How things break: *Fast fracture in slow motion*

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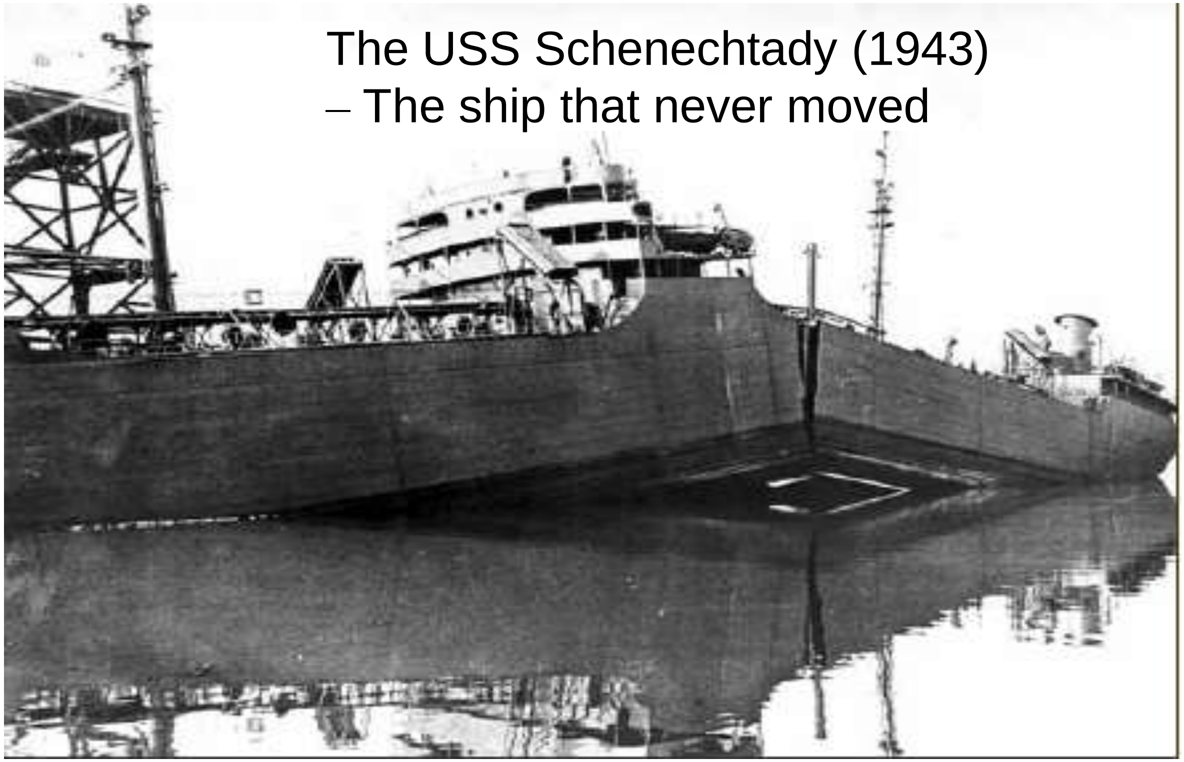


The USS Schenechtady (1943)
– The ship that never moved

Thank you to the **E**uropean **R**esearch **C**ouncil
and the **I**srael **S**cience **F**oundation
for funding this stuff…

Coming up!

Crack primer: Why are cracks interesting –how are they supposed to behave?

When things work …

(When cracks behave as they are supposed to…)

Crack instabilities – when things don't…(a very brief review – wait for next week for more!) (When Nature stops reading the text books…)

Fast Fracture in slow motion

(Dynamic fracture of gels)

A new theory of non-linear fracture mechanics

(A way out of this mess)

Where are we – where are we going?

(The origins of crack instabilities??)

Why Study Fracture ?

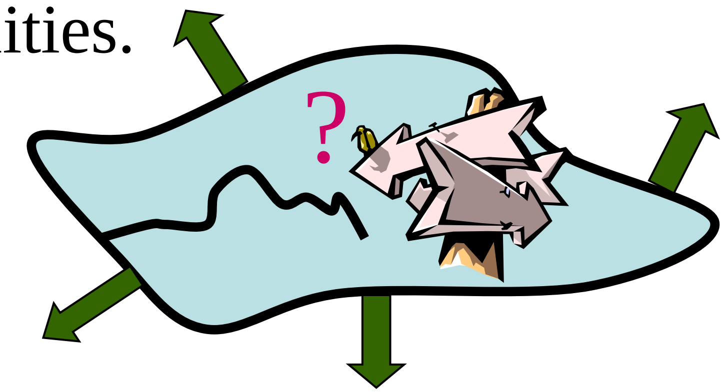
1. Fundamental open question:

Why are some materials brittle (e.g. glass)

…while others are ductile (e.g. iron) ?

2. Theory of dynamic brittle fracture is incomple

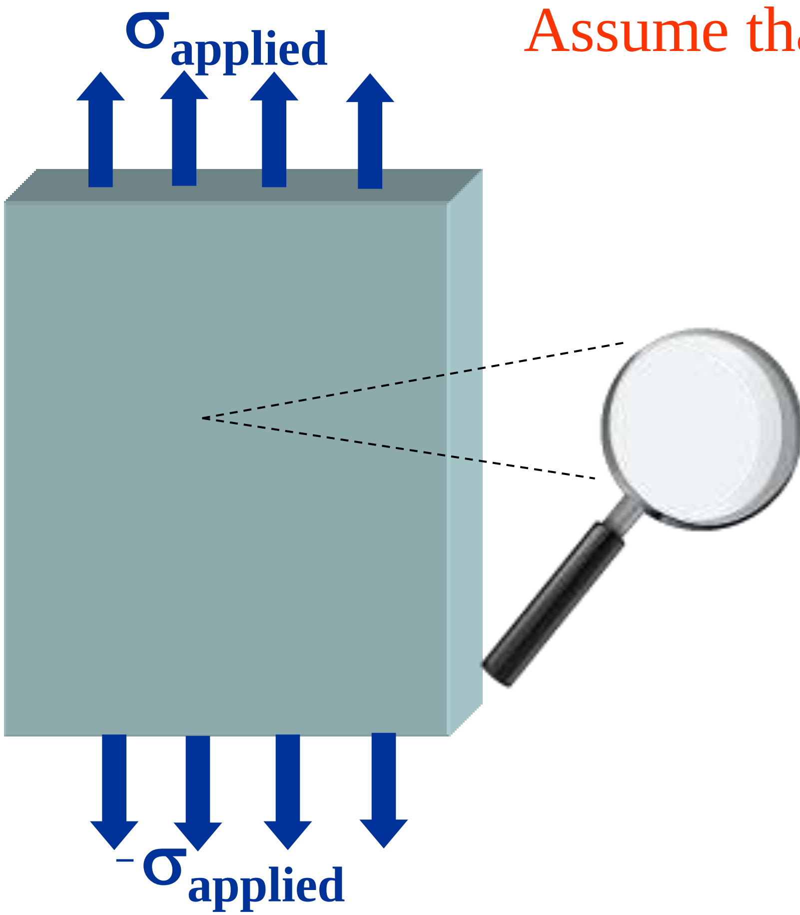
- Important unresolved questions: Criteria for crack path selection and crack instabilities.

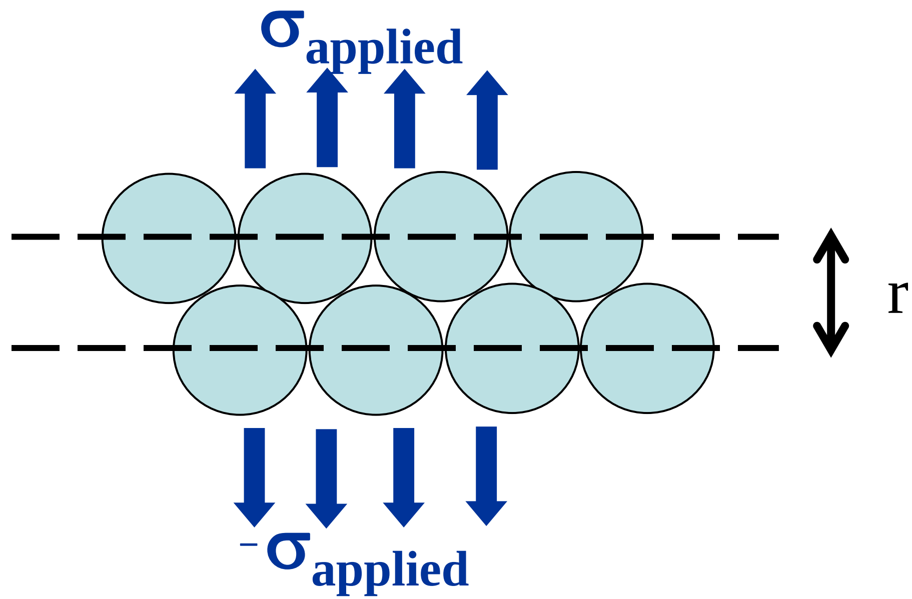


- Discrepancies between theoretical predictions and experimental observations (more on this later).

How Things Break – Naïve Approach

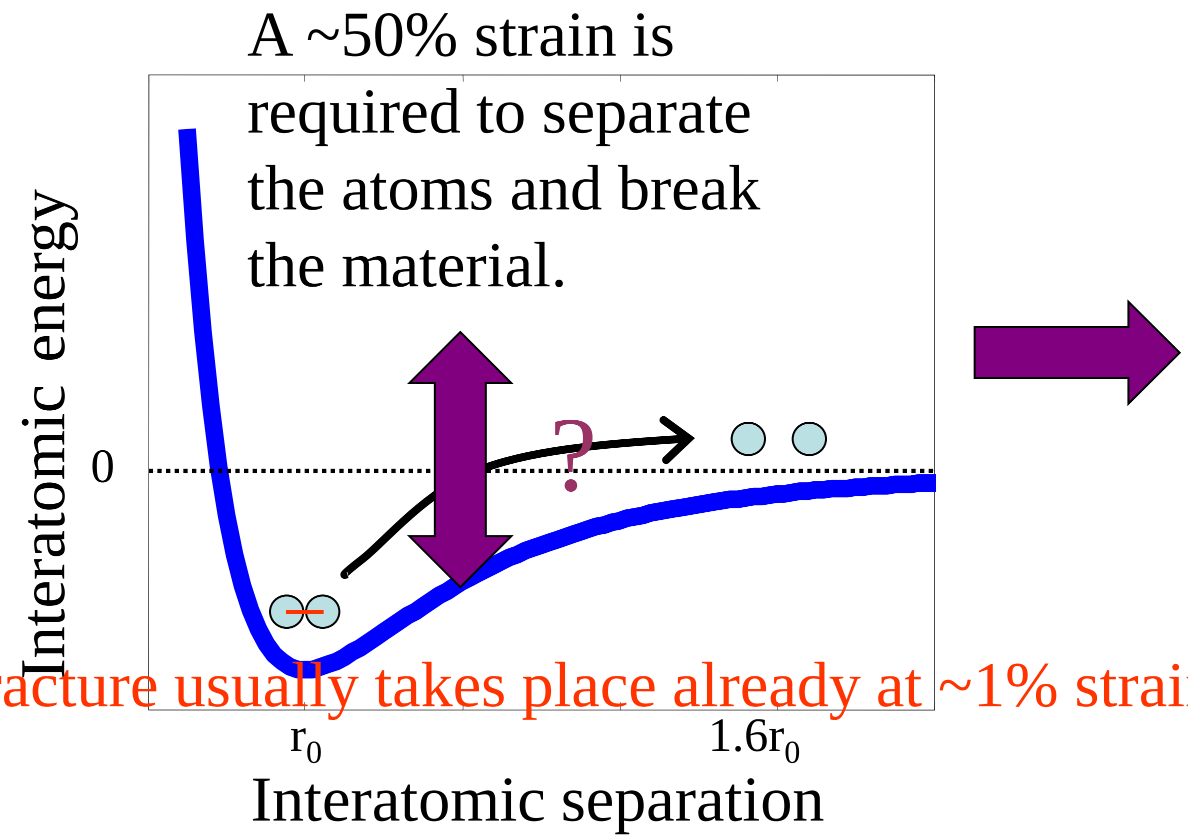
Assume that planes of atoms separate all at once





How Things Break – Naïve Approach

Atoms will dissociate when their binding energy is overcome



Fracture usually takes place already at ~1% strain.

A ~50% strain is required to separate the atoms and break the material.

Something is missing in this picture.

