

# Residual Creases

mechanics of partly (un)folded structures

**Dr Mark Schenk**

Department of Aerospace Engineering, University of Bristol, United Kingdom

[www.markschenk.com/research/](http://www.markschenk.com/research/)

*Workshop: Folding and Creasing of Thin Plate Structures*  
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# Introduction

Mark Schenk

Lecturer in Aerospace Engineering  
University of Bristol, UK

Work done at Surrey Space Centre  
and University of Cambridge.



## Talk outline

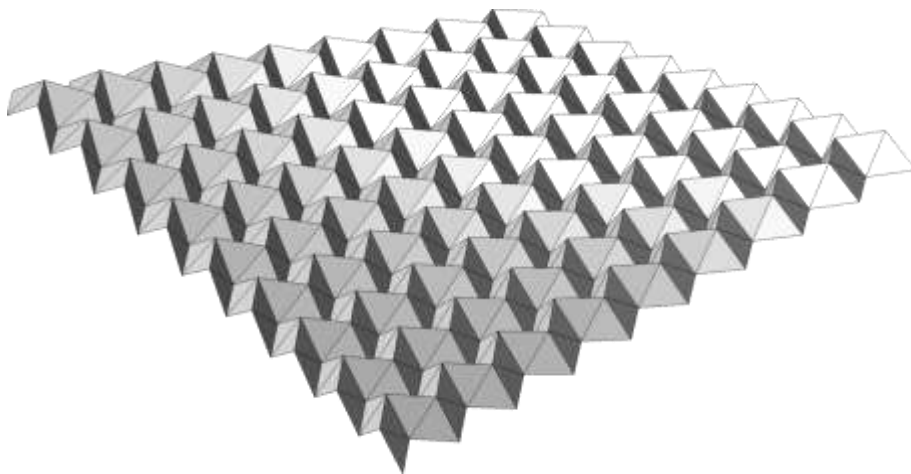
- brief history of mechanics of partly (un)folded structure
- case study of deployable structure with residual creases
- challenges and future directions

# A brief history...

## **partly-folded** or **partly-unfolded** structures

- effect of folds and creases on global structural properties
- dominated by folding kinematics, but requires structural mechanics

examples: morphing structures (compliant shell mechanisms),  
deployable structures with residual creases, *etc.*

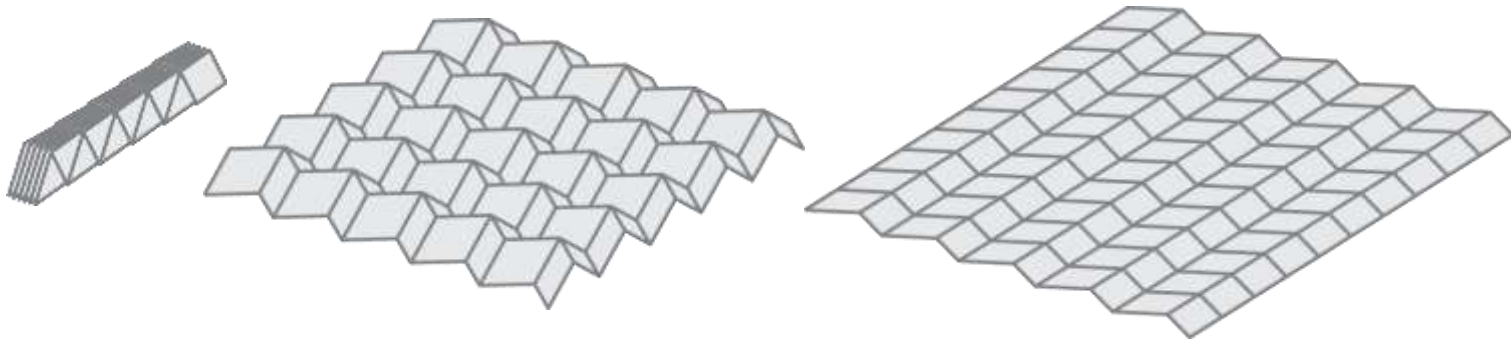


# A brief history...

whistle-stop tour of mechanics of folded structures

## kinematics

- rigid origami (rigid panels and frictionless hinges)
- quadrilateral meshes (Kokotsakis, 1933; Tachi, 2009; Stachel, 2010)
- curved creases (Huffman, 1976; Duncan and Duncan, 1982)



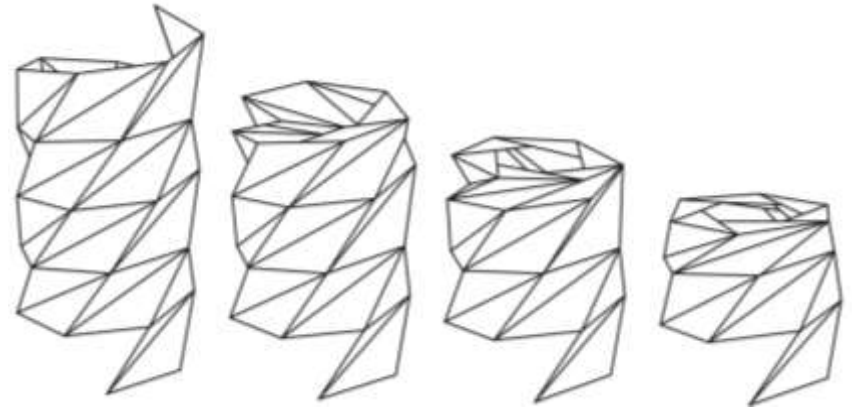
# A brief history...

## mechanics : kinematics + torsion springs at folds

- *linear* torsion springs

### structural analysis

(Resch and Christiansen, 1971;  
Guest and Pellegrino, 1996;  
Okuizumi *et al.*, 2011)



equivalent stiffness of partly-folded sheets (Wei, *et al.* 2013)

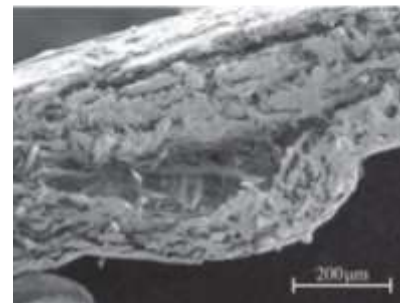
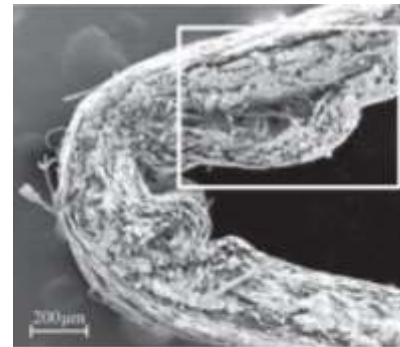
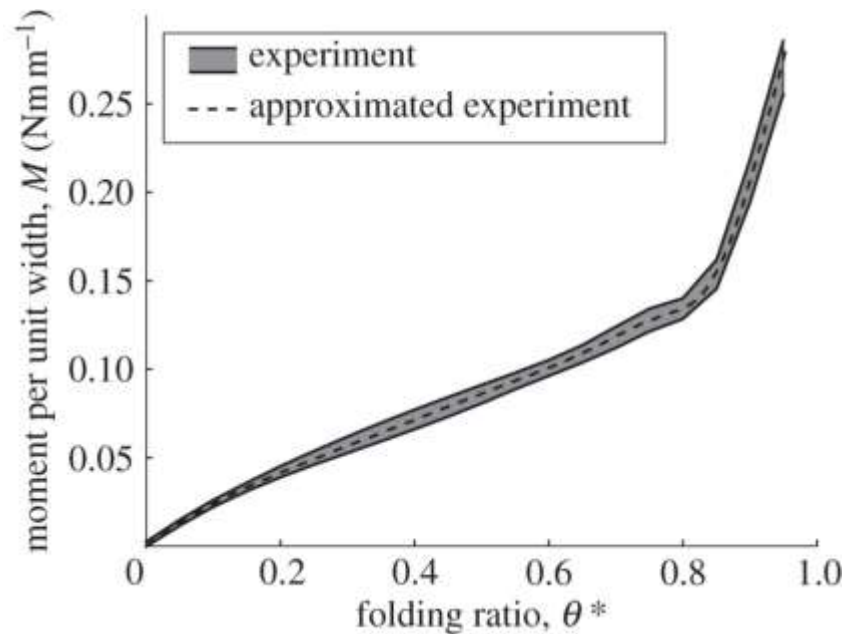
### analysis of multi-stability

