Three Lectures on Interfacial Fracture Mechanics: ESPCI, October 2012

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Lecture #1: Overview. Crack tip fields, Basic solutions, Interface toughness and thin film delamination as an illustration

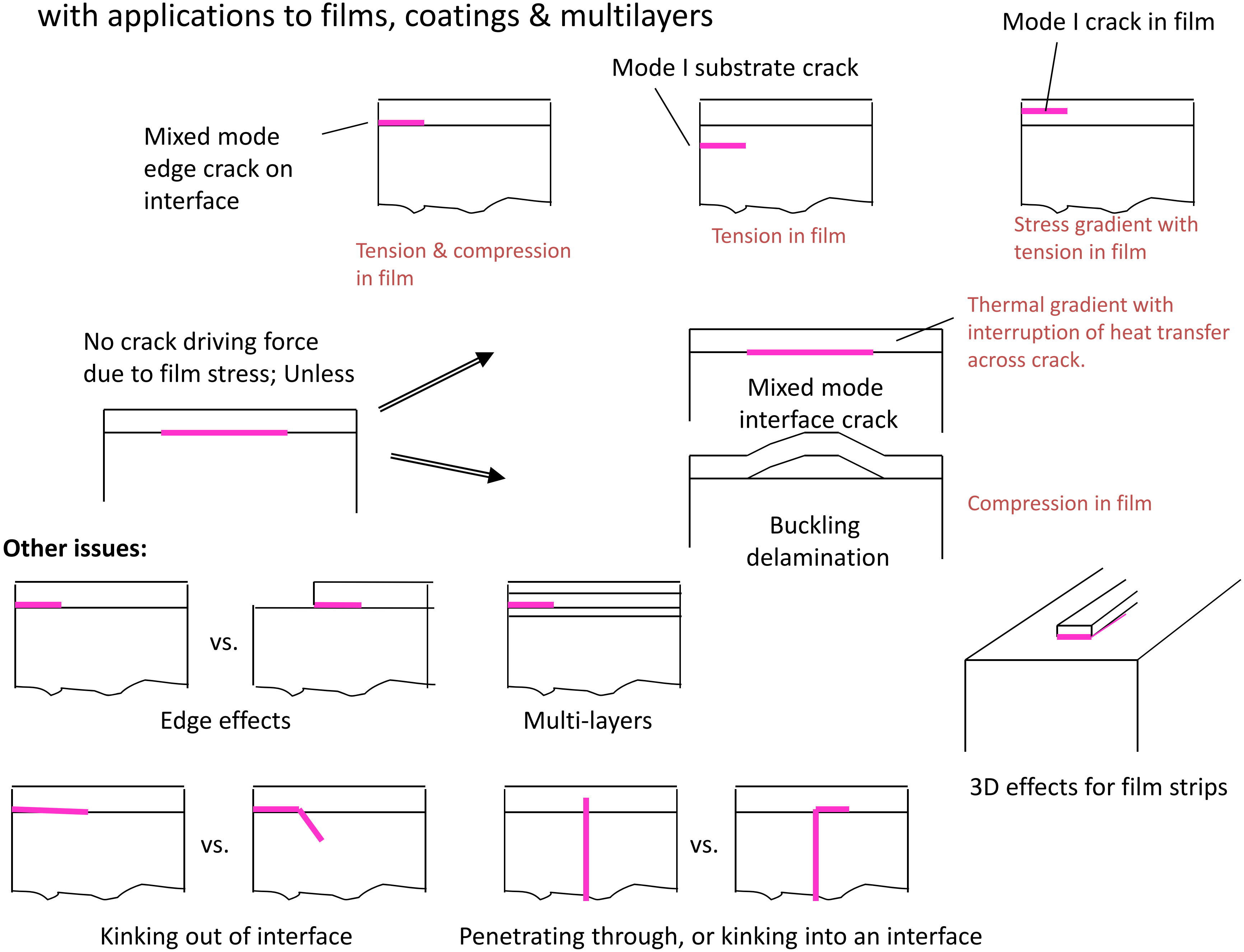
Lecture #2: Special elastic mismatch effects, Origins of mixed mode toughness dependence, Buckling delamination, Thermal barrier coating delamination & interface toughness

Lecture #3: Initiation of delamination vs. steady-state delamination in thin films & 3D effects, Kinking of a crack out of an interface. Cracks approaching an interface: penetration vs. kinking. Various applications.

References will included on the slides.
My own papers are available on my website http://www.seas.harvard.edu/hutchinson
These paper will be referenced in the format (year-paper#), e.g., (2012-3)

VARIOUS KINDS OF DELAMINATION CRACKING WE WILL CONSIDER

with applications to films, coatings & multilayers



Crack Tip Fields for Bilayer Interface Joining Isotropic Elastic Solids (1992-2; pg.72)

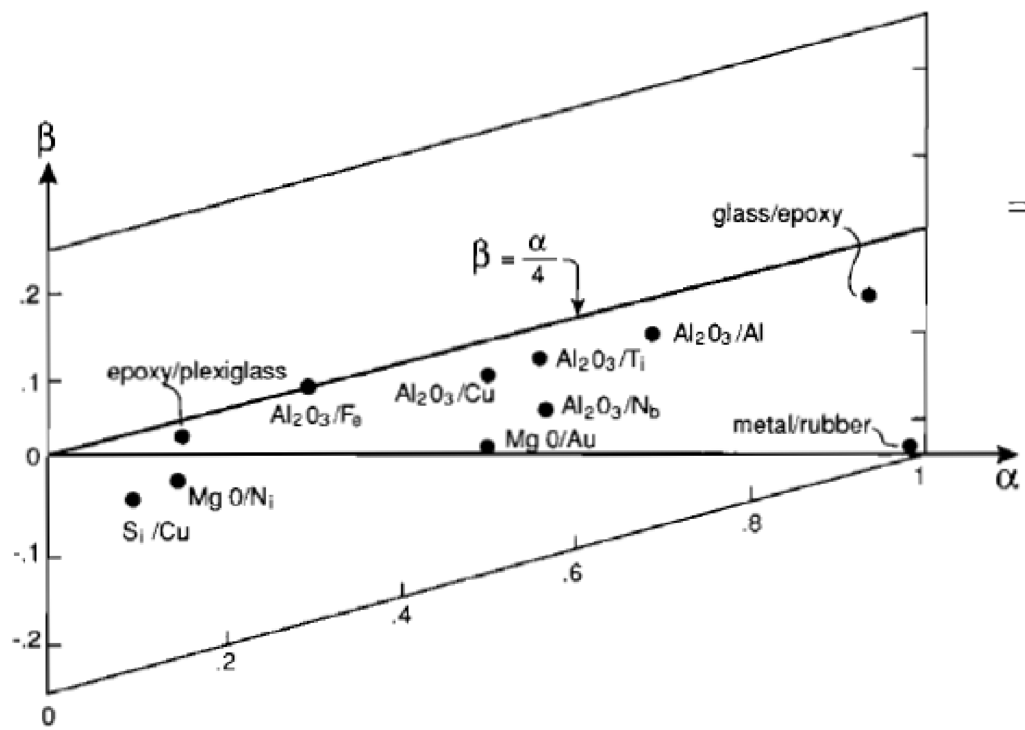
Dundurs parameters (2D elasticity for bilayers)