How Things Break – Crack Approach

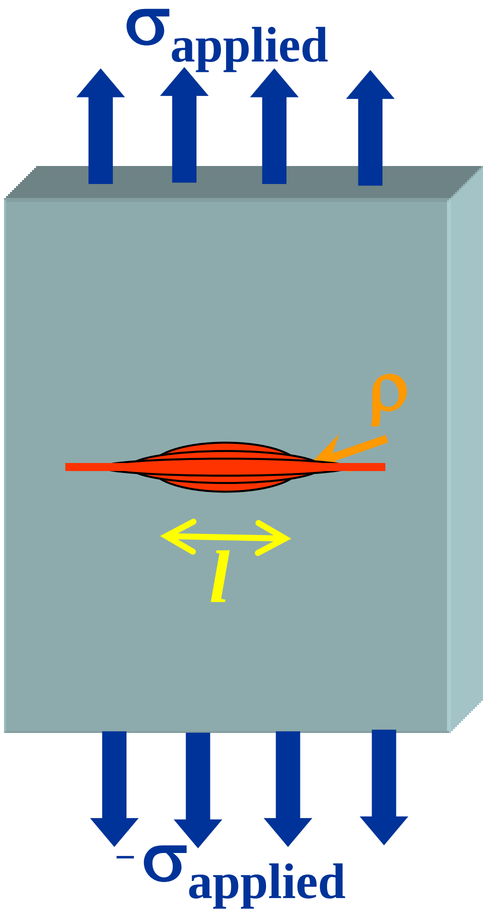
Consider now the same material, this time with an elliptical hole inside:

Inglis (1913)
(uniform applied stress σ) $\sigma_{max} = \sigma_{applied}(1+(2l/\rho))^{1/2}$

The crack acts as a stress "amplifier"

$$\sigma_{\rho \to 0} = \frac{K(\sigma_{applied}, l)}{r^{1/2}}$$

$K(\sigma_{applied}, l) \equiv$ "stress intensity factor"



- The presence of a crack induces a *stress singularity* at its tip
- Bonds are then preferentially broken at the tip of the crack.

The existence of a crack leads to a large decrease in the material's effective strength.

How Things Break – The (canonic) theory of fracture

Sharp cracks ($\rho \rightarrow 0$) + Linear elastic material response

Linear Elastic Fracture Mechanics (LEFM)

Solves the displacement/ stress/ strain fields in a linear elastic medium in the presence of a sharp crack.

Linear Elastic Fracture Mechanics (LEFM)

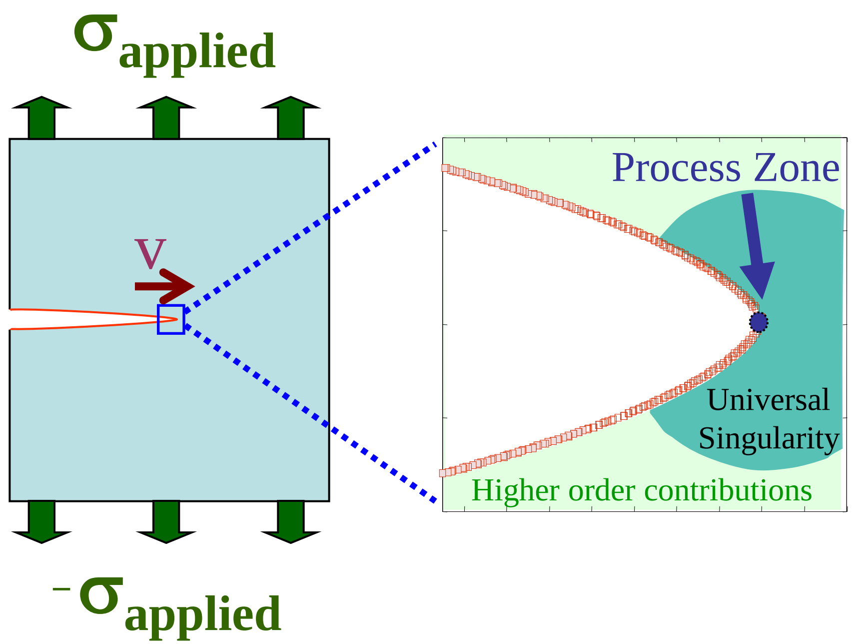
Main assumptions and results

1) Small strains (linear elastic) up to dissipation (the "process zone").

?

2) A universal singularity (K-dominance):

$$\sigma_{ij} = \frac{K(\sigma_{applied}, l; \mathrm{v})}{\sqrt{2\pi r}} \Sigma_{ij}(\theta; \mathrm{v}) + O(1)$$



- K is called the "stress intensity factor"
- Σ_{ij} is a universal function

3) Crack Stability:

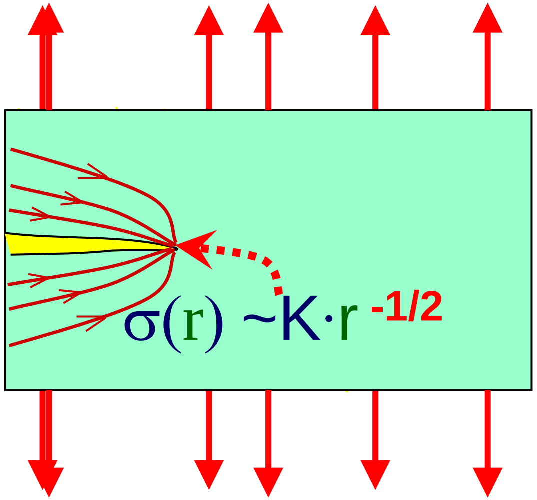
Energy flux to the crack tip = Energy dissipated (***Fracture energy***)

$$G = \Gamma$$

Γ is called the fracture energy and is a material property.

Material Strength:

Material	Theoretical Strength (GPa)	Practical Strength (GPa)	Practical/Theoretical Strength
Iron	50	0.3	0.006
Copper	24-55	0.2	0.005
Titanium	31	0.3	0.009
Silicon	45	0.7	0.01
Glass	37	0.4	0.01
Plexiglass	3	0.05	0.01



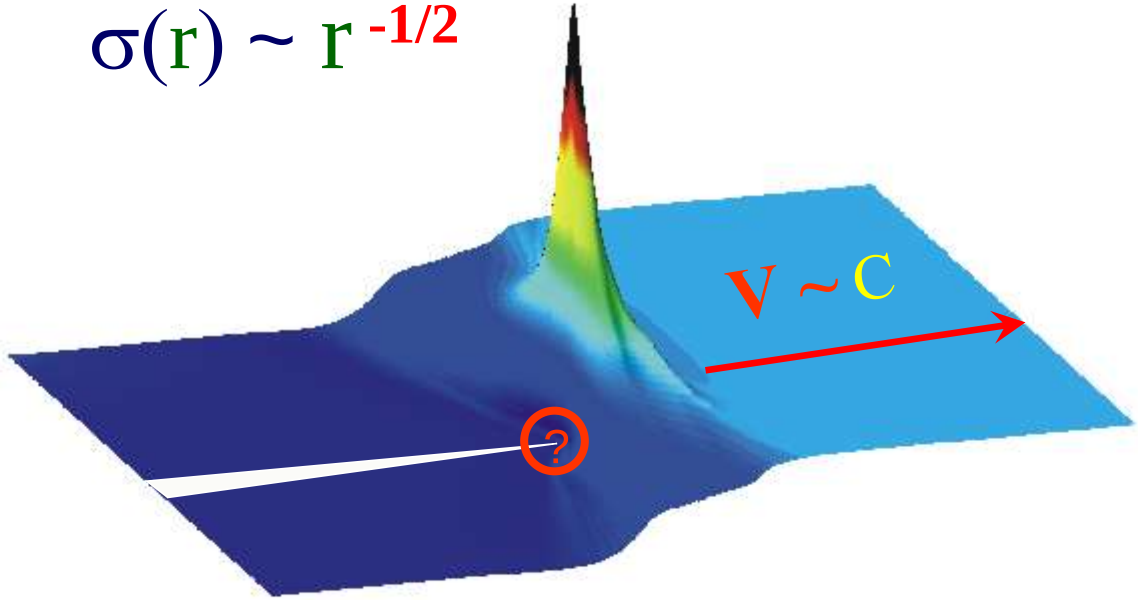
By **focusing** elastic energy into a **stress field singularity** at its tip.

Bonds are preferentially broken at the tip of a crack

The dynamics of propagating cracks are both interesting and difficult to study:

Linear Elastic Fracture Mechanics (LEFM) ⇔ materials are linearly elastic everywhere

$\sigma(r) \sim r^{-1/2}$



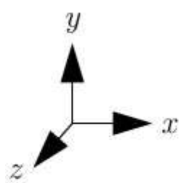
- Singular objects that propagate at v~information (sound) speeds
- Regularization of singularities ⇔ material properties

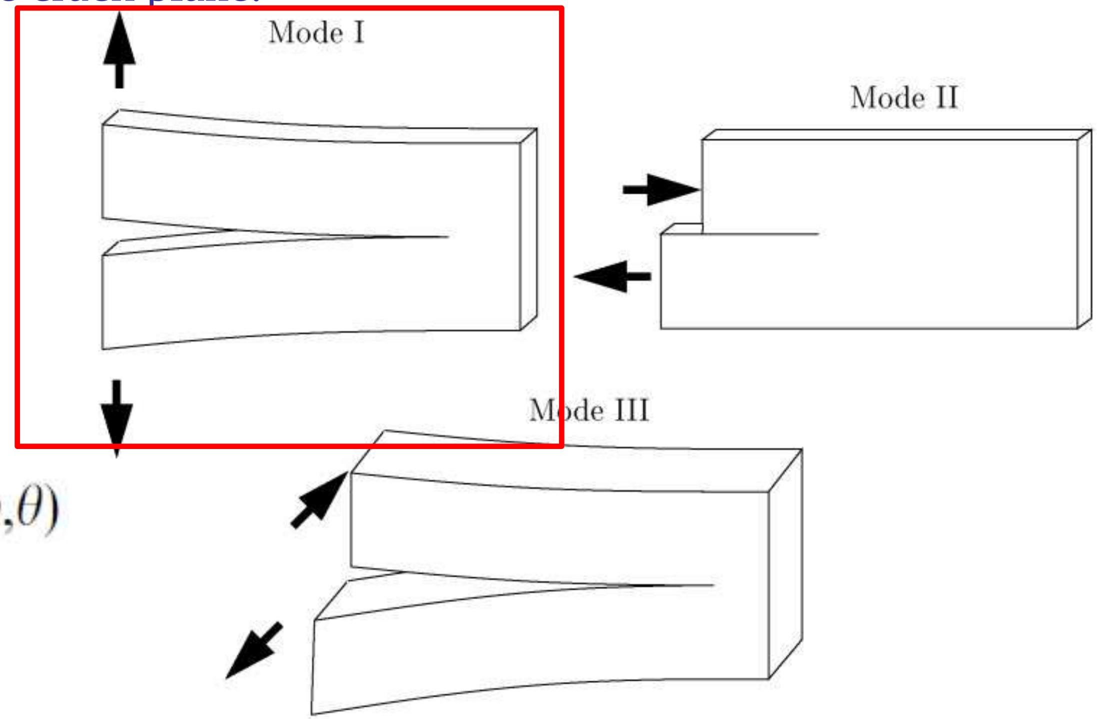
Ductile ?

Brittle ?

Symmetry of The LEFM solution

There are three conventional fracture modes , which are characterized by the symmetry of the loading on the crack plane.