(Hanna, *et al.* 2014; Silverberg, *et al.* 2014; Waitukaitis, *et al.* 2015)

# A brief history...

## mechanics : kinematics + torsion springs at folds

- non-linear torsion springs (Yasuda, *et al.* 2013)

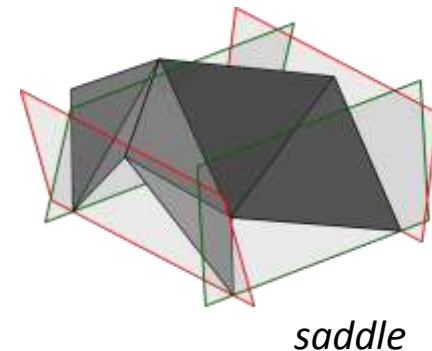
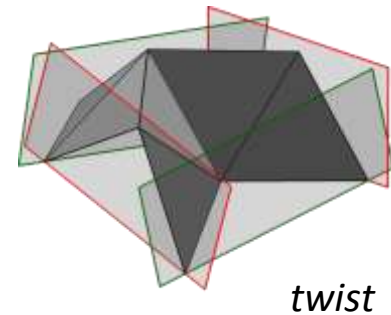
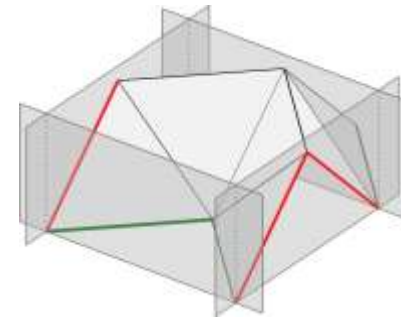
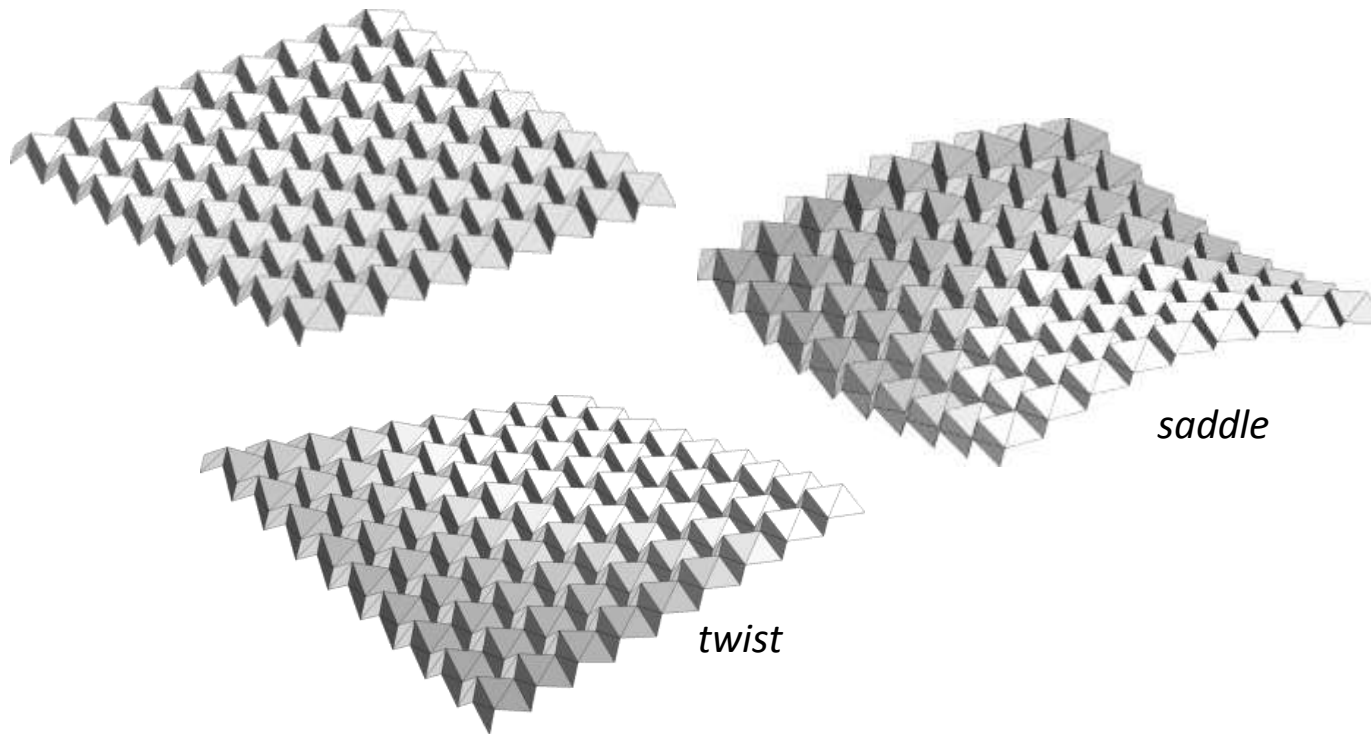