With $\mu = \dfrac{E}{2(1+\nu)}$,

plane strain: $\alpha_D = \dfrac{\bar{E}_1 - \bar{E}_2}{\bar{E}_1 + \bar{E}_2}$, $\beta_D = \dfrac{1}{2}\dfrac{\mu_1(1-2\nu_2) - \mu_2(1-2\nu_1)}{\mu_1(1-\nu_2) + \mu_2(1-\nu_1)}$, $\bar{E} = \dfrac{E}{(1-\nu^2)}$

plane stress: $\alpha_D = \dfrac{\mu_1(1+\nu_1) - \mu_2(1+\nu_2)}{\mu_1(1+\nu_1) + \mu_2(1+\nu_2)}$, $\beta_D = \dfrac{1}{2}\dfrac{\mu_1(1+\nu_1)(1-\nu_2) - \mu_2(1+\nu_2)(1-\nu_1)}{\mu_1(1+\nu_1) + \mu_2(1+\nu_2)}$,

$$\alpha \equiv \alpha_D$$
$$\beta \equiv \beta_D$$



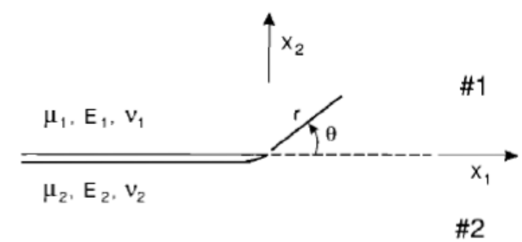
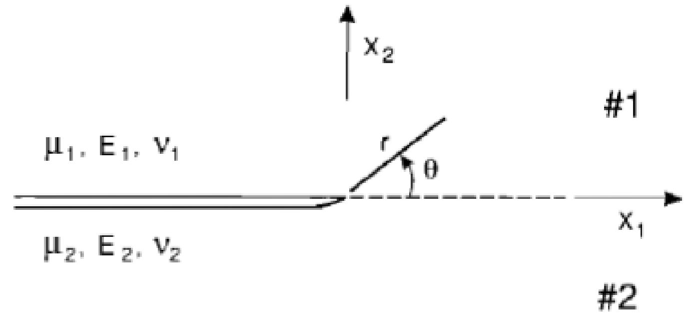


FIG. 6. Values of Dundurs' parameters in plane strain for selected combinations of materials.

Crack Tip Fields for Bilayer Interface Joining Isotropic Elastic Solids (1992-2; pg.72)

$$\sigma_{\alpha\beta} = \mathrm{Re}[Kr^{i\varepsilon}](2\pi r)^{-1/2}\sigma^{\mathrm{I}}_{\alpha\beta}(\theta,\varepsilon) + \mathrm{Im}[Kr^{i\varepsilon}](2\pi r)^{-1/2}\sigma^{\mathrm{II}}_{\alpha\beta}(\theta,\varepsilon)$$

$$K = K_1 + iK_2 \qquad \varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right)$$



On interface ahead of crack:

$$\sigma_{22} = \mathrm{Re}[Kr^{i\varepsilon}](2\pi r)^{-1/2}, \qquad \sigma_{12} = \mathrm{Im}[Kr^{i\varepsilon}](2\pi r)^{-1/2}$$

$$r^{i\varepsilon} = \cos(\varepsilon\ln r) + i\sin(\varepsilon\ln r)$$

Crack opening displacements :

$$\delta_2 + i\delta_1 = \frac{8}{(1+2i\varepsilon)\cosh(\pi\varepsilon)}\frac{(K_1+iK_2)}{E_*}\left(\frac{r}{2\pi}\right)^{1/2} r^{i\varepsilon} \qquad \frac{1}{E_*} = \frac{1}{2}\left(\frac{1}{\bar{E}_1}+\frac{1}{\bar{E}_2}\right)$$

Energy release rate:

$$G = \frac{(1-\beta^2)}{E_*}(K_1^2 + K_2^2)$$

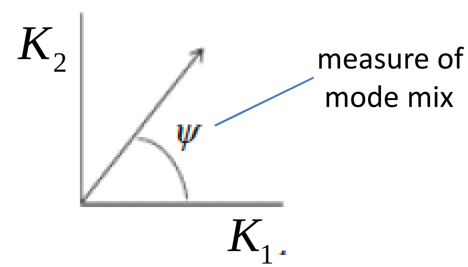
$$\bar{E} = E/(1-\nu^2) \text{ plane strain}$$
$$\bar{E} = E \qquad \text{plane stress}$$

To make life simpler, most of our discussion will take $\beta = 0 \Rightarrow \varepsilon = 0$ but $\alpha \neq 0$

For these cases, the singular stress fields at the tip are identical to those for a homogeneous solid

$$(\sigma_{22},\sigma_{12}) = (K_1,K_2)\frac{1}{\sqrt{2\pi r}}, \quad (\delta_2,\delta_1) = (K_1,K_2)\frac{8}{E_*}\sqrt{\frac{r}{2\pi}}$$

$$G = \frac{1}{E_*}\left(K_1^{\,2} + K_2^{\,2}\right) \ \& \qquad \tan\psi = \frac{K_2}{K_1}$$



Some Basic Bilayer Solutions:

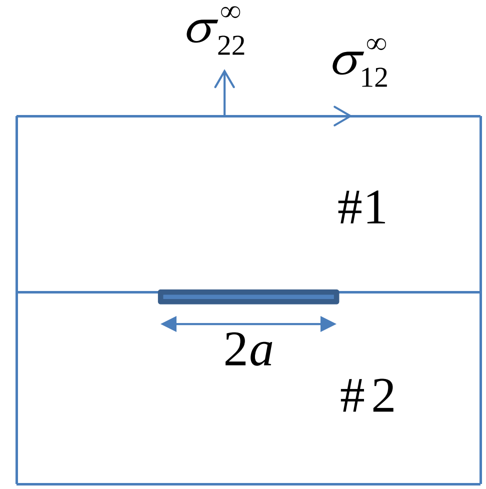
1) crack on an interface between two half spaces: reference cited in (1992-2)

$$K_1 + iK_2 = (\sigma_{22}^{\infty} + i\sigma_{12}^{\infty})(1 + 2i\varepsilon)(\pi a)^{1/2}(2a)^{-i\varepsilon}$$

$$\sigma_{22} = \mathrm{Re}[Kr^{i\varepsilon}](2\pi r)^{-1/2}, \qquad \sigma_{12} = \mathrm{Im}[Kr^{i\varepsilon}](2\pi r)^{-1/2}$$

$$\beta = 0 \Rightarrow \quad (K_1, K_2) = (\sigma_{22}^{\infty}, \sigma_{12}^{\infty})\sqrt{\pi a},$$

$$(\sigma_{22}, \sigma_{12}) = (K_1, K_2)\frac{1}{\sqrt{2\pi r}}, \ (\delta_2, \delta_1) = (K_1, K_2)\frac{8}{E_*}\sqrt{\frac{r}{2\pi}} \quad (r/a << 1)$$



Valid for arbitrary large elastic mismatch with $\alpha \neq 0$ & β=0

For $\beta \neq 0$, $\varepsilon = \frac{1}{2\pi}\ln\left(\frac{1-\beta}{1+\beta}\right)$, near the crack tip:

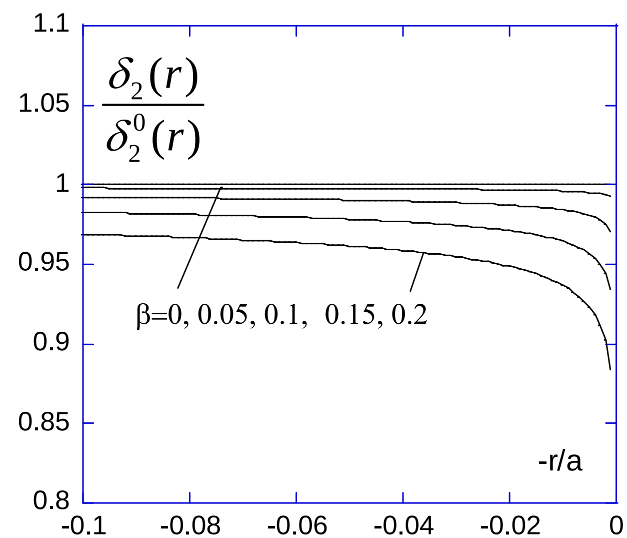
$$\delta_2 + i\delta_1 = \frac{8}{(1 + 2i\varepsilon)\cosh(\pi\varepsilon)}\frac{(K_1 + iK_2)}{E_*}\left(\frac{r}{2\pi}\right)^{1/2} r^{i\varepsilon}$$

$$\delta_2 + i\delta_1 = \frac{8(\sigma_{22}^{\infty} + i\sigma_{12}^{\infty})\sqrt{\pi a}}{\cosh(\pi\varepsilon)E_*}\sqrt{\frac{r}{2\pi}}\left(\frac{r}{2a}\right)^{i\varepsilon}$$

For $\sigma_{12}^{\infty} = 0$,

$$\delta_2(r) = \frac{8}{E_*}\sqrt{\sigma_{22}^{\infty}\pi a}\sqrt{\frac{r}{2\pi}} \times \frac{\cos(\varepsilon \ln(r/2a))}{\cosh(\pi\varepsilon)}$$

$$\equiv \delta_2^0(r) \times \frac{\cos(\varepsilon \ln(r/2a))}{\cosh(\pi\varepsilon)}$$

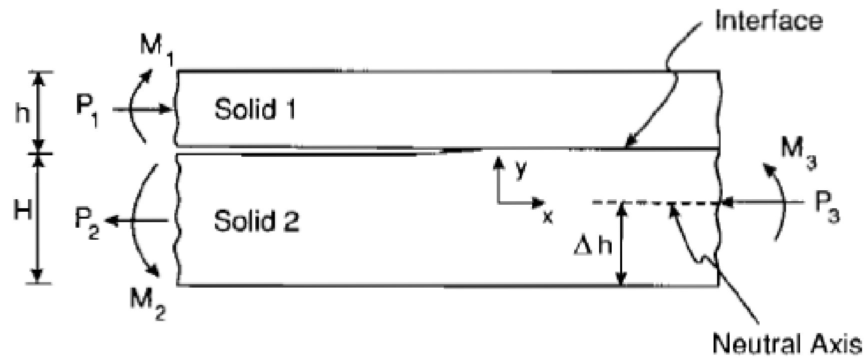


For most problems one can take β=0 to make life simple!!

Some Basic Bilayer Solutions:

2) Crack on an interface between two layers under general loading (1990-1)

The general solution has been obtained and tabulated in (1990-1) see this reference or (1992-2) for full details & numerical tables.

