Physique statistique de la rupture hétérogène (nominalement) fragile

Succès et limites actuelles

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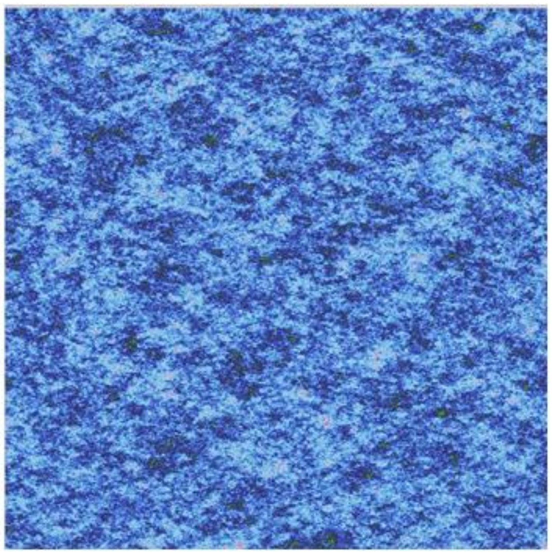
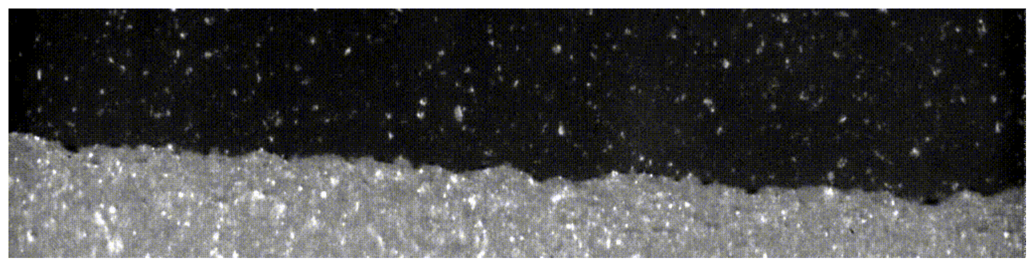


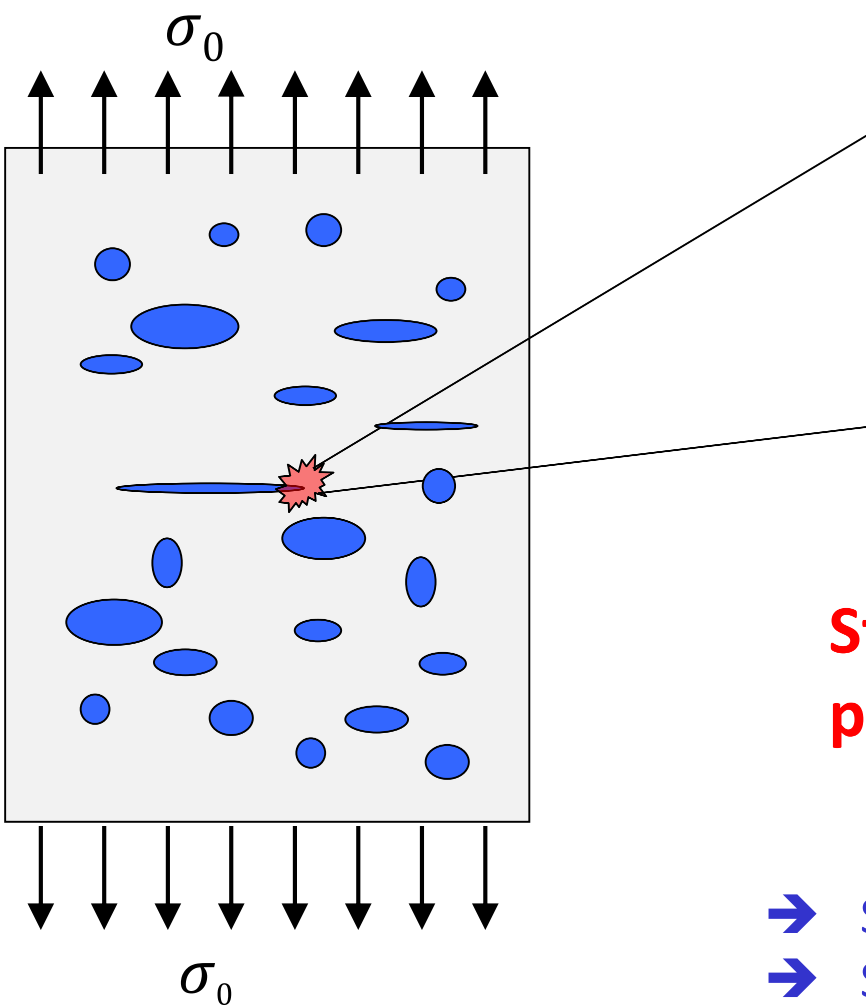


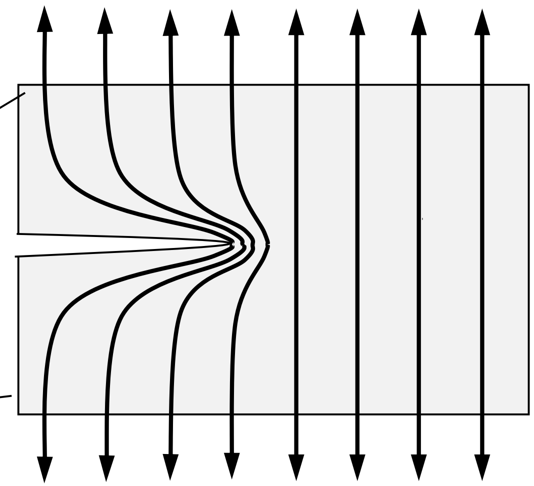
© K.-J. Maloy, S. Santucci, Univ. Oslo

© F. Célarié, LDV, Montpellier

Via discussions with: Jonathan Barés (CEA), Alizée Dubois (CEA), Davy Dalmas (LTDS), Laurent Ponson (IJLRd'A), Véronique Lazarus (FAST), Luc Barbier (CEA), Cindy Rountree (CEA), Stéphane Santucci (Lyon)

Complexity in brittle fracture

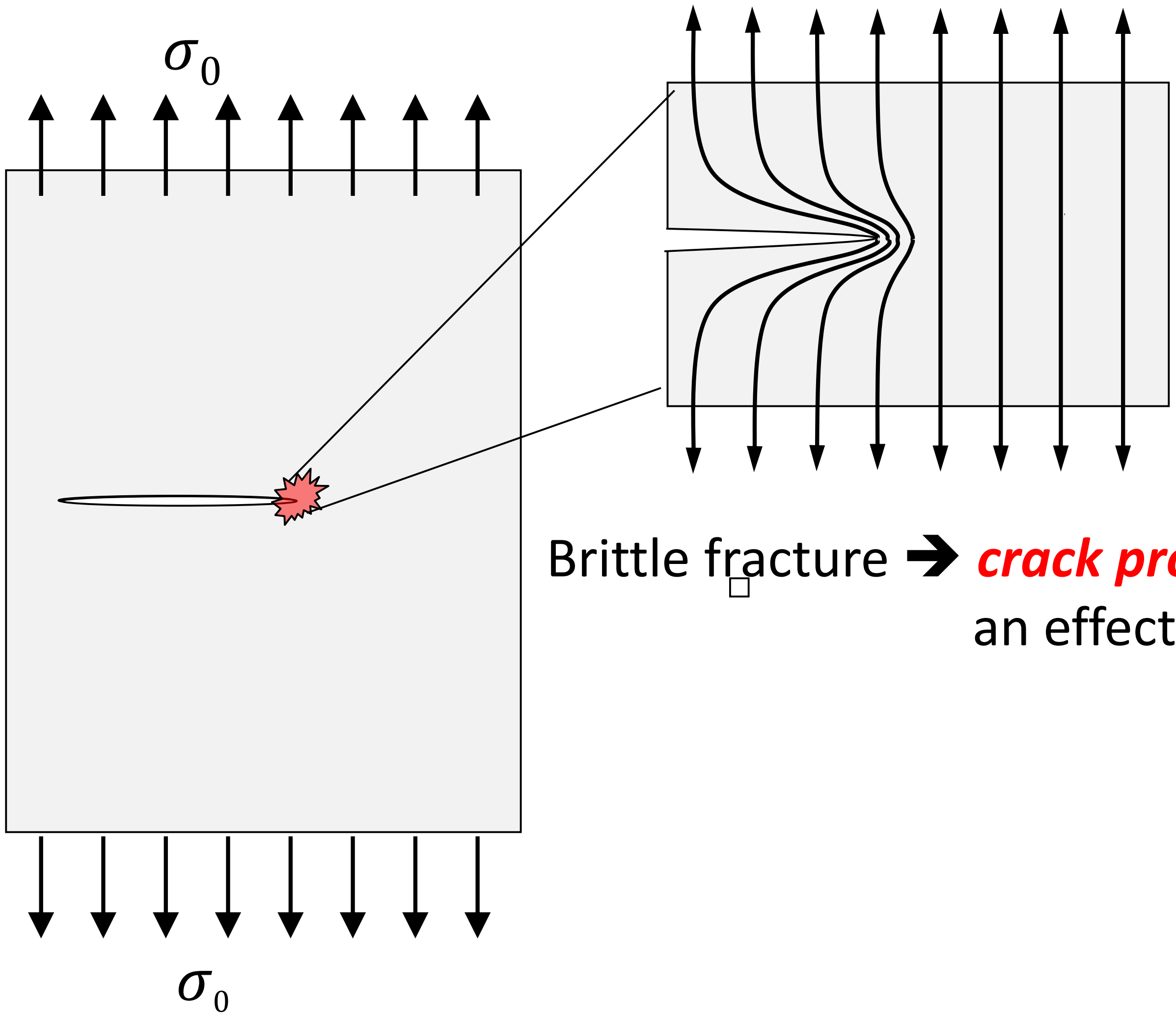




Stress concentration at preexisting cracks/defects

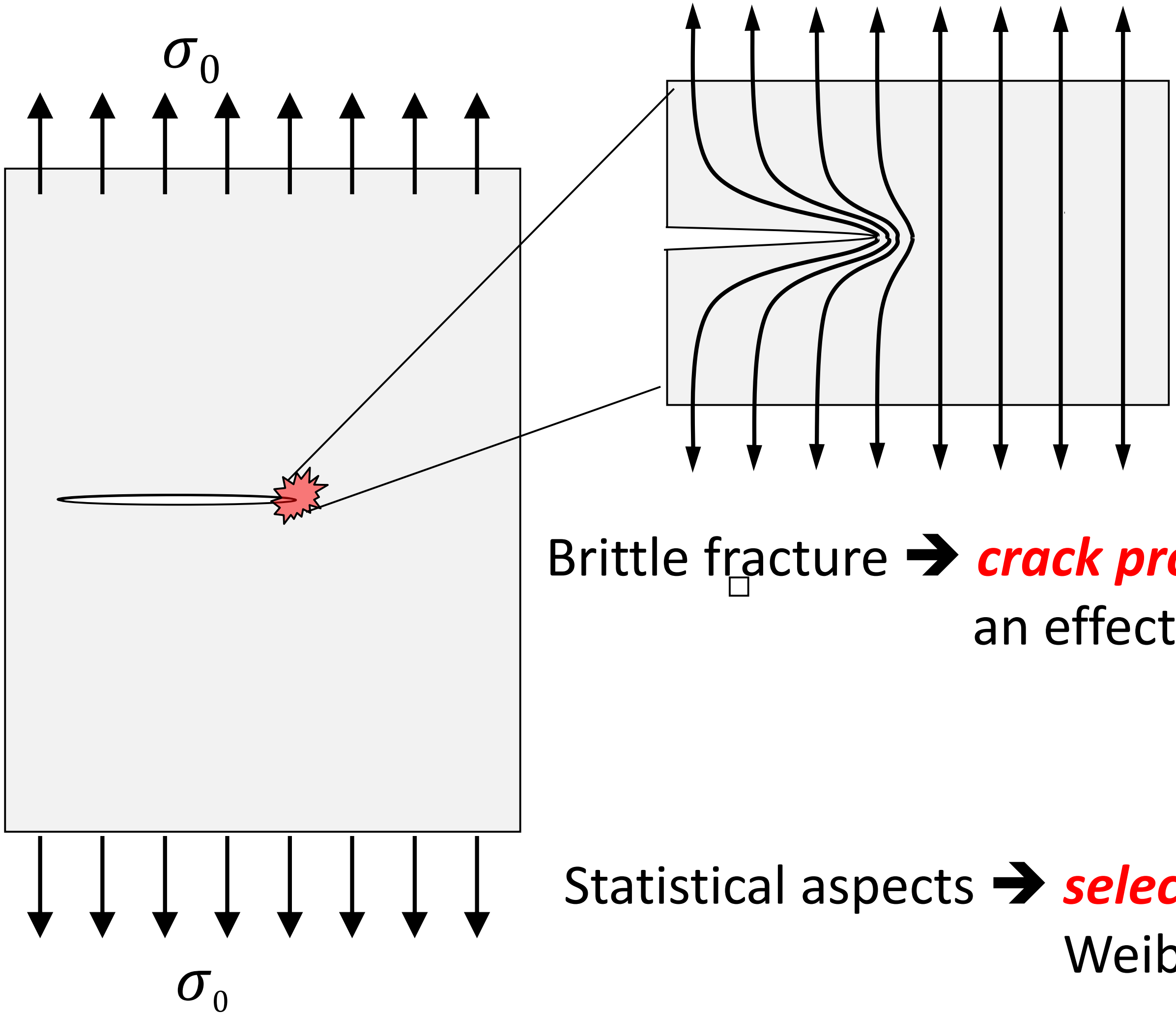
- Sensitivity to microscopic defects
- Statistical aspects at the macroscopic scale
- Size-dependency for strength
- Dominated by extreme events statistics

Continuum engineering approach



Brittle fracture ➔ ***crack propagation problem*** into an effective homogeneous medium

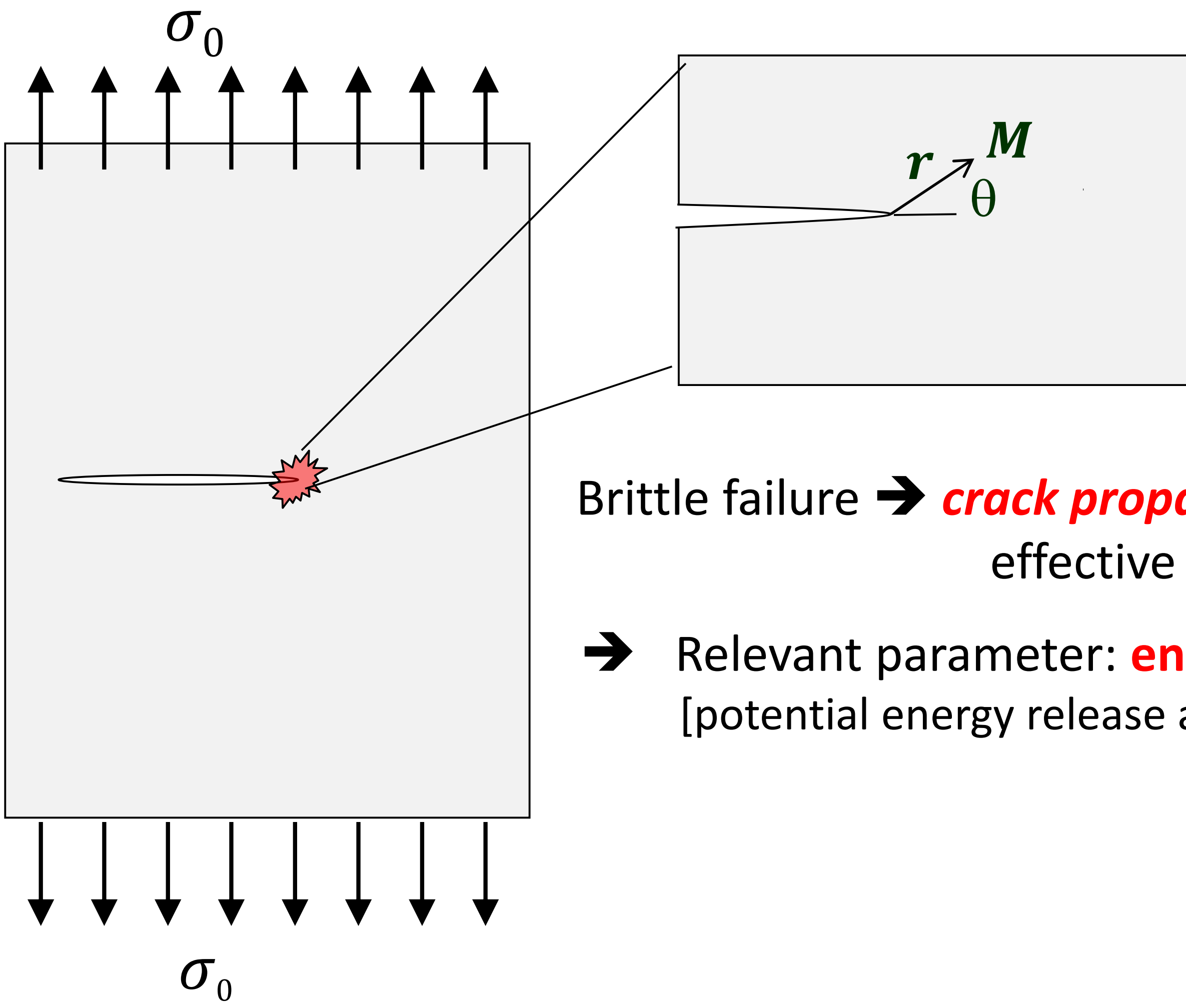
Continuum engineering approach



Brittle fracture ➔ ***crack propagation problem*** into an effective homogeneous medium

Statistical aspects ➔ ***selection of the worst flaw*** via Weibull weakest-link approach

Continuum engineering approach (LEFM)



Near-tip elastic stress singularity

$$\sigma_{ij}(r,\theta) = \frac{K}{\sqrt{2\pi r}} f_{ij}(\theta)$$

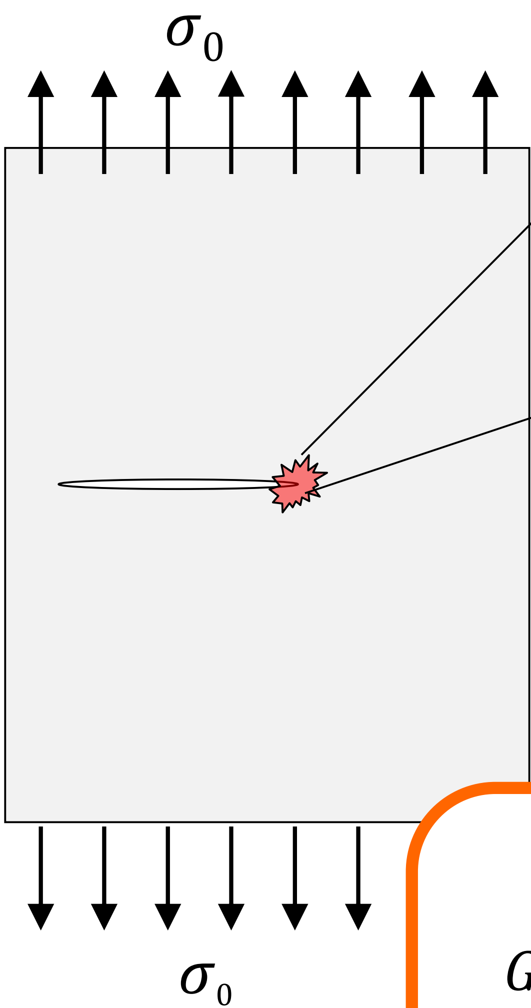
$f_{ij}(\theta)$ universal function

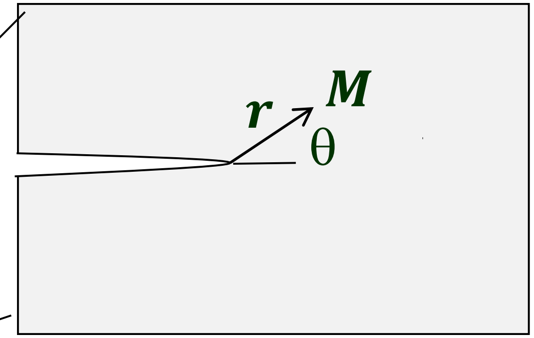
Brittle failure ➔ ***crack propagation problem*** into an effective homogeneous medium

➔ Relevant parameter: **energy release rate $G = K^2/E$**
[potential energy release as crack grows over a unit length]

E. Orowan (1955)
G.R.R. Irwin JAM (1957)

Continuum engineering approach (LEFM)