# A brief history...

## mechanics: deformation between the creases

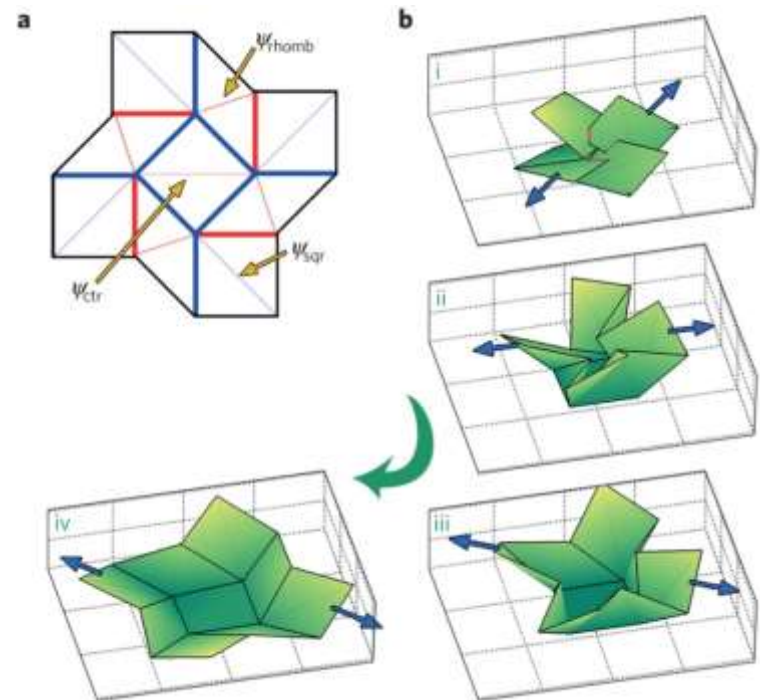
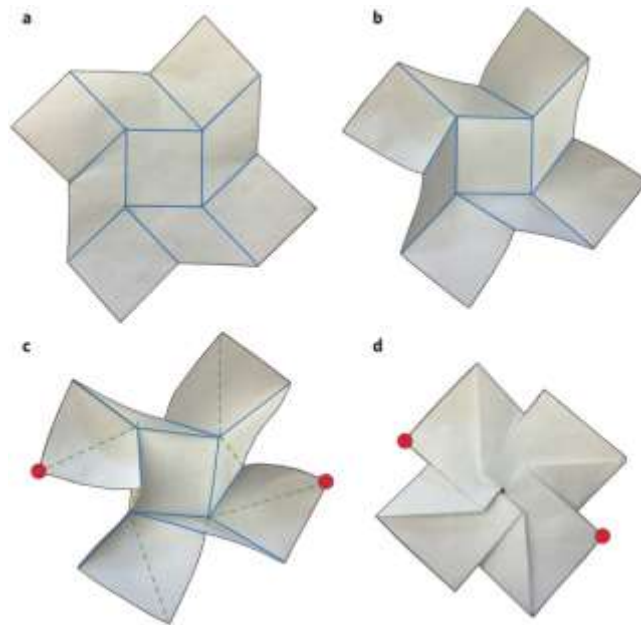
- bending along diagonals of quadrilateral elements (Schenk and Guest, 2010-2013; Wei *et al.*, 2013)
- introduces additional degrees of freedom



# A brief history...

## mechanics: deformation between the creases

recently published work: bending of facets provides additional degrees of freedom and multi-stability (Silverberg *et al.*, 2015)

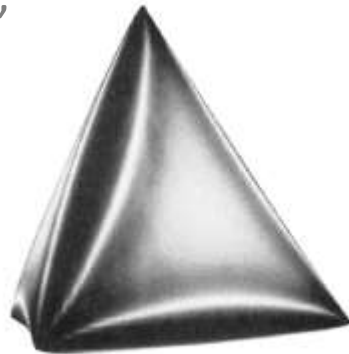




# A brief history...

## mechanics: deformation between the creases

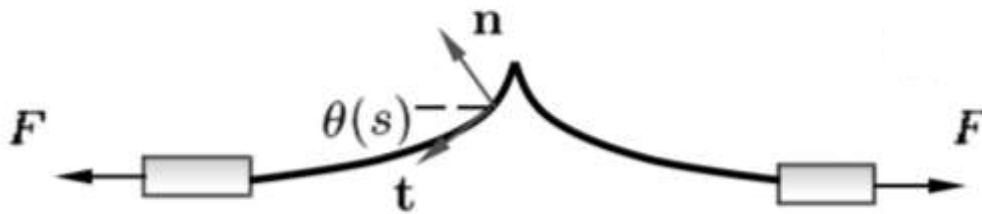
- discrete deformations
  - triangulating quadrilateral panels & linear springs  
(Schenk and Guest, 2010-2013; Silverberg, *et al.* 2014-2015)
  - folding of constrained facets  
(Lobkovsky *et al.*, 1995-1996;  
Filipov *et al.*, 2014)



# A brief history...

## mechanics: elastic deformation between the creases

- continuous deformations : linear hinges with elastic facets
  - engineering beams (Papa and Pellegrino, 2008; Furuya *et al.*, 2005)
  - elastica (MacNeal and Robbins, 1976; Lechenault *et al.* 2014)

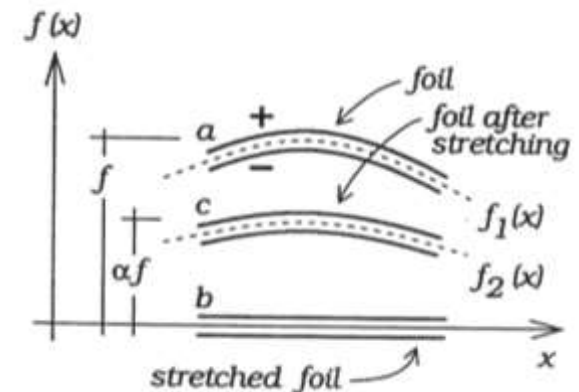
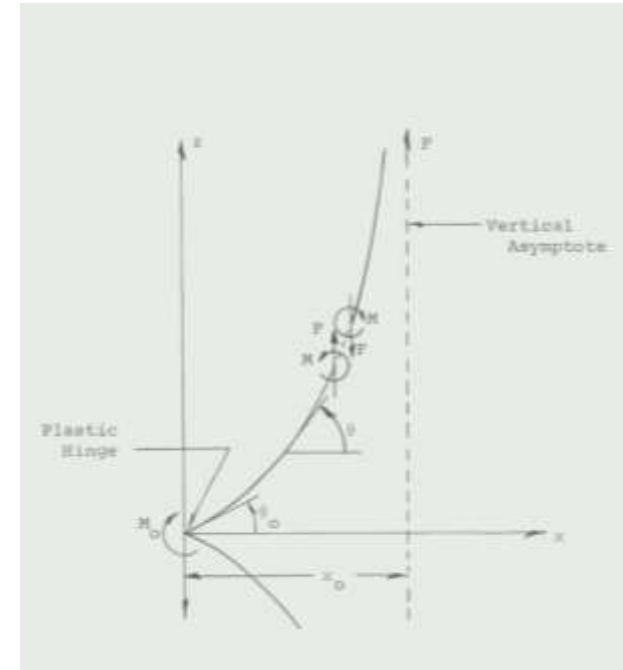


- necessity for curved creases (Dias *et al.*, 2012-2014)

# A brief history...

## mechanics: elasto-plastic modelling

- plastic hinges : manufacturing  
(Duncan and Duncan, 1981; Schenk *et al.*, 2011)
- **plastic** hinges and **elastic** facets  
(MacNeal and Robbins, 1967; Hedgepeth, 1981)
- plastic deformation of a smoothly deformed foil (Greschik, 1996)



# A look towards the future...

A brief look towards the future:

- **elasto-plastic** modelling of the creases and interlying material
- **detailed modelling** of fold lines : moving away from mechanical trends, to accurate (surrogate-)models
- interactions between creases: **vertices!**