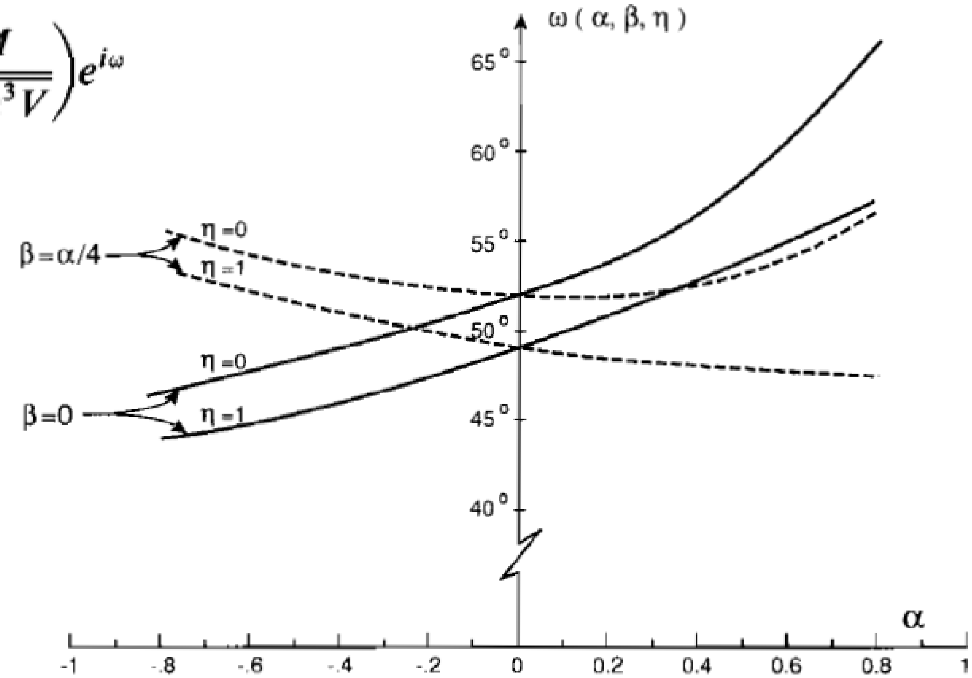


The energy release rate, G, can be obtained by elementary energy accounting because of steady-state character of the problem.

$$G = \frac{1}{2E_1}\left(\frac{P_1^2}{h} + 12\frac{M_1^2}{h^3}\right) + \frac{1}{2E_2}\left(\frac{P_2^2}{H} + 12\frac{M_2^2}{H^3} - \frac{P_3^2}{Ah} - \frac{M_3^2}{Ih^3}\right)$$

$$K_1 + iK_2 = h^{-i\varepsilon}\left(\frac{1-a}{1-\beta^2}\right)^{1/2}\left(\frac{P}{\sqrt{2hU}} - ie^{i\gamma}\frac{M}{\sqrt{2h^3V}}\right)e^{i\omega}$$

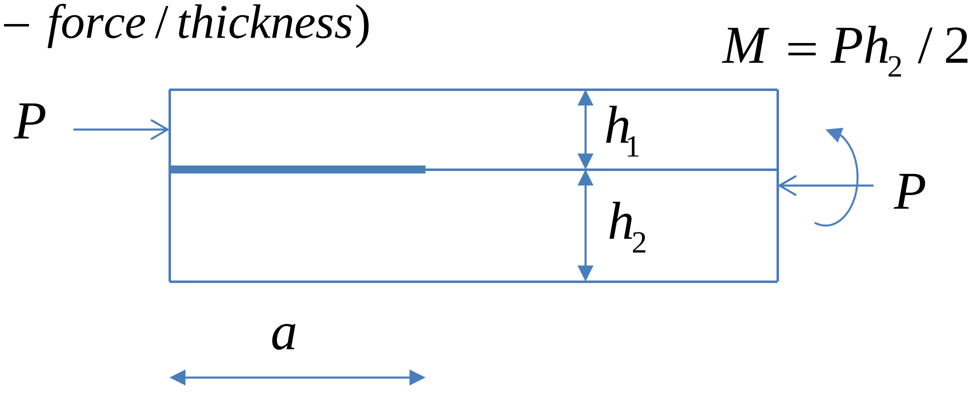
Everything is the above formula is given by simple formulas (1992-1) except $\omega(\alpha,\beta,\eta)$, $\eta = h/H$ which is plotted & tabulated,



An illustration of the energy release rate calculation for a steady-state problem

Energy in the system/thickness (prescribed $P-$ *force / thickness*)

$$\mathrm{E} = \textit{Strain energy} - \textit{potential energy loads}$$
$$= -\textit{Strain energy} \equiv -SE$$



Energy release rate

(energy per area new crack surface):

$$G = -\frac{\partial \mathrm{E}}{\partial a} = \frac{\partial SE}{\partial a} = \left(SE / length\right)_{Behind} - \left(SE / length\right)_{Anead}$$

Here is where steady-state comes in

$$\left(SE / length\right)_{Behind} = \frac{1}{2}\frac{P^2}{\bar{E}h_1} \qquad \left(SE / length\right)_{Ahead} = \frac{1}{2}\frac{P^2}{\bar{E}h_1}\left(\frac{1}{1+h_2/h_1} + 3\frac{\left(h_2/h_1\right)^2}{\left(1+h_2/h_1\right)^3}\right)$$

$$G = \frac{1}{2}\frac{P^2}{\bar{E}h_1}\left(\frac{h_2/h_1}{1+h_2/h_1} - 3\frac{\left(h_2/h_1\right)^2}{\left(1+h_2/h_1\right)^3}\right)$$

$$h_2/h_1 = 1 \Rightarrow \quad G = \frac{1}{16}\frac{P^2}{\bar{E}h_1}; \qquad h_2/h_1 >> 1 \Rightarrow G = \frac{1}{2}\frac{P^2}{\bar{E}h_1}$$

Some special cases of bilayers (1992-2)

Homogeneous material

$E_L \equiv \bar{E}$

$G_I = K_1^{\ 2} / \bar{E}$

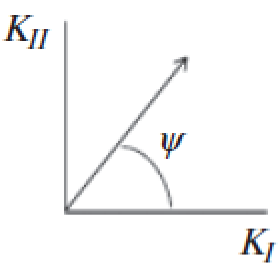
$G_{II} = K_2^{\ 2} / \bar{E}$

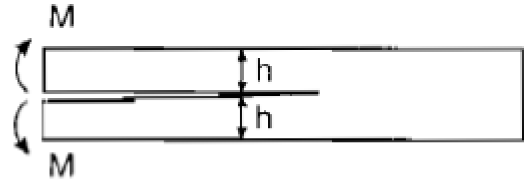
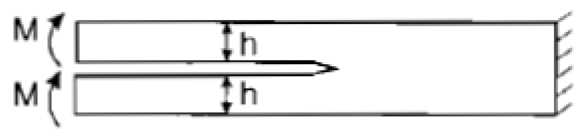
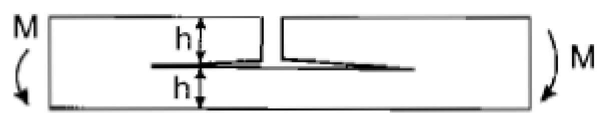
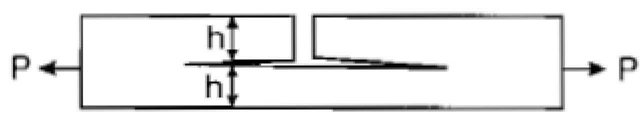
$G = G_I + G_{II}$