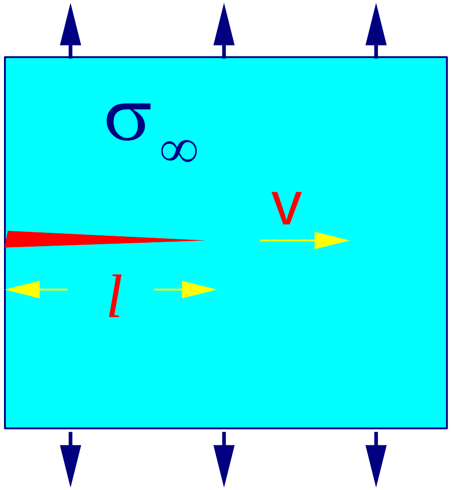




$$\sigma_{i,j} = \sum_{\alpha=1}^{3} K_{\alpha} \frac{1}{\sqrt{2\pi r}} f_{i,j}^{\alpha}(v,\theta)$$

Today we will concentrate on tensile loading ⇨ Mode I

Equation of Motion for an Infinite Plate (Mott 1948)



Assume: 1. Straight line motion
2. Γ= *Constant*
3. Energy is conserved

$$E_{total} = K(l) + P(l) + 2\Gamma l = const = P(l_0) + 2\Gamma l_0$$

use: $\vec{u} \propto l$

$$K(l,\mathrm{v}) = \frac{\rho}{2}\int (\partial_t \vec{u})^2 d^2x = \frac{\rho \mathrm{v}^2}{2}\int (\partial_x \vec{u})^2 d^2x = \boxed{\mathrm{v}^2 l^2 C_1}$$

$$P(l) = \int\left(\partial_\alpha \vec{u}_\beta \sigma_{\alpha\beta}\right) d^2x = \boxed{-l^2 C_2}$$

$$E_{total} = C_1 \mathrm{v}^2 l^2 + C_2 l^2 + 2\Gamma l = C_2 l_0^2 + 2\Gamma l_0$$

Fracture initiation (Griffith) condition:

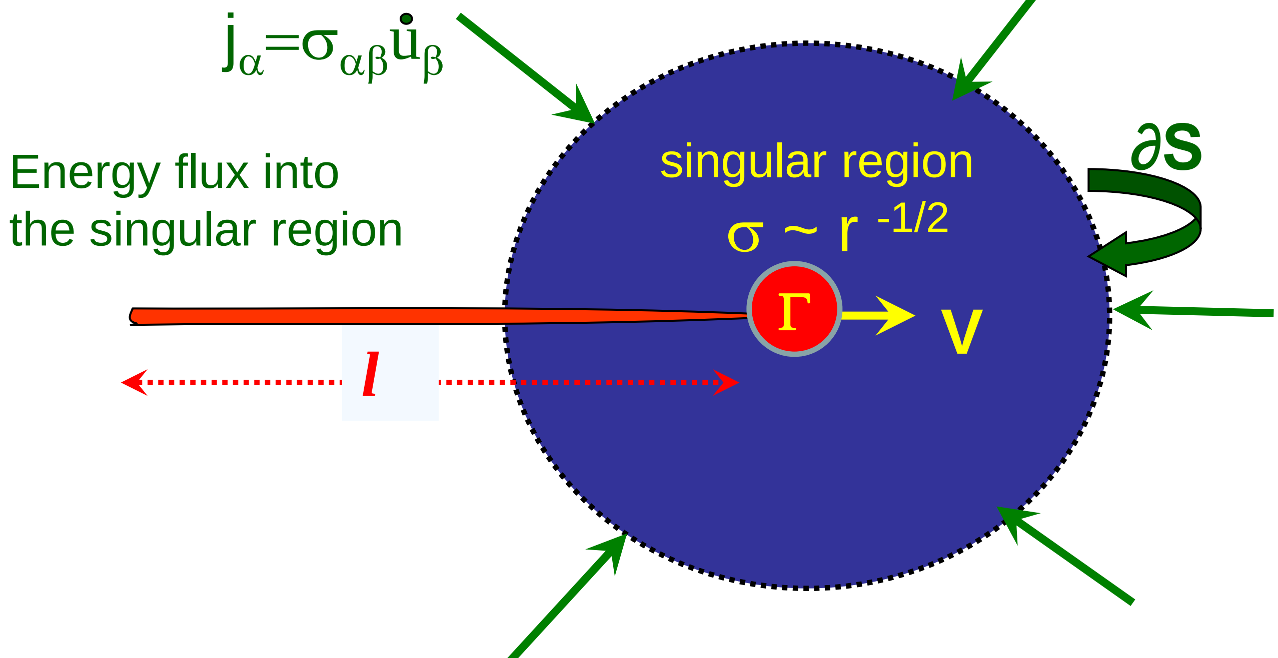
Released Pot. Energy= Energy to create surface

$$\left.\frac{-\partial P}{\partial l}\right|_{l_0} = 2\Gamma \Leftrightarrow C_2 l_0 = \Gamma$$

$$\mathrm{v} = Const \cdot \left(1 - {l_0}/{l}\right) \quad \Longrightarrow \quad \mathrm{v} = V_{Rayleigh} \cdot \left(1 - \frac{Const \cdot \Gamma}{\sigma_\infty^2 \, l}\right)$$

(L. B. Freund 1972)

Local energy balance ➔ the same equation of motion for a crack:



Geometry and Loading specific:
elastic energy released by a static crack

Dissipation
"fracture energy"

$$\text{Total Energy flux} = \oint_{\partial S} j_\alpha n_\alpha = G(l,\sigma)\cdot(1-v/V_R) = \Gamma$$

Universal kinematic term

Energy into crack tip = dissipation at the tip

$$\boxed{v = V_R[1-\Gamma/G(l,\sigma)]}$$

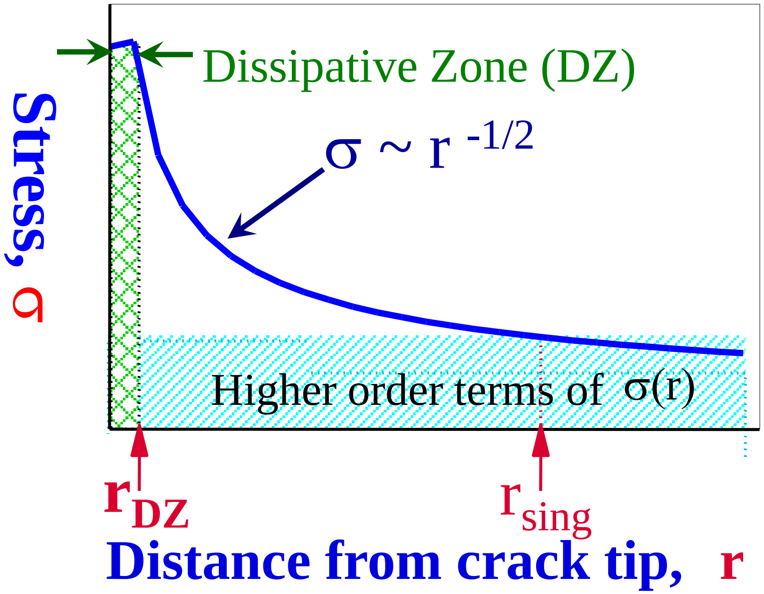
Freund, Eshelby, Kostrov ,Willis ~ 1972

Characteristics of the Equation of motion

$$v = V_R \cdot (1 - \Gamma / G(l, \sigma))$$

Conditions:

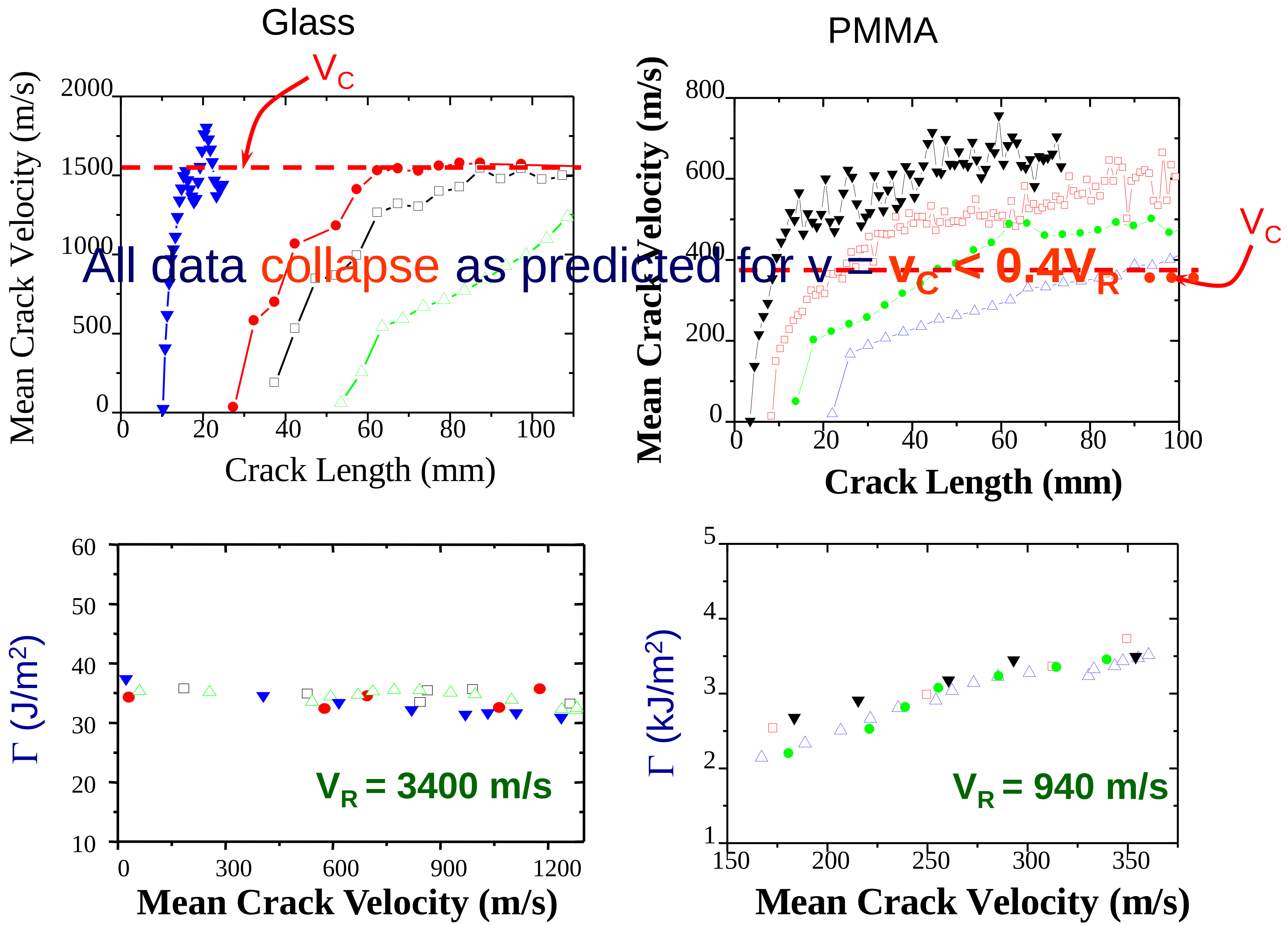
- Linear elasticity until Dissipative Zone
- Single crack
- $r_{DZ} << r << r_{sing}$
- Infinitely large medium – no "history" dependence or ***memory***



Characteristics:

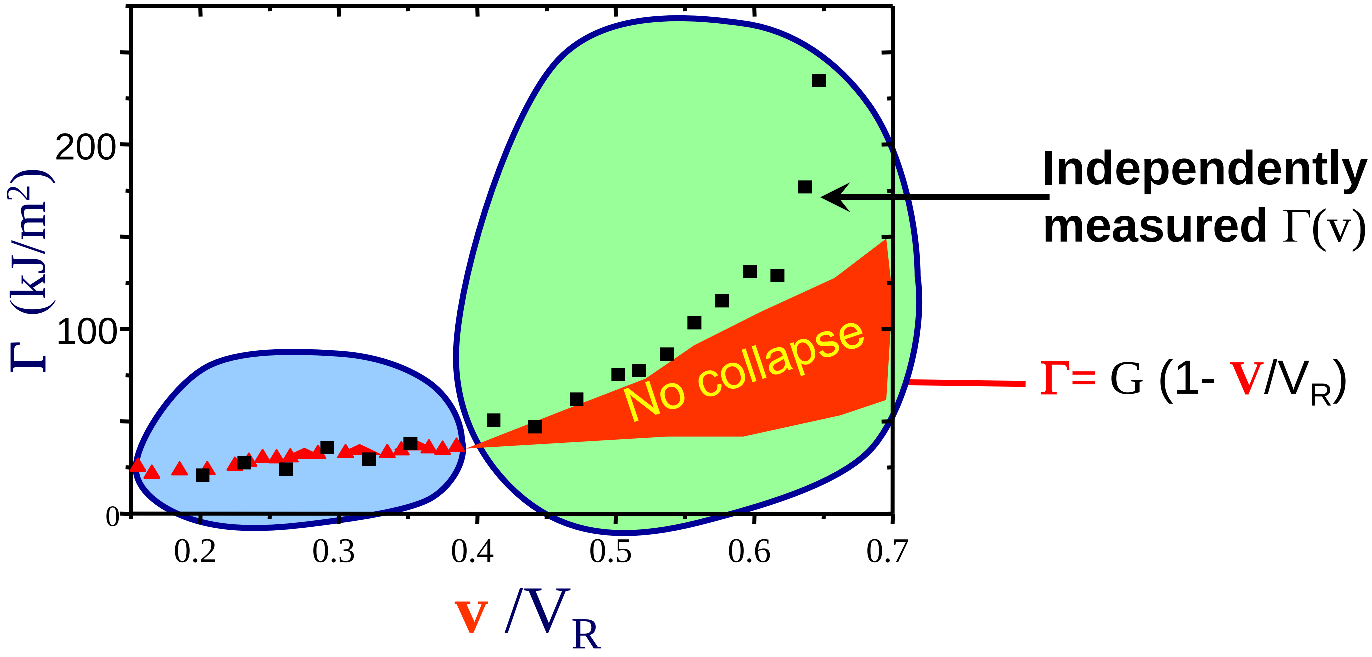
- V_R = Asymptotic crack speed
- V is first order in time => a crack has no inertia
  => it should jump instantaneously to the value dictated by Γ and G

Testing the equation of motion: $v = V_R \cdot (1 - \Gamma / G(l,\sigma)) \Leftrightarrow \Gamma = G(l,\sigma)(1 - v/V_R)$



E. Sharon and J. F. Nature **397**, 333 (1999)

What happens for higher crack velocities?

