La mue du crabe (élasticité, défauts et plis dans les systèmes cellulaires bidimensionnels)

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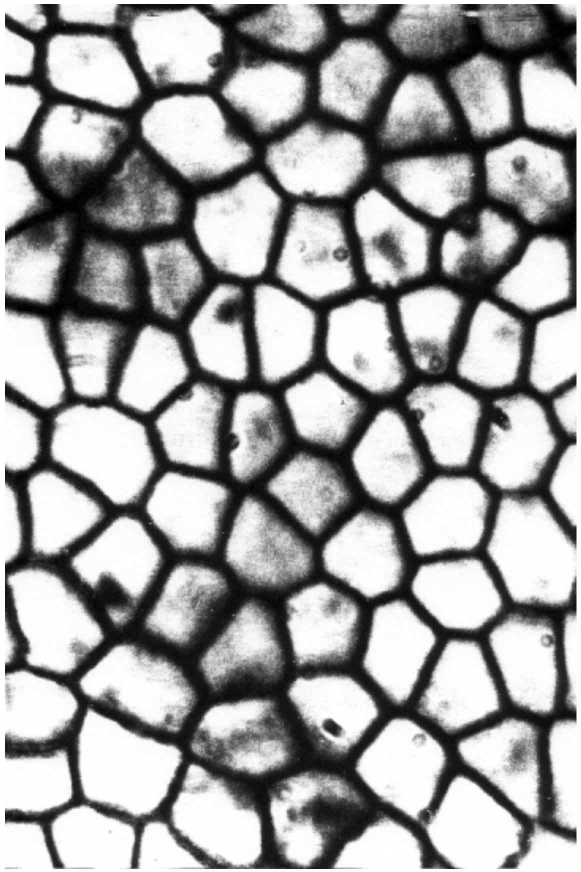
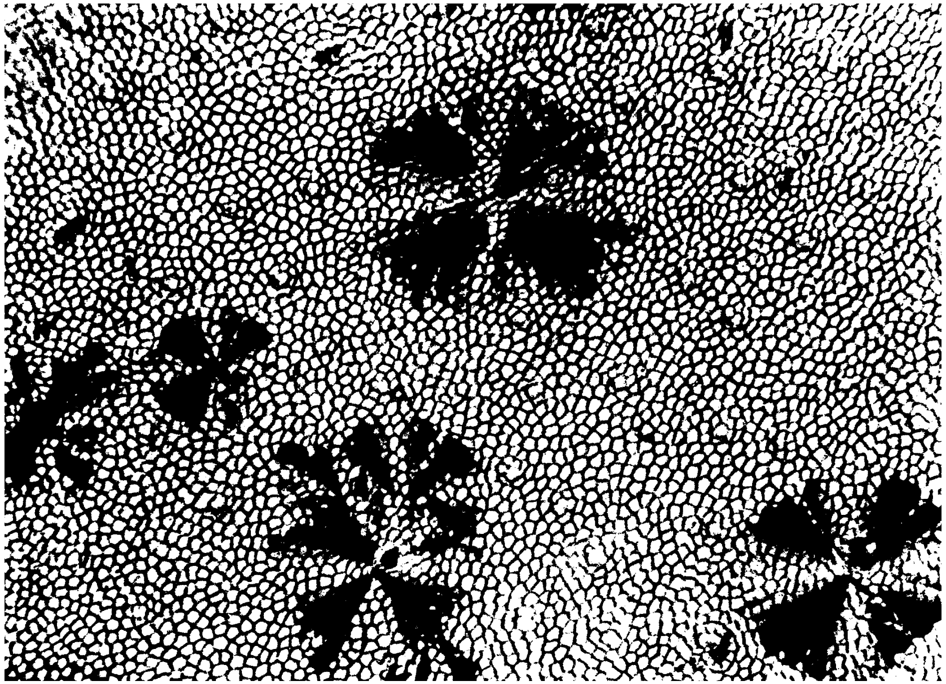
Natural history of the crab (thanks Y. Bouligand)

