete scenario



Residual stresses in a polymer due to oxydation



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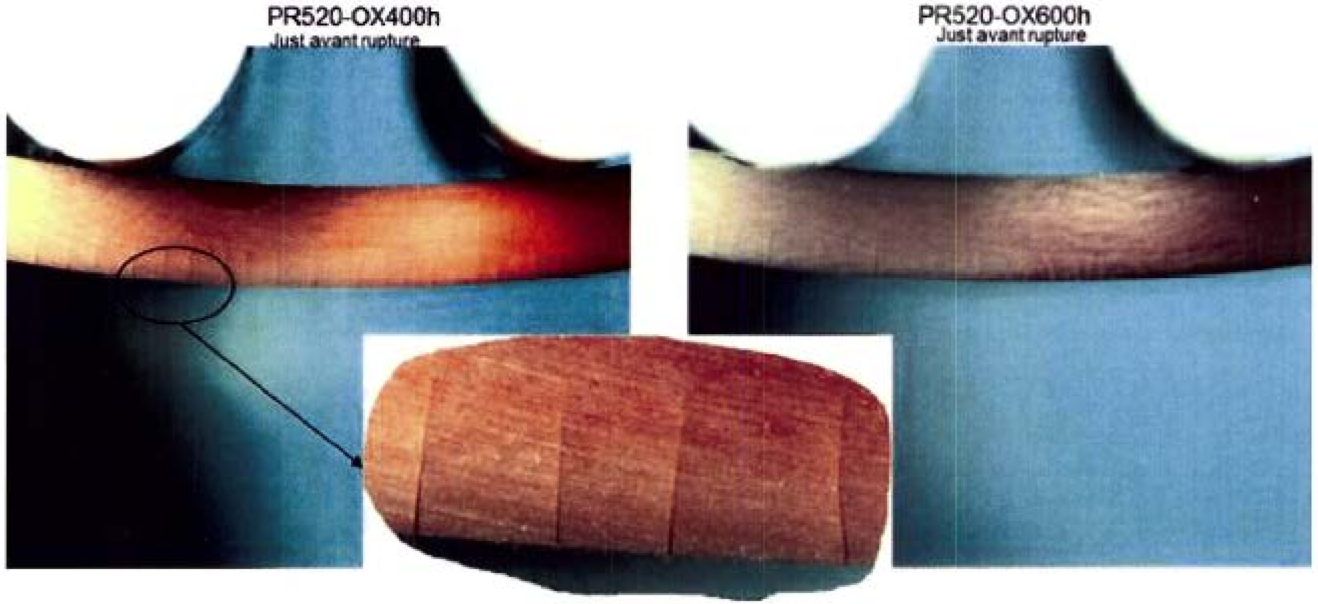
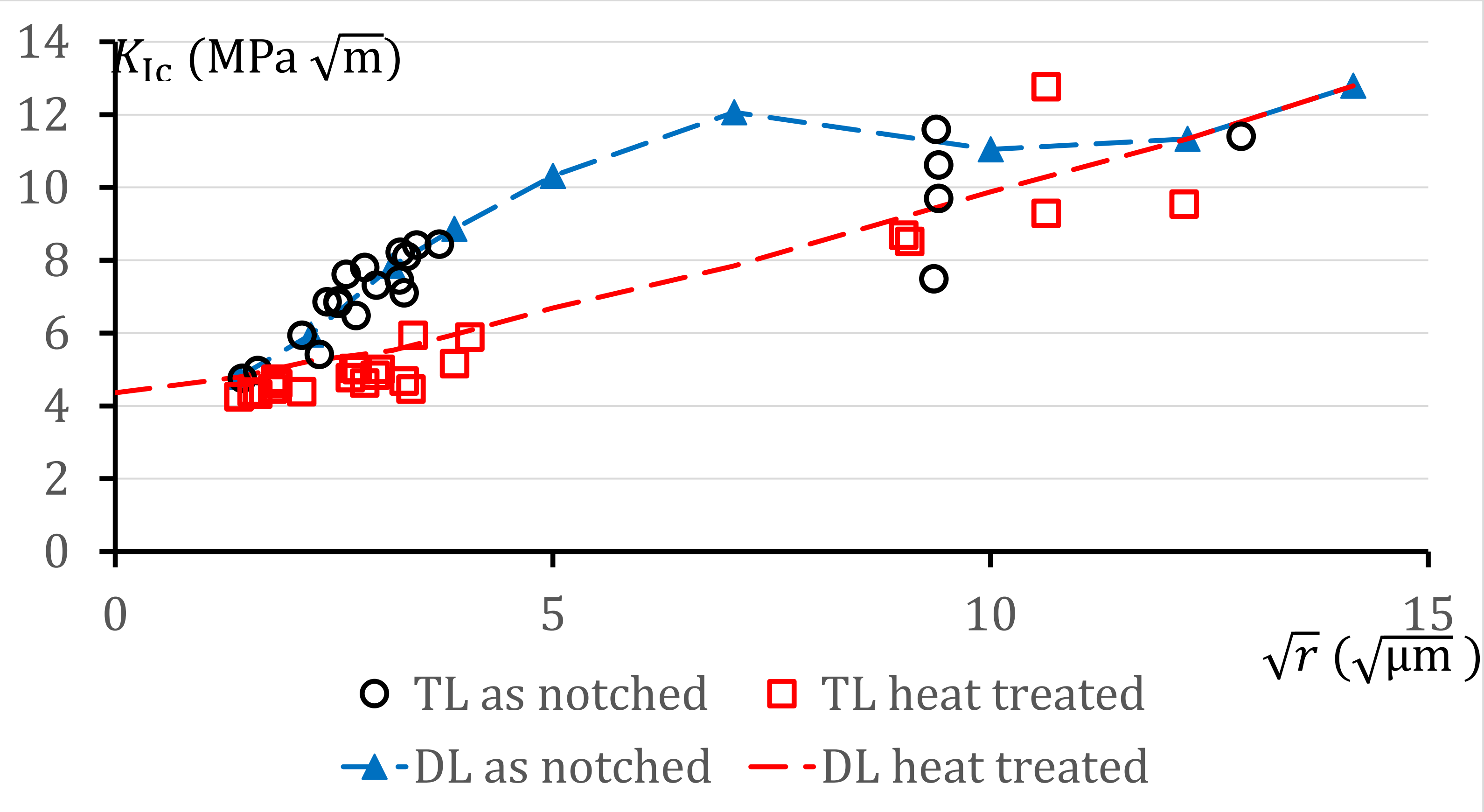


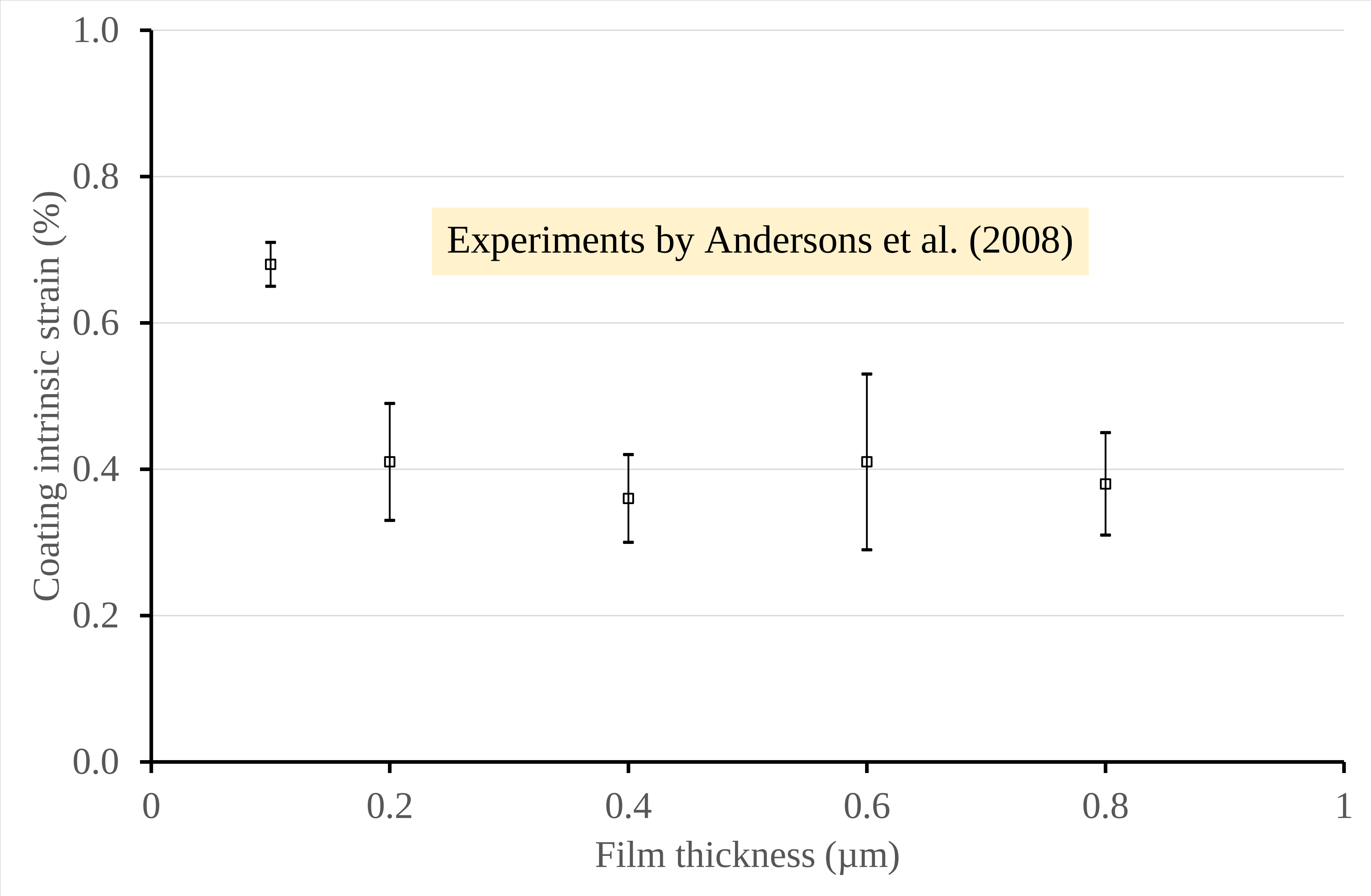
The challenge here is that in the oxydized layer, the Young modulus, the tensile strength and the toughness vary. Again, only pure FE calculations are allowed. A comparison with experiments (Pannier et al., 2014; Leguillon et al., 2016) shows a good agreement, both in terms of applied load at initiation of crack(s) and in terms of cracks density.

Residual stresses in zirconia due to phase change

Grinding a notch in zirconia produces big stresses at the notch which induces a phase change and generates high compressive residual stresses (around 1800 MPa) in a thin layer of a few micrometers (< 10 µm). They are released by a heat treatment.