$\tan\psi = \dfrac{K_2}{K_1}$

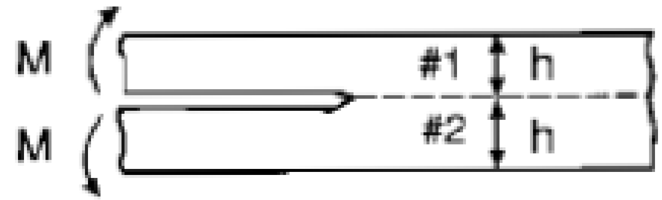
$\psi = 0^0 \Leftrightarrow$ mode I

$\psi = \pm 90^0 \Leftrightarrow$ mode II



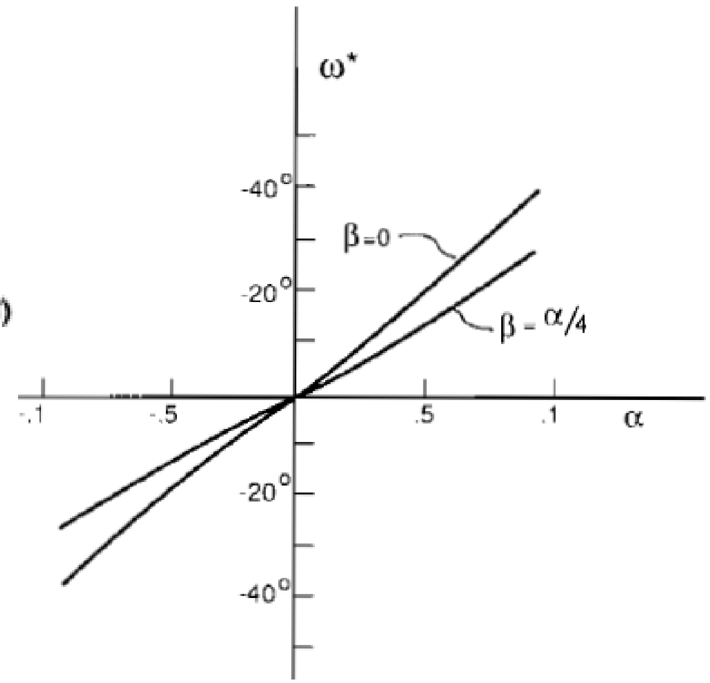
	Specimen	$\mathcal{G}_{\mathrm{I}}$	$\mathcal{G}_{\mathrm{II}}$	ψ
a)		$\frac{12M^2}{E_L h^3}$	0	0^o
b)		0	$\frac{9M^2}{E_L h^3}$	-90^o
c)		$\frac{3M^2}{E_L h^3}$	$\frac{9M^2}{4E_L h^3}$	36.9^o
d)		$\frac{3P^2}{4E_L h}$	$\frac{P^2}{E_L h}$	53.1^o

Crack on an interface between two materials of equal thickness (1992-2)



$$G = \frac{1}{2}\left(\frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}\right) 12 M^2 h^{-3}$$

$$K_1 + iK_2 = 2\sqrt{3}\, M h^{-3/2 - i\varepsilon}(1 - \beta^2)^{-1/2} e^{i\omega^*(\alpha,\beta)}$$



Recall: $\varepsilon = \frac{1}{2\pi} \ln\left(\frac{1-\beta}{1+\beta}\right)$

We will revisit this example later

Some special cases of bilayers (1987-1) , (1992-2)

Homogeneous material

$$G = \frac{1}{2\bar{E}}\left(P^2 h^{-1} + 12 M^2 h^{-3}\right)$$

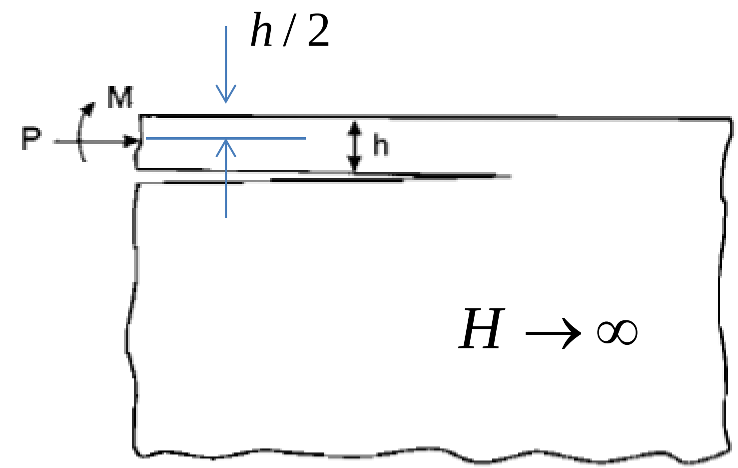
$$K_I = \frac{1}{\sqrt{2}}[Ph^{-1/2}\cos\omega + 2\sqrt{3}\,Mh^{-3/2}\sin\omega]$$

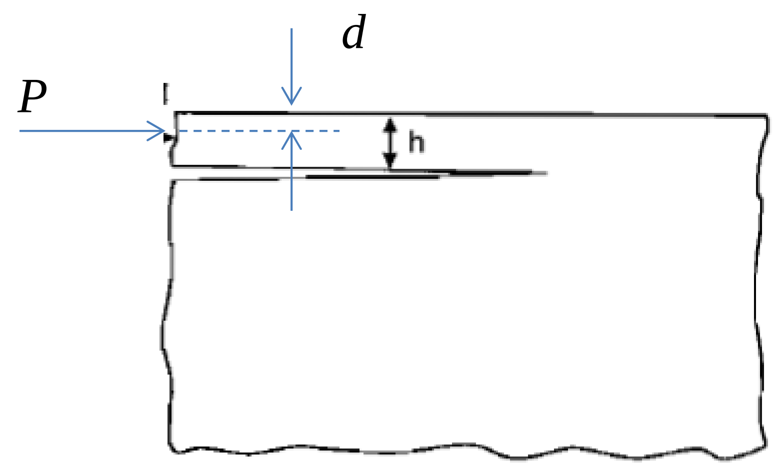
$$K_{II} = \frac{1}{\sqrt{2}}[Ph^{-1/2}\sin\omega - 2\sqrt{3}\,Mh^{-3/2}\cos\omega]$$