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[potential energy release as crack grows over a unit length]

Griffith criterion for (slow) crack growth

$G < \Gamma$ ➔ immobile crack

$G \geq \Gamma$ ➔ crack moving with $v \sim G - \Gamma$
(when v<<sound speed)

Γ fracture energy (material constant)
[energy dissipated to create crack surface of unit area]

E. Orowan (1955)
G.R.R. Irwin JAM (1957)

Complements

Shortcomings in continuum fracture theory…

Predictions

Continuous crack growth

Smooth Fracture surfaces

Shortcomings in continuum fracture theory…

Predictions

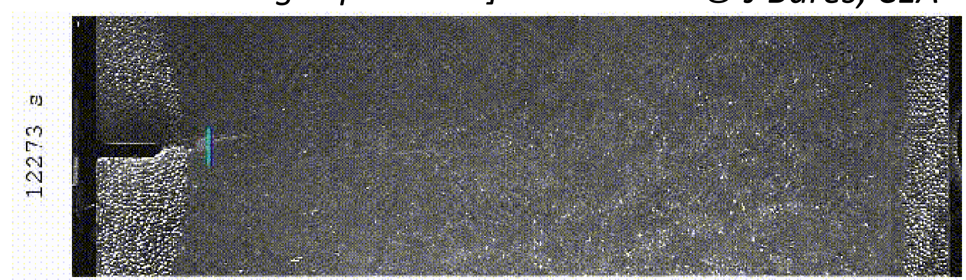
Continuous crack growth

Smooth Fracture surfaces

Observations:

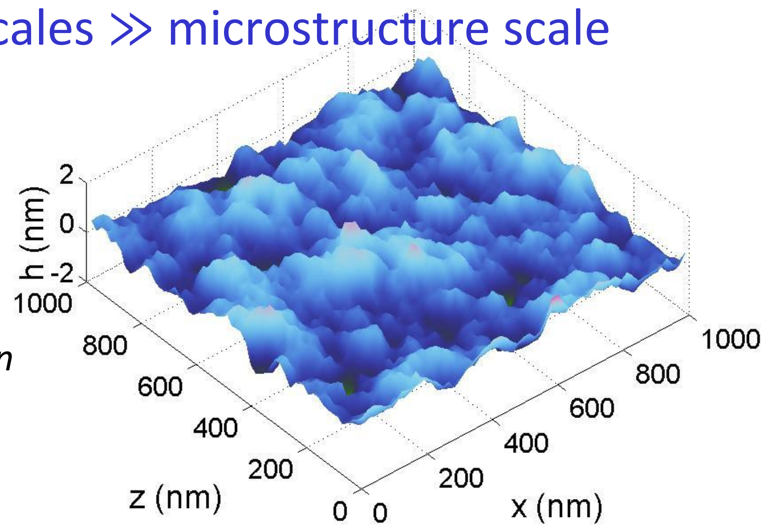
[sometimes] jerky dynamics for cracks

[here crack slowly driven in an artificial rock 5 hours long experiment] *© J Barés, CEA*

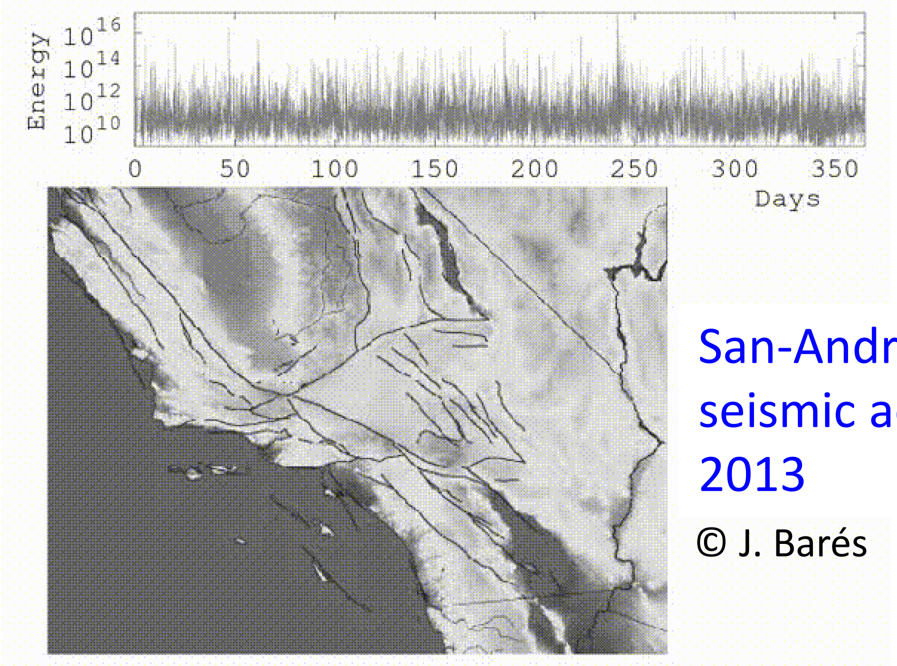


fracture surfaces Rough at scales ≫ microstructure scale

[here Silica glass broken upon stress corrosion]



On jerky dynamics in cracks

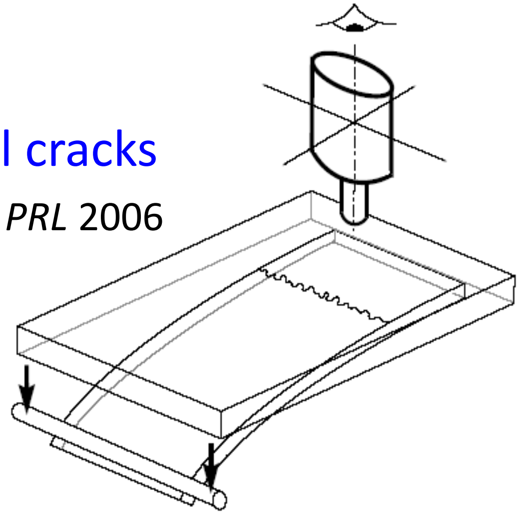


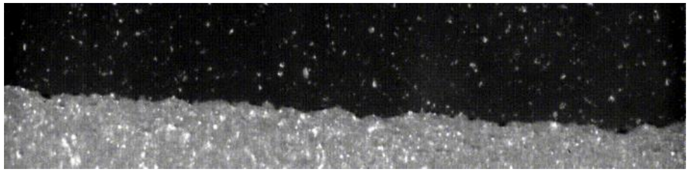
San-Andreas:
seismic activity
2013

© J. Barés

Interfacial cracks

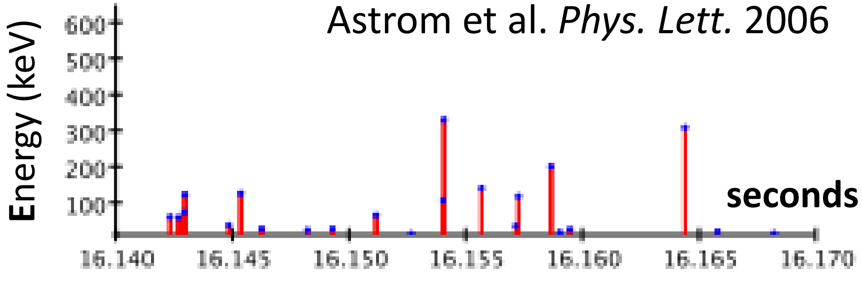
Maloy et al. *PRL* 2006

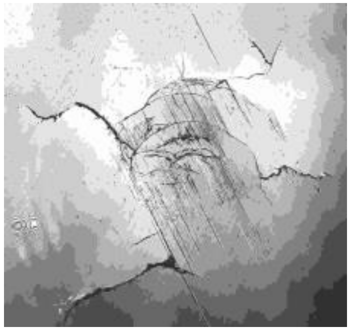




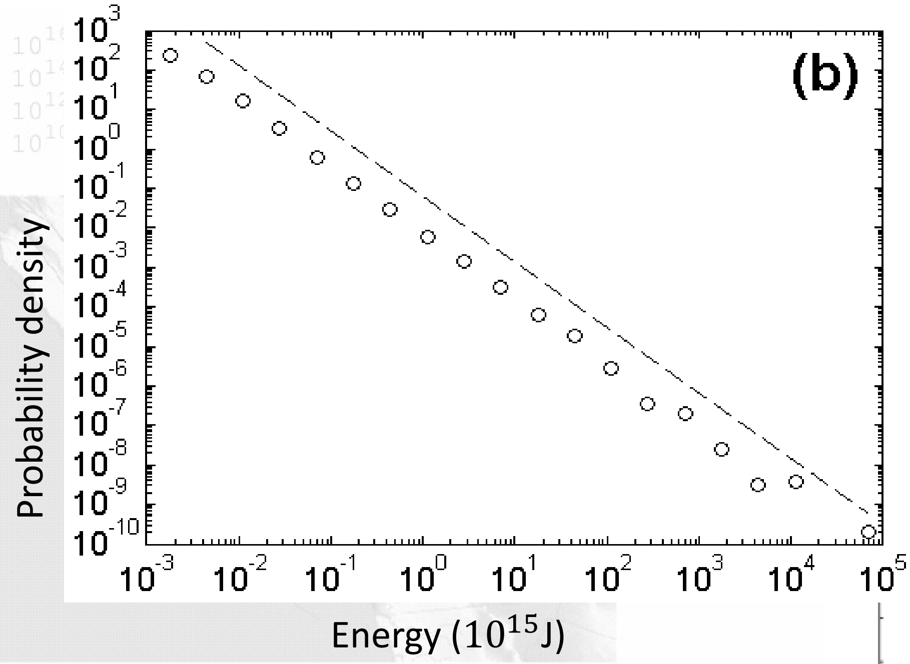
Activity in saphir monocrystals at 10mK (CRESST experiment)

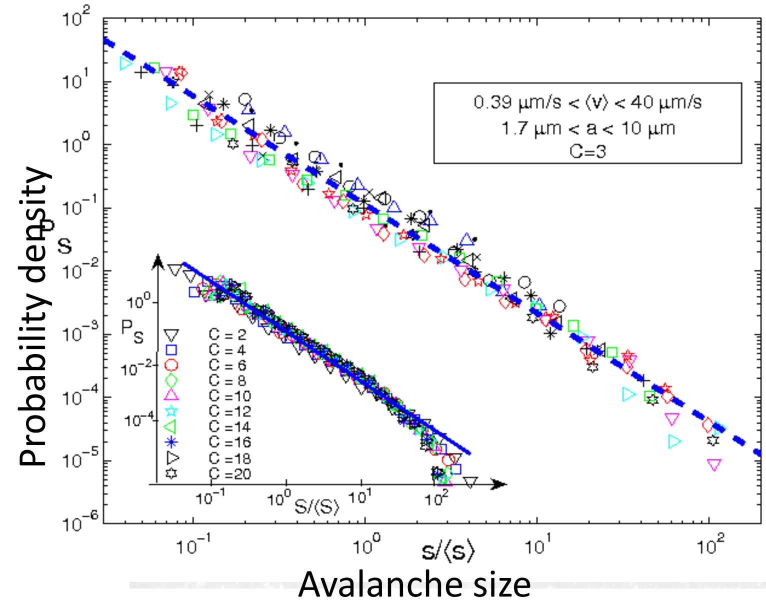
Astrom et al. *Phys. Lett.* 2006

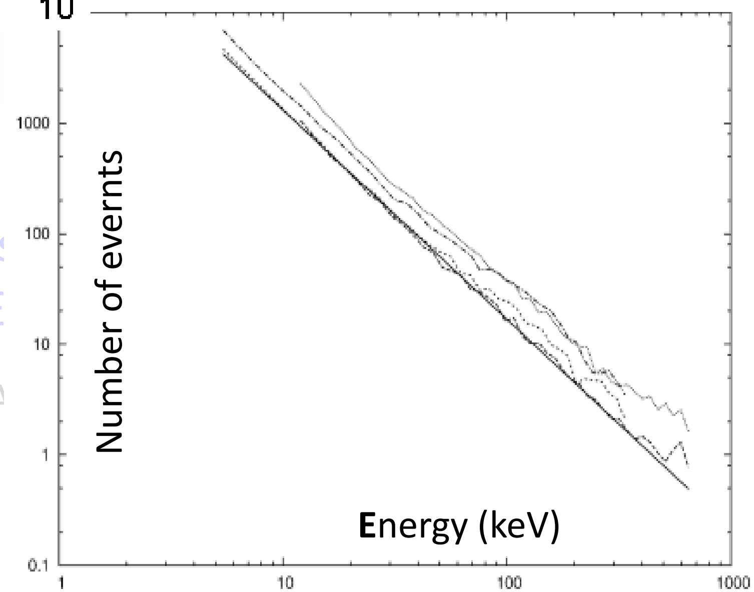




On jerky dynamics in cracks







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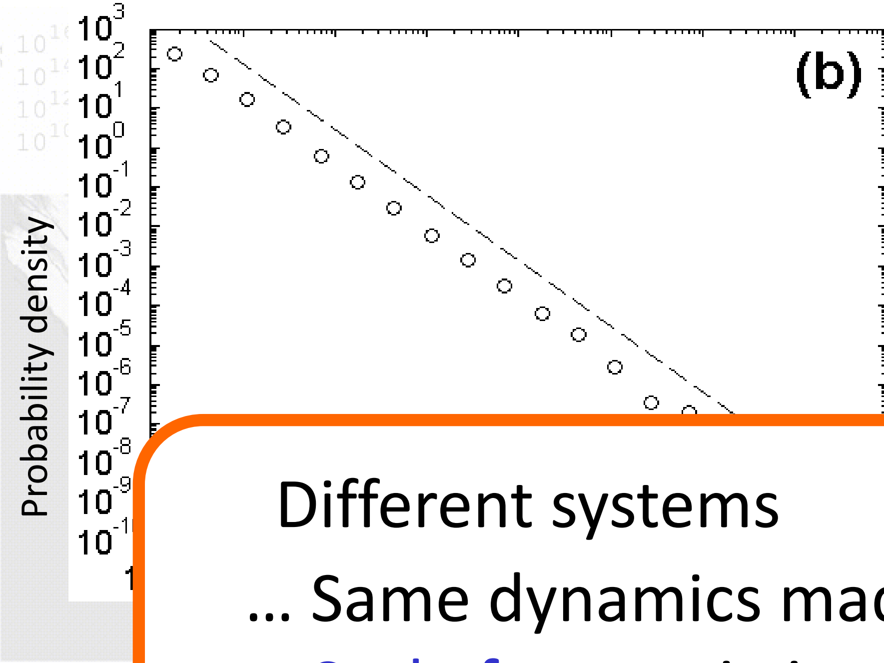
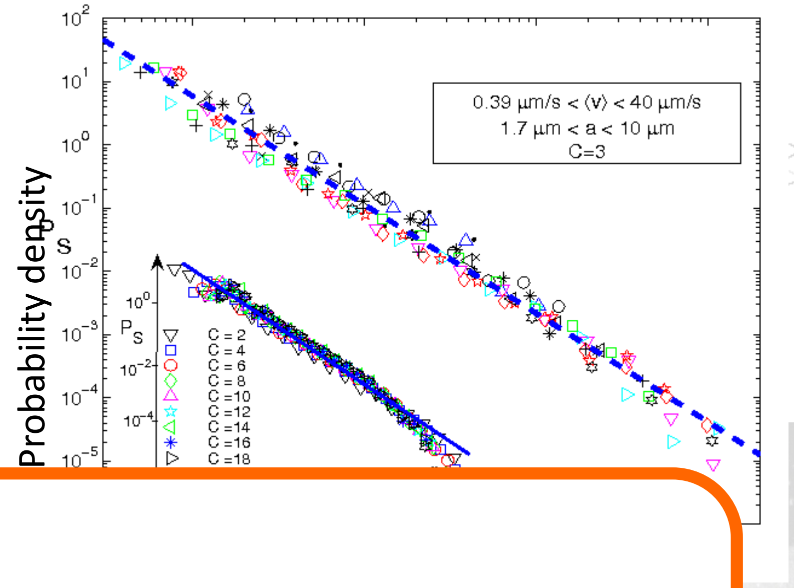
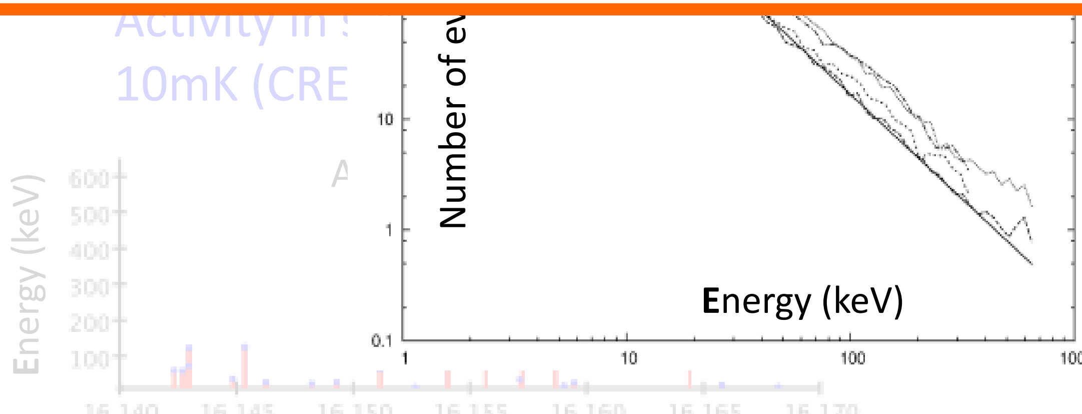




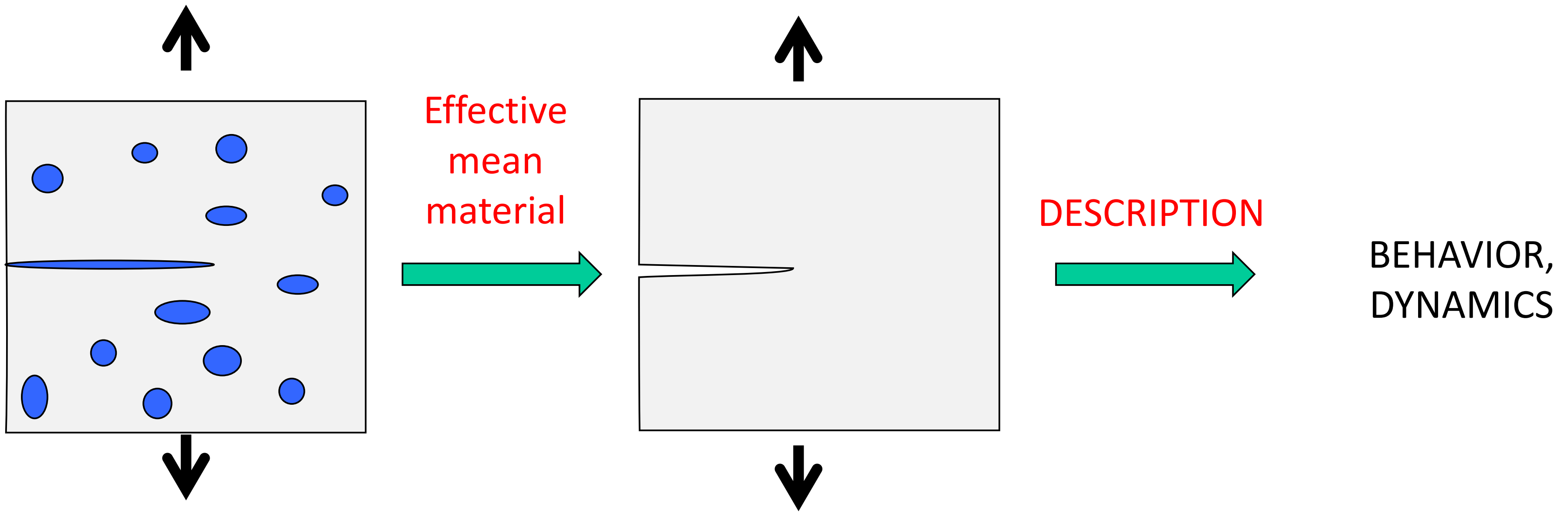
Different systems

… Same dynamics made of discrete impulses

… Scale free statistics (power-law)