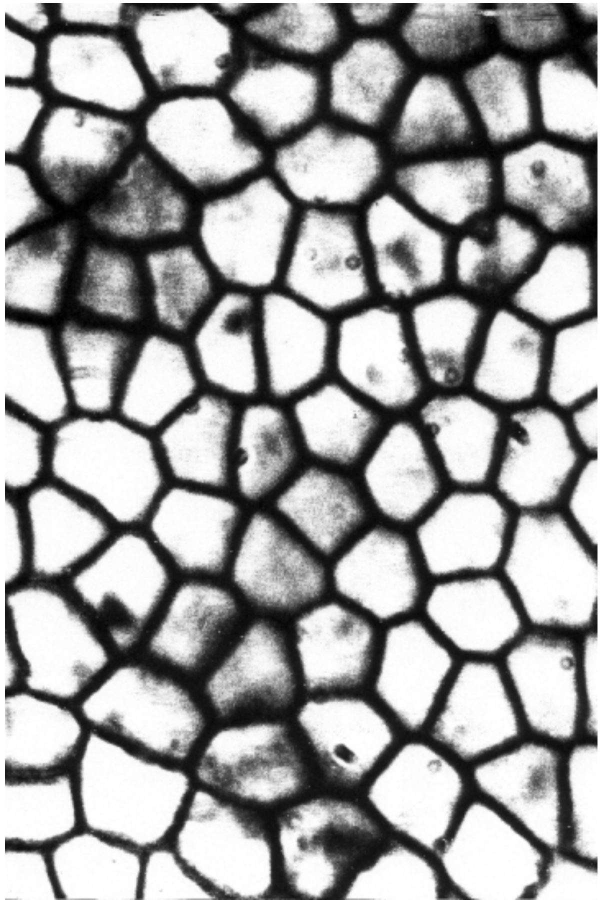
- Before moulting, the crab builds up a new cuticle inside its old one, conformal copy but larger and all wrinkled (folds). Mechanism ?
- hides, moults, drinks, swells, and waits for its new cuticle to mineralize into a shell
- biological tissues, $\lambda >> \mu$, soft under shear (M. Fink)
- cf. foams (cellular material, incompressible but soft under shear) or weaved tissue



- New cuticle is a 2D foam
- cells = polygons, $\langle n \rangle = 6$ (–12 pentagons for closing cuticle)
- Where are the (scars of the) folds?

The crab's new cuticle shortly after moulting resembles any two-dimensional foam. Right : detail.
(Picture by M.M. Giraud-Guille).