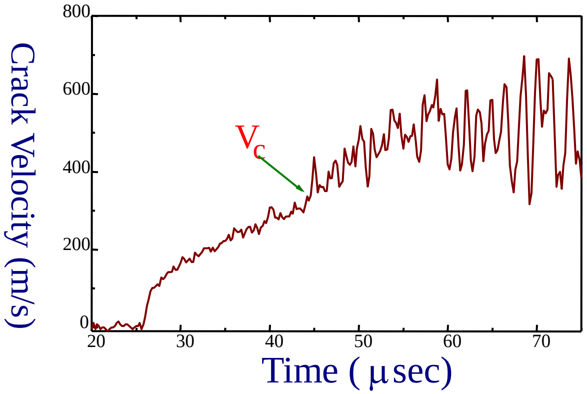


For **v < 0.4V$_R$** excellent agreement with the equation of motion

When v > V$_R$ **something is wrong!**

The explanation:

The Micro-Branching Instability



At a critical velocity, V_C, the motion of a crack becomes unstable

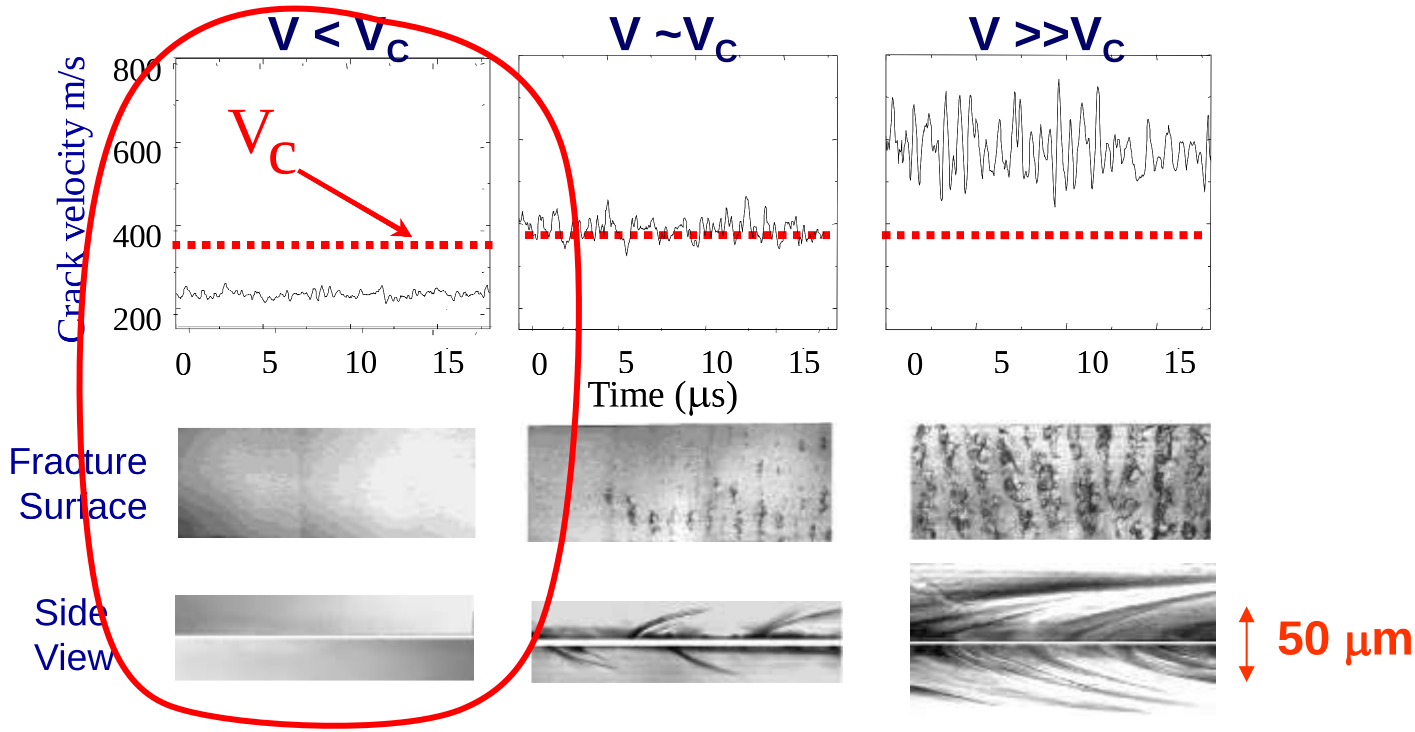
At this point:

Velocity oscillates

Structure is formed on the fracture surface

Instability Mechanism :

Local (micro-) crack branching



The equation of motion holds only for single cracks!

Theoretical approaches to the instability

Cracks are frustratingly stable to any perturbation – no instability observed analytically
(*J. Langer, M. Marder, J. R. Rice, M. Ada-Bedia, M. Ben-Amar,I. Procaccia….)*

Crack instability similar to experiments observed when studied on a lattice as well as in MD, finite element simulations, and phase-field models
(*M. Marder, D. Kessler, H. Levine, L. Sander, A. Needleman, F. Abraham, P. Gumbsch, H. Gao, Vashishita, Kalia, Ortiz,* …)

In both finite element and lattice calculations :

Micro-branching goes away when grid sizes are taken to zero!
(*M. Marder, M. Falk, A. Needleman*…)

To understand fracture - we need to know what is happening within/around the "dissipative region" in the vicinity of a crack's tip!!

- Empirical ways to model the dissipative zone
  - Phase field models (*I. Aranson, D. Kessler, H. Levine, A. Karma, H. Henry, V. Hakim* …
  - Cohesive Zone models (*Barenblatt, Dugdale, Needleman, H. Gao, Ortiz,…*
- Attempts to develop a fundamental theory of amorphous materials
  - (*J. Langer, M. Falk, I. Procaccia, E. Bouchbinder, J. L. Barrat, A. Lemaitre*…

Why is fracture such a hard problem to "break"?

Basically a wide range of time and length scales are coupled…

Rapid propagation:

Material	V_R (m/s)
Glass	3300
PMMA	930
Homolite	900
Brittle steels	3500

Small length scales:

Material	Dissipative zone size
Glass	1-10 nm
PMMA	< 1-5 μm
Homolite	1 - 10μm
. . .	

Can we (experimentally) circumvent these difficulties?

“Yes we can!” – break “Jello”!

Fracture of polyacrylamide gels: sound velocity reduced by **2-3 orders of magnitude**

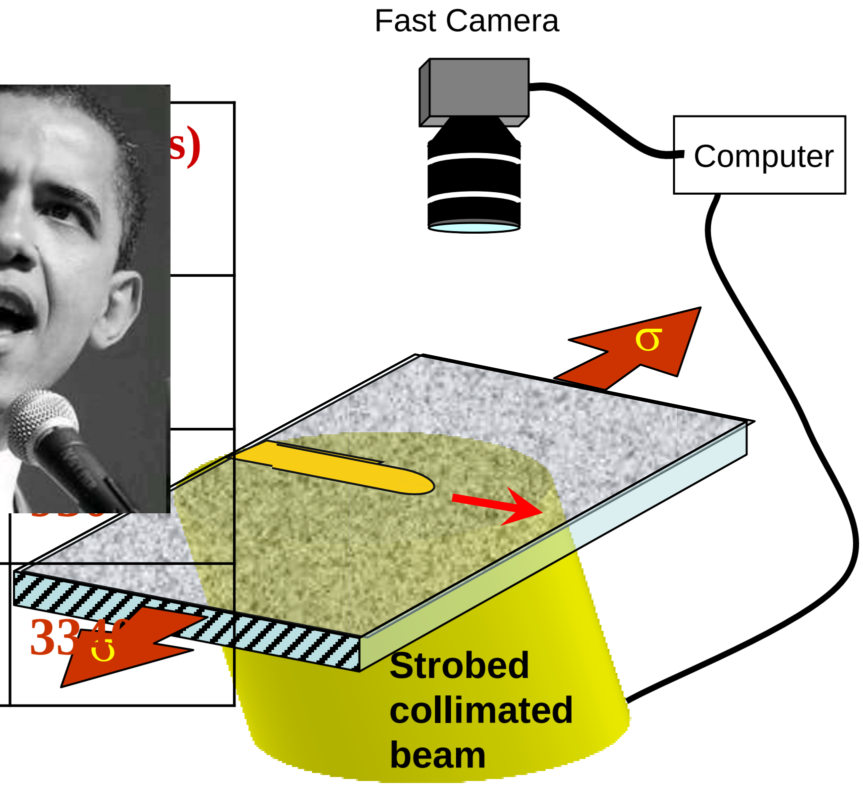
High speed visualizatio… measurements of:

- *Crack tip opening pro…*
- *Displacement field a…*

(by “particle tracking” i…

Material	**Y… M… (k…**		**…s)**
Gel 13.8 % acrylamide 2.6% bis-acrylamide	10…		
PMMA	3…		**33…**
Soda-Lime glass	70,000,000	0.22	**334…**

Visualization system



Sample dimensions: **150 × 150 × 0.1 mm**

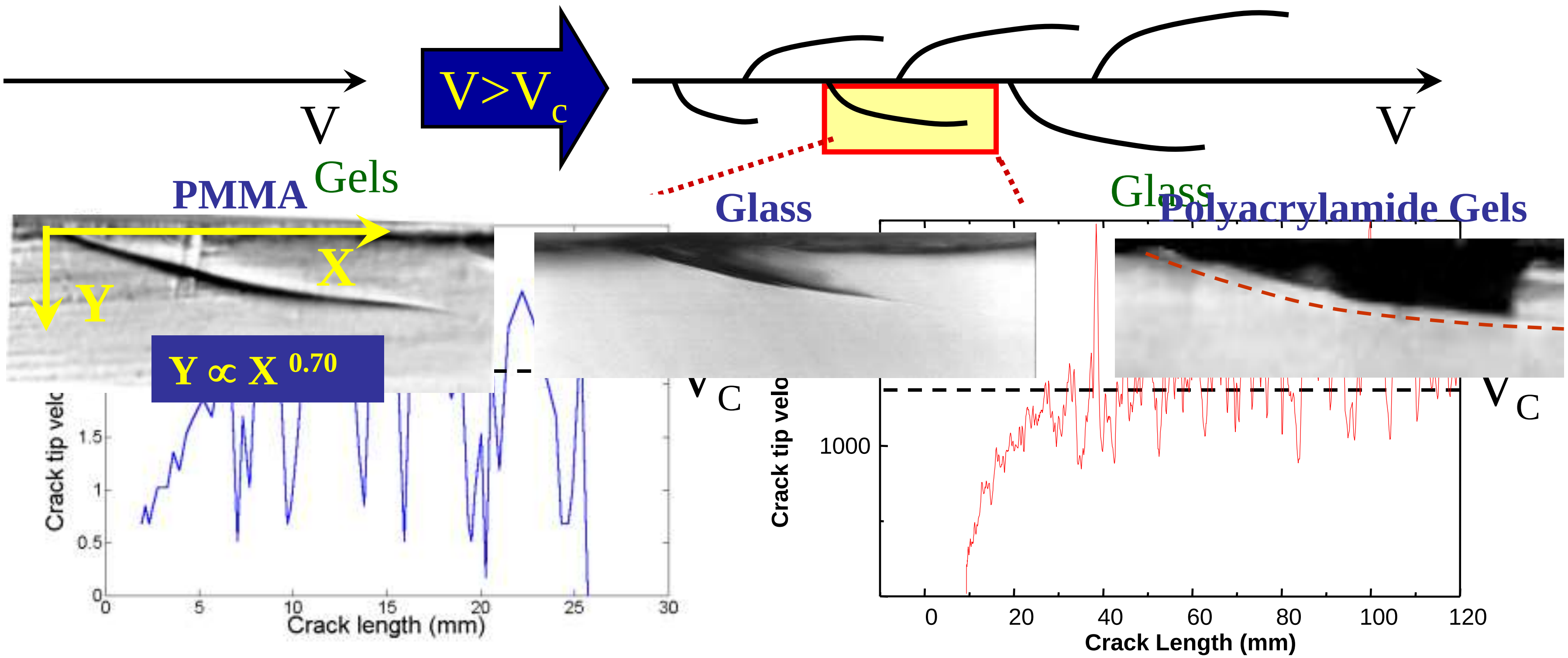
Frame rate: **5000 frames/sec**

(~1 frame per 0.5-1 mm crack advance)

Brittle fracture in Gels is **identical** to that of other brittle amorphous materials:

Micro-branching in Gels:

- At a critical velocity a single crack becomes unstable to frustrated micro-branches
- Microbranches have the ***same functional form*** as in other brittle materials