**Case Study:**

Deployable Space Structures



## InflateSail: technology demonstration mission

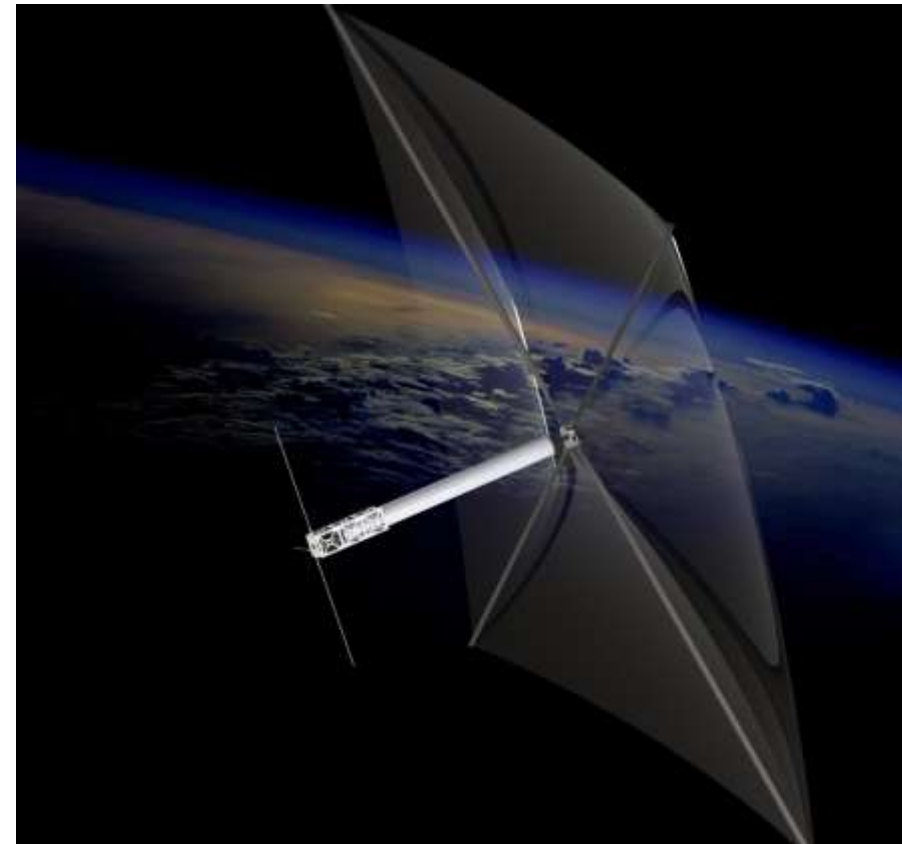
3U CubeSat (100 x 100 x 340 mm)

- deploy 10 m<sup>2</sup> gossamer sail
- deploy 1 m long inflatable mast

Objective: demonstrate feasibility of  
drag augmentation to de-orbit  
satellites from LEO

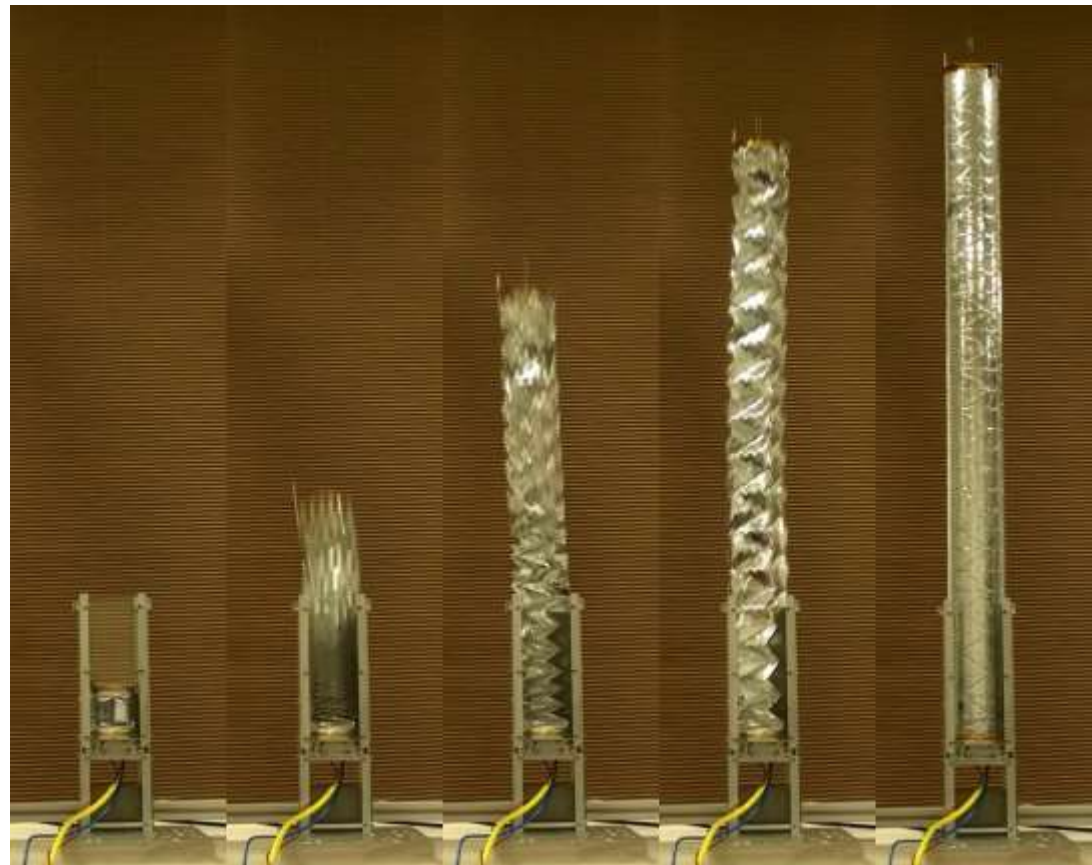
EU FP7 funding / QB50 launch

PI: Prof. Vaios Lappas



# Inflatable Origami Mast

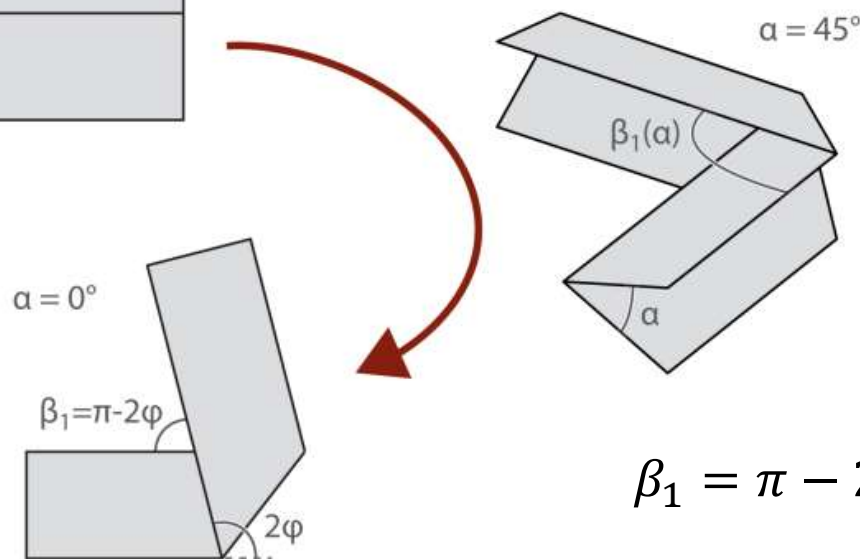
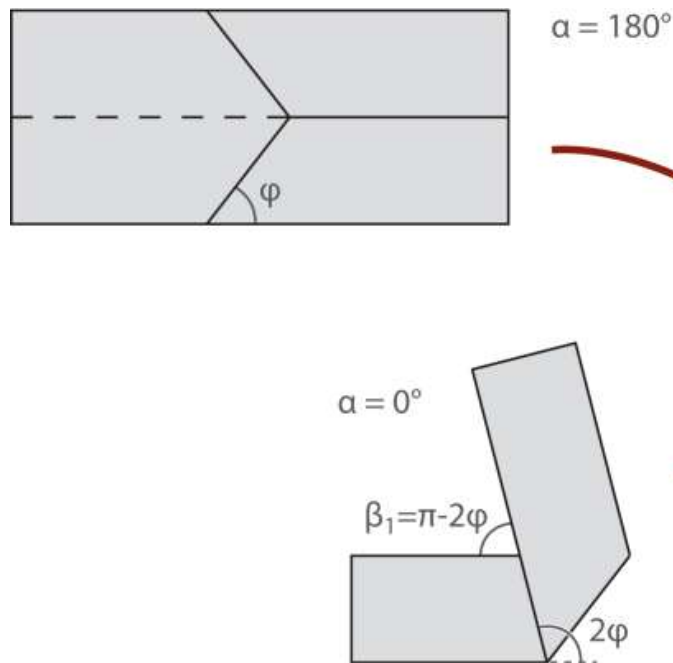
## Deployable Origami Mast ([video](#))