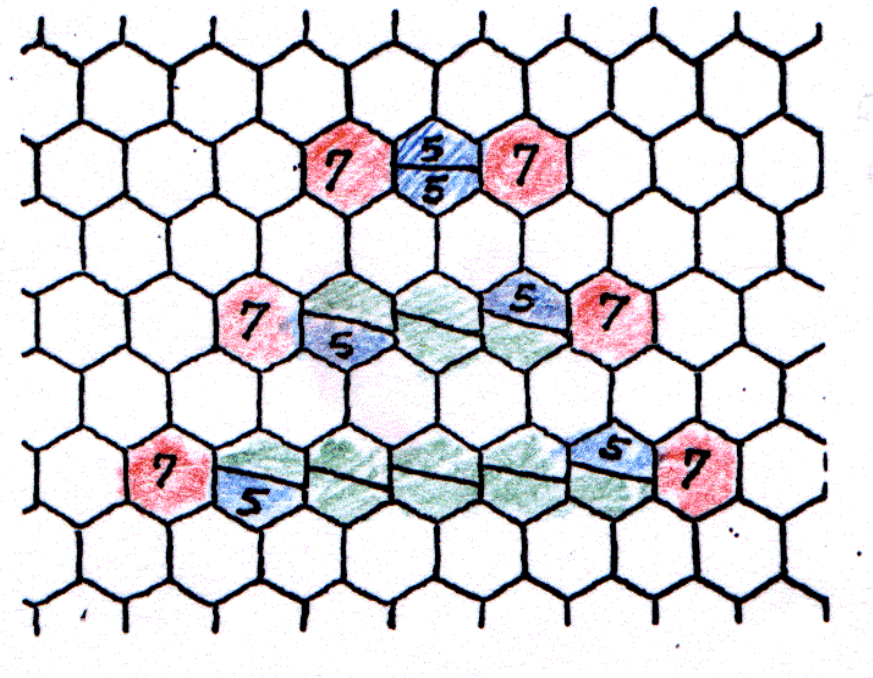


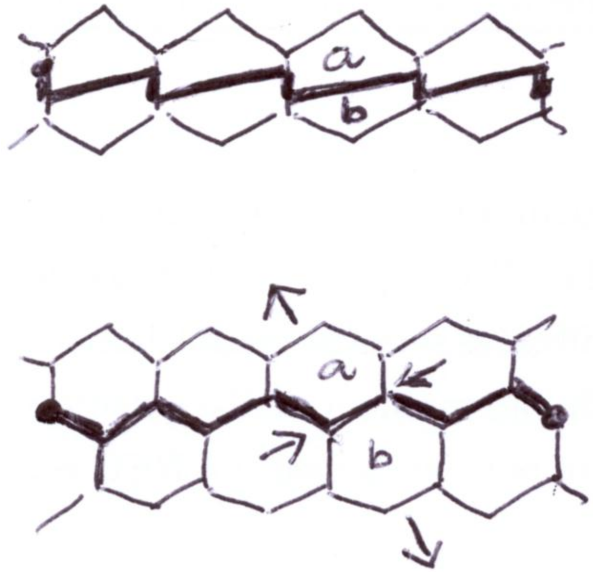
Extra matter in incompressible foam (additional layer of cells) produced by successive mitoses, i.e. by climb of one dislocation (5/7) from the other. If the first division is symmetrical (6→5/5), the tissue remains topologically flat (fold).

Cell division: **nucleation** of two dislocations 5/7

Dislocation climbs: extra matter with some (Poisson) shear

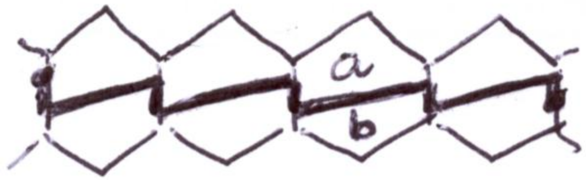
Fold (in Λ or V)

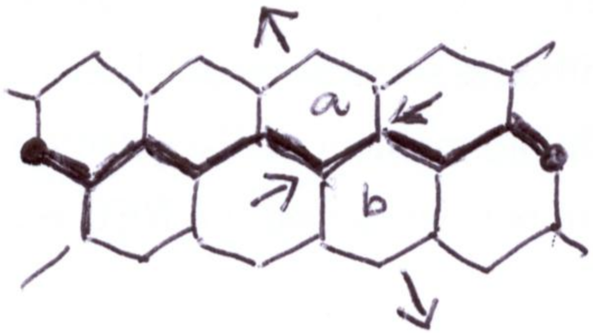


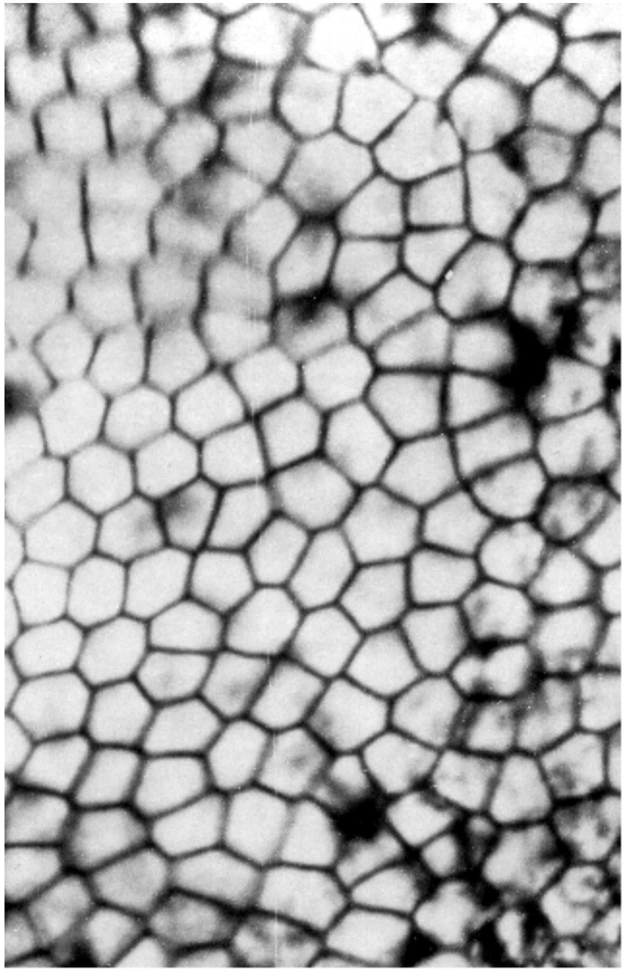


Biological tissues

- soft tissue, macroscopically continuous and isotropic
- Lamé λ et μ, with $\mu << \lambda$
- shear
- cell cannot grow (λ too large), **must** divide
- division = shear (Poisson)