$$\omega = 52.1^o$$

Note: Mode I (K_{II} =0) if

$$\frac{Ph}{M} = \frac{2\sqrt{3}}{\tan\omega} = 2.70$$





$d / h = 0.13 \Leftrightarrow$ mode I

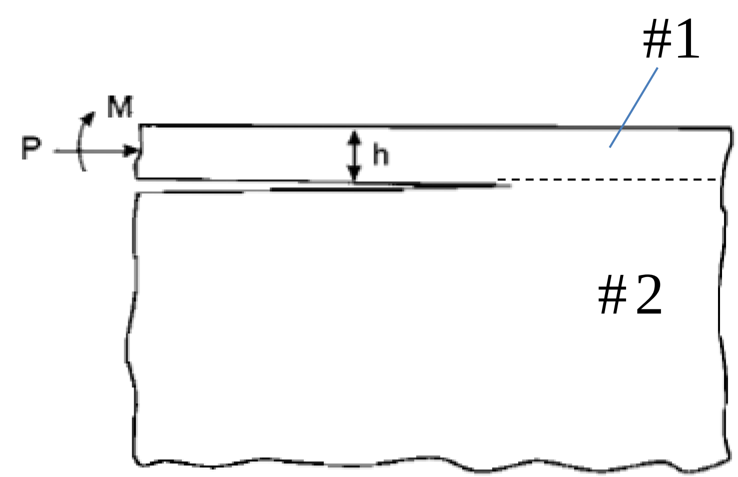
Some special cases of bilayers: film on an infinitely deep substrate (1992-2)

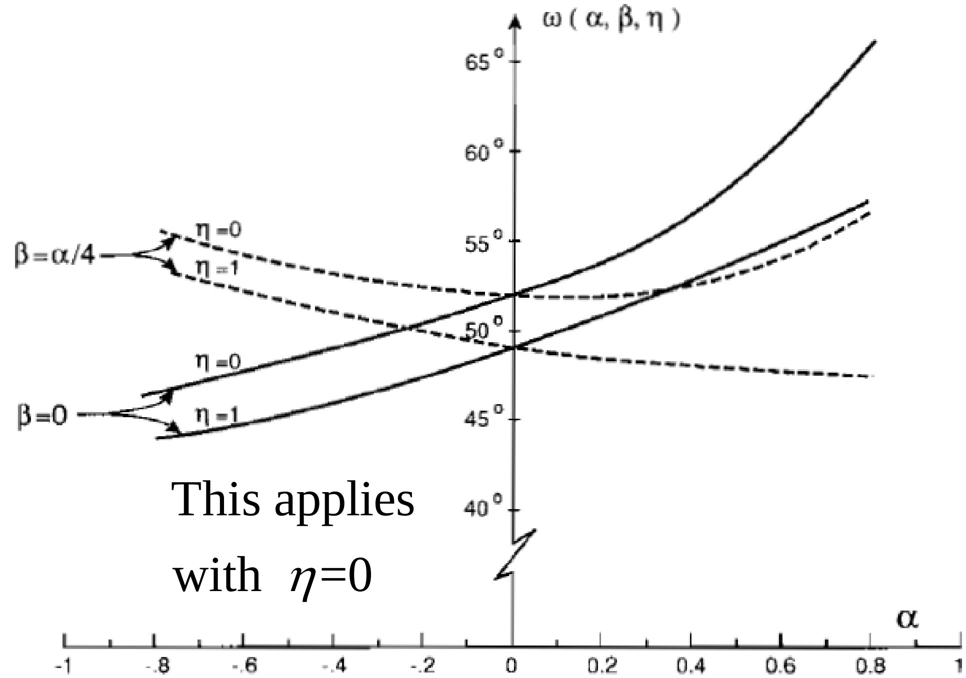
$$G = \frac{1}{2\bar{E}_1}\left(P^2 h^{-1} + 12M^2 h^{-3}\right)$$

$$K_{\mathrm{I}} = \frac{1}{\sqrt{2}}\left[P h^{-1/2}\cos\omega + 2\sqrt{3}\,M h^{-3/2}\sin\omega\right]$$

$$K_{\mathrm{II}} = \frac{1}{\sqrt{2}}\left[P h^{-1/2}\sin\omega - 2\sqrt{3}\,M h^{-3/2}\cos\omega\right]$$

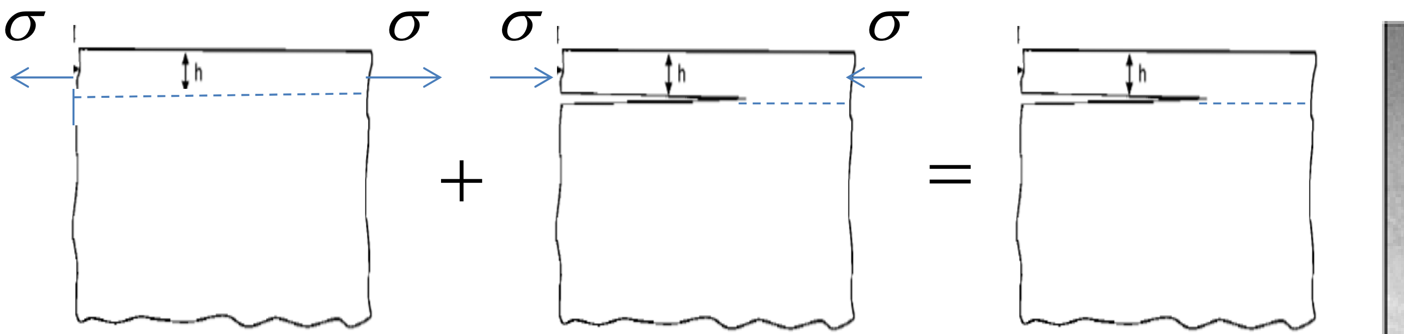
$$\omega = 52.1^o \;\text{ if }\; \alpha = 0 \;\&\; \beta = 0$$





This applies with η=0

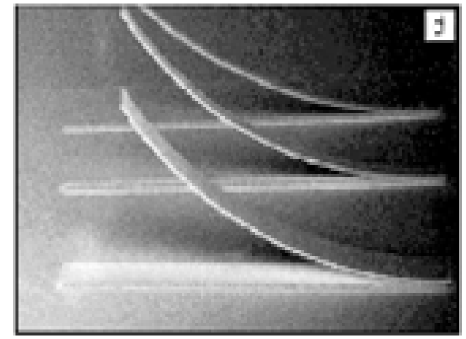
An example: Uniformly stressed thin film on deep substrate