Continuum approach : principle



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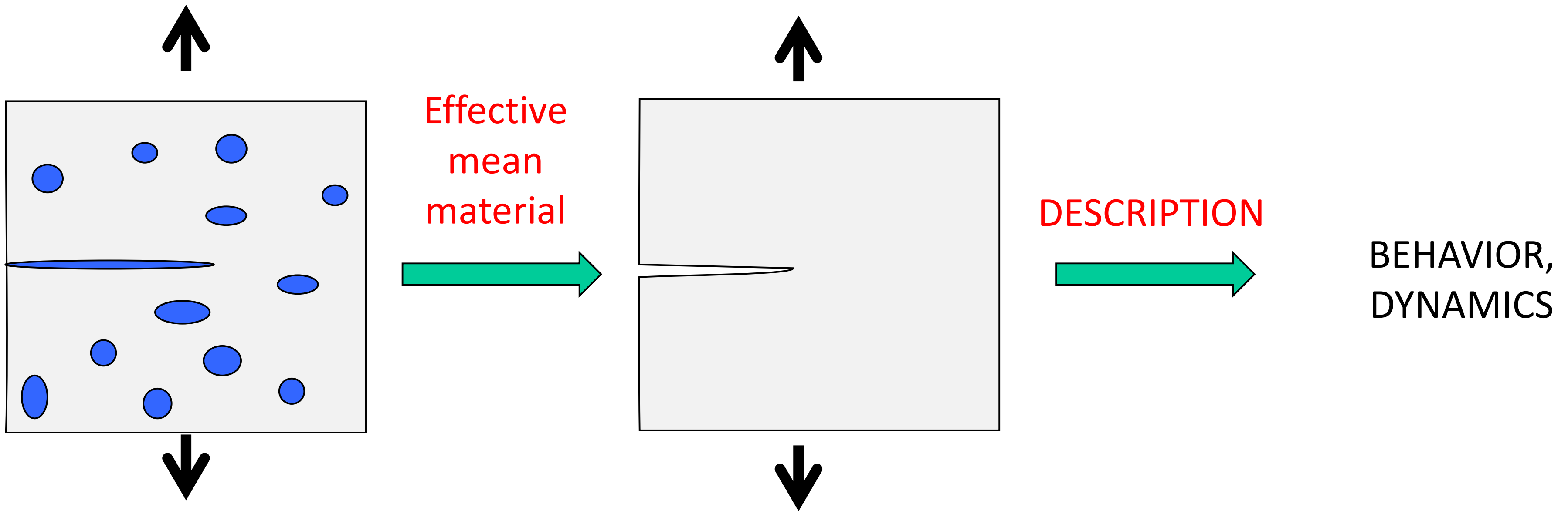
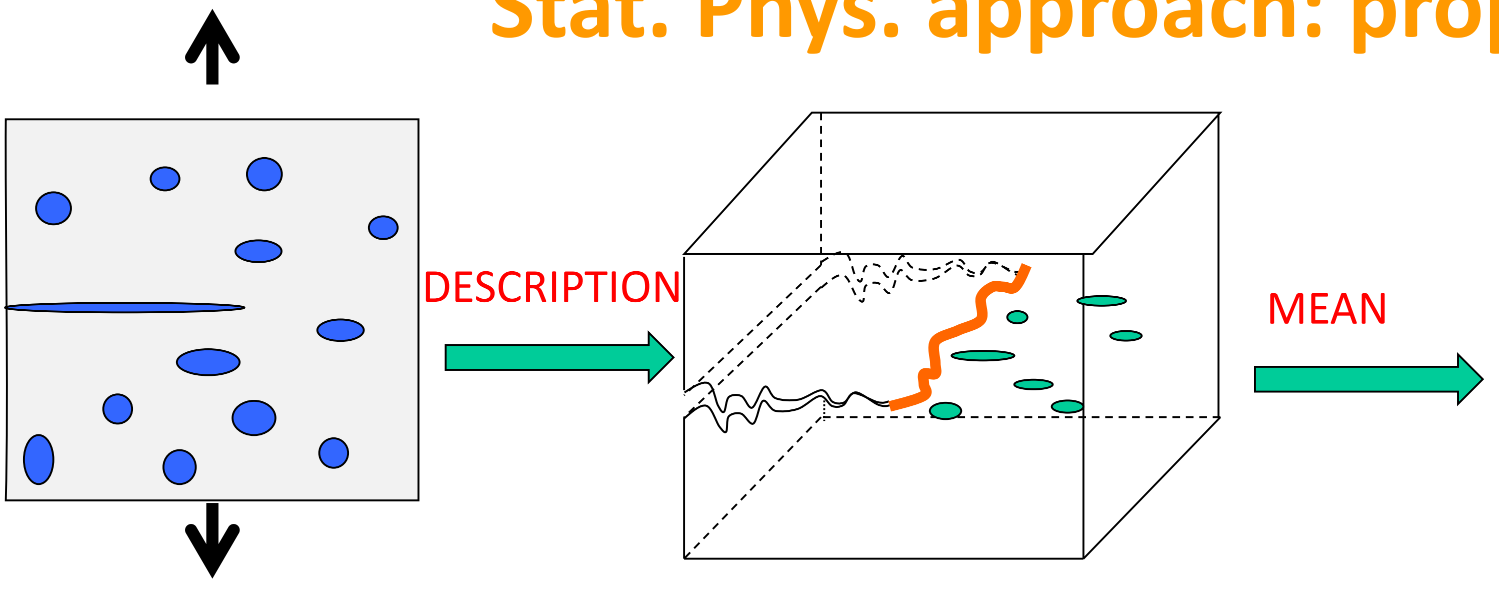
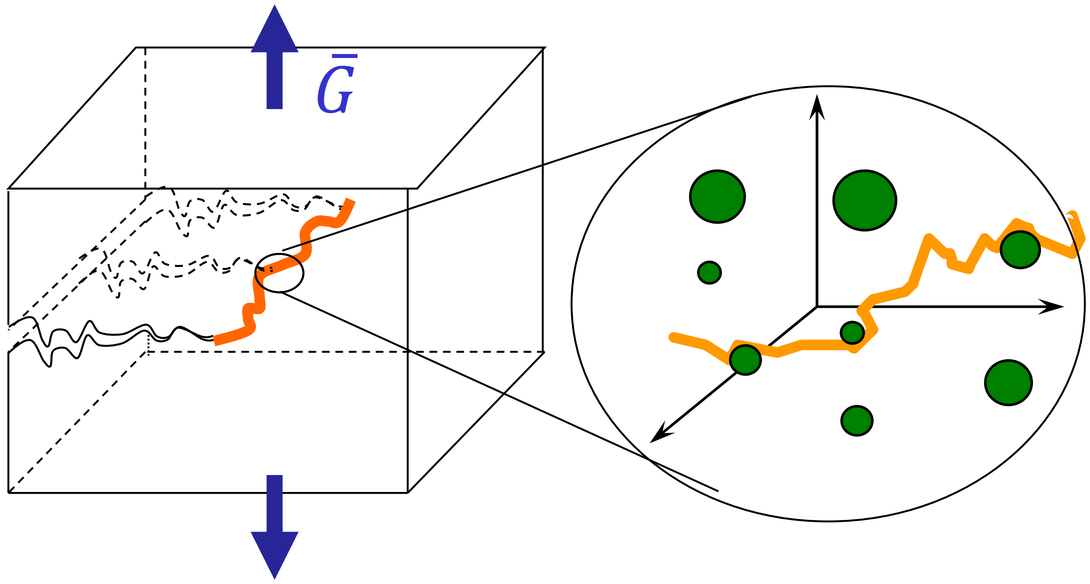


Stat. Phys. approach: proposition



Formalism



Fracture mechanics

Elastic material

Speed $_{local}$ $\propto (G - \Gamma)_{local}$

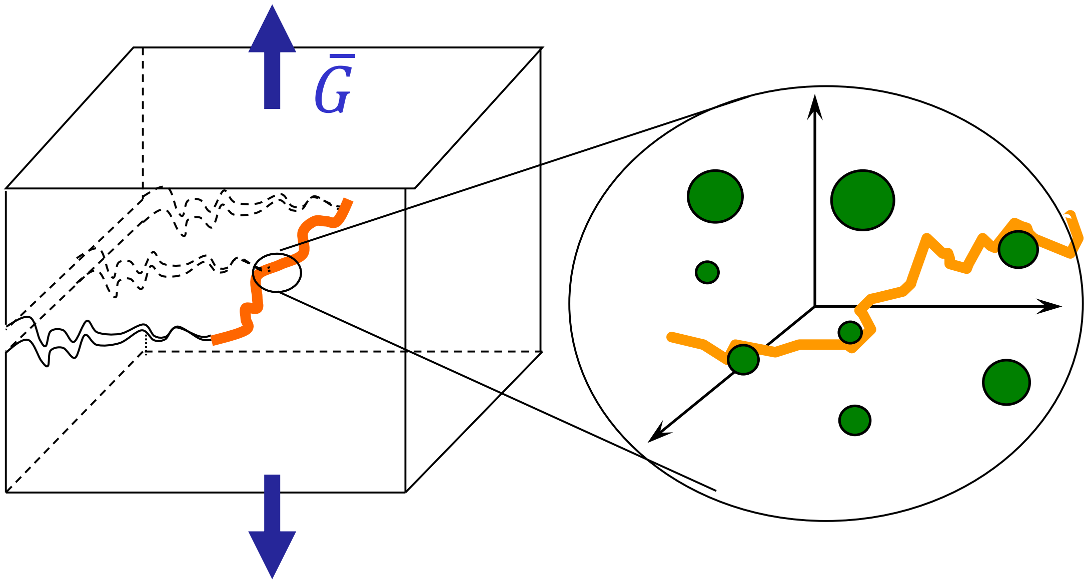
Local shear $\equiv 0$

+

Spatially-distributed fracture energy due to microstructure disorder

Coupling between front corrugation and local energy release/local SIF

Formalism



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System of two *stochastic* equations (Langevin-like), describing crack propagation

- Quenched noise *explicitly account for the microstructure disorder*
- (Long range) *interaction kernel account for retroaction of front distortion onto local loading*

Path equation

➔ Capture the statistics of fracture roughness

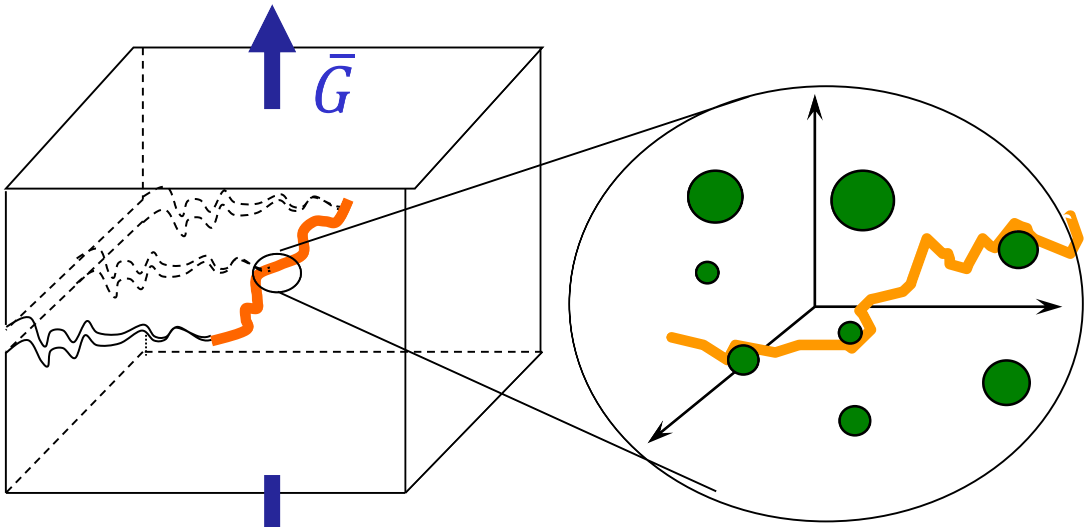
Equation of motion

➔ Capture the statistical aspects associated with intermittent dynamics

Details

Gao and Rice JAM(1989), Bouchaud et al. PRL (1993), Schmittbuhl et al. (1995), Ramanathan et al. PRL (1997)…

Formalism



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Equation of motion

Interaction kernel
Coupling between local G and front shape f(z,t)

$$H(\{f\}(t),z) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(z',t)-f(z,t)}{(z'-z)^2}dz'$$

Rice JAM (1985)

Local speed

$$\mu\frac{\partial f}{\partial t} = \bar{G} - \bar{\Gamma} + \bar{G}H(\{f\}(t),z) + \gamma(f,x)$$

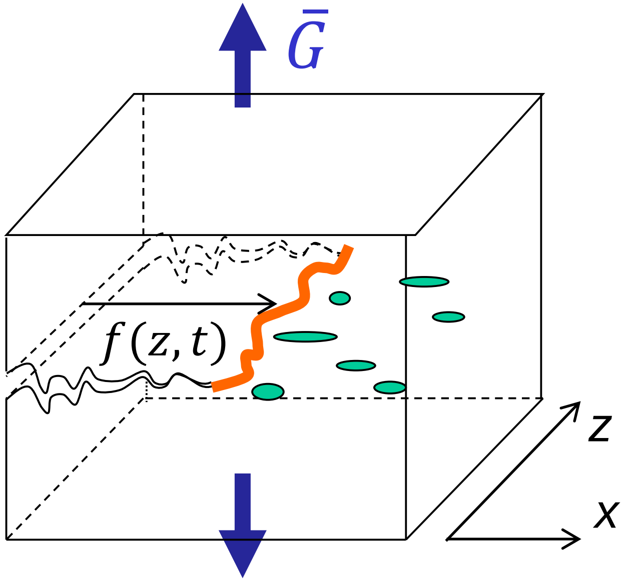
Mean energy release rate

Mean fracture energy

Spatially distributed ***disorder term*** for fracture energy

Gao and Rice JAM(1989), Bouchaud et al. PRL (1993), Schmittbuhl et al. (1995), Ramanathan et al. PRL (1997)…

Formalism : On the origine of intermittency

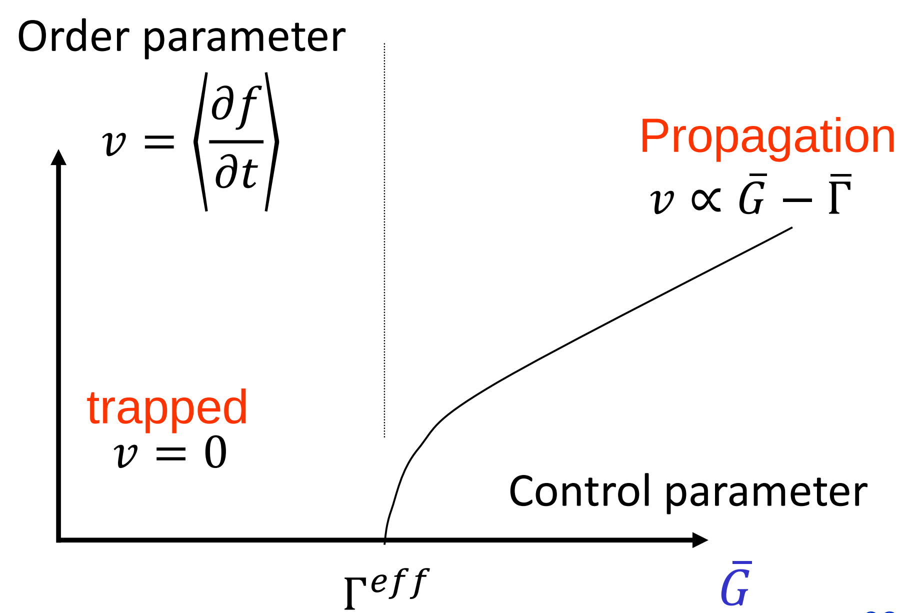


Equation of motion

Crack front ⟷ Elastic line in a random potential

Existence of critical depinning transition

Narayan & Fisher PRL 1992 , Ertas & Kardar, PRE, 1994



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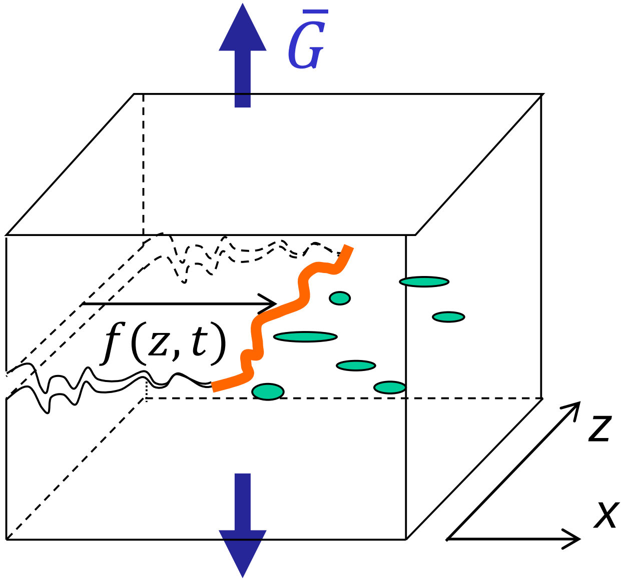

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In all fracture situations with crackling dynamics

Self-ajustment of $\bar{G}$ around Γ^{eff}

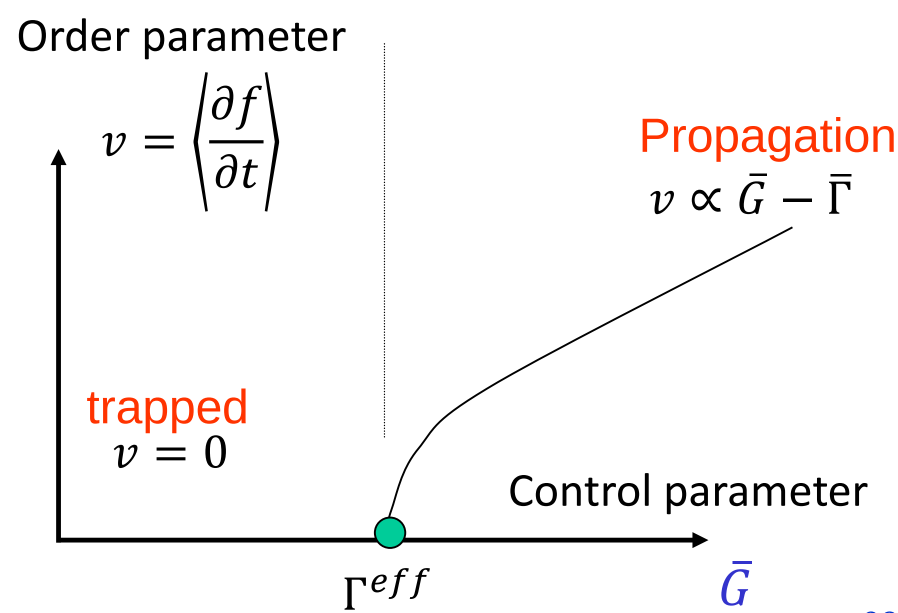
DB et al. PRL (2008)

Imposed displacement rate

$\bar{G}$ ↗ avec $\langle f \rangle$

Effect of the effective compliance

$\bar{G}$ ↘ avec $\langle f \rangle$



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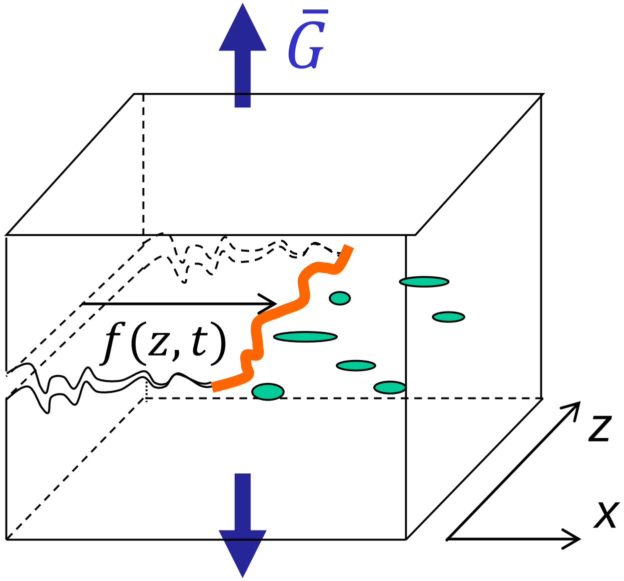