# Inflatable Origami Mast

## Kinematics : Rigid Origami

building block: reverse fold



$$\alpha \in [0, \pi]$$

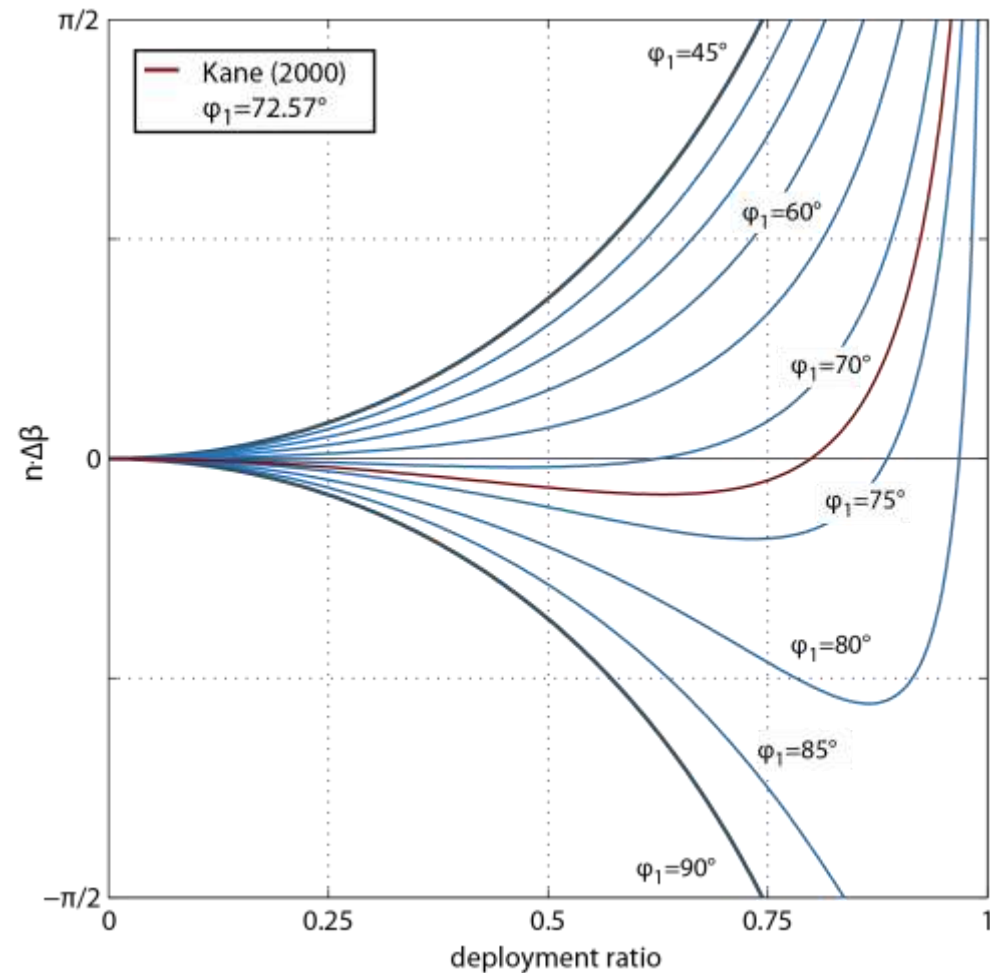
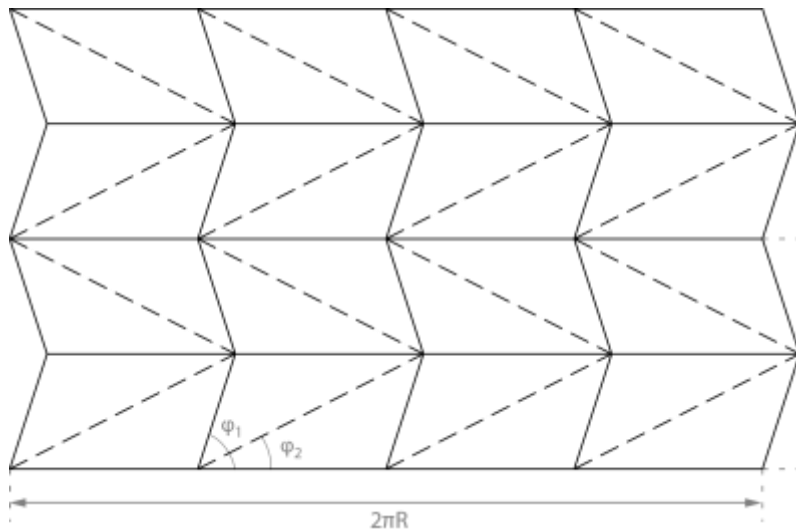
$$\beta \in [\pi - 2\varphi, \pi]$$

$$\beta_1 = \pi - 2 \cdot \text{atan}\left(\cos \frac{\alpha}{2} \cdot \tan \varphi\right)$$