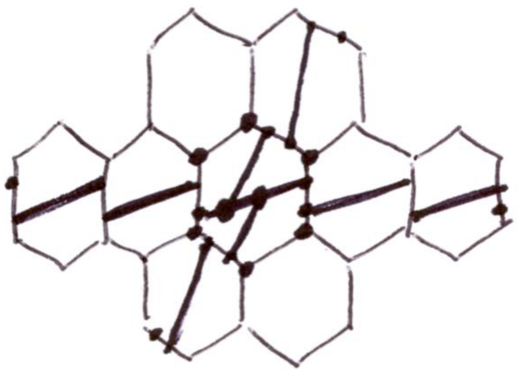




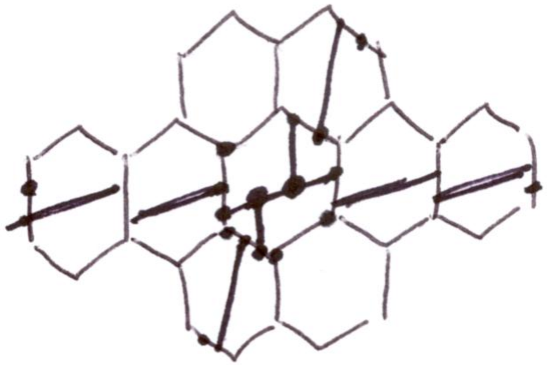


One can observe a few folds =
consecutive cellular divisions.

Crossing of two folds: nonsingular



6/6\6/6

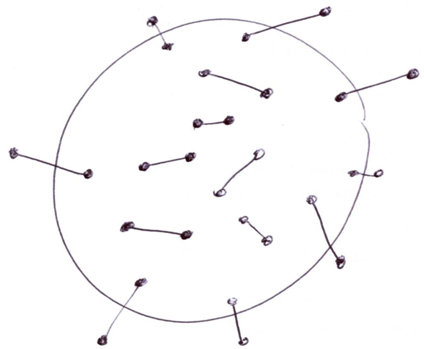


5\7/7\5 = two dislocations « tete-bèche
reduces to 6/6\6/6 by T1 flip

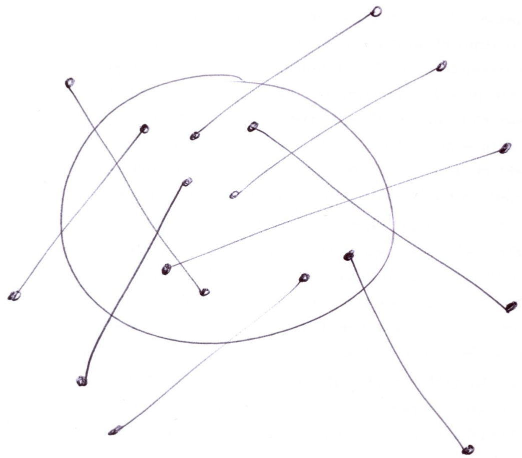
given that fold is either V (valley) or Λ (syncline), nonsingular crossing is ΛΛΛV or VVVΛ, i.e monkey-saddle

Crossing of folds: monkey saddle Λ (syncline), V (valley),V,V

Folds: short or long? $\mathbf{b} = \sum \mathbf{b_i}$, $|\mathbf{b}_i| = 1$



- **Short folds** $N \sim L$ cut by Burgers contour
- $|\mathbf{b}| \sim \sqrt{N} \sim \sqrt{L}$
- **lower energy cost for same added material**



- **Long folds** $N \sim L^2$ cut by Burgers contour
- $|\mathbf{b}| \sim \sqrt{N} \sim L$