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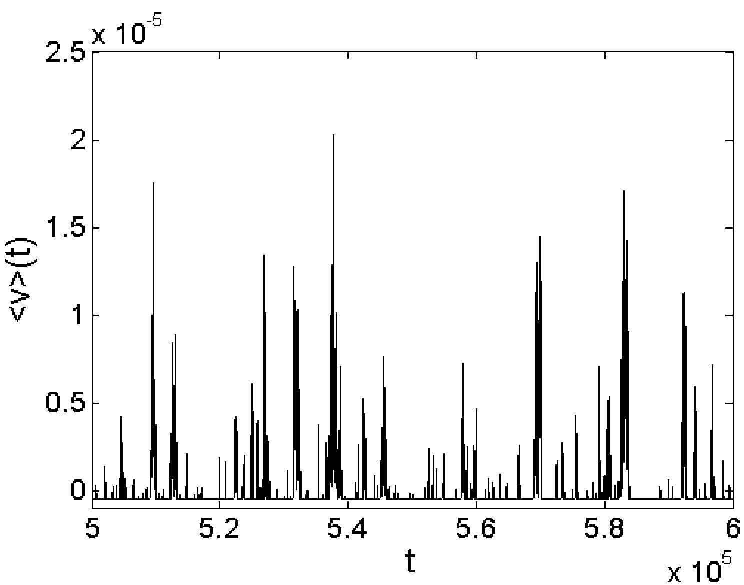
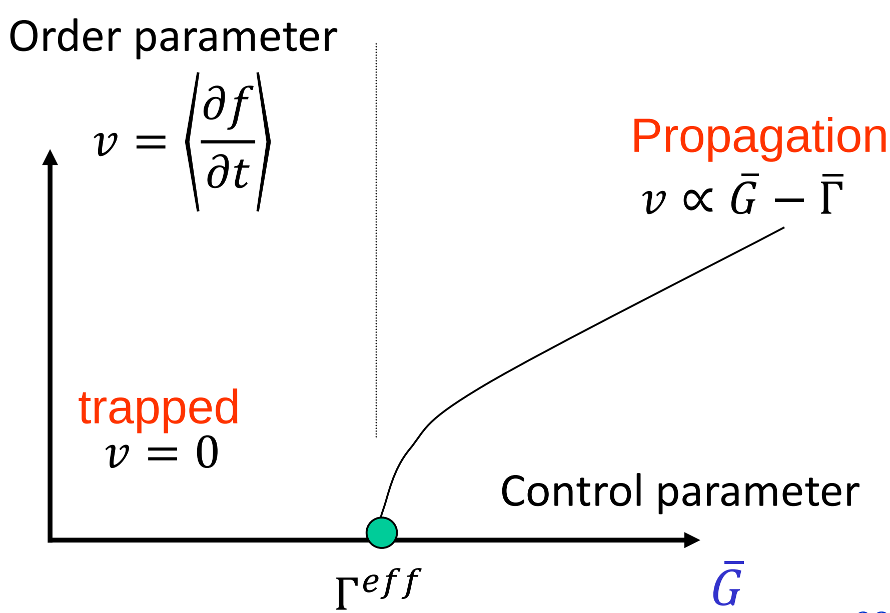
➔ giant speed fluctuations





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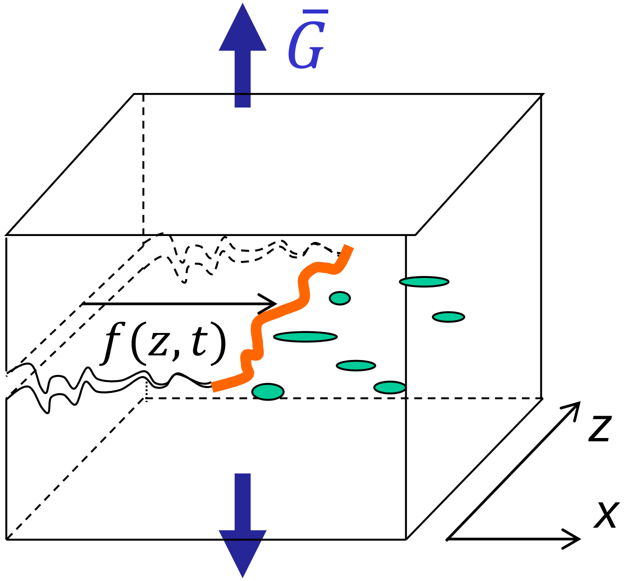

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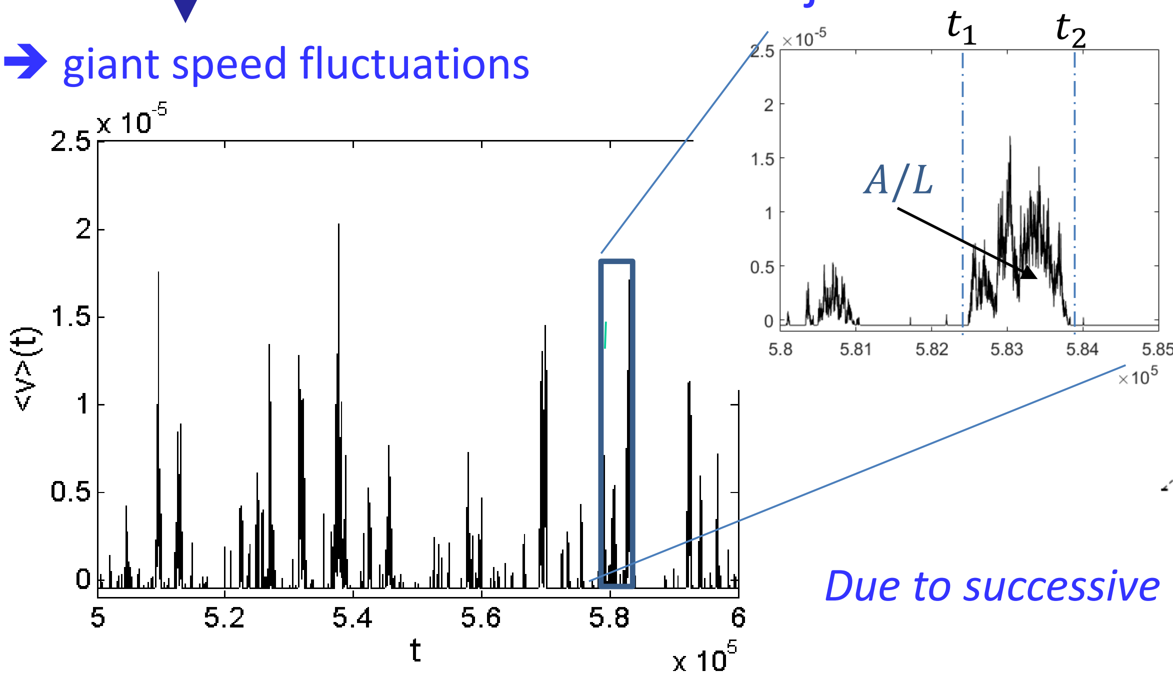
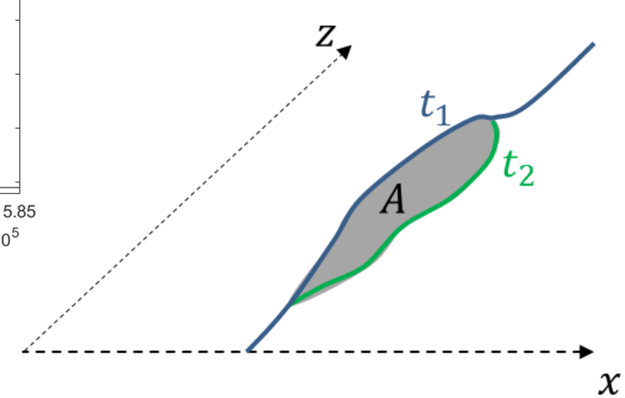
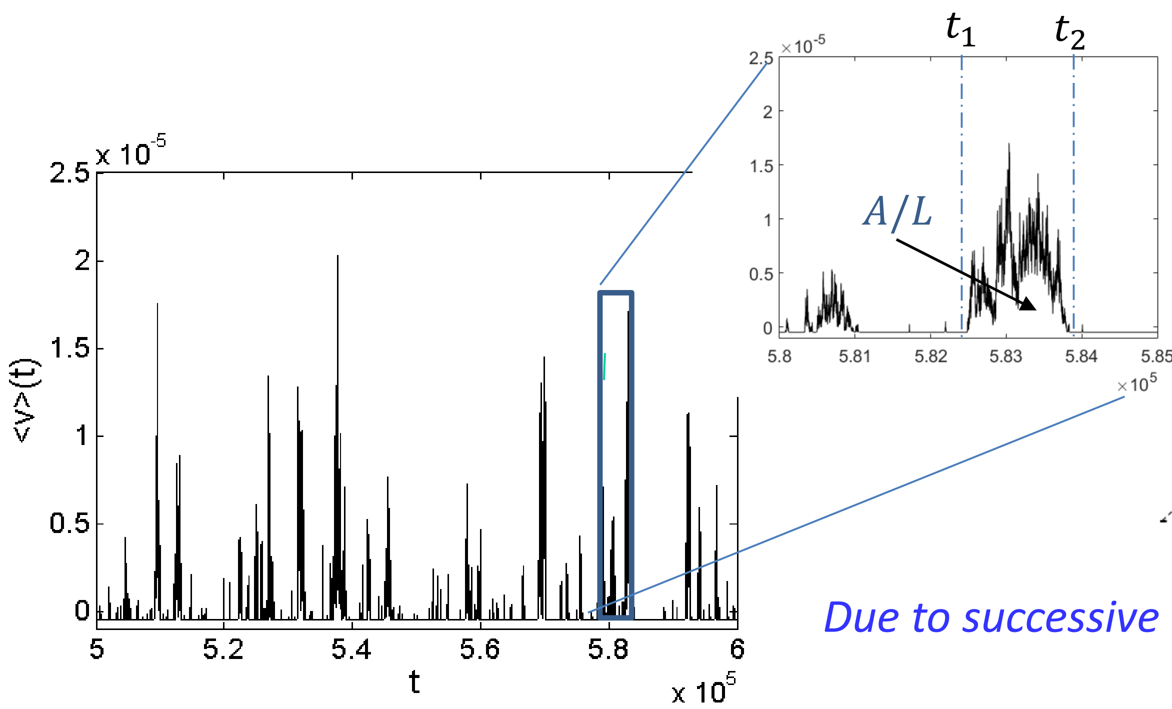
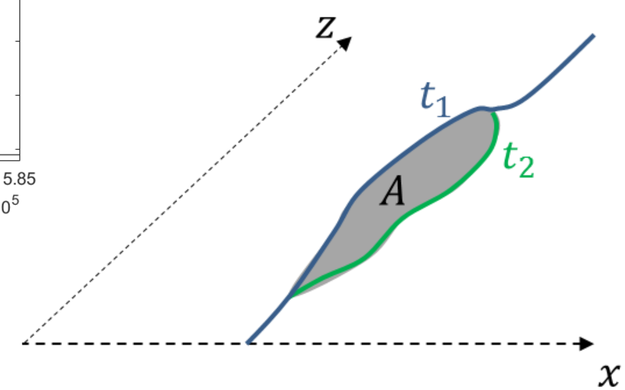




Due to successive depinning avalanches…

Formalism : On the origine of intermittency





Due to successive depinning avalanches…

Variety of universal & predictable scale invariant features, e.g.:

- Distribution of size & duration: $P(A) \sim A^{-\tau} f_A\left(\frac{A}{A_0}\right)$, $P(D) \sim D^{-\alpha} f_D\left(\frac{D}{D_0}\right)$
  with known $\tau \approx 1.28$, $\alpha \approx 1.5$, f_A(u), f_B(u)
  Ertas & Kardar 1994 PRE, Rosso et al. 2009

- Average pulse shape:
  Laurson et al. Nat. Com. 2013, Dobrinevski et al. EPL 2014, Tiery et al. EPL 2016

- Self-affine nature of f and associated roughness exponent $\zeta \approx 0.39$
  Rosso et al. PRE 2002, Duemmer & Krauth J.Stat. Mech. 2007

….

Outline

I. Heterogeneous fracture and elastic line formalism

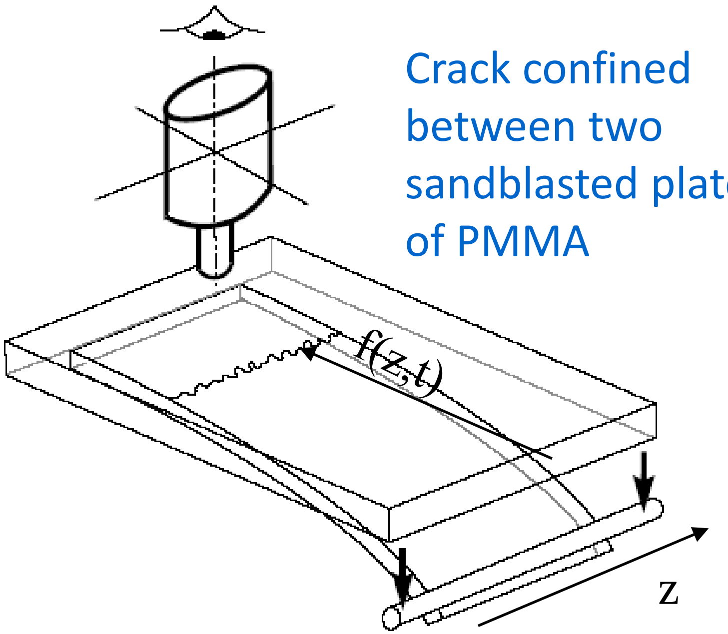
- Two key ingredients: interaction kernel H and spatial disorder γ
- Crack onset ≡ depinning transition
- Jerky dynamics 🡐 self-adjustment of forcing around critical value
- Variety of universal (and predictable) scale invariant features and scaling laws

II. Confrontation to 2D interfacial fracture experiments

III. Confrontation to 3D fracture

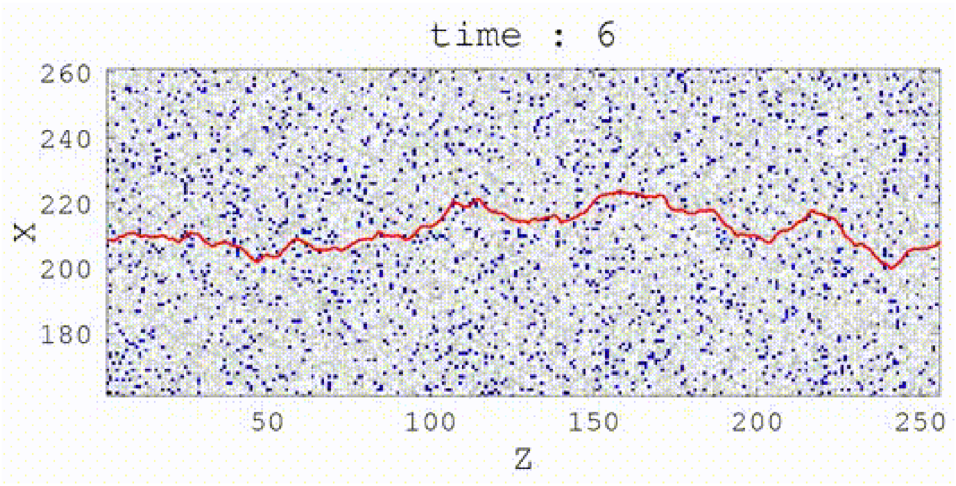
IV. Other avenues for extension

Comparison with 2D Oslo peeling experiment



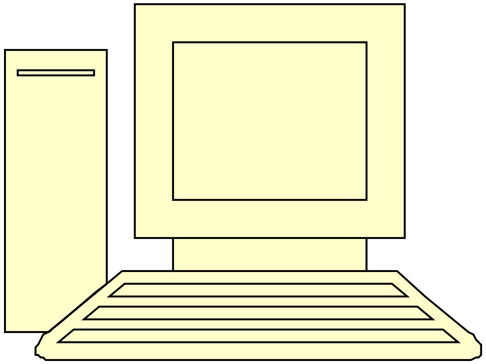
Crack confined between two sandblasted plates of PMMA

Maloy et al, PRL, 2006



Bares 2014, Pindra & Ponson PRE 2017

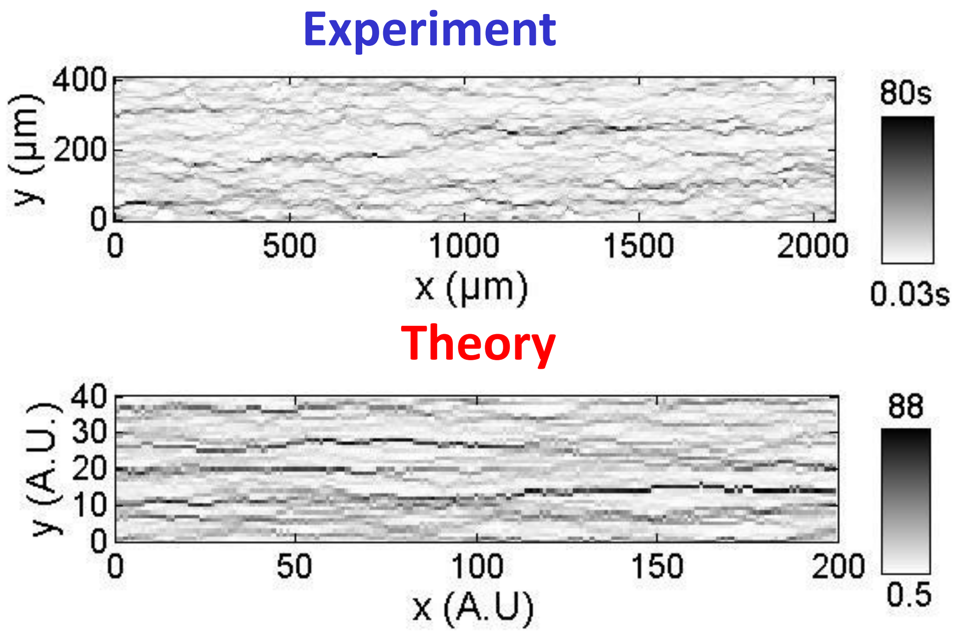
Numerical simulation of the elastic line model



Comparison with 2D Oslo peeling experiment

Activity map W(x,y)
Maloy et al, PRL 2006

Time spent by the front in(x,y)



Comparison with 2D Oslo peeling experiment

Activity map W(x,y)
Maloy et al, PRL 2006

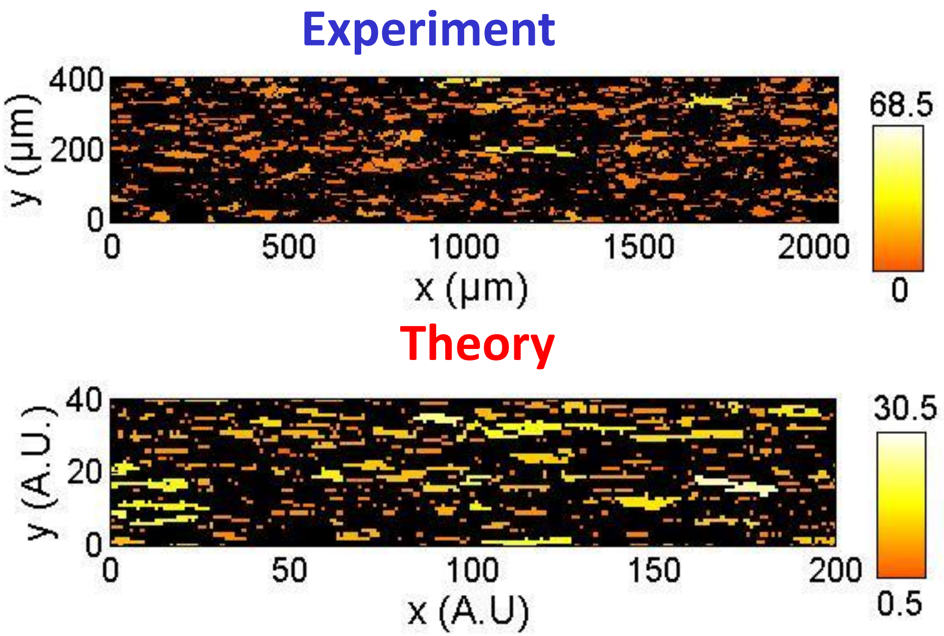
Time spent by the front in(x,y)