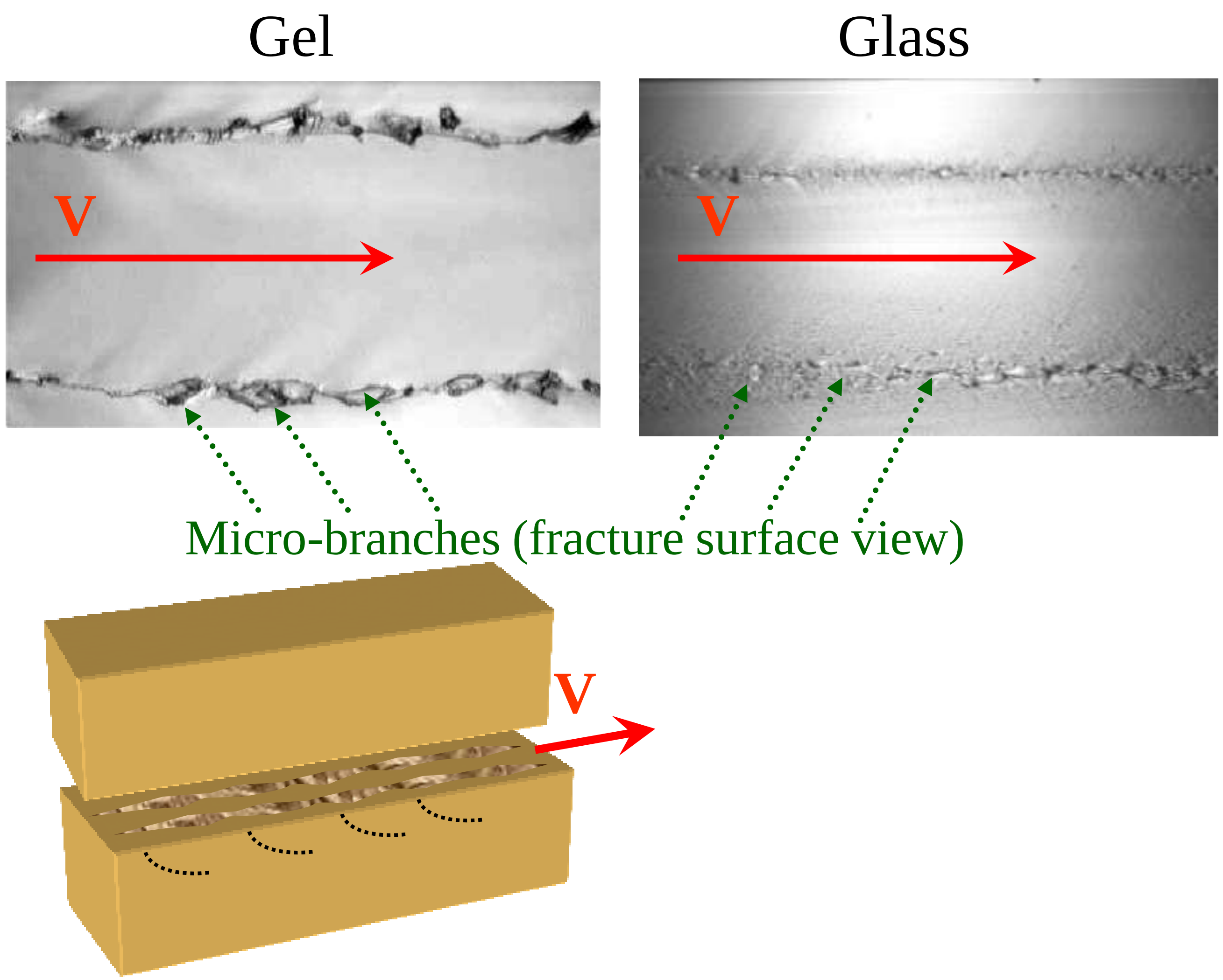


A. Livne, G. Cohen and J. F., Phys. Rev. Lett. **94,** 224301 (2005)

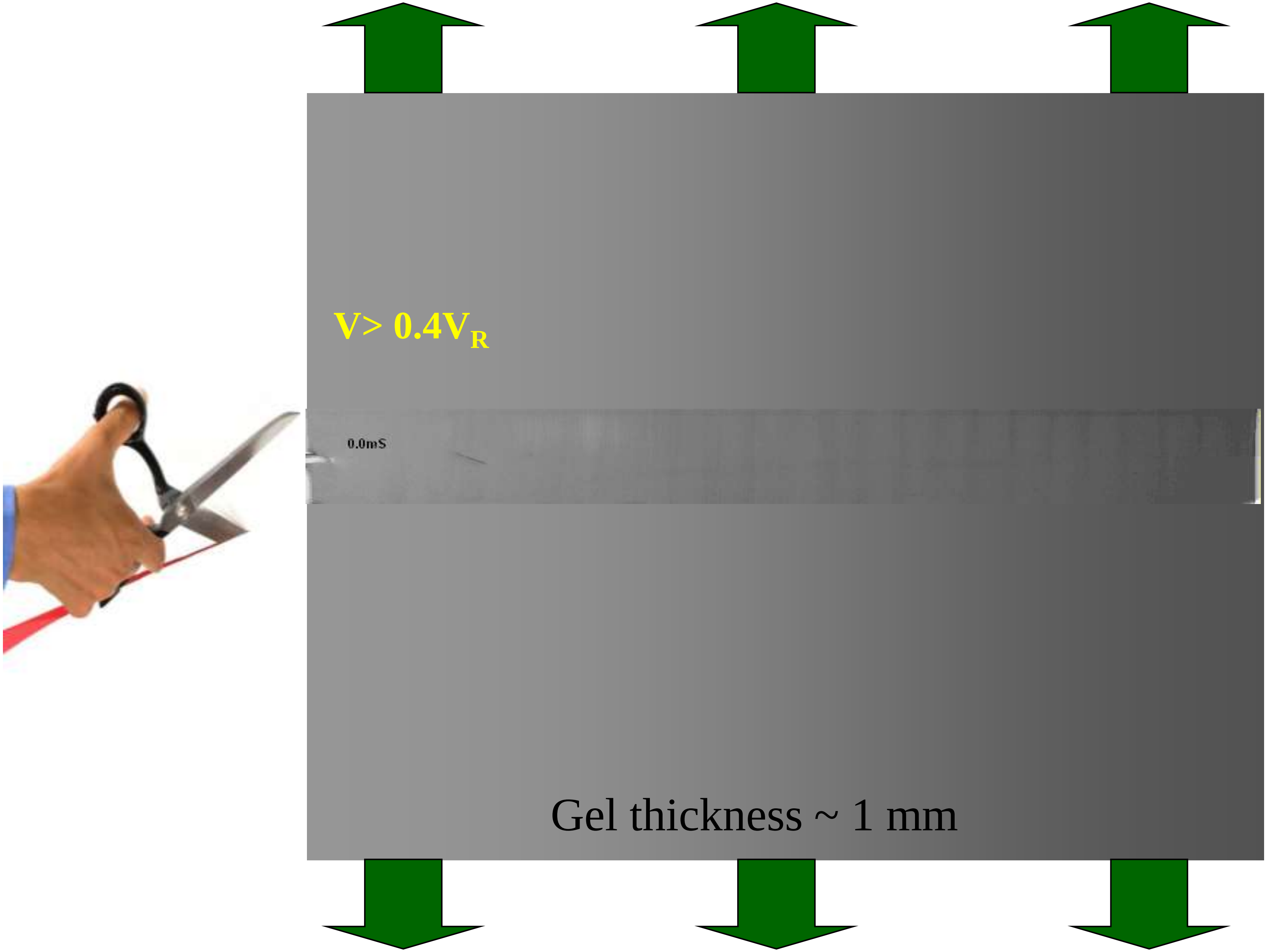
A. Livne, O. Ben-David, and J. F., Phys. Rev. Lett. **98**, 124301 (2007)

“3D” behavior

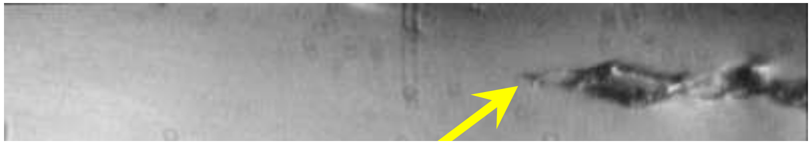
In both Gels and Glass micro-branches are formed in directed lines aligned parallel to the propagation direction.



Micro-branching limits V to ~0.5V_R



Suppression of micro-branching ➔ ??

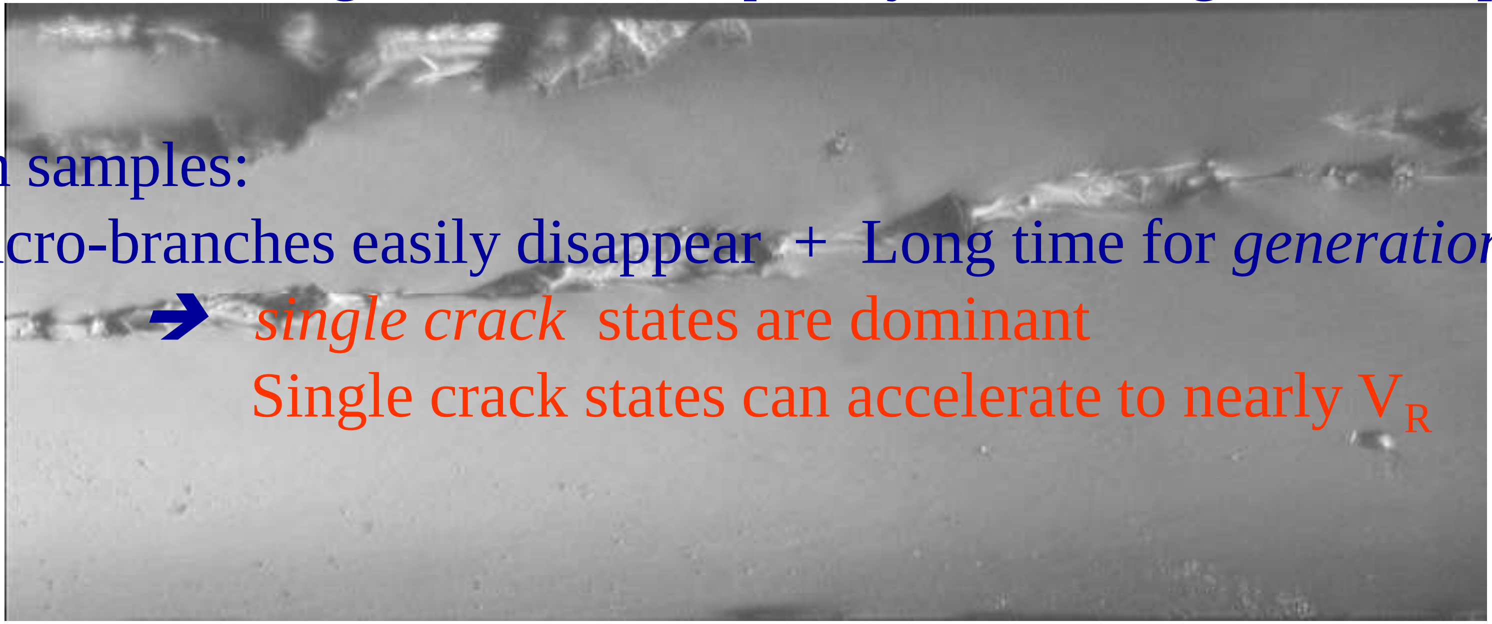


Micro-branches (in gels):

- are *noise-triggered*
- Have a *minimum width, ΔZ*
- *Disappear* when reaching a sample edge

When the sample width, h → ΔZ

- Number of effective noise sources reduced as $h/\Delta Z \rightarrow 1$
- Branch-lines that *are* generated will quickly find an edge and disappear



➔For thin samples:

Micro-branches easily disappear + Long time for *generation*

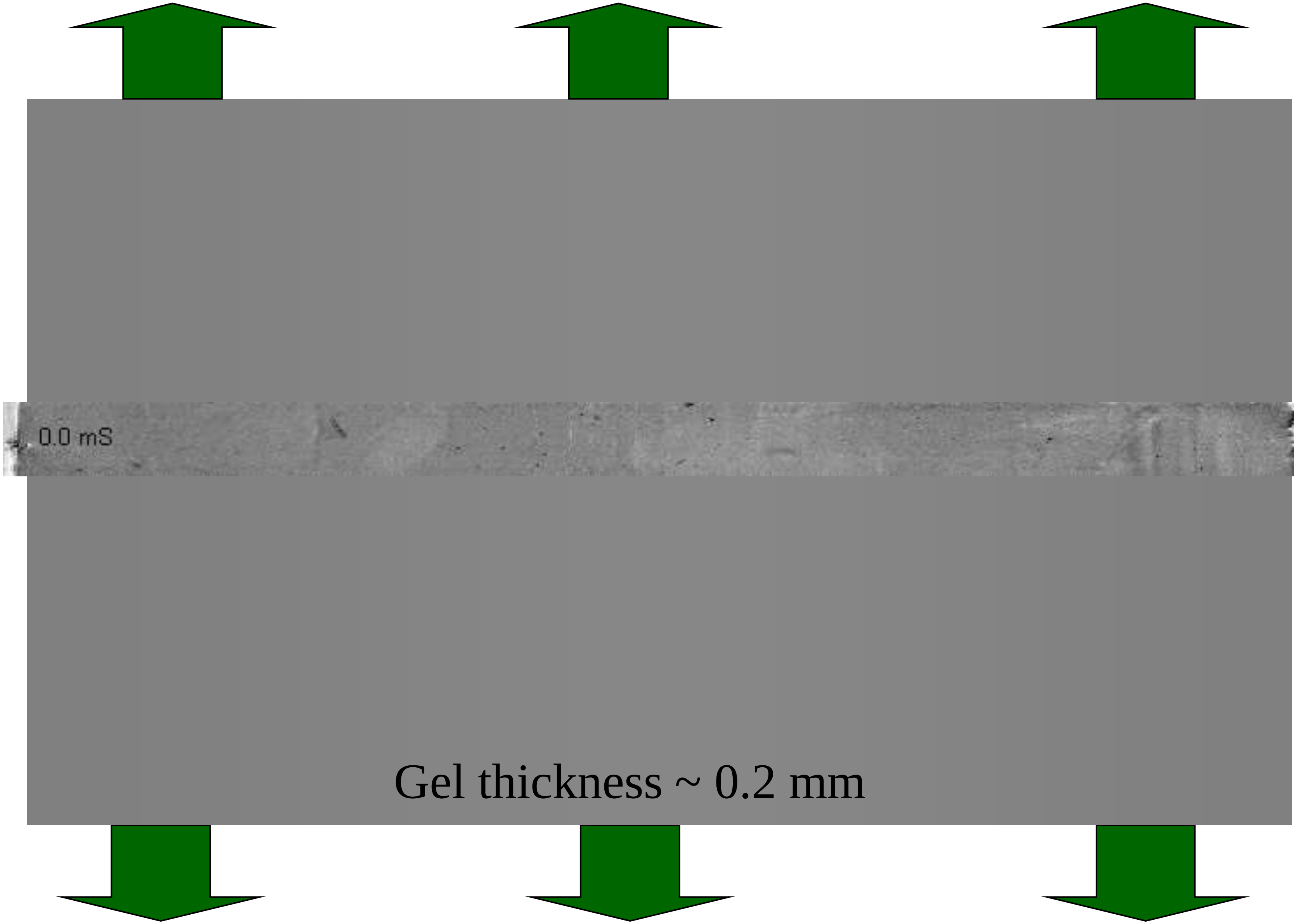
➔ *single crack* states are dominant

Single crack states can accelerate to nearly V_R

For very thin gels (thickness ~ minimum micro-branch width):

we can ***suppress*** the instability…

A **new** oscillatory instability appears at v ~ $0.9V_R$!



As a first step in understanding all of this complex stuff:

Let's take a closer look at "simple" cracks

- Suppress instabilities (making the material's thickness ~ 100μm)
  Compare to **equation of motion** for $v < 0.9C_S$

- Measure the structure of the ***near crack-tip region***
  PIV ⇔ ***measure*** the ***singular*** displacement field

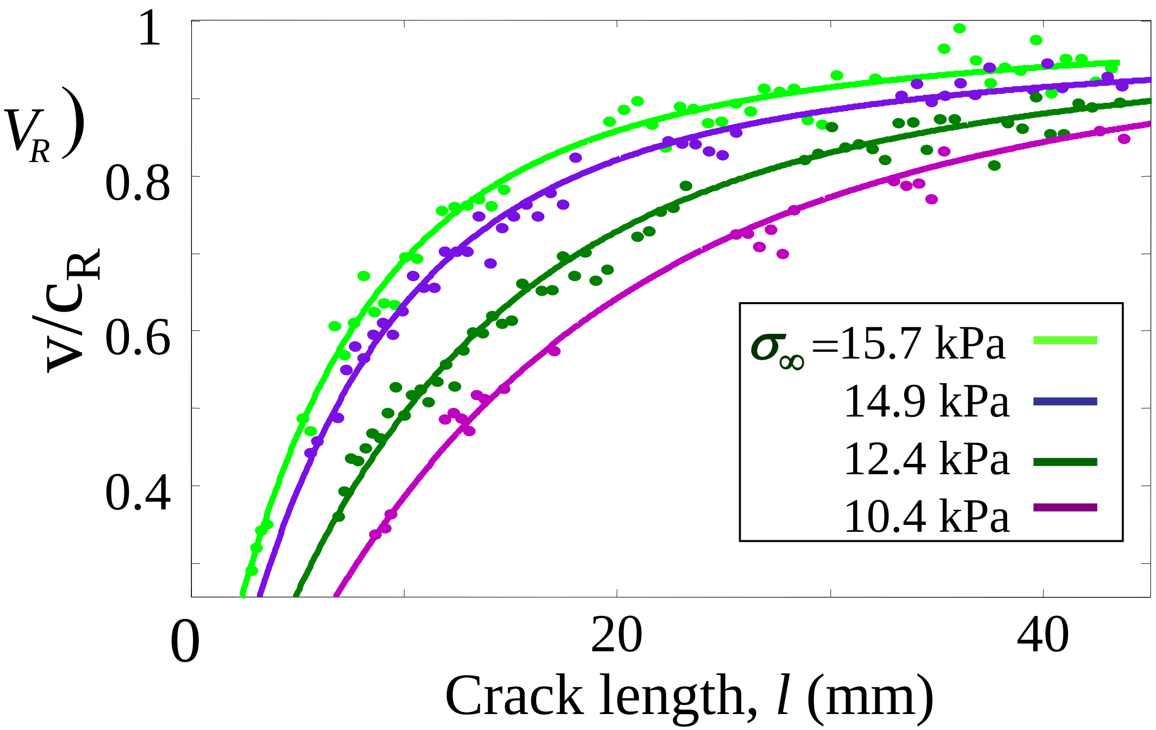
Let's check the equation of motion for ***simple cracks***: G=Γ

Simple crack in an ***infinite medium*** for constant stress, σ:

$$v = V_R \cdot (1-\Gamma / G(l,\sigma)) \Leftrightarrow \Gamma = G(l,\sigma)\,(1-v/V_R)$$

$$\Gamma = G \approx \frac{1-\nu^2}{E}\frac{8l}{\pi}\sigma_\infty^2\left(1-v/V_R\right)$$

L.B. Freund (1990)



The equation of motion works perfectly for an ***infinite system!***

Does the (Eq. motion) **G=Γ** work for ***different*** geometries?

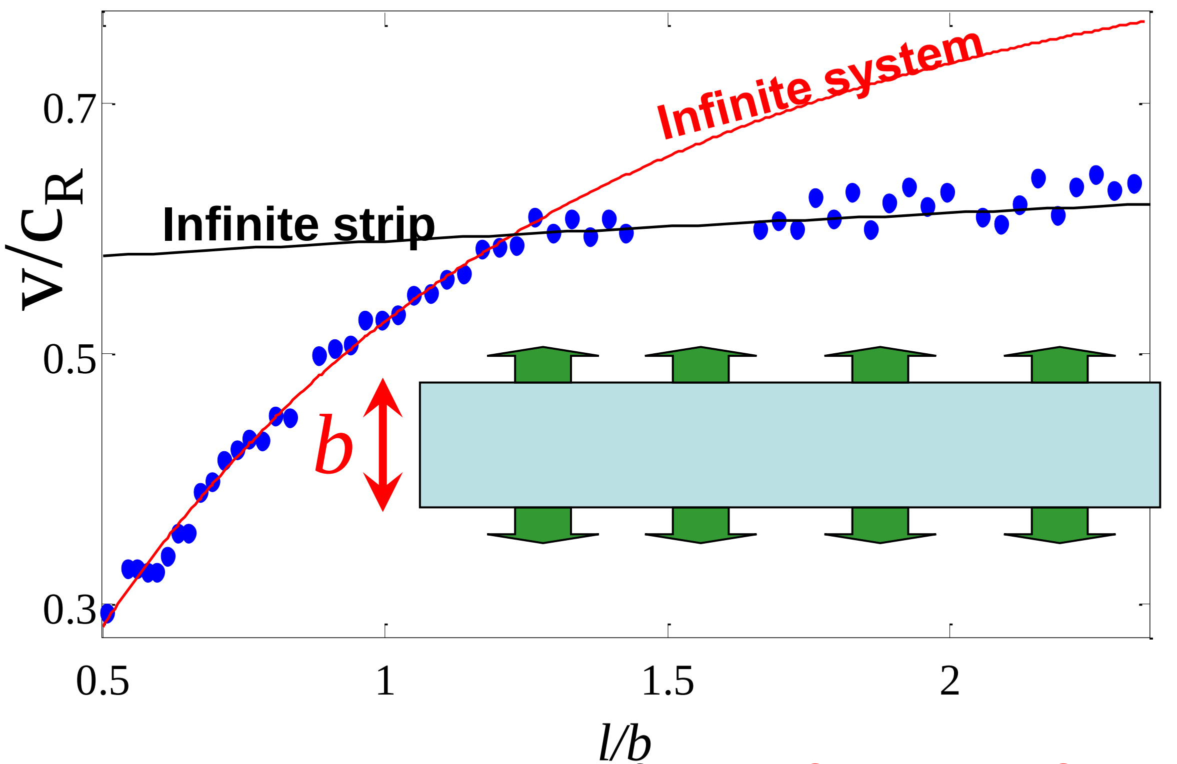
T. Goldman, A. Livne, J. Fineberg, Phys. Rev. Lett. **104**, 1144301(2010).

Under different loading conditions: Cracks in an ***infinite strip***

M. Marder, Phys. Rev. Lett. **66**, 2484 (1991).

$$\Gamma = G = W\,[1-\dot{v}\cdot \boldsymbol{f(v)}] \sim W\left[1-\frac{\dot{v}\cdot b}{c_l^2\underbrace{[1-(v/c_R)^2]^2}_{\text{Effective } Mass\ \to\ \infty \text{ as } v\ \to\ c_R}}\right]$$

➔cracks attain a ***singular inertia*** as $v \to c_R$



Dynamics change when cracks feel the ***finite size of system***

But Γ=G still describes the equation of motion of a simple crack!

T. Goldman, A. Livne, J. Fineberg, Phys. Rev. Lett. **104**, 1144301(2010).

Energy Balance ⇔ excellent ***agreement*** with LEFM predictions…

What more do we need?

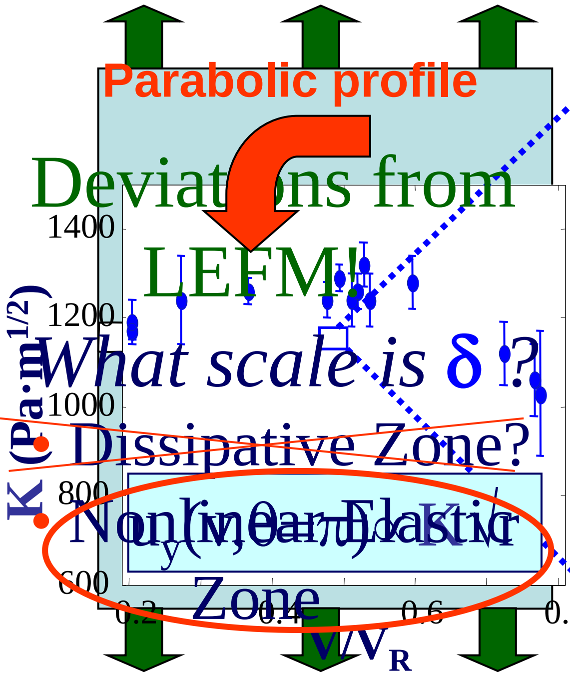
While we are already here…

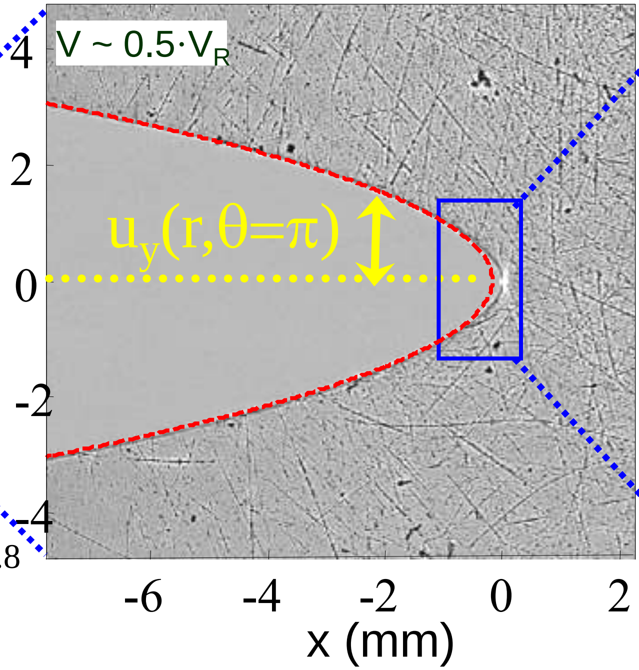
Let's look at the ***predictions for the singular fields*** at ***crack tips***

