How things break: *Fast fracture in slow motion*

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The USS Schenechtady (1943)
– The ship that never moved

Thank you to the European Research Council
and the Israel Science Foundation
for funding this stuff…
Coming up!

Crack primer: Why are cracks interesting – how are they supposed to behave?

When things work …
  (When cracks behave as they are supposed to…)

Crack instabilities – when things don’t…( a very brief review – wait for next week for more!)  (When Nature stops reading the text books…)

Fast Fracture in slow motion
  (Dynamic fracture of gels)

A new theory of non-linear fracture mechanics
  (A way out of this mess)

Where are we – where are we going?
  (The origins of crack instabilities??)
Why Study Fracture?

1. Fundamental open question: Why are some materials brittle (e.g. glass) …while others are ductile (e.g. iron) ?

2. Theory of dynamic brittle fracture is incomplete:
   - Important unresolved questions: Criteria for crack path selection and crack instabilities.
   - Discrepancies between theoretical predictions and experimental observations (more on this later).
How Things Break – Naïve Approach

Assume that planes of atoms separate all at once
Atoms will dissociate when their binding energy is overcome. A ~50% strain is required to separate the atoms and break the material. Fracture usually takes place already at ~1% strain.

Something is missing in this picture.
How Things Break – Crack Approach

Consider now the same material, this time with an elliptical hole inside:

Inglis (1913)
(uniform applied stress $\sigma$)

$$\sigma_{\text{max}} = \sigma_{\text{applied}} \left(1 + \left(\frac{2l}{\rho}\right)\right)^{1/2}$$

The crack acts as a stress “amplifier”

$$\sigma_{\rho} \rightarrow 0 = \frac{K(\sigma_{\text{applied}}, l)}{r^{1/2}}$$

$K(\sigma_{\text{applied}}, l) \equiv \text{“stress intensity factor”}$

- The presence of a crack induces a stress singularity at its tip
- Bonds are then preferentially broken at the tip of the crack.

The existence of a crack leads to a large decrease in the material’s effective strength.
How Things Break – The (canonic) theory of fracture

Sharp cracks \( (\rho \to 0) \) + Linear elastic material response

Linear Elastic Fracture Mechanics (LEFM)

Solves the displacement/ stress/ strain fields in a linear elastic medium in the presence of a sharp crack.
Linear Elastic Fracture Mechanics (LEFM)

Main assumptions and results

1) Small strains (linear elastic) up to dissipation (the “process zone”).

\[ \sigma_{ij} = \frac{K(\sigma_{\text{applied}}, l; v)}{\sqrt{2\pi r}} \Sigma_{ij}(\theta; v) + O(1) \]

- \( K \) is called the “stress intensity factor”
- \( \Sigma_{ij} \) is a universal function

2) A universal singularity (K-dominance):

3) Crack Stability:

Energy flux to the crack tip = Energy dissipated (Fracture energy)

\[ G = \Gamma \]

\( \Gamma \) is called the fracture energy and is a material property.
By focusing elastic energy into a **stress field singularity** at its tip. Bonds are preferentially broken at the tip of a crack.

### Material Strength:

<table>
<thead>
<tr>
<th>Material</th>
<th>Theoretical Strength (GPa)</th>
<th>Practical Strength (GPa)</th>
<th>Practical/Theoretical Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iron</td>
<td>50</td>
<td>0.3</td>
<td>0.006</td>
</tr>
<tr>
<td>Copper</td>
<td>24-55</td>
<td>0.2</td>
<td>0.005</td>
</tr>
<tr>
<td>Titanium</td>
<td>31</td>
<td>0.3</td>
<td>0.009</td>
</tr>
<tr>
<td>Silicon</td>
<td>45</td>
<td>0.7</td>
<td>0.01</td>
</tr>
<tr>
<td>Glass</td>
<td>37</td>
<td>0.4</td>
<td>0.01</td>
</tr>
<tr>
<td>Plexiglass</td>
<td>3</td>
<td>0.05</td>
<td>0.01</td>
</tr>
</tbody>
</table>

\[
\sigma(r) \sim K \cdot r^{-1/2}
\]
The dynamics of propagating cracks are both interesting and difficult to study:

Linear Elastic Fracture Mechanics (LEFM) ⇔ materials are linearly elastic everywhere:

\[ \sigma(r) \sim r^{-1/2} \]

- Singular objects that propagate at \( v \sim \text{information (sound) speeds} \)
- Regularization of singularities ⇔ material properties

Ductile ?
Brittle ?
Symmetry of The LEFM solution

There are three conventional fracture modes, which are characterized by the symmetry of the loading on the crack plane.

\[
\sigma_{i,j} = \sum_{\alpha=1}^{3} K_\alpha \frac{1}{\sqrt{2\pi r}} f_{i,j}^\alpha(v, \theta)
\]

Today we will concentrate on tensile loading
Equation of Motion for an Infinite Plate (Mott 1948)

Assume:
1. Straight line motion
2. \( \Gamma = \text{Constant} \)
3. Energy is conserved

\[
E_{\text{total}} = K(l) + P(l) + 2\Gamma l = \text{const} = P(l_0) + 2\Gamma l_0
\]

use: \( \vec{u} \propto l \)

\[
K(l, v) = \frac{\rho}{2} \int \left( \partial_t \vec{u} \right)^2 d^2x = \frac{\rho v^2}{2} \int \left( \partial_x \vec{u} \right)^2 d^2x = V^2 l^2 C_1
\]

\[
P(l) = \int \left( \partial_{\alpha} \vec{u} \right) \left( \sigma_{\alpha\beta} \right) d^2x = -l^2 C_2
\]

\[
E_{\text{total}} = C_1 v^2 l^2 + C_2 l^2 + 2\Gamma l = C_2 l_0^2 + 2\Gamma l_0
\]

Fracture initiation (Griffith) condition:

Released Pot. Energy = Energy to create surface

\[
-\frac{\partial P}{\partial l} \bigg|_{l_0} = 2\Gamma \iff C_2 l_0 = \Gamma
\]

\[
V = \text{Const} \cdot \left( 1 - \frac{l_0}{l} \right) \quad \rightarrow \quad V = V_{\text{Rayleigh}} \cdot \left( 1 - \frac{\text{Const} \cdot \Gamma}{\sigma^2_{\infty} l} \right)
\]

(L. B. Freund 1972)
Local energy balance \( \rightarrow \) the same equation of motion for a crack:

\[
j_{\alpha} = \sigma_{\alpha\beta} \dot{u}_{\beta}
\]

Energy flux into the singular region

\( \sigma \sim r^{-1/2} \)

Geometry and Loading specific:

elastic energy released by a static crack

\[
j_{\alpha} n_{\alpha} = G(l, \sigma) \cdot (1 - v/V_R) = \Gamma
\]

Dissipation

“fracture energy”

Energy into crack tip = dissipation at the tip

Universal kinematic term

\[
v = V_R \left[ 1 - \Gamma / G(l, \sigma) \right]
\]

Freund, Eshelby, Kostrov, Willis ~ 1972
Characteristics of the Equation of motion

\[ v = V_R \cdot (1 - \frac{\Gamma}{G(l, \sigma)}) \]

**Conditions:**
- Linear elasticity until Dissipative Zone
- Single crack
- \( r_{DZ} \ll r \ll r_{\text{sing}} \)
- Infinitely large medium – no “history” dependence or memory

**Characteristics:**
- \( V_R \) = Asymptotic crack speed
- \( v \) is first order in time => a crack has no inertia
  => it should jump instantaneously to the value dictated by \( \Gamma \) and \( G \)
Testing the equation of motion: \( v = V_R \cdot (1 - \Gamma / G(l, \sigma)) \) \( \iff \Gamma = G(l, \sigma) \cdot (1 - v/V_R) \)

All data collapse as predicted for \( v = v_C < 0.4V_R \)

For $v < 0.4V_R$ excellent agreement with the equation of motion

When $v > V_R$ something is wrong!
At a critical velocity, $V_C$, the motion of a crack becomes unstable.