the enclosed angle  $\beta_1$  is a strictly increasing function of  $\alpha$

# Inflatable Origami Mast

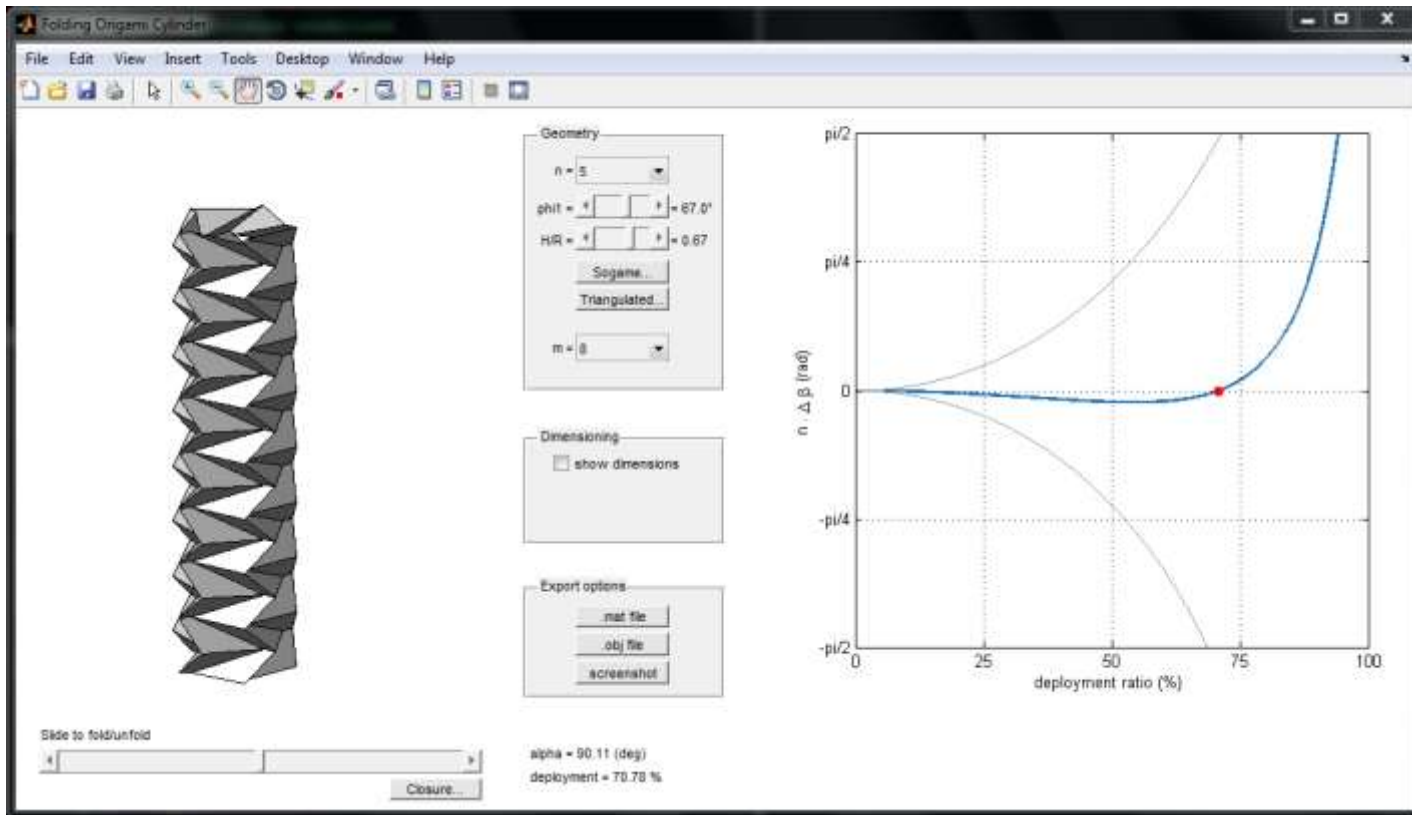
## Kinematics : Compatibility



Deployment ratio:  $\frac{L}{L_{max}} = \sin \frac{\alpha}{2}$

# Inflatable Origami Mast

## Parametric Design Tool



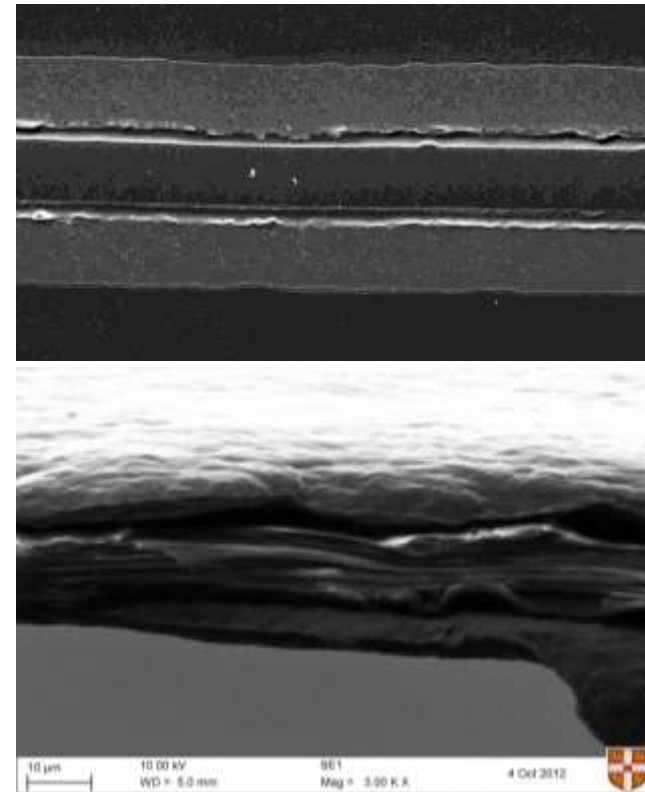
<http://www.markschenk.com/research/#software>



# Inflatable Origami Mast

## Inflatable Boom : Membrane Material

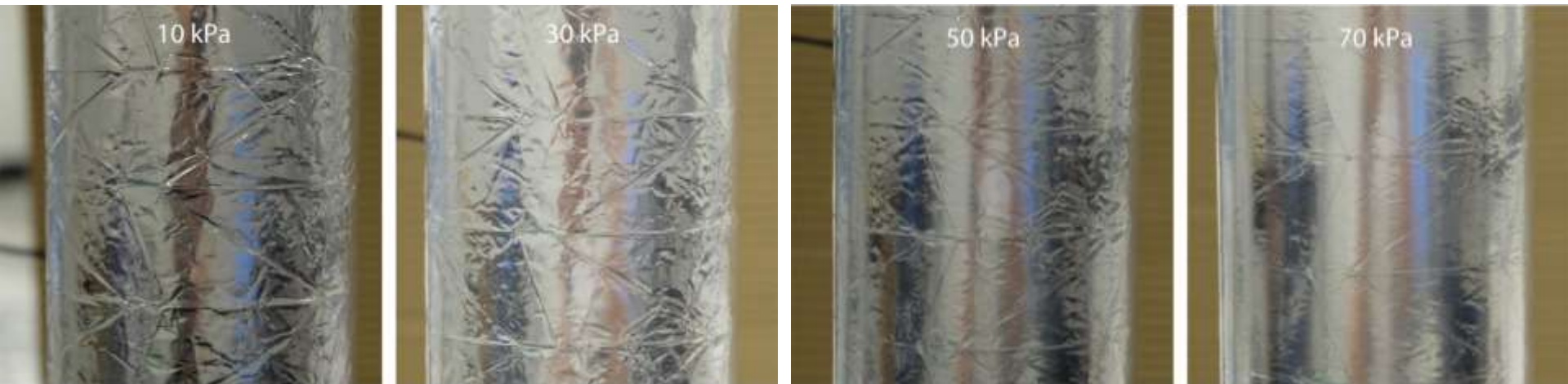
- laminate of Aluminium-Mylar-Aluminium (14.5/16/14.5  $\approx$  45 $\mu$ m)
- the Aluminium layers provides stiffness, and polymer layer adds toughness
- limited thickness of membrane due to plastic deformation (folding and rigidisation)



# Inflatable Origami Mast

## Inflatable Boom : Strain-rigidisation

- ensure long-term structural performance after deployment
- strain-rigidisation : permanently remove folding creases by plastic deformation of aluminium-polymer laminate