Avalanche definition

Zones où W(z,x) < C<W>

Avalanche duration

= last time spent in avalanche
- first time in avalanche



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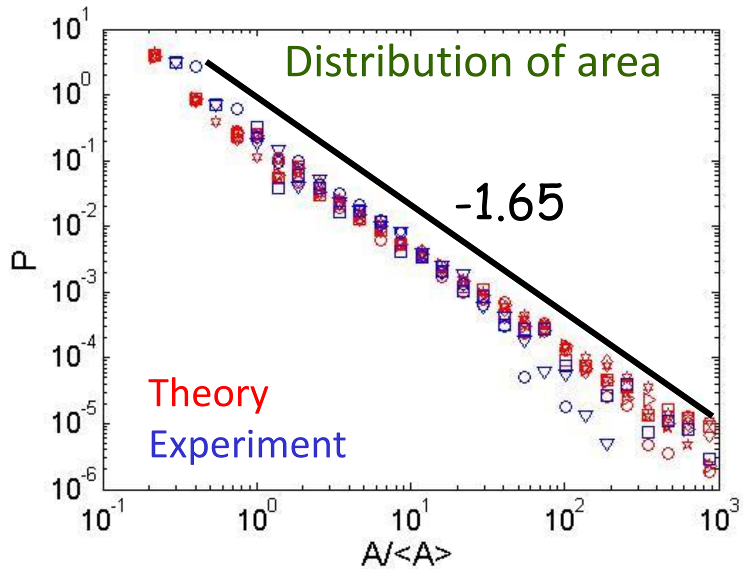
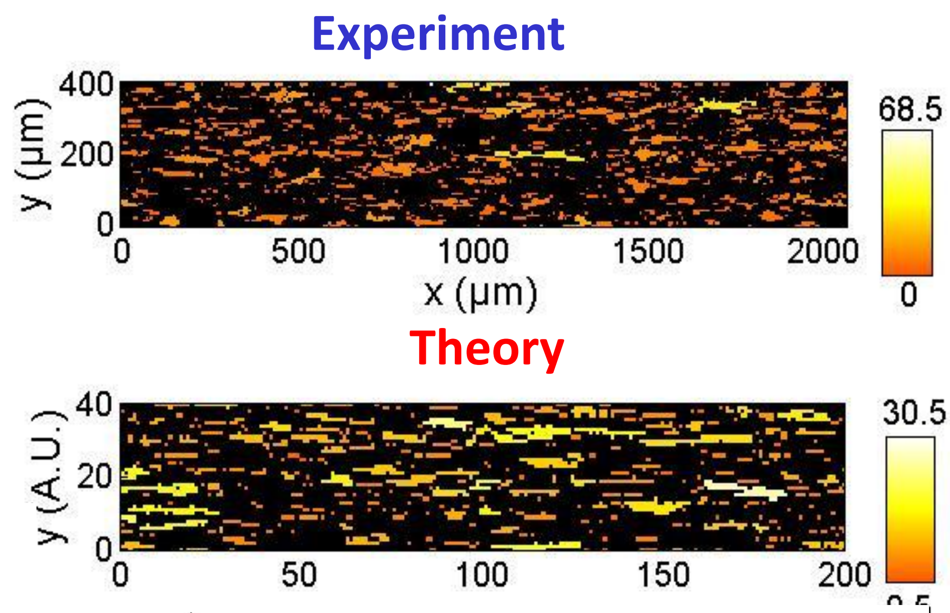
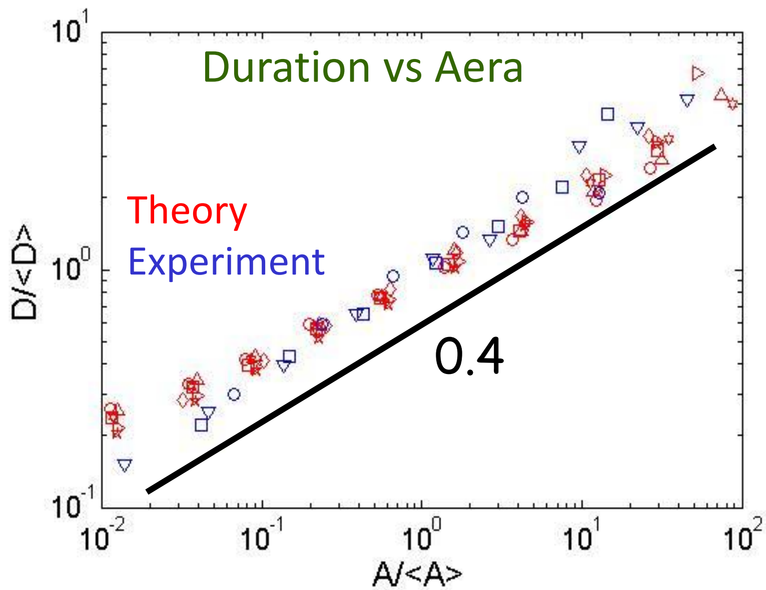
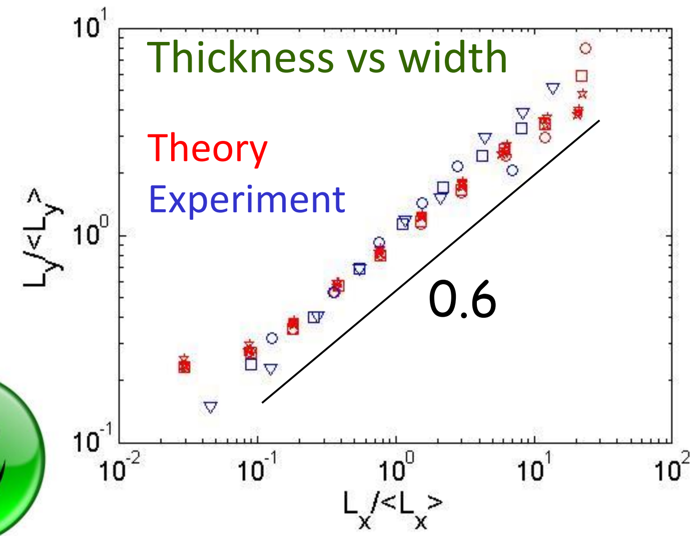


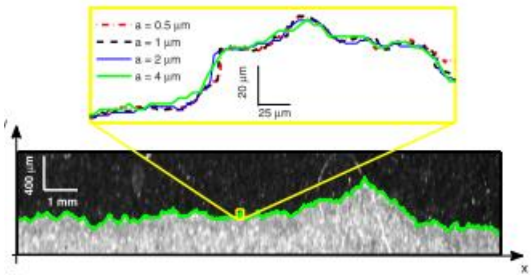


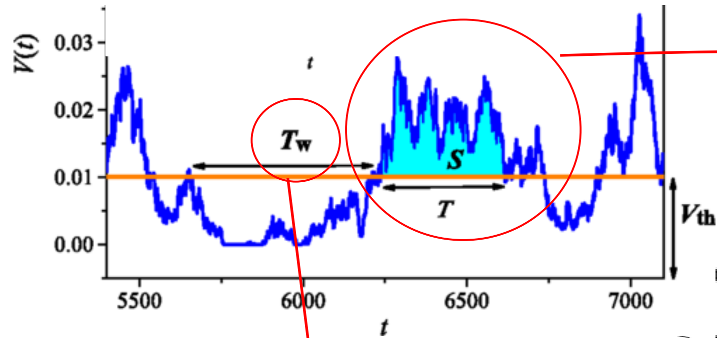




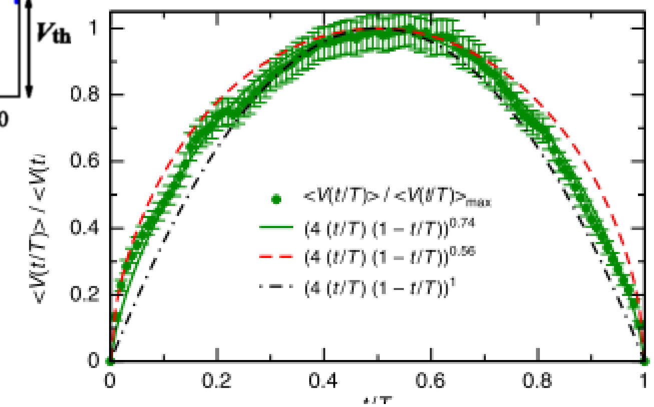
Comparison with 2D Oslo peeling experiment

Self-affine fronts, roughness exponent compatible with theory

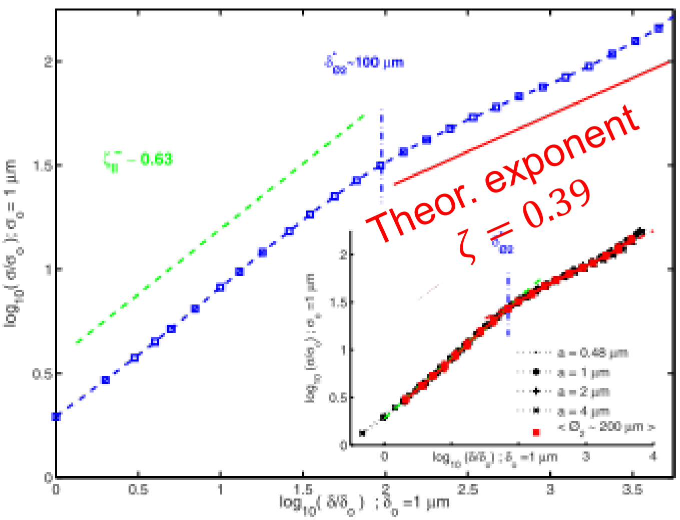




Mean pulse shape also

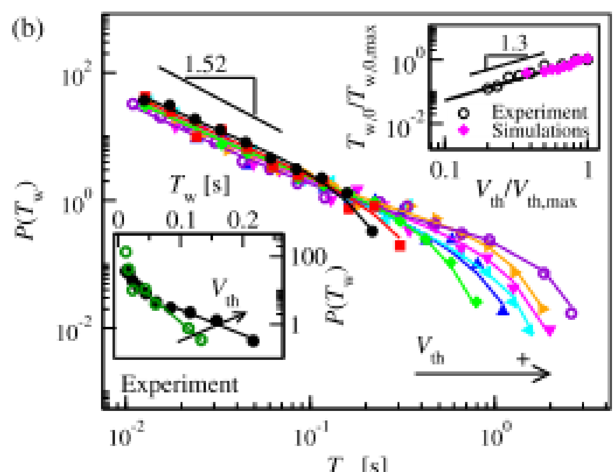


Laurson et al. Nat. Com 2013



Santucci et al. EPL 2010

Statistics of inter-event time also



Janicevicz et al. PRL 2016

Beyond scaling features
… kernel refinement and quantitative tests

$$\mu \frac{\partial f}{\partial t} = \bar{G} - \bar{\Gamma} + \bar{G} H(\{f\}, z) + \gamma(z, f)$$

- **1st order+semi-infinite:**

$$H(\{f\}, z) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(z', t) - f(z, t)}{(z' - z)^2} dz'$$

Rice JAM (1985):

Recent kernel refinement:

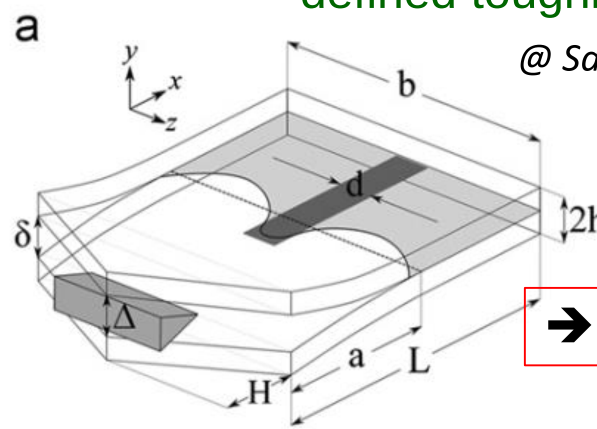
- **2nd order+semi-infinite**
  Adda Bedia et al. 2006,
  Leblond et al. EFM 2012:
- **1st order+thin plate**
  Legrand et al IJF 2011
- **…**

Beyond scaling features
… kernel refinement and quantitative tests

$$\mu \frac{\partial f}{\partial t} = \bar{G} - \bar{\Gamma} + \bar{G} H(\{f\}, z) + \gamma(z, f)$$

Careful experiment with well defined toughness defect

@ Saint-Gobain, IJLRd'A…



➔ Davy's talk

- **1st order+semi-infinite:**

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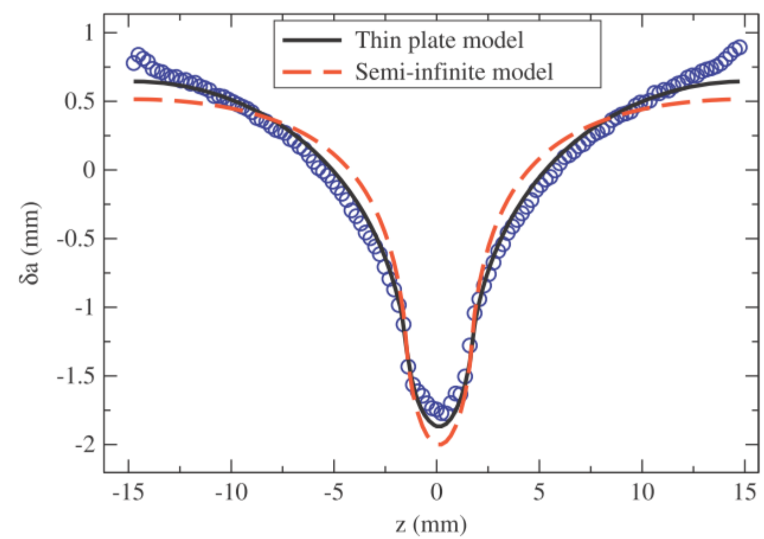


Dalmas et al. (JMPS) 2009,
Patinet et al. (JMPS) 2013

Outline

I. Heterogeneous fracture and elastic line formalism

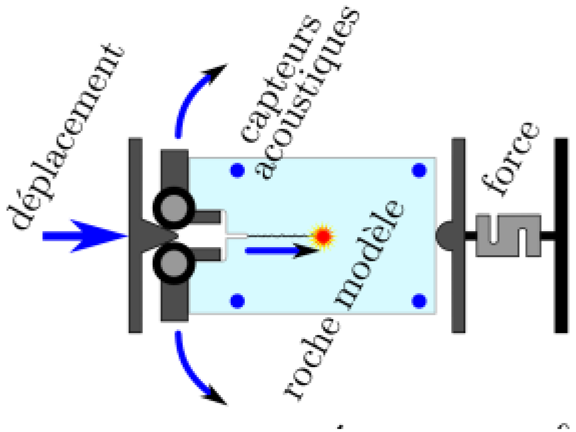
II. Confrontation to 2D interfacial fracture experiments

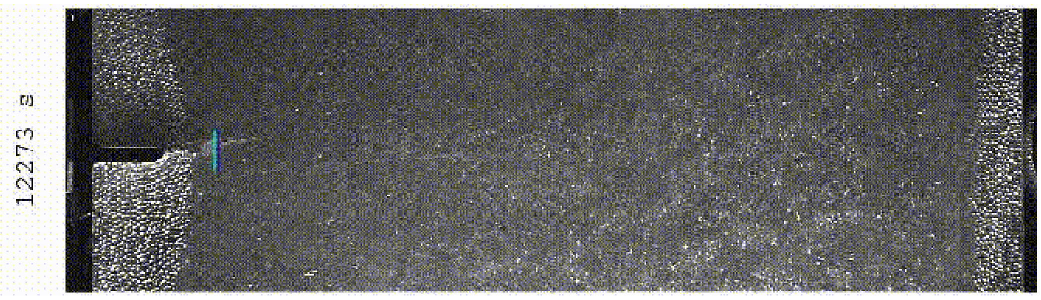
- Many of the (analytically or theoretically) predicted scale invariant features, scaling laws, pulse shapes… verified in (Oslo) 2D interfacial experiment
- Availability of more refined interaction kernels
  … describing quantitatively 2D single-defect well-controlled experiments

III. Confrontation to 3D fracture

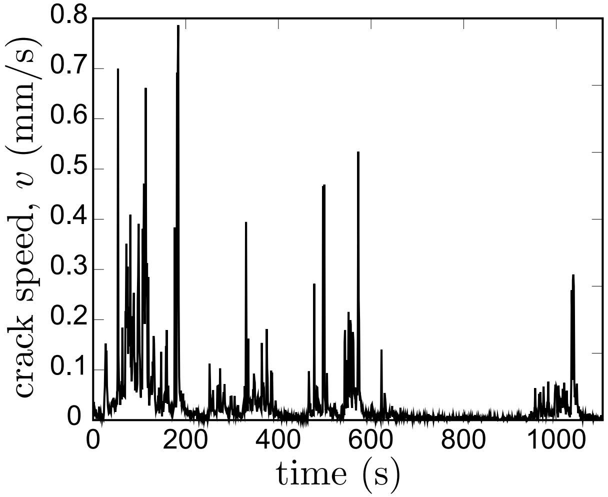
IV. Other avenues for futur extension

Comparison with 3D fracture experiments



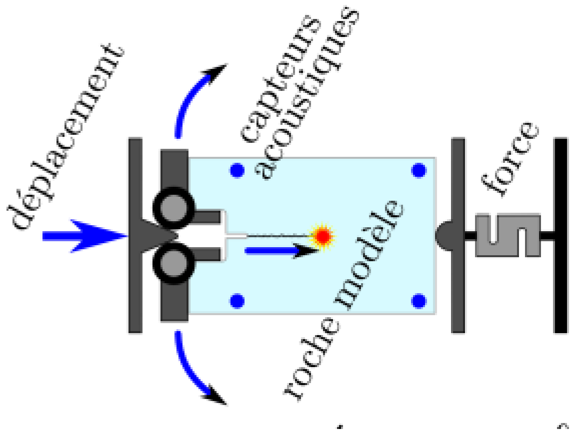


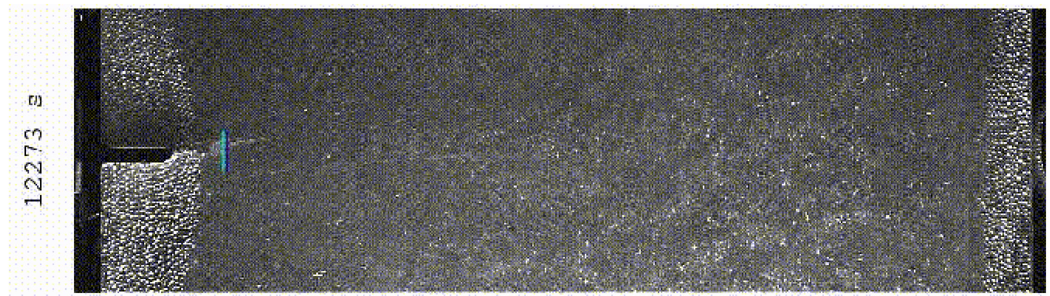
© J. Barés @ CEA



Bares PhD, 2013, Bares et al. PRL 2014

Comparison with 3D fracture experiments





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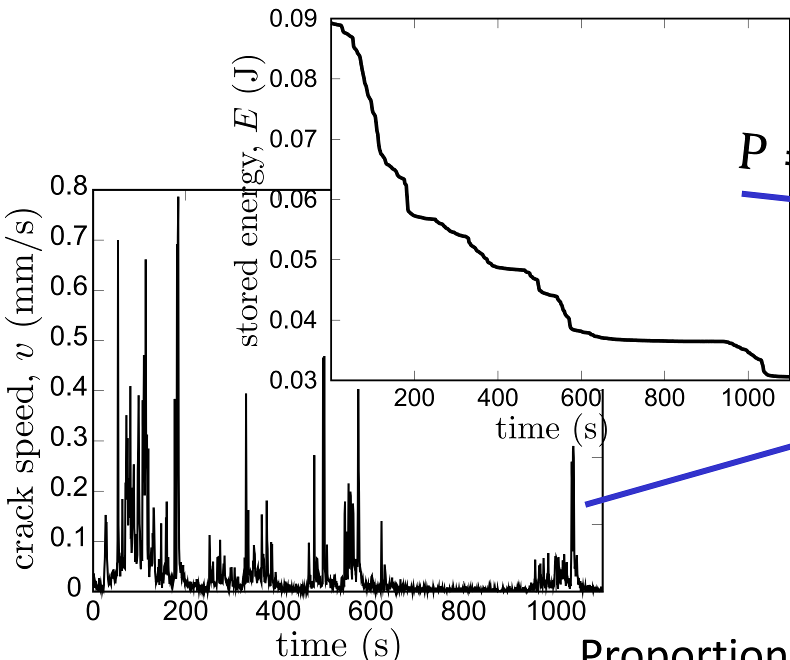
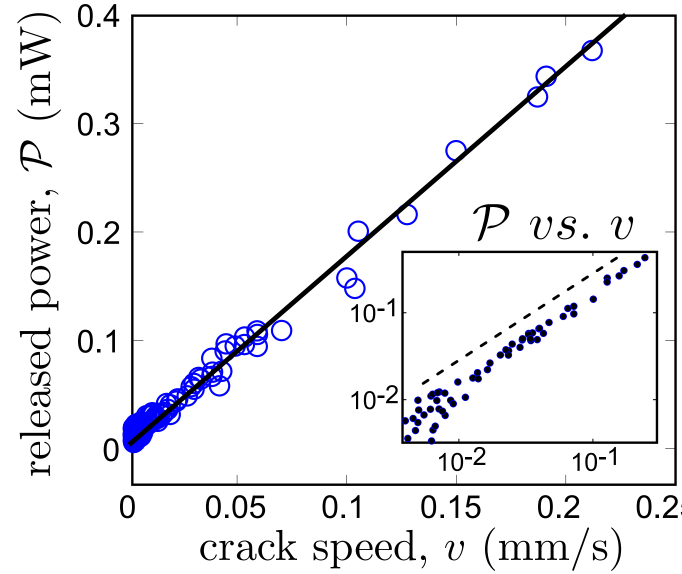