At this point:
- Velocity oscillates
- Structure is formed on the fracture surface

Instability Mechanism:
- Local (micro-) crack branching

The Micro-Branching Instability

The equation of motion holds only for single cracks!
Theoretical approaches to the instability

Cracks are frustratingly stable to any perturbation – no instability observed analytically
(J. Langer, M. Marder, J. R. Rice, M. Ada-Bedia, M. Ben-Amar, I. Procaccia, …)

To understand fracture, we need to know what is happening within/around the “dissipative region” in the vicinity of a crack’s tip!
Crack instability similar to experiments observed when studied on a lattice as well as in MD, finite element simulations, and phase-field models
(M. Marder, D. Kessler, H. Levine, L. Sander, A. Needleman, F. Abraham, P. Gumbsch, H. Gao, Vashishita, Kalia, Ortiz, …)

Empirical ways to model the dissipative zone

Empirical ways to model the dissipative zone


Microbranching goes away when grid sizes are taken to zero!
(M. Marder, M. Falk, A. Needleman, …)

In both finite element and lattice calculations:

Cohesive Zone models (Barenblatt, Dugdale, Needleman, H. Gao, Ortiz, …)

Attempts to develop a fundamental theory of amorphous materials

(J. Langer, M. Falk, I. Procaccia, E. Bouchbinder, J. L. Barrat, A. Lemaitre, …)
Why is fracture such a hard problem to “break”? Basically a wide range of time and length scales are coupled…

**Rapid propagation:**

<table>
<thead>
<tr>
<th>Material</th>
<th>$V_R$ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>3300</td>
</tr>
<tr>
<td>PMMA</td>
<td>930</td>
</tr>
<tr>
<td>Homolite</td>
<td>900</td>
</tr>
<tr>
<td>Brittle steels</td>
<td>3500</td>
</tr>
</tbody>
</table>

**Small length scales:**

<table>
<thead>
<tr>
<th>Material</th>
<th>Dissipative zone size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glass</td>
<td>1-10 nm</td>
</tr>
<tr>
<td>PMMA</td>
<td>&lt; 1-5 μm</td>
</tr>
<tr>
<td>Homolite</td>
<td>1 - 10μm</td>
</tr>
</tbody>
</table>

Can we (experimentally) circumvent these difficulties?
“Yes we can!” – break “Jello”!
Fracture of polyacrylamide gels: sound velocity reduced by 2-3 orders of magnitude

High speed visualization enables direct measurements of:

- Crack tip opening profile
- Displacement field at each point (by “particle tracking” imposed scratch patterns)

<table>
<thead>
<tr>
<th>Material</th>
<th>V (m/s)</th>
<th>Poisson ratio</th>
<th>Young's Modulus (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gel</td>
<td>5.6</td>
<td>0.5</td>
<td>10^6</td>
</tr>
<tr>
<td>13.8 % acrylamide</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.6 % bis-acrylamide</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMMA</td>
<td>930</td>
<td>0.35</td>
<td>3,900,000</td>
</tr>
<tr>
<td>Soda-Lime glass</td>
<td>70,000,000</td>
<td>0.22</td>
<td>70,000,000</td>
</tr>
</tbody>
</table>

Sample dimensions: **150 × 150 × 0.1 mm**
Frame rate: **5000 frames/sec**
( ~1 frame per 0.5-1 mm crack advance)
Brittle fracture in Gels is identical to that of other brittle amorphous materials:

Micro-branching in Gels:
- At a critical velocity a single crack becomes unstable to frustrated micro-branches
- Microbranches have the same functional form as in other brittle materials