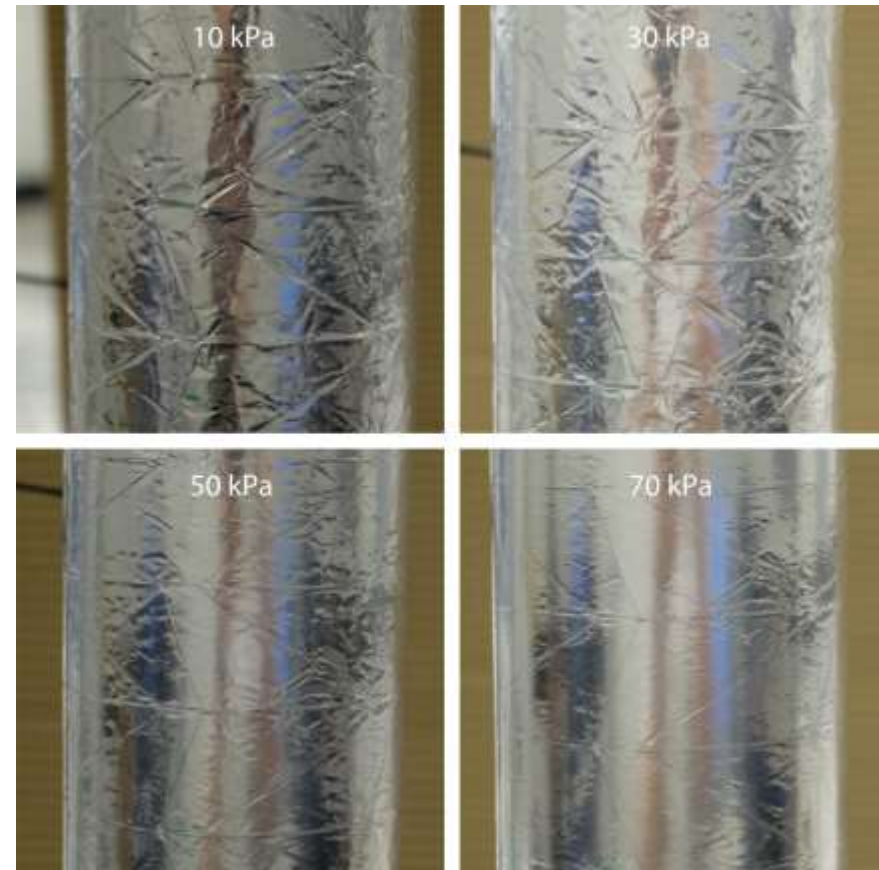


# Inflatable Origami Mast

## Inflatable Boom : Strain-rigidisation

- investigate efficacy : measure boom against inflation pressures
- first natural frequency as indication of stiffness:  $EI \propto \omega_n^2$
- theoretical rigidisation pressure

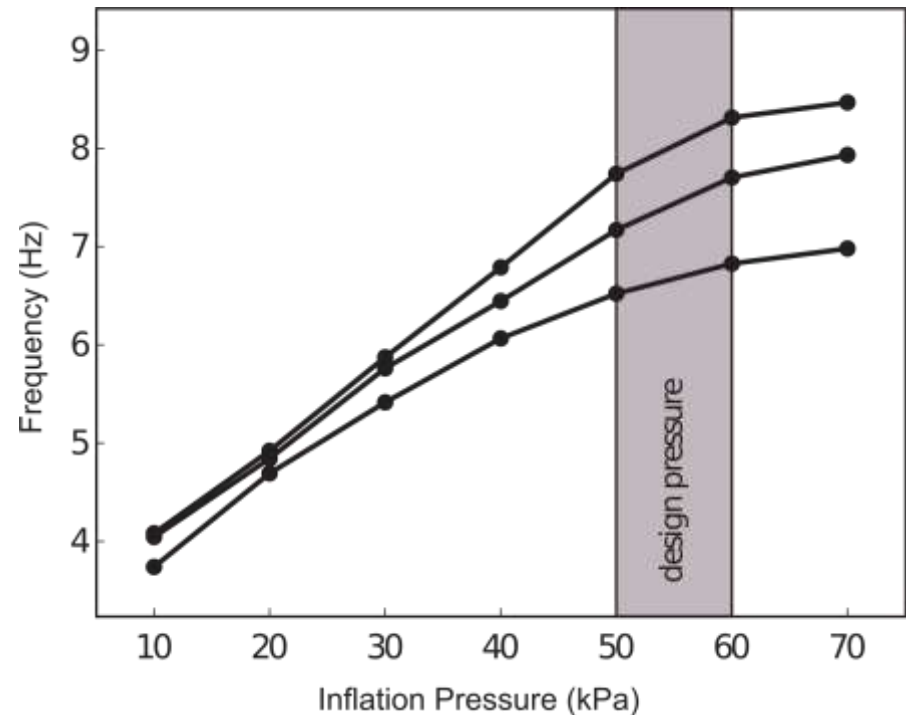
$$P = \sqrt{\frac{4}{3}} \frac{\sigma_{yt}}{R} \approx 50 - 60 \text{ kPa}$$



# Inflatable Origami Mast

## Inflatable Boom : Strain-rigidisation

- recovery up to approximately 50% of theoretical stiffness
- Experimental! No predictions of stiffness (or strength) due to residual creases



- numerical prediction, *e.g.* feed imperfections into FEM?

# Inflatable Origami Mast

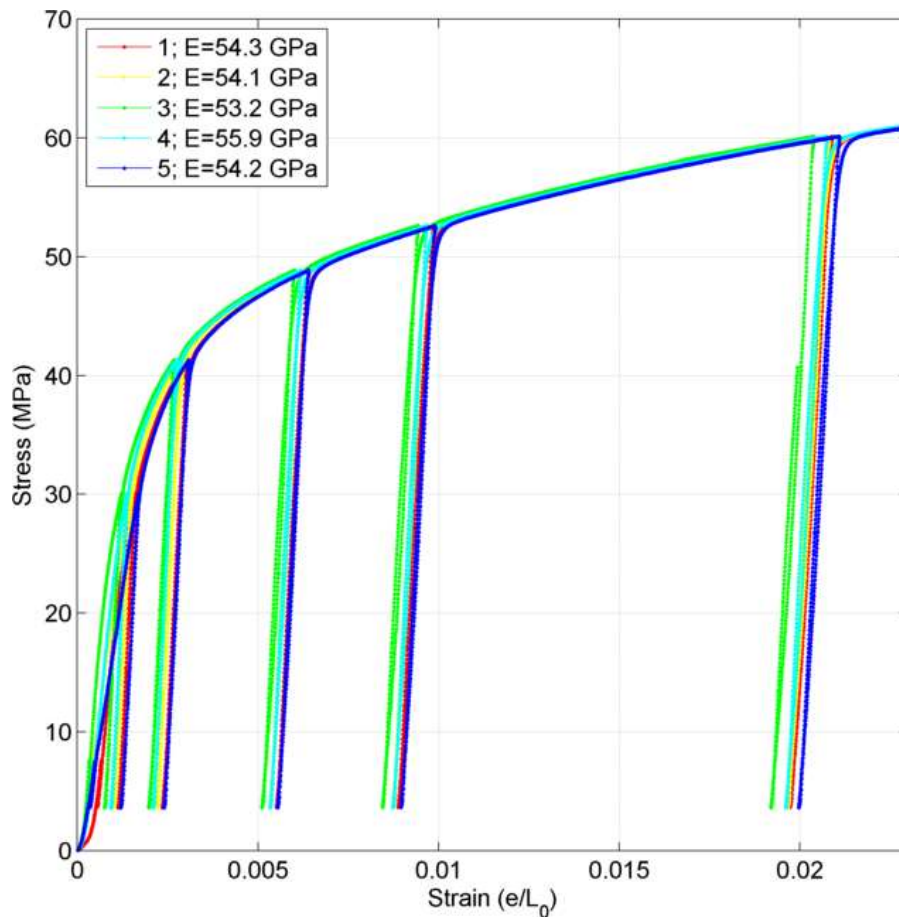
## Inflatable Boom : Material Characterisation