Before cracking

Auxiliary problem producing stress intensity factors

Final cracked interface



$$P = \sigma h, \;\; M = 0 \;\Rightarrow$$

$$G = \frac{\sigma^2 h}{2\bar{E}_1}$$

$$K_I = \sigma\sqrt{h}\cos\omega/\sqrt{2} \overset{\beta=0}{=} 0.434\sigma\sqrt{h}, \;\; K_{II} = \sigma\sqrt{h}\sin\omega/\sqrt{2} \overset{\beta=0}{=} 0.558\sigma\sqrt{h}/\sqrt{2} \;\; (\psi = 52.1^o)$$

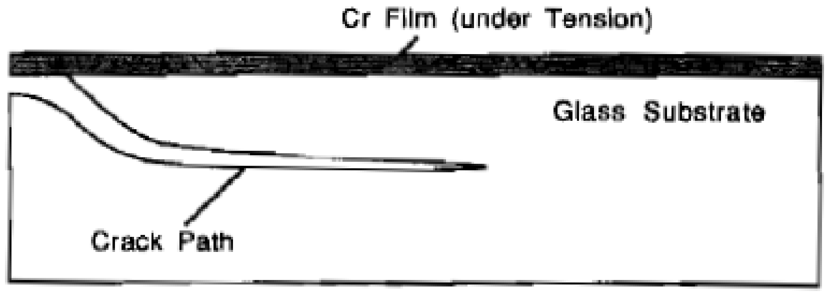
This applies for **tensile stress** in film (σ>0) with $K_I > 0$:

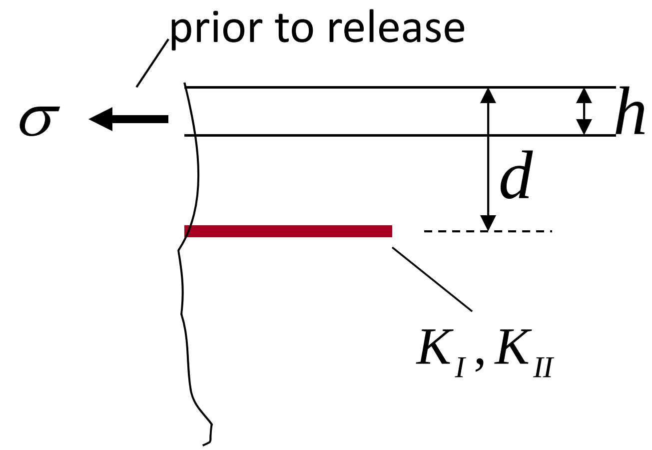
If the film stress is **compressive** ($\sigma < 0$), the formula predicts $K_I < 0$

the formula is not valid--the problem is approximately mode II

Closed, mode II crack. Only valid if friction is neglected. $\sigma < 0: \quad K_I = 0, \quad K_{II} = -0.707\sigma\sqrt{h}$

Mode I cracking in substrate driven by tensile stresses in film or coating (1989-2, 1992-2)





Neglecting elastic mismatch the basic solution gives

$$K_I = \frac{\sigma h}{\sqrt{2d}}\left(\cos\omega + \sqrt{3}\left((d-h)/d\right)\sin\omega\right)$$

$$K_{II} = \frac{\sigma h}{\sqrt{2d}}\left(\sin\omega - \sqrt{3}\left((d-h)/d\right)\cos\omega\right)$$

Depth of mode I crack

$$K_{II} = 0 \text{ with } \omega = 52.1^o \Rightarrow \frac{d}{h} = 3.86, \quad K_I = 0.586\sigma\sqrt{\mathrm{h}} \text{ and } \quad G = 0.343\sigma^2 h/\bar{E}$$

Compare with mixed mode delamination along interface

$$K_I = 0.434\sigma\sqrt{\mathrm{h}}, \; K_I = 0.556\sigma\sqrt{\mathrm{h}} \quad \text{and} \quad G = 0.50\sigma^2 h/\bar{E}$$

Substrate delamination as mode I crack propagation is observed in systems where the interface is relatively tough and the substrate is brittle. The stress in the film or coating must be in tension. No mode I path exists in the substrate if the stress is compression.

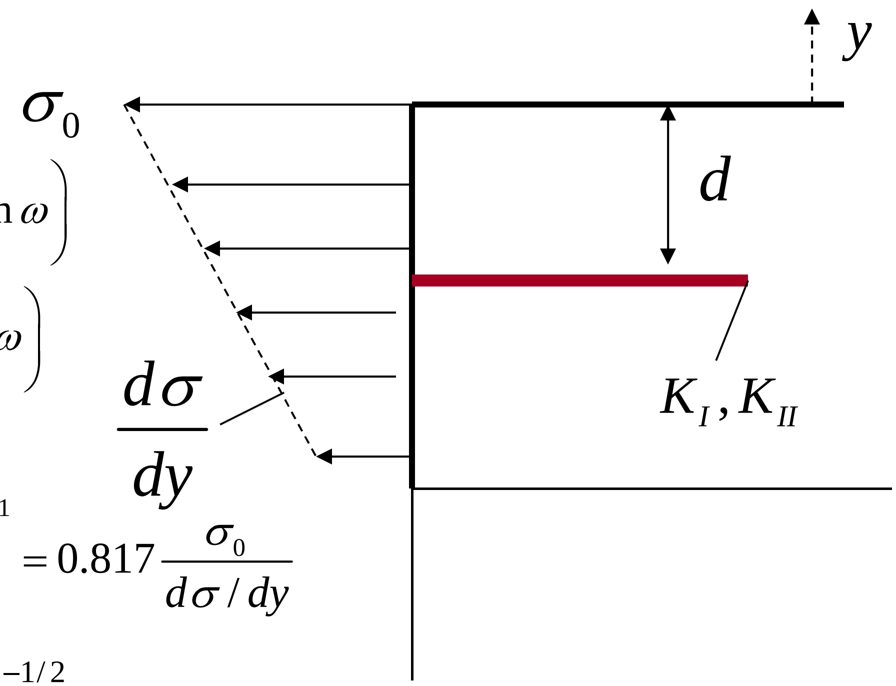
See Drory, Thouless & Evans (1988) for experimental observations for metal/glass systems.