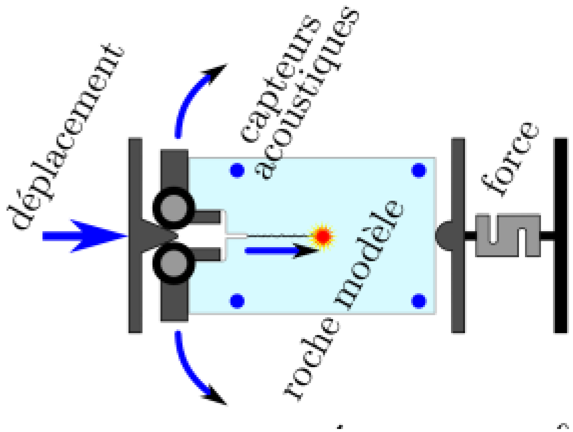
$P = dE/dt$

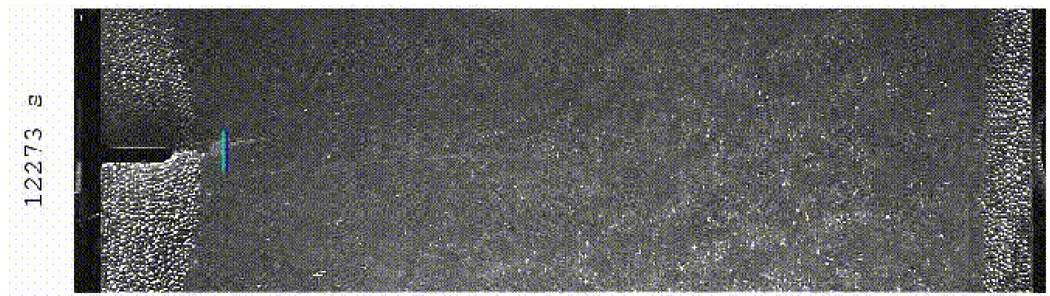


Proportionality between P and v ➔ Material constant $\Gamma \sim 100 \mathrm{J/m^2}$

Bares PhD, 2013, Bares et al. PRL 2014

Comparison with 3D fracture experiments





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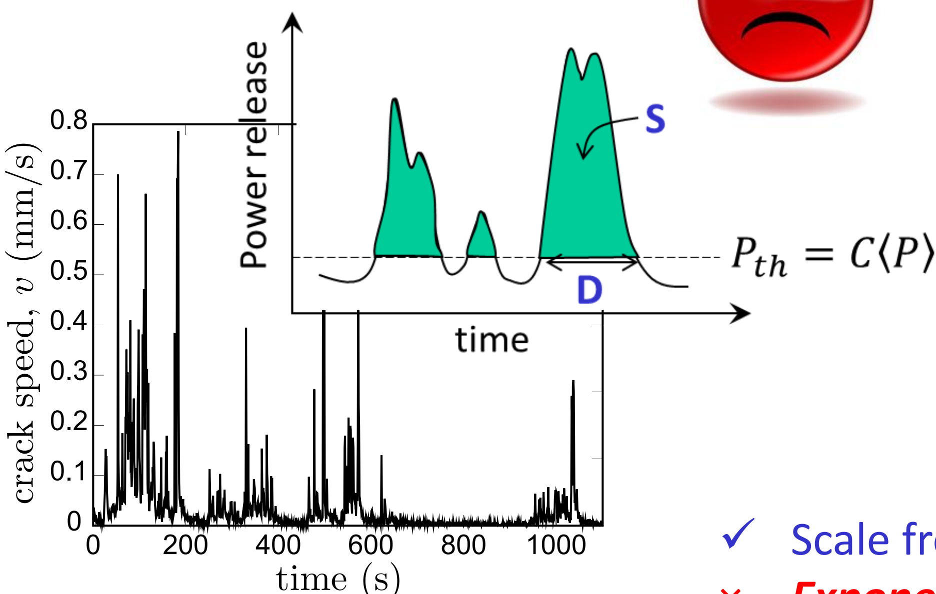
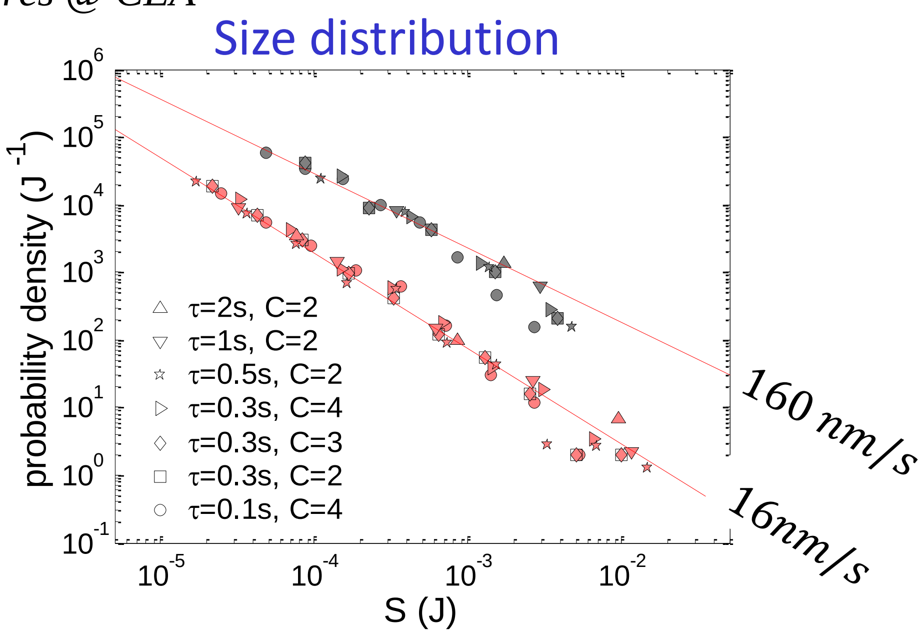


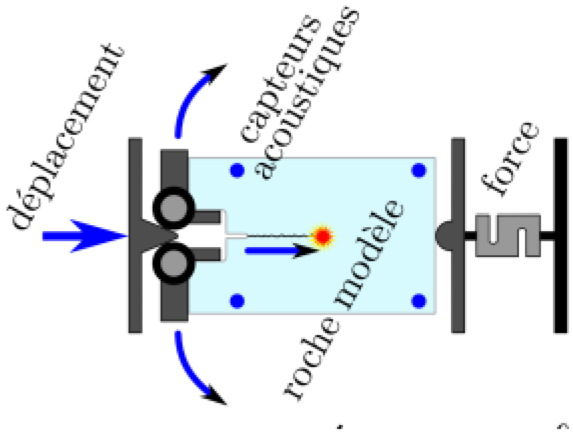


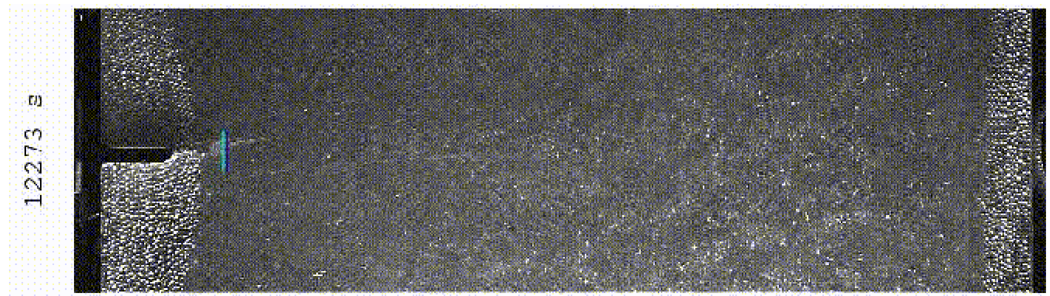
✓ Scale free statistics as expected in depinning approach

× ***Exponents different from that expected***

Bares PhD, 2013, Bares et al. PRL 2014

Comparison with 3D fracture experiments





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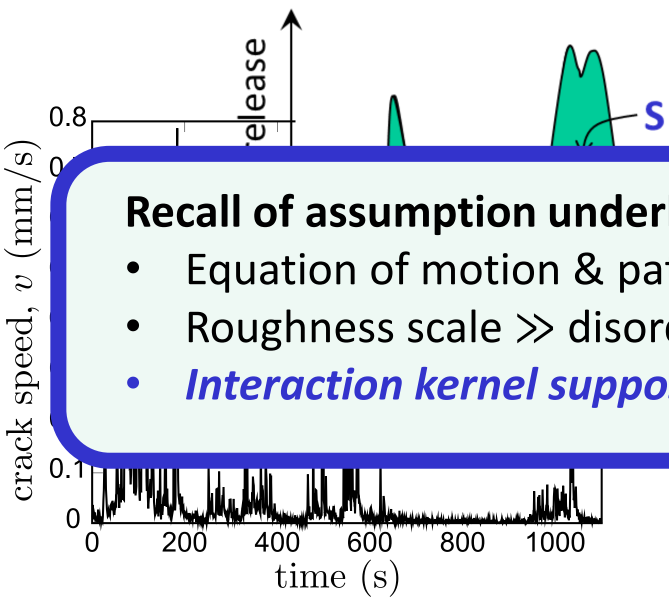
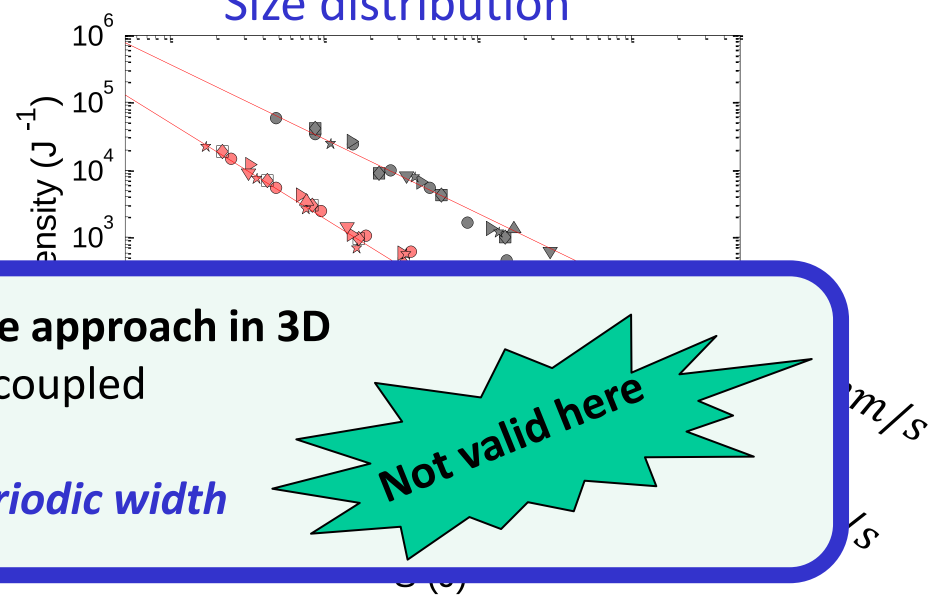




Recall of assumption underlying elastic line approach in 3D

- Equation of motion & path equation decoupled
- Roughness scale ≫ disorder scale ℓ
- ***Interaction kernel supposes infinite/periodic width***

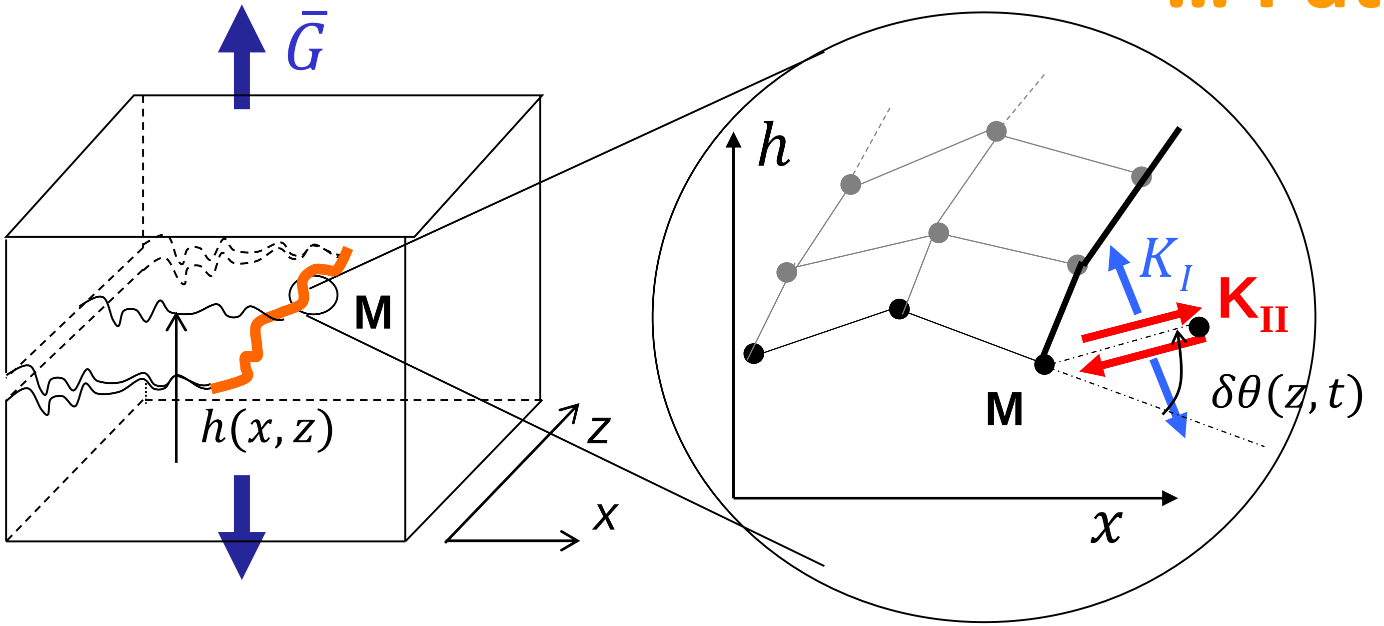
Not valid here

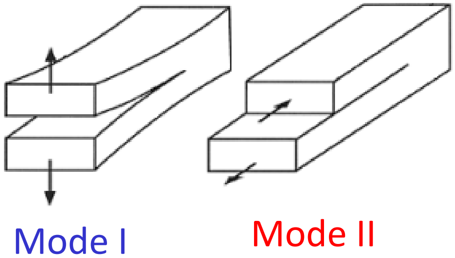
✓ Scale free statistics as expected in depinning approach

× ***Exponents different from that expected***

Bares PhD, 2013, Bares et al. PRL 2014

Out-of-plane roughness in 3D fractured solids … Path equation





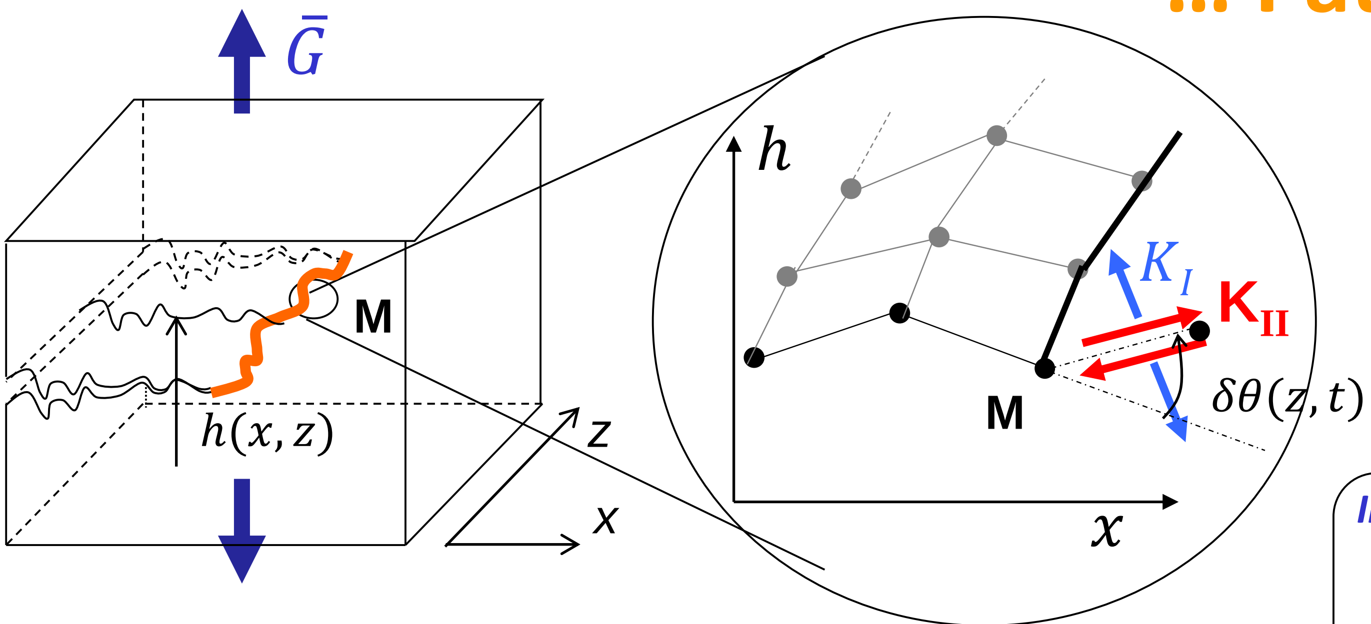
Local symmetry principle

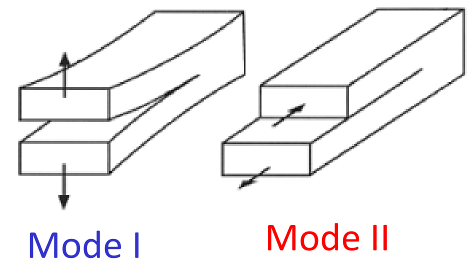
$K_{II}(M) = 0$

Goldstein & Salganic (1974)

Ramanathan et al. PRL 1997, DB et al. PRL 2006, Bares et al. Front. Phys. 2014

Out-of-plane roughness in 3D fractured solids … Path equation





Interaction kernel provided:
roughness slope <<1
Roughness wavelength <<L

Movchan et al. IJSS (1998)

Local symmetry principle

$K_{II}(M) = 0$

Goldstein & Salganic (1974)

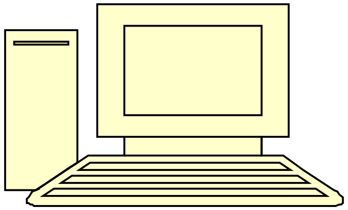
$$2\ell \frac{\partial^2 h}{\partial x^2} + \frac{\partial h}{\partial x} = A \int_{-\infty}^{\infty} \frac{h(x,z') - f(x,z)}{(z'-z)^2} dz' + \gamma(x,z,h)$$

Disorder length-scale

Spatially distributed ***Disorder term***

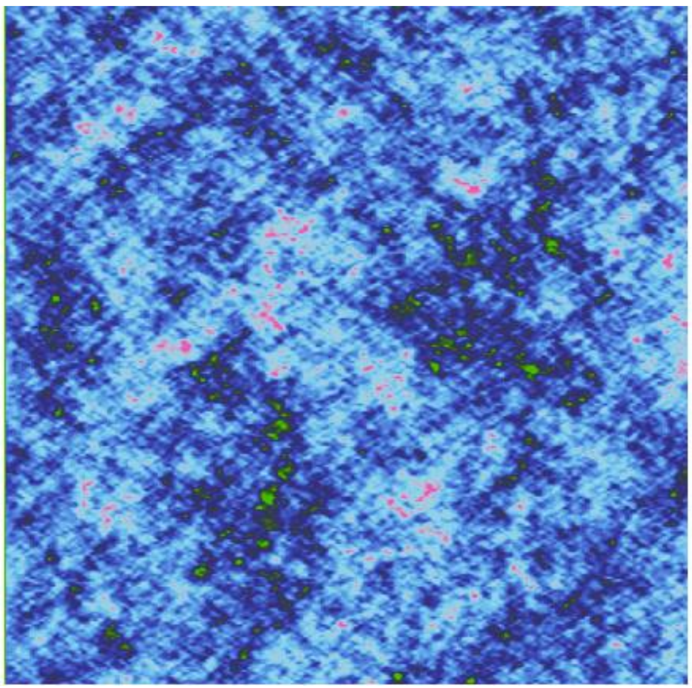
Ramanathan et al. PRL 1997, DB et al. PRL 2006, Bares et al. Front. Phys. 2014