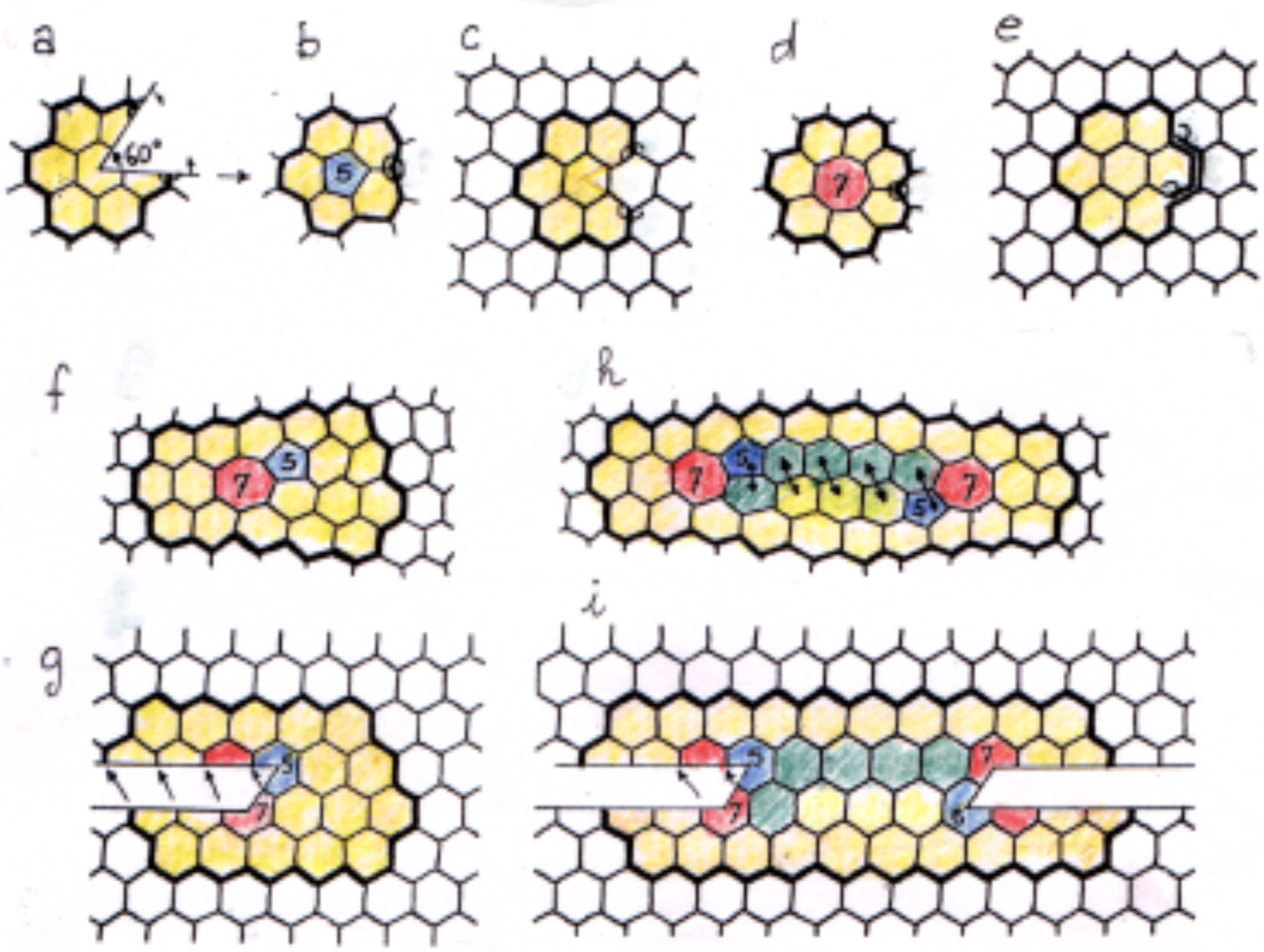
Conclusions

- Topological defects cause large deformations
- endogeneous, isotropic and uniform growth

- distribution of folds (short or long) to be verified

Topological defects and Burgers contour (thick line) in a **cellular network**.
Disclination (5) b→ c, or (7) d→e (curvature)
Dislocation (5/7) f→ g (torsion) Burgers vector (single arrow).
Extra matter h→ i, through successive cell divisions, i.e through dislocation

clim



b) Topological defects (in color: 5/7, 5, 7) in a)

Folds dashed. Thick lines: Burgers contours

c) Representation in the reference state

a b c