Mode I cracking within a film or coating driven by stress gradients

Basic solution gives:

$$K_I = \frac{1}{\sqrt{2}}\sigma_0 d^{1/2}\cos\omega + \frac{1}{\sqrt{2}}\frac{d\sigma}{dy}d^{3/2}\left(-\cos\omega + \frac{1}{2\sqrt{3}}\sin\omega\right)$$

$$K_{II} = \frac{1}{\sqrt{2}}\sigma_0 d^{1/2}\sin\omega + \frac{1}{\sqrt{2}}\frac{d\sigma}{dy}d^{3/2}\left(\sin\omega + \frac{1}{2\sqrt{3}}\cos\omega\right)$$



Mode I crack:

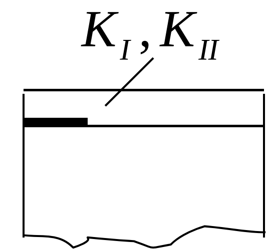
$$K_{II} = 0 \;\; (\omega = 52.1^o) \;\Rightarrow\; d = \frac{\sigma_0}{d\sigma/dy}\left(1 + \frac{\cot\omega}{2\sqrt{3}}\right)^{-1} = 0.817\frac{\sigma_0}{d\sigma/dy}$$

$$K_I = 0.657\sigma_0 d^{1/2} = 0.594\sigma_0^{\;3/2}(d\sigma/dy)^{-1/2}$$

For a mode I crack to exist within layer with linear stress variation:

$$\begin{array}{l}\sigma_0 > 0 \quad \& \\ d\sigma/dy > 0.817\sigma_0/H\end{array} \quad \Longrightarrow \quad d = 0.817\frac{\sigma_0}{d\sigma/dy} \;\; \& \quad G = 0.353\frac{\sigma_0^{\;3}}{\overline{E}\,d\sigma/dy}$$

Interface toughness—the role of mode mix (1992-2)



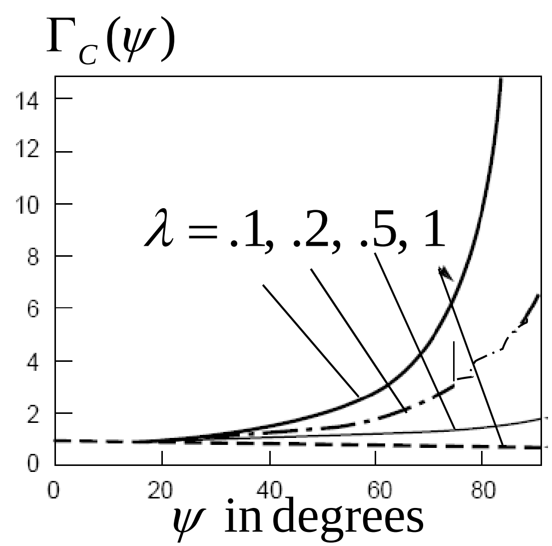
Experimental finding: The energy release rate required to propagate a crack along an interface generally depends on the mode mix, often with larger toughness the larger the mode II component

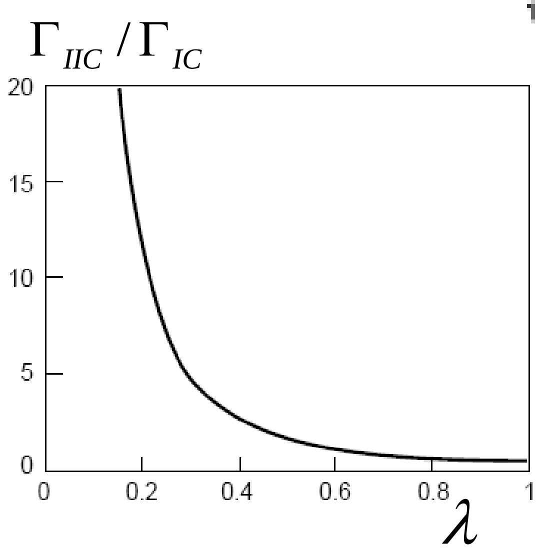
Interface Toughness: $\Gamma_C(\psi)$

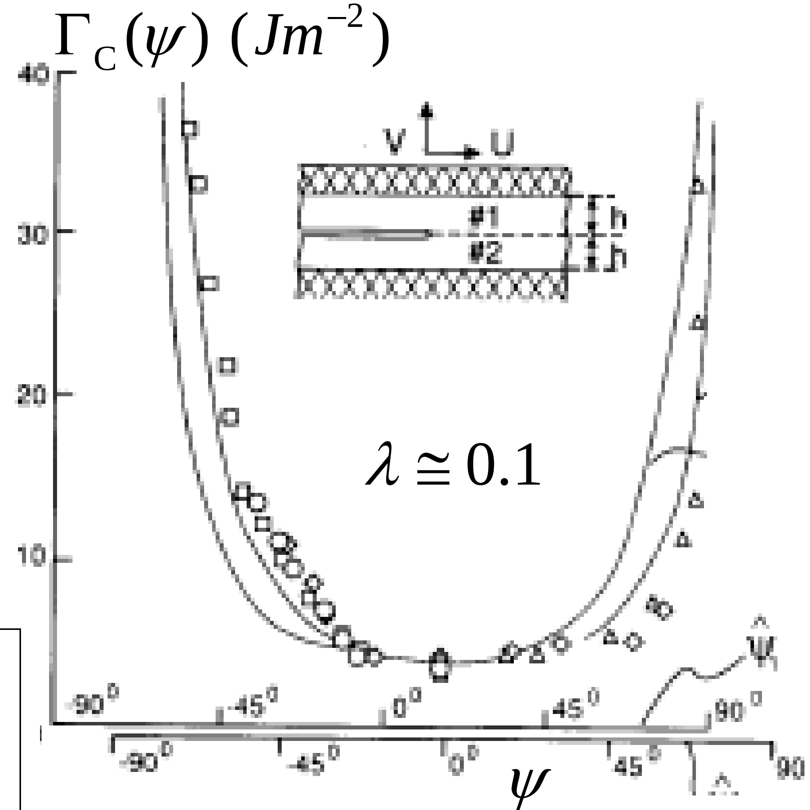
Propagation condition: $G = \Gamma_C(\psi)$

A phenomenological interface toughness law

$$\Gamma_C(\psi) = \Gamma_{IC}\left(1 + \tan^2((1-\lambda)\psi)\right)$$







Liechti & Chai (1992) data for an epoxy/glass interface.

$\lambda = 1 \Rightarrow$ no mode dependence

$\lambda << 1 \Rightarrow$ significant mode dependence