Out-of-plane roughness in 3D fractured solids
Path equation vs. fractography (in glass)

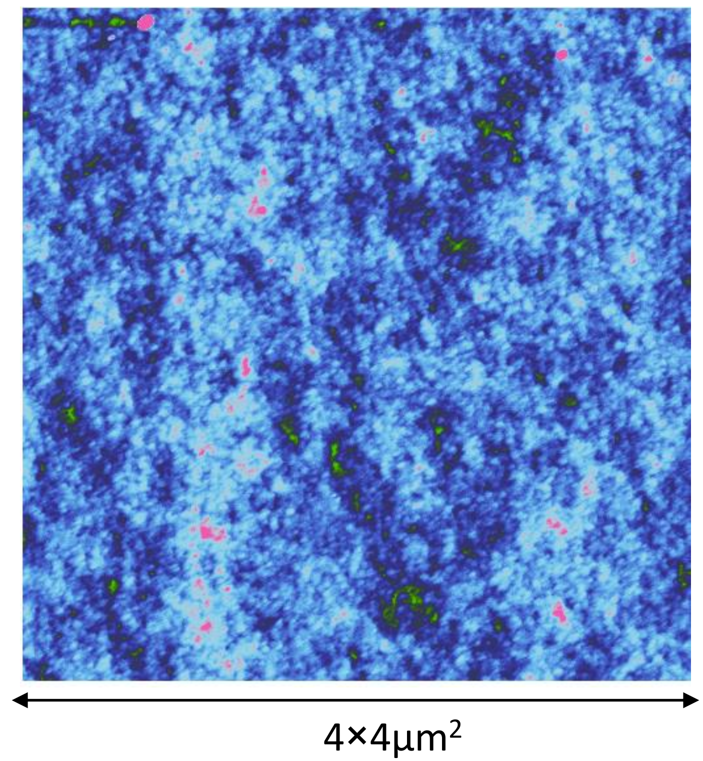


Surface simulated from the path equation predicted in the stochastic formalism

AFM image on a silica glass broken under stress corrosion
© C. Rountree, M. Barlet



J. Barés et al. Front. Phys. (2014)



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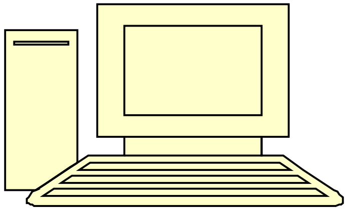
Path equation vs. fractography (in glass)



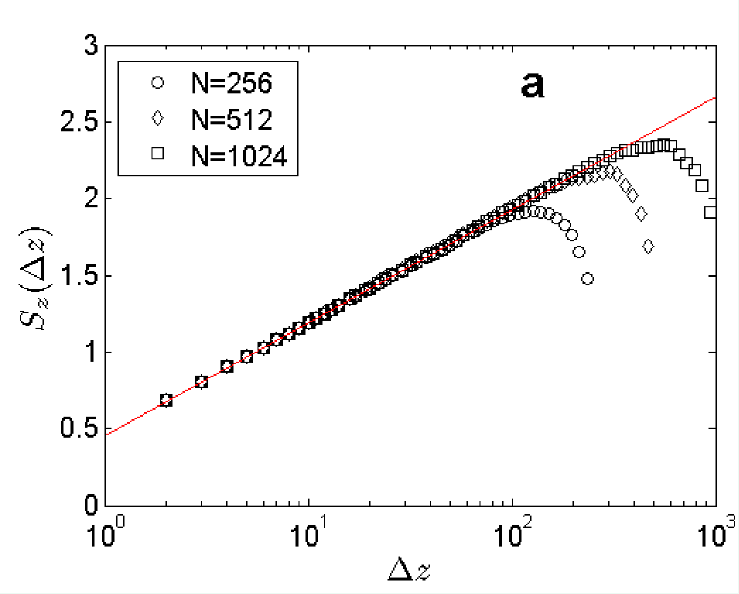
Surface simulated from the path equation predicted in the stochastic formalism

AFM image on a silica glass broken under stress corrosion
© C. Rountree, M. Barlet

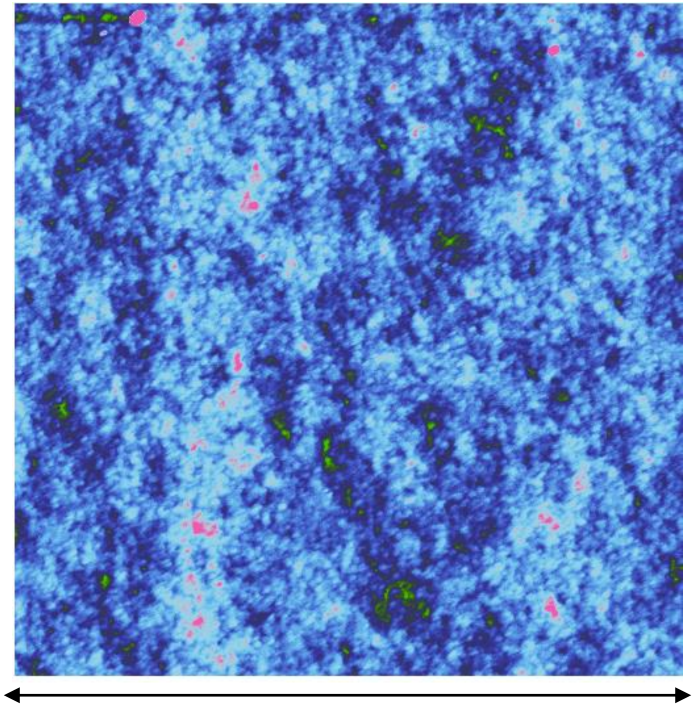
Prediction *e.g.* on structure function

$$S(\Delta z) = \left\langle \left(h(z+\Delta z) - h(z) \right)^2 \right\rangle$$

$$S \approx A\ell^2 \log(B\Delta z/\ell)$$

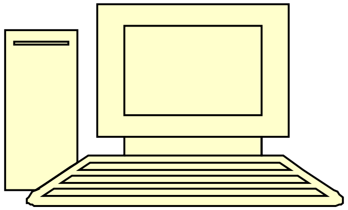


Ramanathan et al. PRL 1997,



$4\times4\mu m^2$

Out-of-plane roughness in 3D fractured solids
Path equation vs. fractography (in glass)



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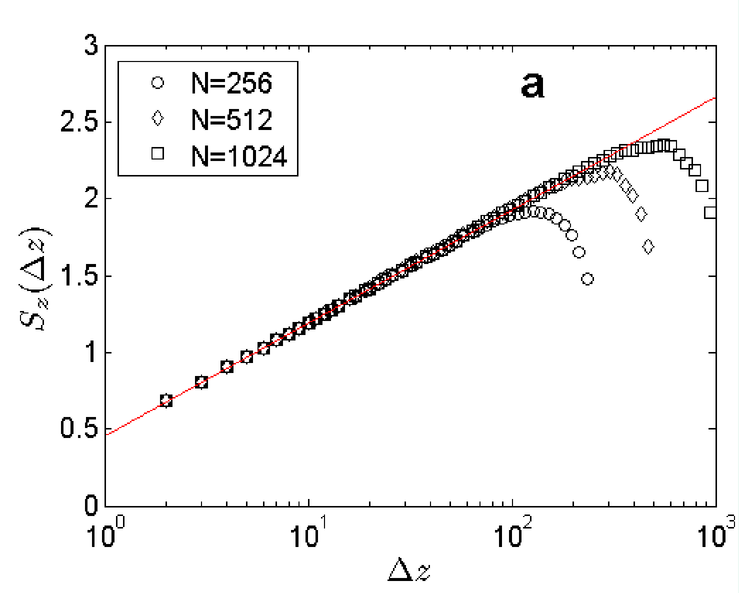
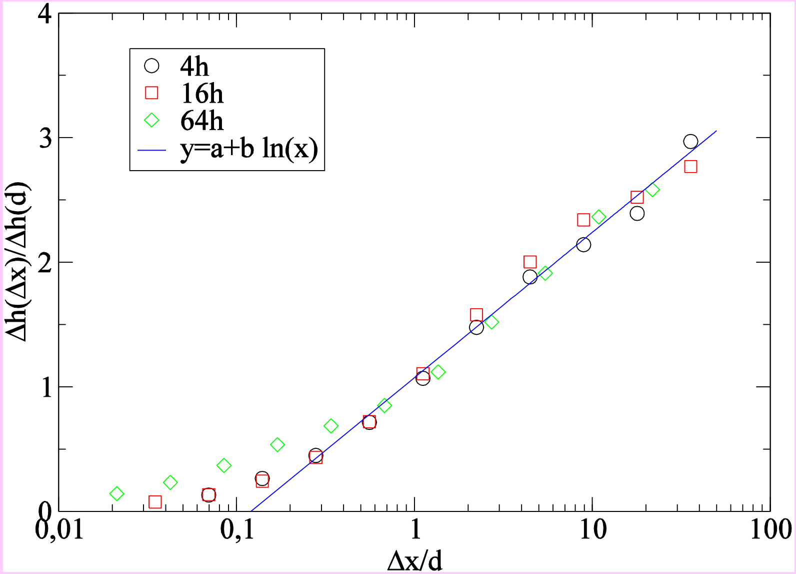


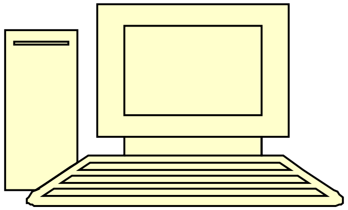
Ramanathan et al. PRL 1997,

Verified on phase separated glass



Dalmas et al. PRL (2008)

Out-of-plane roughness in 3D fractured solids
Path equation vs. other fracto. Exp.



Surface simulated from the path equation predicted in the stochastic formalism

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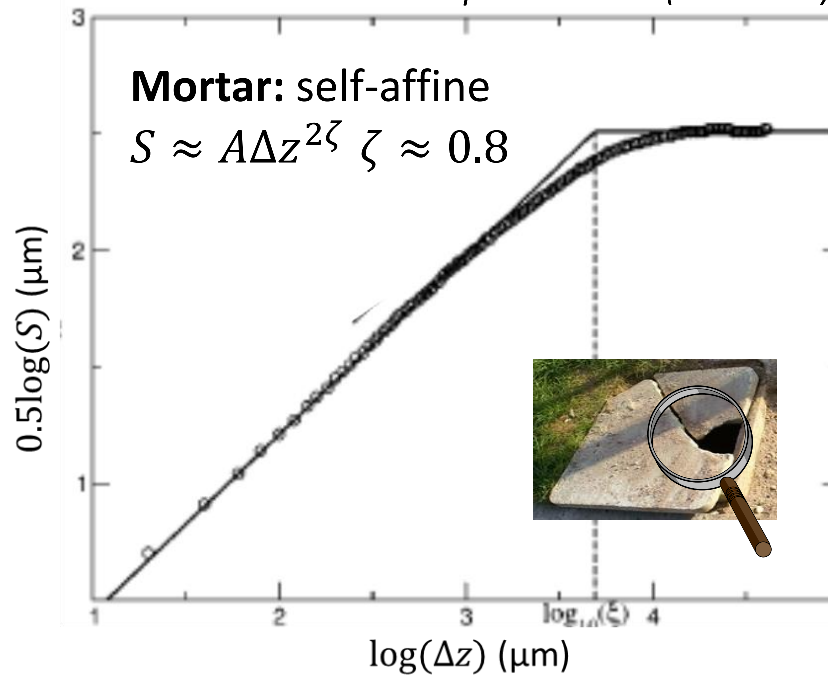
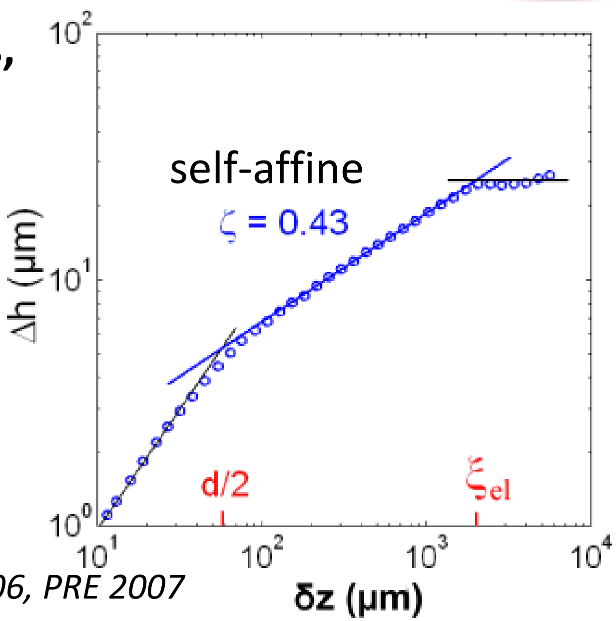
© Stéphane Morel (Bordeaux)



Sintered beads, sandstone,

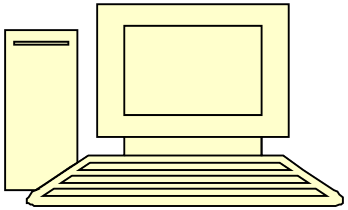


Ponson et al. PRL 2006, PRE 2007

… And many more reporting self-affine, rather than logarithmic out-of-plane roughness

Bouchaud EPL 1990, Maloy et al. PRL 1992, Bouchaud JPCM (1997), DB & Bouchaud Phys. Rep (2011)…

Out-of-plane roughness in 3D fractured solids
Path equation vs. other fracto. Exp.



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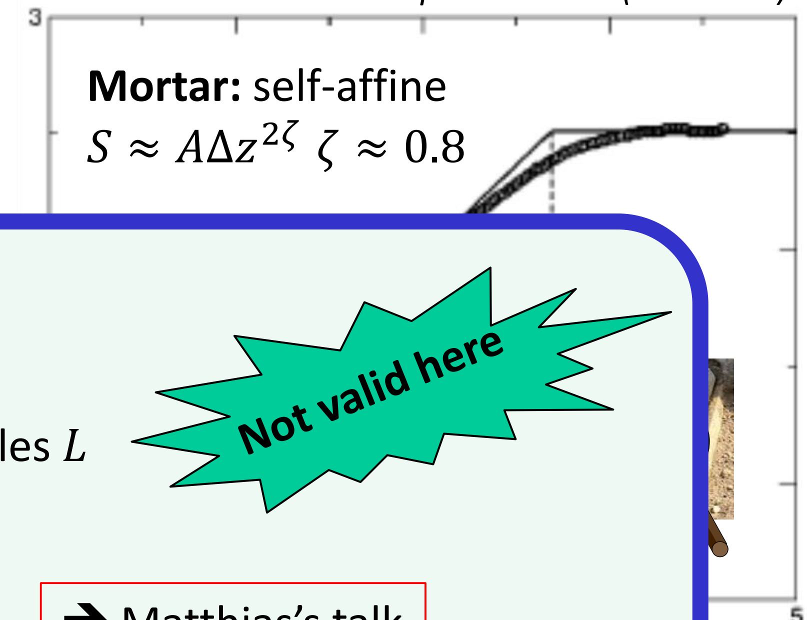


Mortar: self-affine
$S \approx A\Delta z^{2\zeta}$ $\zeta \approx 0.8$

Prediction *e.g.* on structure function

Recall of assumption underlying path equation:

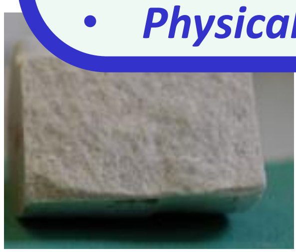
- Roughness slope $\ll 1$
- Roughness scale $\gg$ disorder scale ℓ
- Roughness wavelength $\ll$ geometry/loading scales L
- ***Principle of local symetry*** ➔ $K_{II} = 0$

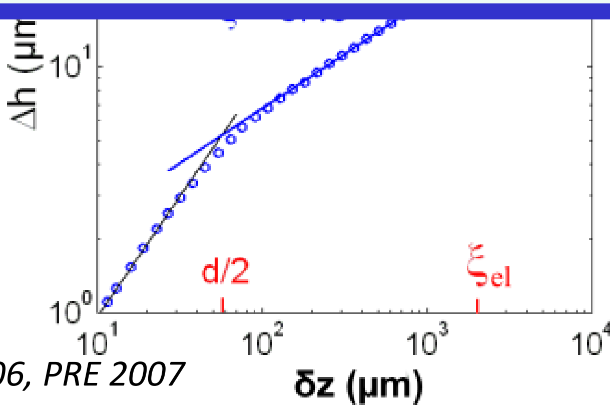
Not valid here

Other difficulty

- ***Physical nature of the disorder term?***

➔ Matthias's talk





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Outline

I. Heterogeneous fracture and elastic line formalism

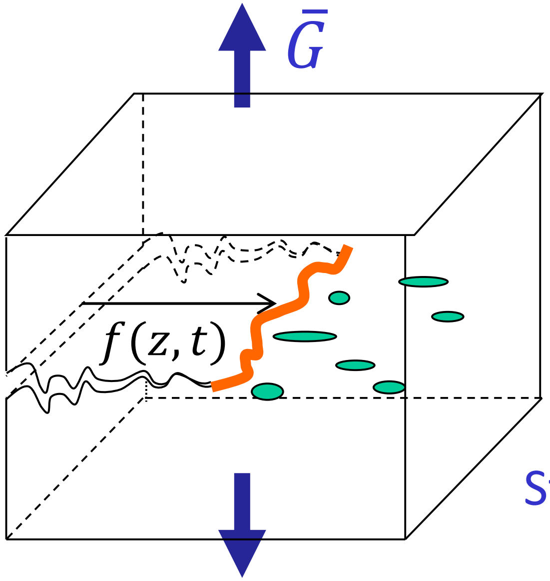
II. Confrontation to 2D interfacial fracture experiments

III. Confrontation to 3D fracture

- Crackling dynamics also observed in 3D crack gorwth
- Despite huge fluctuations, $v(t) \sim dE/dt$ ➔ consistent with depinning approach
- Scale invariant feature in qualitative, not quantitative agreement with elastic line formalism
- Same for the morphological scaling feature of out-of-plane roughness
- Both interaction kernel and spatial disorder not easy to interpret in path equation

IV. Other avenues for future extension

Toward elastodynamic line models of fracture

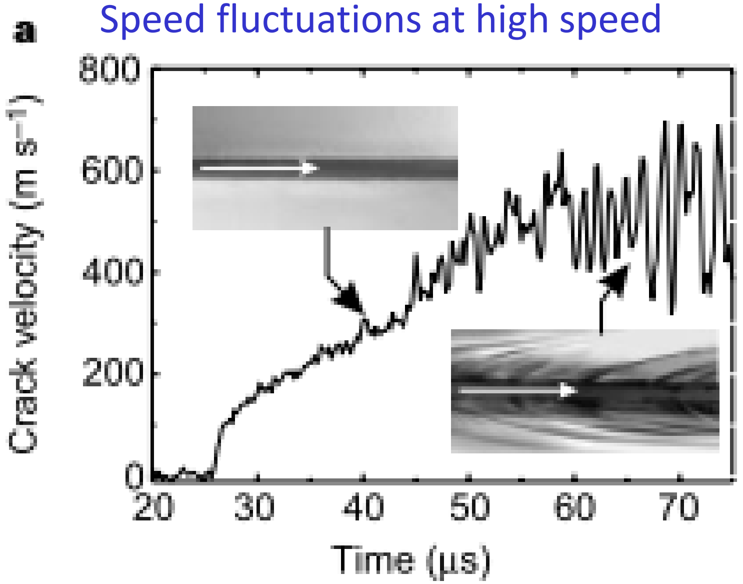


$$\mu \frac{\partial f}{\partial t} = \bar{G} - \bar{\Gamma} + \bar{G} H(\{f\}(t)) + \gamma(z, f)$$

Till now elastic line models restricted to slow cracks, with ***interaction kernels*** derived from elastostaticity

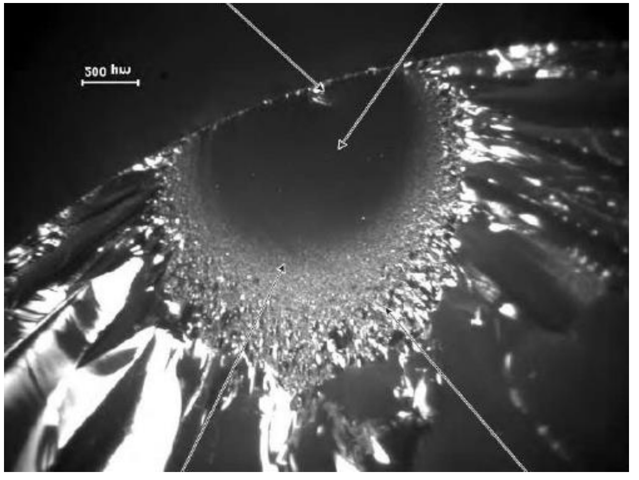
Still many instability/high fluctuations arise at high speed