Thermal residual stresses in sub-micron films



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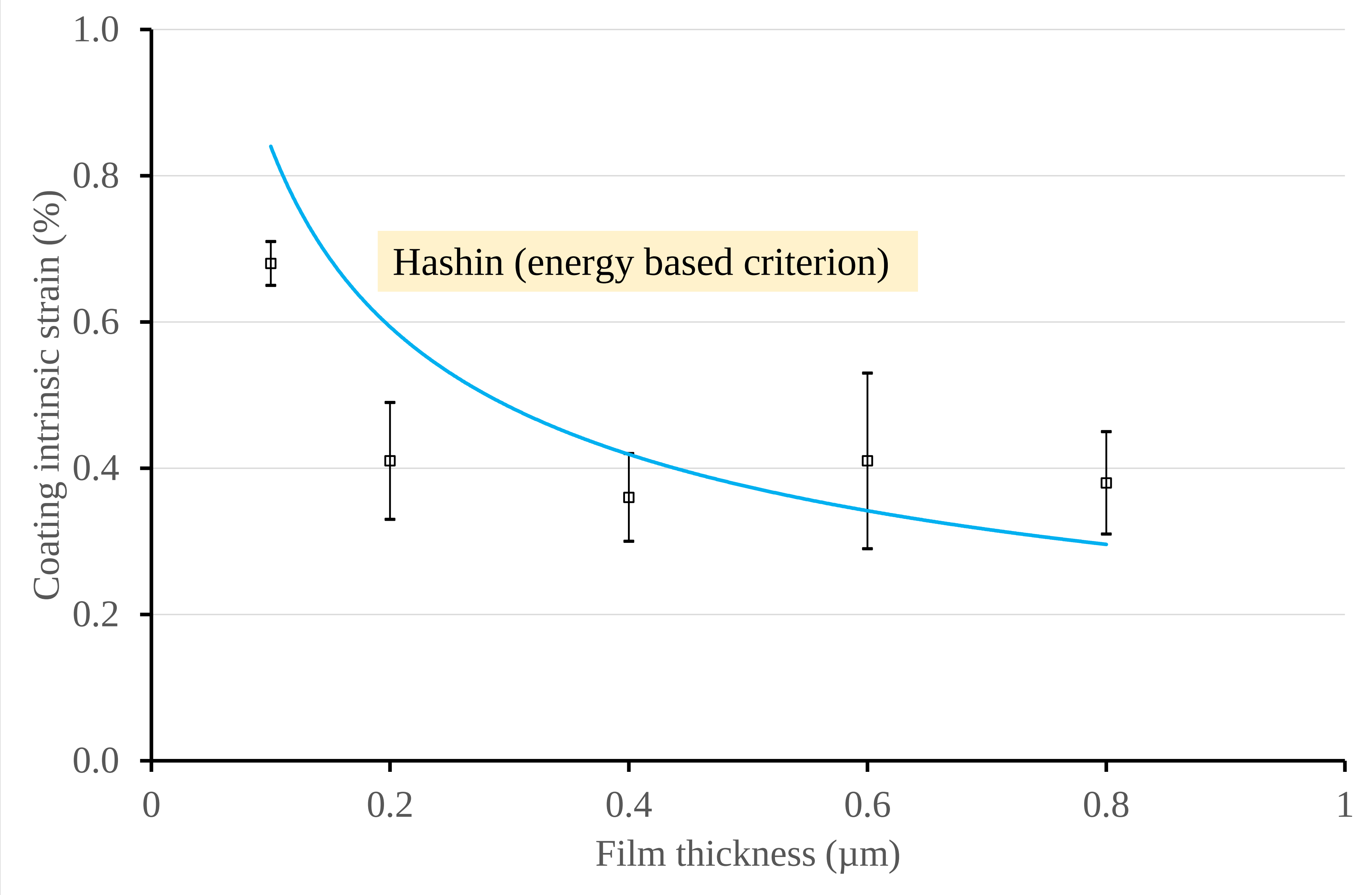