“3D” behavior

In both Gels and Glass micro-branches are formed in directed lines aligned parallel to the propagation direction.
Micro-branching limits $V$ to $\sim 0.5V_R$

$V > 0.4V_R$

Gel thickness $\sim 1$ mm
**Suppression of micro-branching ➔ ??**

Micro-branches (in gels):
- are *noise-triggered*
- Have a *minimum width, \( \Delta Z \)
- *Disappear* when reaching a sample edge

*When the sample width, \( h \rightarrow \Delta Z \)*
- Number of effective noise sources reduced as \( h/\Delta Z \rightarrow 1 \)
- Branch-lines that *are* generated will quickly find an edge and disappear

➔ For thin samples:
- Micro-branches easily disappear ➔ Long time for generation
  ➔ *single crack* states are dominant
- Single crack states can accelerate to nearly \( V_R \)
For very thin gels (thickness ~ minimum micro-branch width): we can suppress the instability…

A new oscillatory instability appears at \( v \sim 0.9 V_R \)!

Gel thickness \( \sim 0.2 \text{ mm} \)
As a first step in understanding all of this complex stuff:

Let’s take a closer look at “simple” cracks

• Suppress instabilities (making the material’s thickness ~ 100μm)
  Compare to equation of motion for v< 0.9C_S

• Measure the structure of the near crack-tip region
  PIV ↔ measure the singular displacement field
Let's check the equation of motion for **simple cracks**: $G = \Gamma$

**Simple crack** in an **infinite medium** for constant stress, $\sigma$:

$$v = V_R \cdot \left(1 - \frac{\Gamma}{G(l, \sigma)}\right) \iff \Gamma = G(l, \sigma) \left(1 - \frac{v}{V_R}\right)$$

$$\Gamma = G \approx \frac{1 - \nu^2}{E} \frac{8l}{\pi} \sigma_\infty^2 \left(1 - \frac{v}{V_R}\right)$$

L.B. Freund (1990)

The equation of motion works perfectly for an **infinite system**! Does the (Eq. motion) $G = \Gamma$ work for **different** geometries?

Under different loading conditions: Cracks in an *infinite strip*

\[ \Gamma = G = W \left[ 1 - \dot{v} \cdot f(v) \right] \sim W \left[ 1 - \frac{\dot{v} \cdot b}{c_1^2 [1 - (v/c_R)^2]^2} \right] \]

**Effective Mass** \( \rightarrow \infty \) as \( v \rightarrow c_R \)

\( \Rightarrow \) cracks attain a *singular inertia* as \( v \rightarrow c_R \)

**Dynamics change when cracks feel the finite size of system**

*But \( \Gamma = G \) still describes the equation of motion of a simple crack!*

Energy Balance $\Leftrightarrow$ excellent *agreement* with LEFM predictions...

What more do we need?

While we are already here...

Let’s look at the *predictions for the singular fields at crack tips*

$$
\sigma_r \to 0 = \frac{K(\sigma_{\text{applied}}, l)}{r^{1/2}}
$$

$K(\sigma_{\text{applied}}, l) \equiv \text{“stress intensity factor”}$
Crack tip shape: *a look at the fields* surrounding the crack tip

Deviations from LEFM!

What scale is $\delta$?

Dissipative Zone?

Nonlinear Elastic Zone

Crack tip shape: *a look at the fields* surrounding the crack tip

Parabolic profile

$V \sim 0.5 \cdot V_R$

$u_y(r, \theta = \pi)$

$\sigma(r) \propto \delta \sqrt{r}$

$u_y(r) \propto K \sqrt{r}$

$\Gamma = K^2/E \cdot A(v)$

Fracture energy $\Gamma$ (J/m$^2$)

$\delta$ (mm)

$V/C_S$

$V/C_S$

$V/C_S$

$V/C_S$
Comparison with LEFM – strain field: $\varepsilon_{yy}(r, \theta=0)$

Approaching the tip takes us **Beyond LEFM**!