Speed fluctuations at high speed



Sharon et al. Nature (1999)

Transition to mist/hackle fracture surface at high speed



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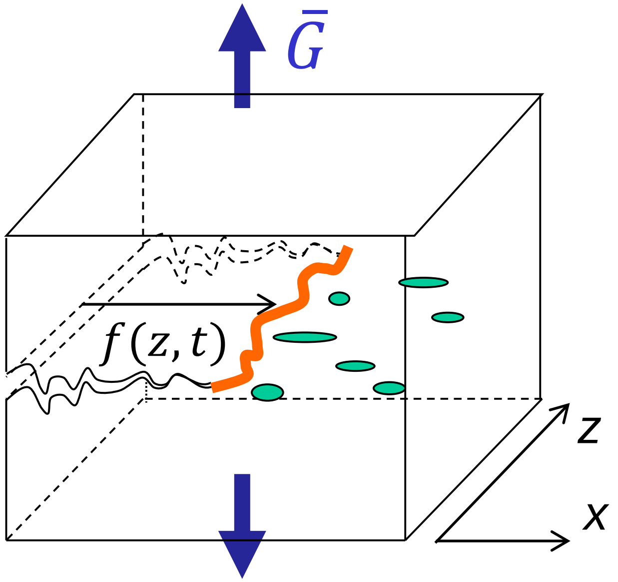
Toward elastodynamic line models of fracture
A priori possible…



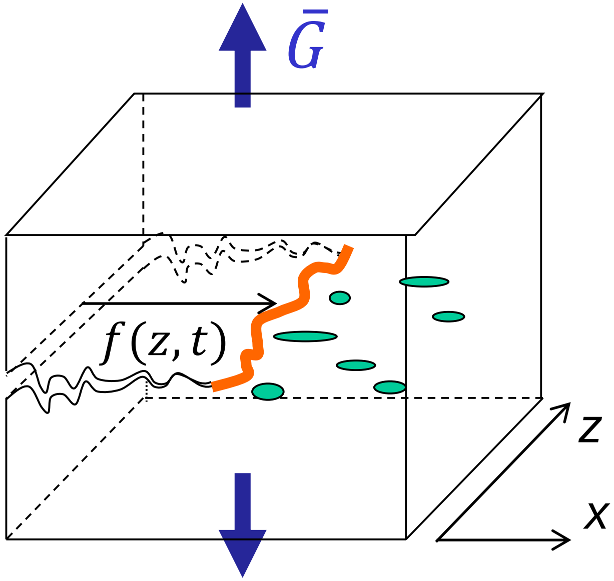
$$\mu \frac{\partial f}{\partial t} = \bar{G} - \bar{\Gamma} + \bar{G} H(\{f\}(t)) + \gamma(z,f)$$

Availibility of elastodynamics kernels

Movchan & Willis JMPS 1995, 1997, Ramanathan Fisher PRL 1997

$$H_{dyn}(\{f\}) = \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} dz' \, g(z-z', t-t') \frac{\partial f}{\partial t}$$

Toward elastodynamic line models of fracture
A priori possible…



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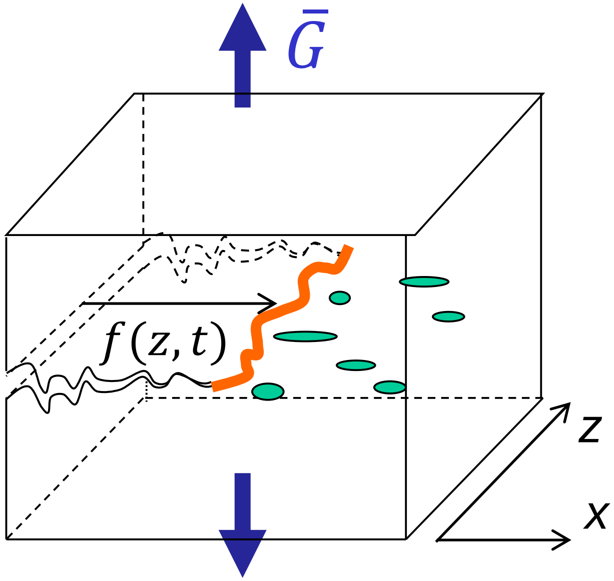
$$H_{dyn}(\{f\}) = \int_{-\infty}^{t} dt' \int_{-\infty}^{\infty} dz' \, g(z-z', t-t') \frac{\partial f}{\partial t}$$

$$\mathscr{G}(z,t) = \left(\frac{2\, c_r\, t}{\pi\, z^2} \frac{H(\sqrt{c_r^2 - v_0^2}\, t - |z|)}{(\sqrt{c_r^2 - v_0^2}\, t^2 - z^2)^{1/2}} - \frac{c_d\, t}{\pi\, z^2} \frac{H(\sqrt{c_d^2 - v_0^2}\, t - |z|)}{(\sqrt{c_d^2 - v_0^2}\, t^2 - z^2)^{1/2}} \right) - \sqrt{\frac{2}{\pi}} \int_{c_s}^{c_d} \theta(\eta, cd, cs) \frac{\eta^2 + v_0^2}{\eta^2 - v_0^2}\, t \, \frac{H(\sqrt{\eta^2 - v_0^2}\, t - |z|)}{(\sqrt{\eta^2 - v_0^2}\, t^2 - z^2)^{3/2}} \mathrm{d}\eta$$

© A. Dubois

Toward elastodynamic line models of fracture

A priori possible...



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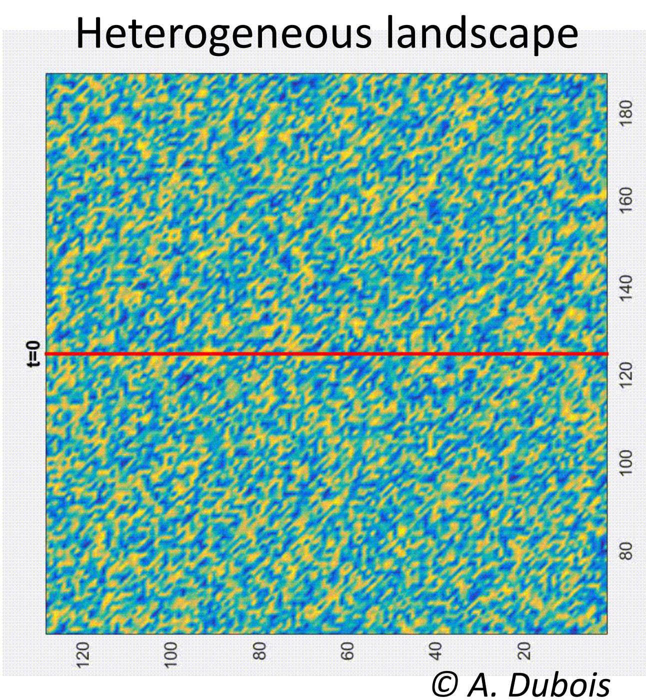
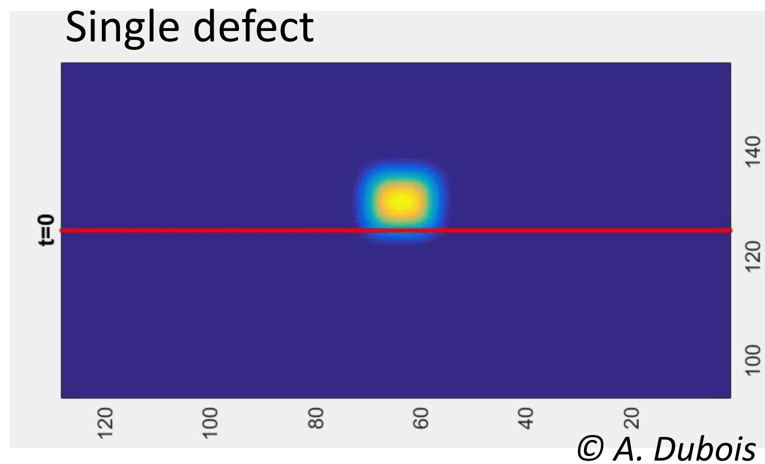




... but difficult:

Memory ➔ lack of stationary regime

Problem of wave reflections in finite width...

In a nutshell:

- ***Interface growth model*** and paradigm of ***depinning transition*** provide a promising framework to nominally brittle fracture of heterogeneous materials: able to explain ***crackling dynamics*** and (out-of-plane) ***roughness***

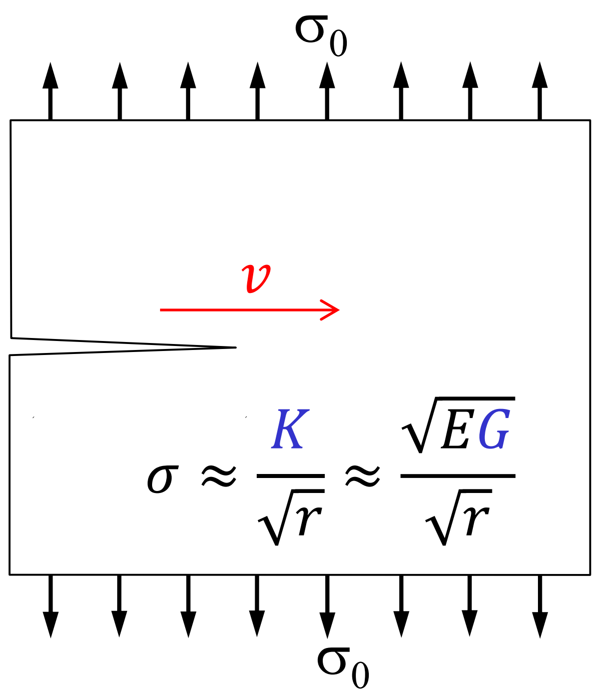
- ***Quantitative agreement*** between theoretical/numerical predictions in situations where ***microstructure scale*** $\ll$ ***observation zone*** $\ll$ ***structure scale*** [not so clear otherwise]

A wish: An interaction kernel for finite width?

- Some questions arises when out-of-plane roughness is considered: Meaning of disorder ? Importance of neglected terms?

- ***A wish:*** extension to dynamic crack growth using ***elastodynamic interaction kernels.*** A priori available, but for infinite/periodic width & the *question of finite width* all the more important there

Continuum fracture mechanics (LEFM)
A complete description



Where ??

Goldstein and Salganik (1974) Cotterell & Rice (1980)

Path selected so that shear stress vanishes

When ??

$$G > \Gamma$$

At which speed ??

Eshelby (1969), Kostrov (1966) and Freund (1972)

$$A(v)G = \Gamma$$

« relativistic » factor:

$$A(v) \sim (1 - v/C_R)$$

C_R Rayleigh wave speed

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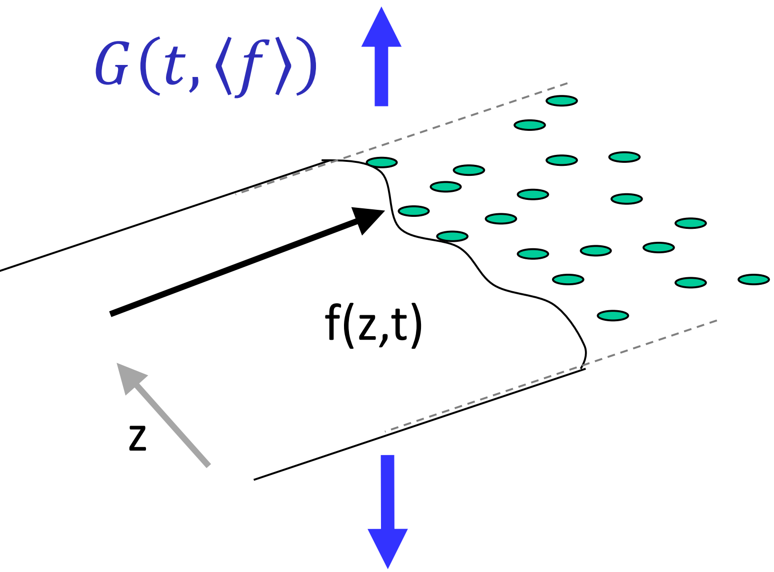
Crack dynamics in presence of disorder



Equation of motion

Ingredients

Elastostatic framework

2D geometry

Non-correlated spatial disorder

Local equation of motion

$$\frac{\partial f}{\partial t}(M) = G(M) - \Gamma(M)$$

Schmittbuhl et al., PRL (1995), Ramanathan et al. PRL, (1997), Bonamy et al PRL, 2008

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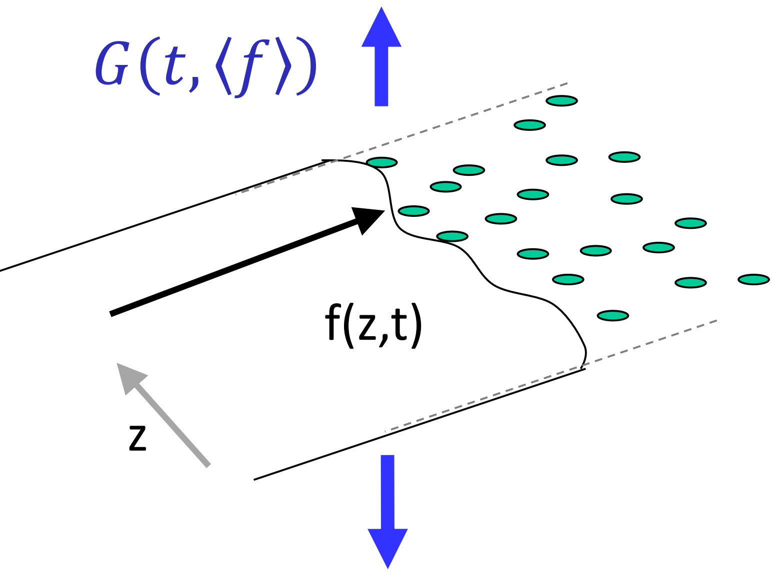
Crack dynamics in presence of disorder



Equation of motion

Ingredients

Elastostatic framework

2D geometry

Non-correlated spatial disorder

Local equation of motion

$$\frac{\partial f}{\partial t}(M)=G(M)-\Gamma(M)$$

Global compatibility

Local disorder

Rice (85),

$$G(M)=G\times\left[1+\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(z',t)-f(z,t)}{(z'-z)^2}dz'\right]+o(f)$$

$$\Gamma(M)=\overline{\Gamma}+\gamma(M)$$

Schmittbuhl et al., PRL (1995), Ramanathan et al. PRL, (1997), Bonamy et al PRL, 2008

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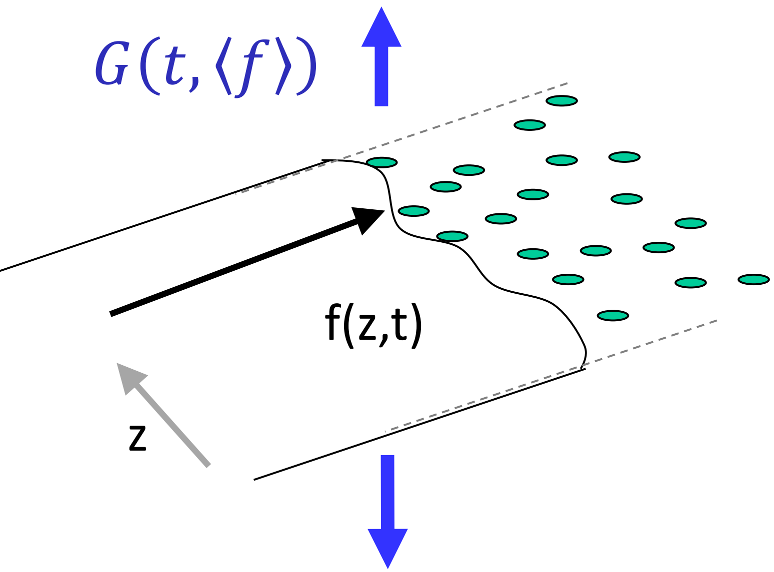
Crack dynamics in presence of disorder



Equation of motion

Ingredients

Elastostatic framework

2D geometry

Non-correlated spatial disorder

Local equation of motion

$$\frac{\partial f}{\partial t}(M) = G(M) - \Gamma(M)$$

Global compatibility

Local disorder

Rice (85),

$$\Gamma(M) = \bar{\Gamma} + \gamma(M)$$

$$G(M) = G \times \left[1 + \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2} dz' \right] + o(f)$$

Stable crack
Imposed displacement rate

$$G \approx \bar{\Gamma} + \dot{G}t - G'\langle f \rangle$$

Schmittbuhl et al., PRL (1995), Ramanathan et al. PRL, (1997), Bonamy et al PRL, 2008

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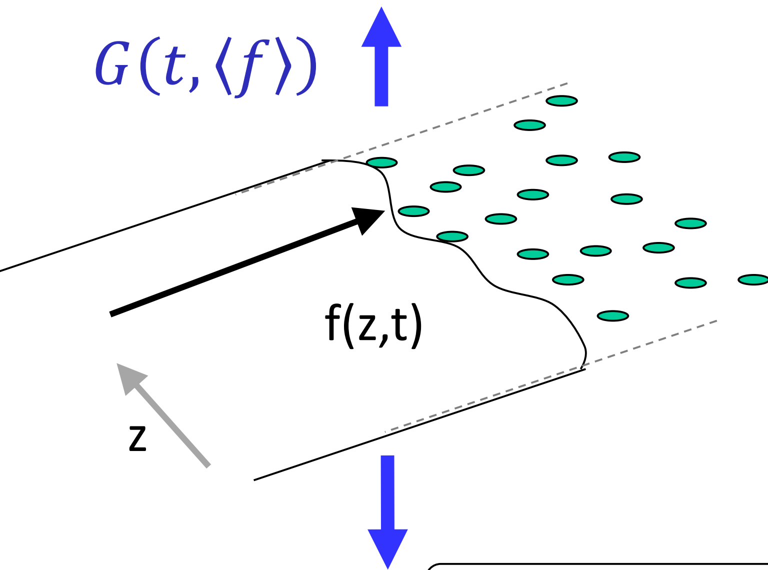
Crack dynamics in presence of disorder



Equation of motion

Ingredients

Elastostatic framework

2D geometry

Non-correlated spatial disorder

Coupling between local SIF and overall front geometry

Elastic relaxation

$$\frac{\partial f}{\partial t} = \dot{G}t - G'\langle f \rangle + \frac{\bar{\Gamma}}{2\pi}\int_{-\infty}^{\infty} \frac{f(z',t) - f(z,t)}{(z'-z)^2}\,dz' + \gamma(z,f)$$

Forcing speed

Quenched disorder due to microstructure

Schmittbuhl et al., PRL (1995), Ramanathan et al. PRL, (1997), Bonamy et al PRL, 2008

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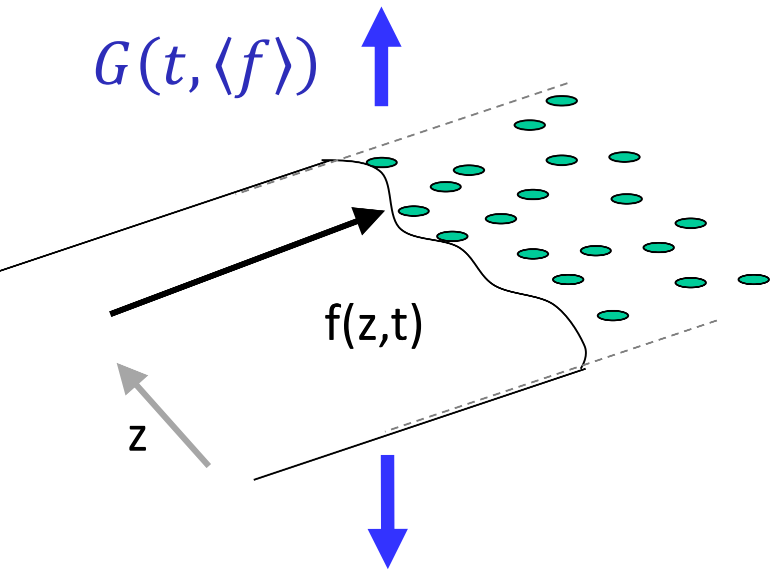
Crack dynamics in presence of disorder



Equation of motion

Ingredients

Elastostatic framework

2D geometry

Non-correlated spatial disorder

Elastic manifold with long range interaction propagating within a random potential

Schmittbuhl et al., PRL (1995), Ramanathan et al. PRL, (1997), Bonamy et al PRL, 2008

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