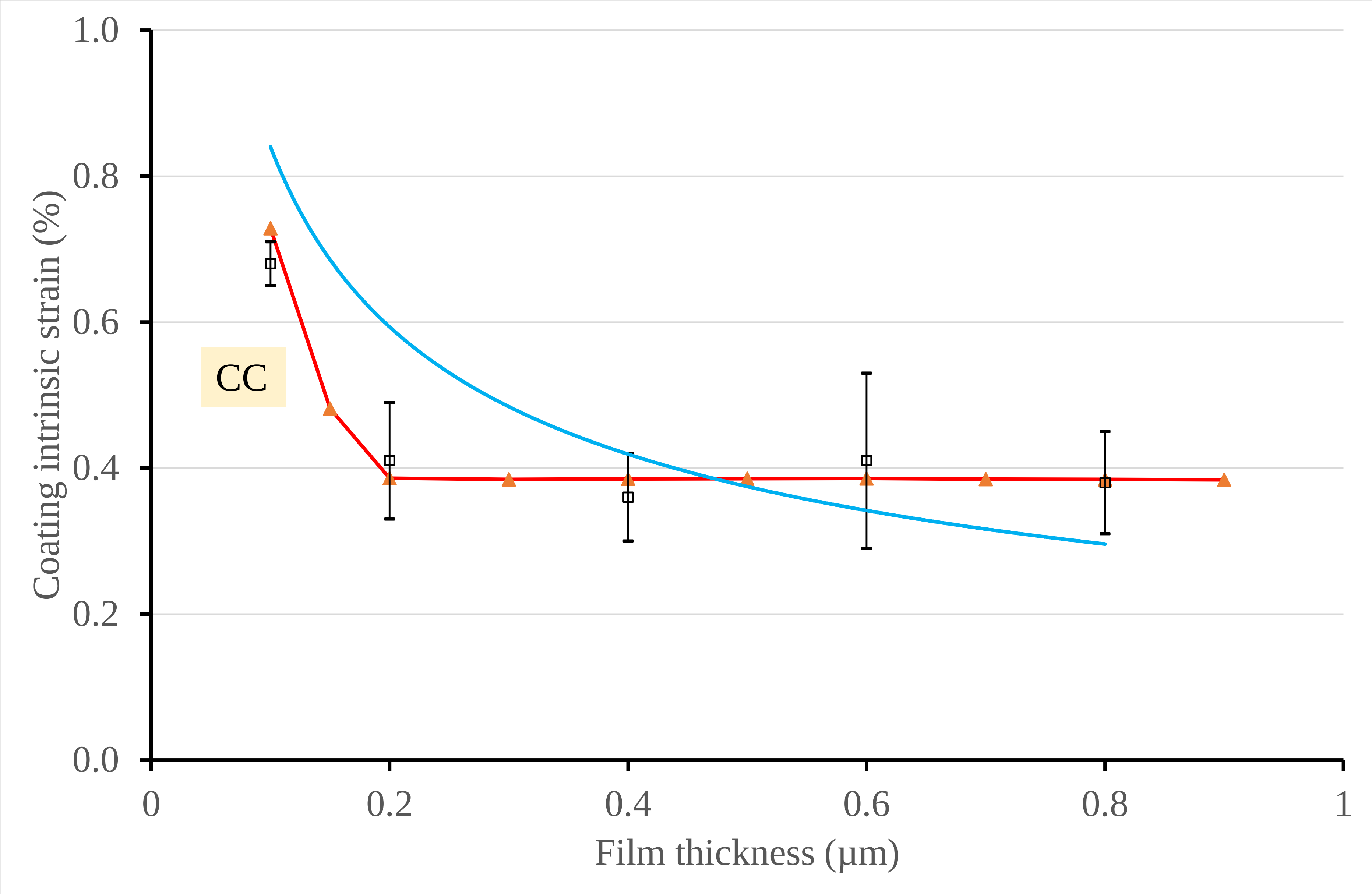
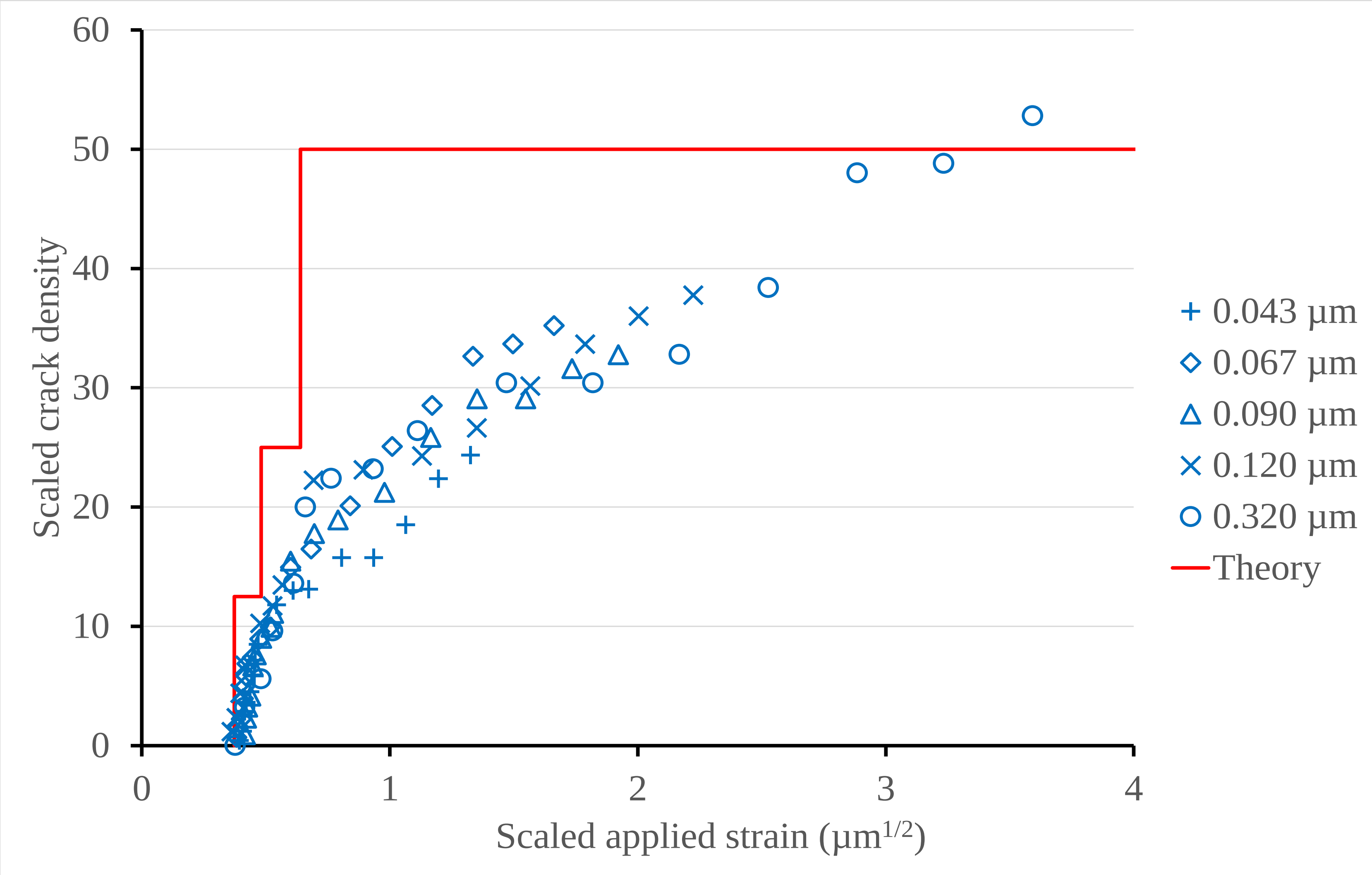


Thermal residual stresses in sub-micron films





Scaled crack density
0
10
20
30
40
50
60
0
1
2
3
4
Scaled applied strain (μm1/2)
0.043 μm
0.067 μm
0.090 μm
0.120 μm
0.320 μm
Theory