- Nonlinear zone surrounding crack tip
  distortion of crack-tip profile
- $u_x \Leftrightarrow K ; u_y \Leftrightarrow K' ; \quad K \neq K'$
- $\varepsilon_{yy}$ is far from LEFM predictions
In gels:

1. Stress-strain nonlinearity is *known*
2. Nonlinearity is *significant* at *moderate* (10-20%) strains:
   - *Extend* LEFM to weakly *nonlinear elasticity*

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E. Bouchbinder, A. Livne and J. Fineberg,, JMPS **57**, 1568-1577 (2009)
Idea of the Theory

When do nonlinearities become relevant?

$$\sigma/\mu = \varepsilon - \varepsilon^2 \Theta(0.1)$$

Ratio of nonlinear/linear term $\sim \varepsilon \sim \varepsilon_{LEFM} \sim \frac{K}{2\mu (2\pi)^{1/2}}$

Define a critical strain, $\Delta$, delineating the importance of nonlinearity

Order $\Delta$ problem = LEFM

Order $\Delta^2$ problem:

$$\mu \nabla^2 \vec{u}^{(2)} + 3 \mu \nabla (\nabla \cdot \vec{u}^{(2)}) + \frac{\mu \ell \bar{g}(\theta; \nu)}{r^2} = \rho\nu^2 \partial_{xx} \vec{u}^{(2)}$$

+ B.C.’s (zero stress crack faces) + loading

Inhomogeneous term has the generic form:

$$\partial(\partial u^{(1)} \partial u^{(1)})$$

Perform a controlled expansion in $\Delta$ of the fields and constitutive relation

Interatomic separation

Interatomic separation $r_0 \sim 1.6r_0$
Excellent (no free parameter) agreement with measurements

Leading strain nonlinearity is $1/r$

\[
\varepsilon_{yy}(r,0,v) = \frac{C_1 K}{r^{1/2}} + \frac{C_2 K^2}{r} + C_3 \frac{T}{E}
\]

A *dynamic length-scale* exists (no length-scale in LEFM)

\[
l_{nl}(v) \propto \Gamma/E
\]
\[ \Delta = \frac{K_I}{4\mu (2\pi l_{nl})^{1/2}} \]

\[ D = \Delta \ll 1 \]

\[ D = \Delta \sim O(0.1) \]

\[ D = \Delta >> 1 \]

\[ \text{Asymptotic zone} \]

\[ \text{Weakly nonlinear zone} \]

\[ \text{Strongly nonlinear zone} \]
We now have: 

An **excellent** understanding of simple dynamic cracks.

Good... 

Better ... 

Great!

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What scales are important for crack instabilities?

**Micro-branching**

The theory also supplies a dynamic scale: $l_{nl}$

**Oscillatory instabilities v>0.9C_S**

`LEFM has no intrinsic scales…
Instabilities generate dynamic scales`

$\Rightarrow$ **Instabilities might be described with scales $\sim l_{nl}$ given by the weakly nonlinear theory !??!**


Next Step…
Utilize this new theoretical framework to go *beyond* single-crack dynamics
(e.g. *Is this sufficient to understand the origin of crack instabilities*..)
Summary:

We now understand “simple” cracks in brittle materials!

- Equation of motion ⇔ Energy Balance
- Singular fields near crack tip ⇔ **nonlinear elasticity** matters
  - **Worse** singularity
  - New **intrinsic** nonlinear length scale, \( l_{nl} \)

What’s next:

*Understanding crack instabilities...using \( l_{nl} \). (Next week)*

- The dynamics of crack instabilities and crack branching?
  - ⇔ perturb the **correct** “ground state”!

- Beyond brittle materials ⇔ The near-tip stress field structure: How does *microscopic* structure affect *macroscopic* behavior?
Thank You!