- accurately measuring the material properties ( $E$  and  $\sigma_Y$ ) was challenging!
- ASTM E-345 : *“Standard Test Methods of Tension Testing of Metallic Foil”*
- define a yield point by 0.2% proof stress  $\sigma_Y$ ; however, tensile tests gave inconsistent results and a very low  $E$  for Aluminium foils: 15-25 GPa

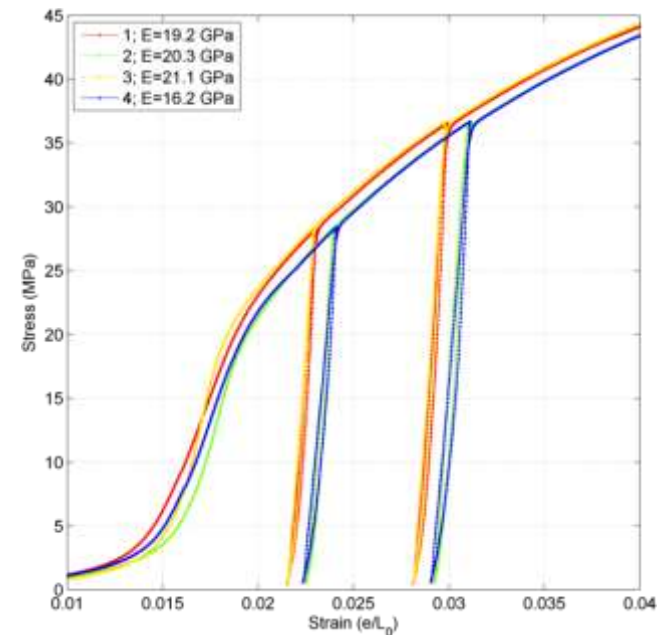


# Inflatable Origami Mast

cause: (negligibly) non-flat material!



	Young's Modulus	Yield Strength
elastic unloading	$55.4 \pm 5.63$ GPa	$49.3 \pm 6.50$ MPa
Initial linear region	$24.8 \pm 6.13$ GPa	N/A

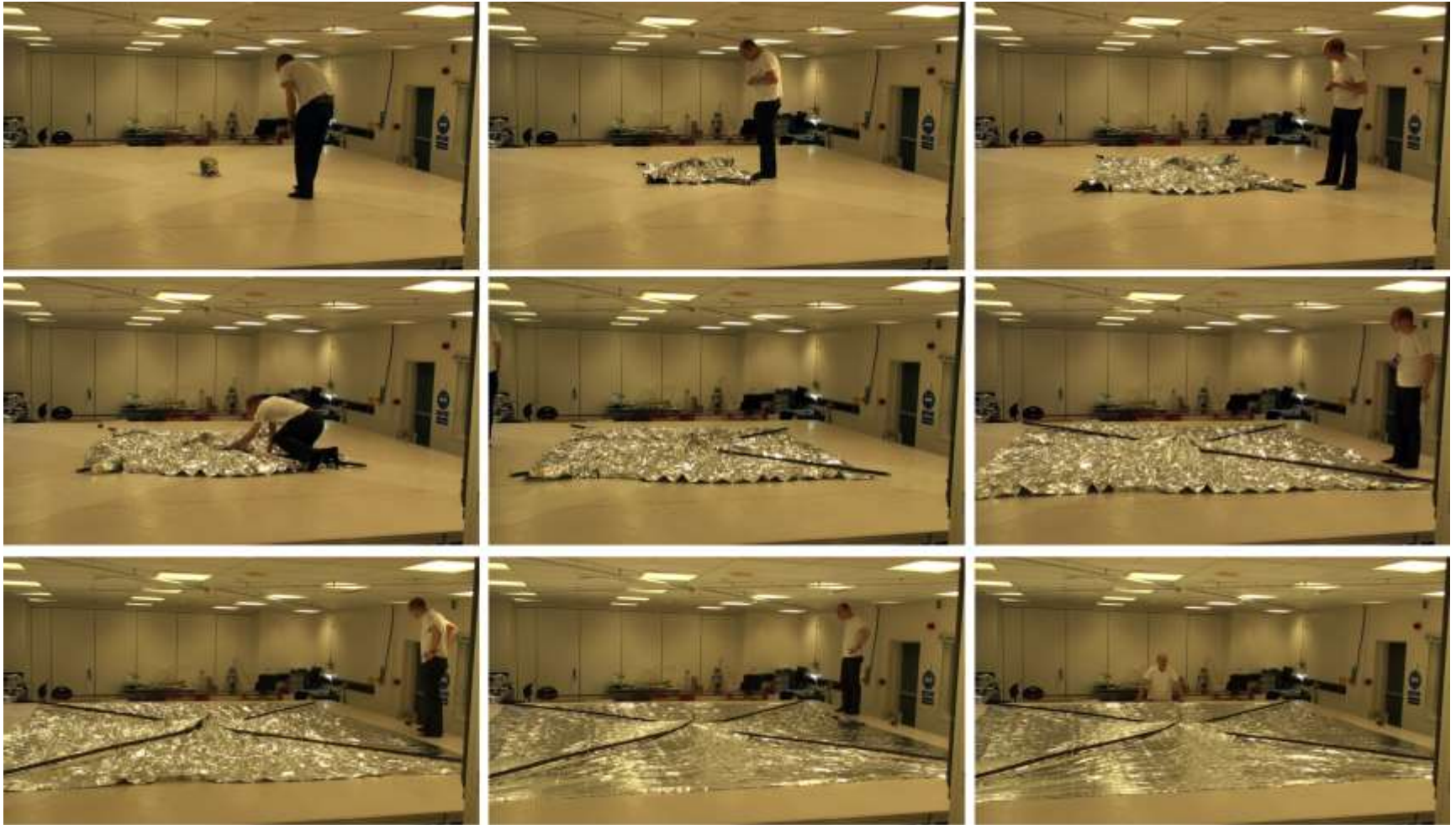


Charly Knight (2014)

# Inflatable Origami Mast

- Lessons learned:
  - measuring material properties of thin membranes is tricky!
  - easy to find trends that give good approximations (power laws, *etc.*) but hard to get accurate numbers.
  - account for non-flatness of the base material (low effective  $E$ )
  - common measurement techniques for the hinge stiffness

# Gossamer Sails



Fernandez, Schenk, Prassinou, Lappas and Erb (2013), "*Deployment Mechanisms of a Gossamer Satellite Deorbiter*" 15th European Space Mechanisms and Tribology Symposium 2013 (ESMATS 2013), 25–27<sup>th</sup> September 2013, Noordwijk, The Netherlands.

# Gossamer Sails

Residual creases in membranes:

- flatness vs reflectivity
- low in-plane modulus
- uncertain deployed dimensions
- increased bending stiffness (IKAROS mission)





# Conclusions

## Conclusions & Future Work

- elasto-plastic deformations (folds *and* facets)
- interactions between creases (vertices)
- accurately measuring the hinge stiffness (not order of magnitude) : develop common measurement techniques
- incorporating low-level analysis into high-level models



# Questions

Thank you!

Mark Schenk ([M.Schenk@bristol.ac.uk](mailto:M.Schenk@bristol.ac.uk))

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