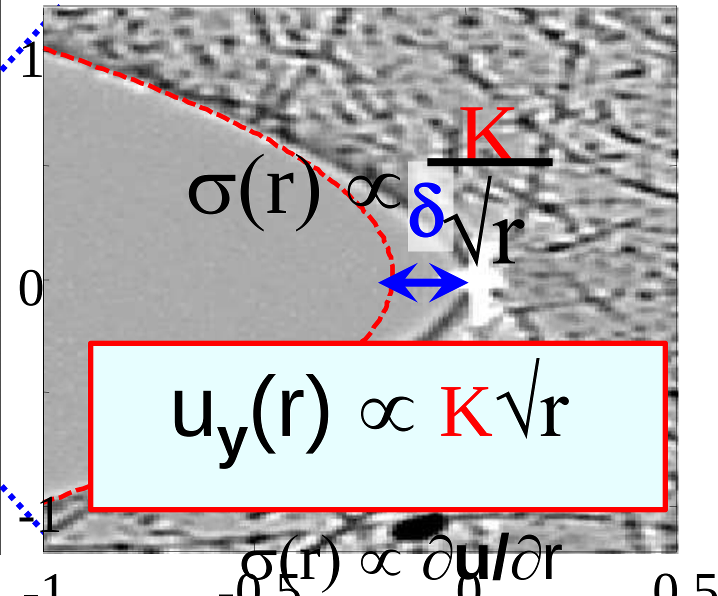
$$\boldsymbol{\sigma}_{\mathbf{r \to 0}} = \frac{K(\boldsymbol{\sigma}_{\text{applied}},\ l\ )}{\mathbf{r}^{\mathbf{1/2}}}$$

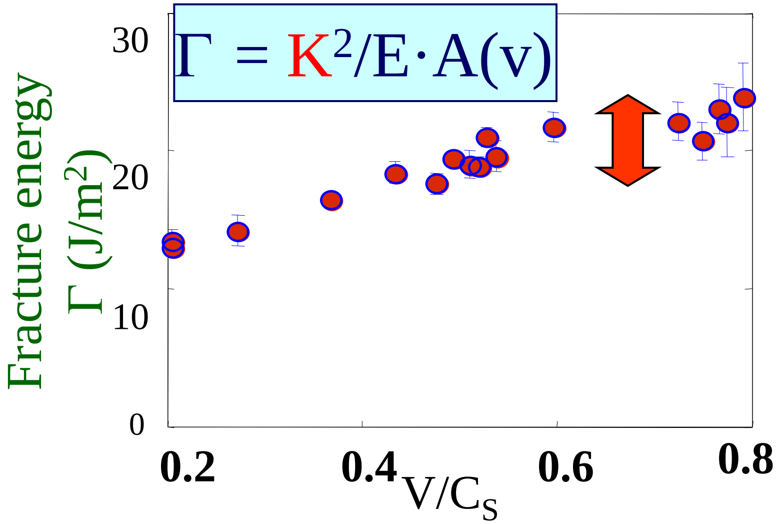
$K(\boldsymbol{\sigma}_{\text{applied}}, l) \equiv$ "stress intensity factor"

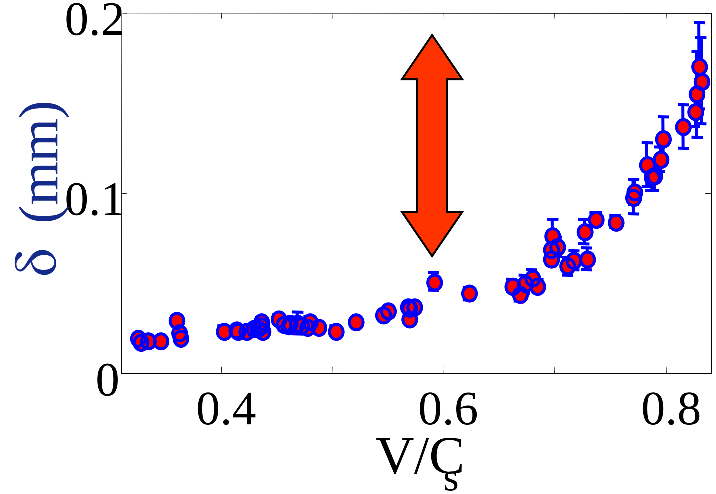
Crack tip shape: *a look at the fields surrounding the crack tip*



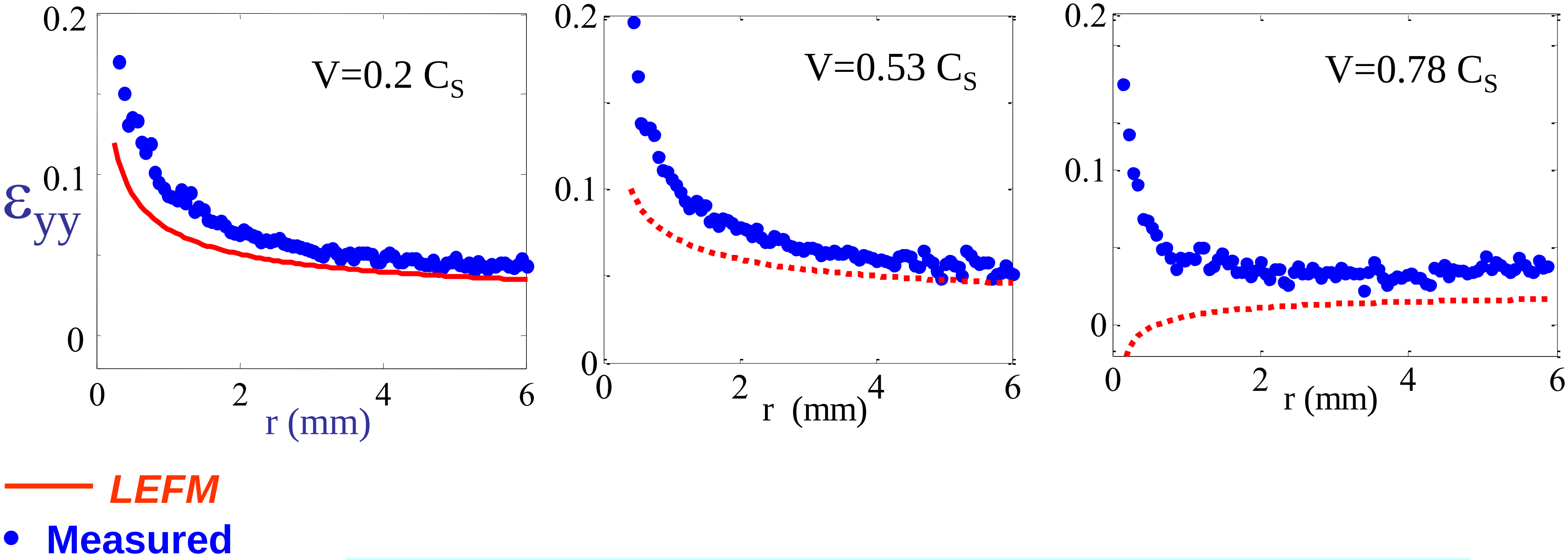








Comparison with LEFM – strain field: $\varepsilon_{yy}(r,\theta=0)$

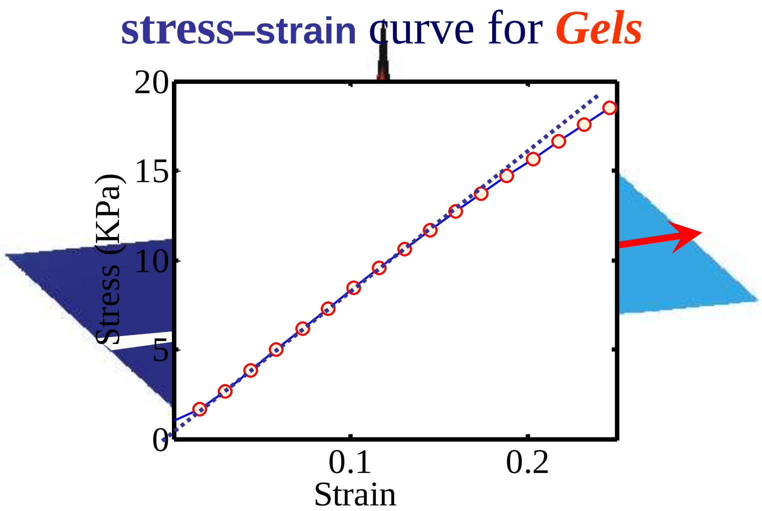


Approaching the tip takes us <u>Beyond LEFM!</u>

- **Nonlinear zone** surrounding crack tip *distortion* of crack-tip profile
- $u_x \Leftrightarrow K$; $u_y \Leftrightarrow K'$; **$K \neq K'$**
- ε_{yy} is **far** from LEFM predictions

Sketch of a New Theoretical framework near the crack tip:

Near a singularity ➔ high strains ➔ no material is linearly elastic!



In gels:

1. Stress-strain nonlinearity is ***known***
2. Nonlinearity is ***significant*** at ***moderate*** (10-20%) strains:

➔ ***Extend*** LEFM to weakly *nonlinear elasticity*

A. Livne, E. Bouchbinder and J. F., Phys. Rev. Lett. **101,** 264301 (2008)

E. Bouchbinder, A. Livne and J. F., Phys. Rev. Lett. **101,** 264302 (2008)

E. Bouchbinder, A. Livne and J. Fineberg,, JMPS **57**, 1568-1577 (2009)

Idea of the Theory

Perform a controlled expansion in Δ of the *fields* and *constitutive* relation

When do nonlinearities become ***relevant***?

$\sigma/\mu = \varepsilon - \varepsilon^2 + \ldots$ $\sim O(0.1)$

$$\vec{u}(r,\theta) = \boxed{\Delta\,\vec{u}^{(1)}(r,\theta}$$

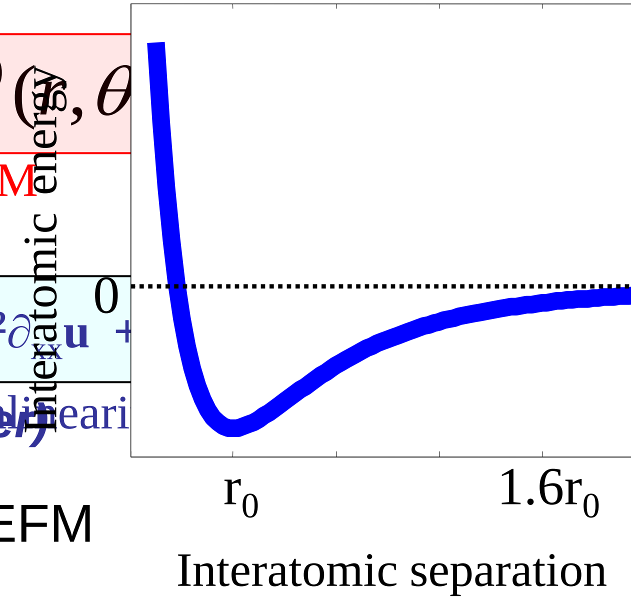
LEFM

Ratio of nonlinear/linear term ~ $\varepsilon \sim \varepsilon_{LEFM} \sim \frac{K}{2\mu(2\pi r)^{1/2}}$

Solve $\nabla\cdot\sigma = \rho\partial_{tt}\mathbf{u} = \rho\,\mathbf{v}^2\partial_{xx}\mathbf{u}\ +$

Define a critical strain, Δ, **delineating** the importance of nonlineari

(order by order)



Order Δ problem = LEFM

Order Δ^2 problem:

$\Delta = \frac{K}{4\ldots}$

$$\mu\nabla^2\vec{u}^{(2)} + 3\mu\nabla(\nabla\cdot\vec{u}^{(2)}) + \frac{\mu\,\ell\,\vec{g}(\theta;v)}{r^2} = \rho v^2\partial_{xx}\vec{u}^{(2)}$$

+ B.C.'s (zero stress crack faces) + loading

inhomogeneous term has the generic form:

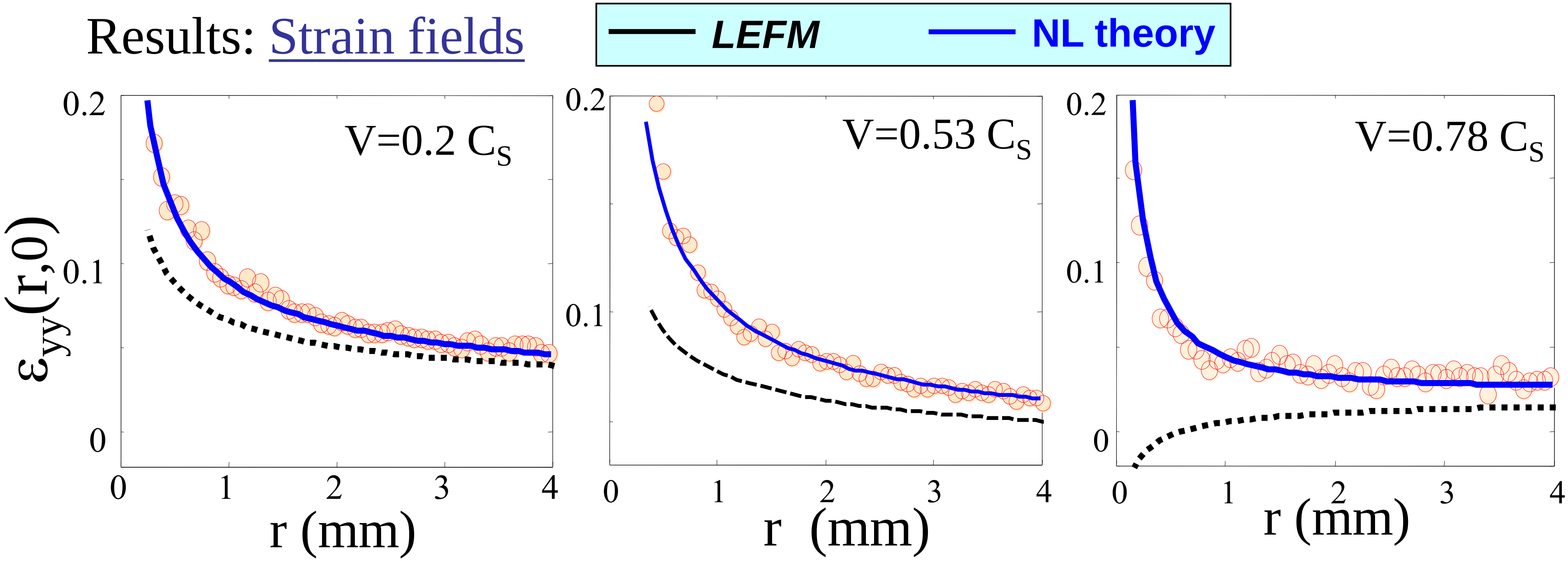
$\partial(\partial u^{(1)}\,\partial u^{(1)})$

Nonlinearity b…

The length *sca…* ***…mically***

(in gels…

Results: Strain fields



- Excellent (no free parameter) agreement with measurements
- Leading strain nonlinearity is 1/r !

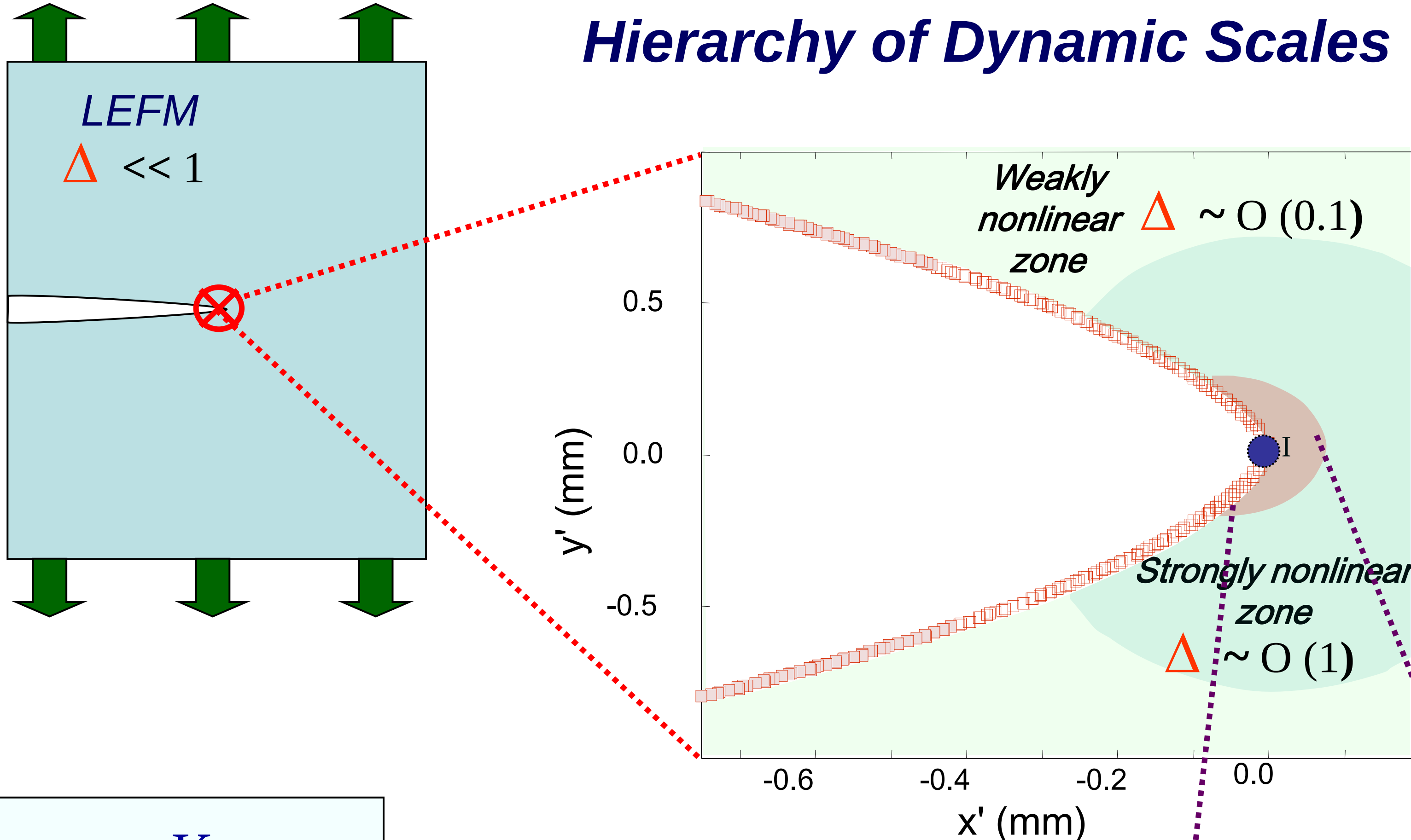
$$\varepsilon_{yy}(r,0,v) = \boxed{\frac{C_1 K}{r^{1/2}}} + \boxed{\frac{C_2 K^2}{r}} + C_3 \cdot T/E$$

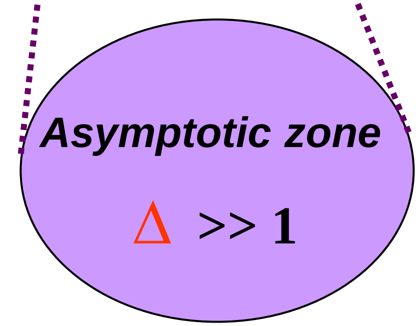
LEFM NL term

- A ***dynamic length-scale*** exists (no length-scale in LEFM)

$$l_{nl}(v) \propto \Gamma/E$$

Hierarchy of Dynamic Scales

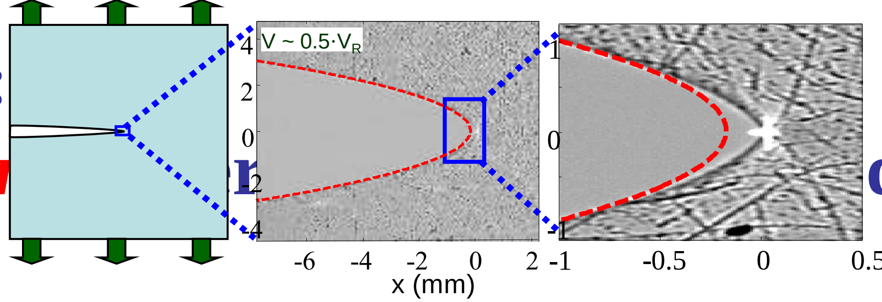


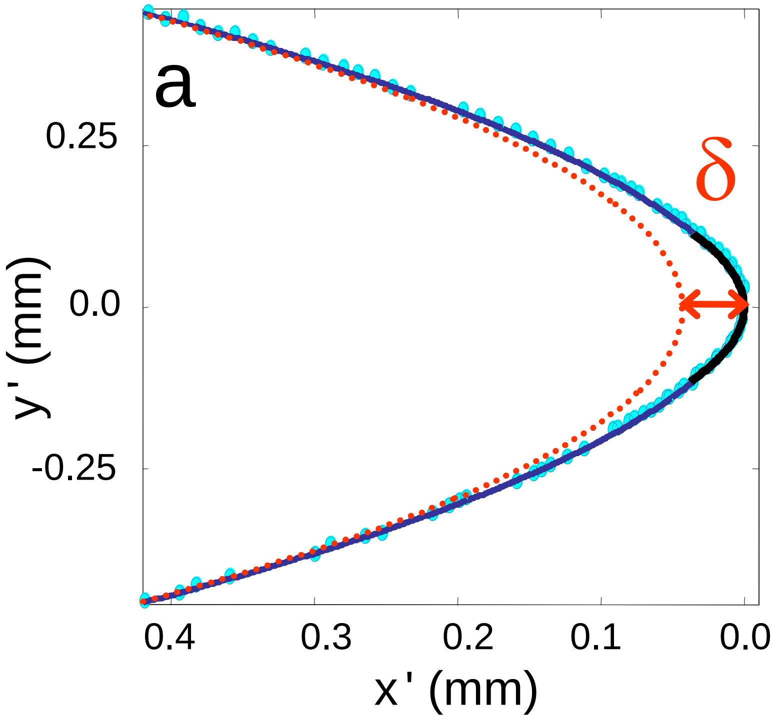


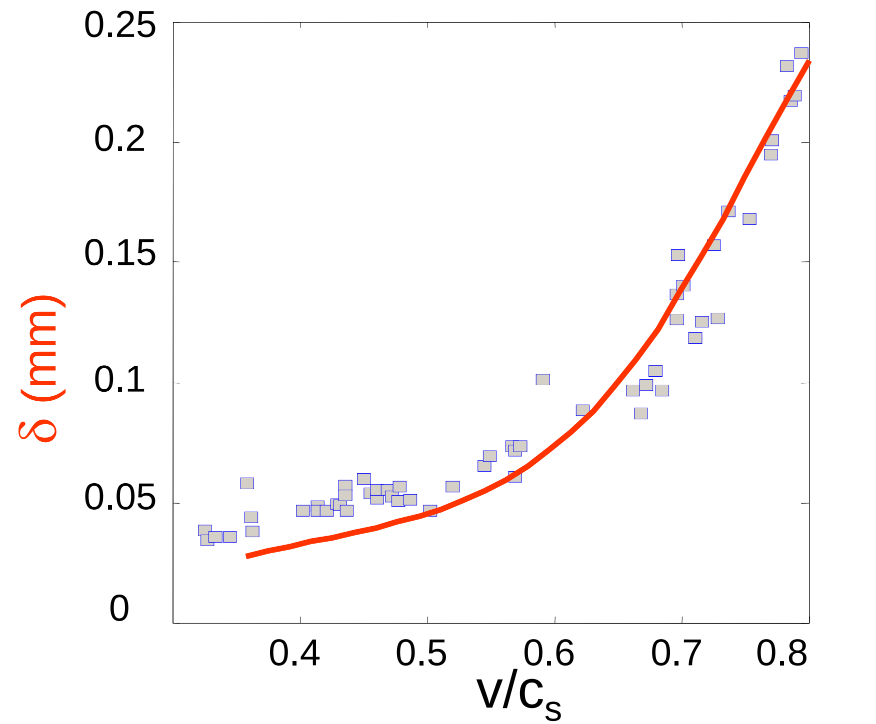
$$\Delta = \frac{K_I}{4\mu\,(2\pi l_{nl})^{1/2}}$$

We now have:

An *excellen* ... dynamic cracks







Good…

Better …

Great!

- - - *LEFM*

NL theory

Δ >> 1 theory

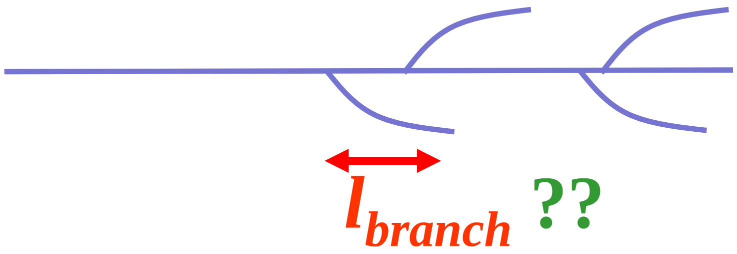
E. Bouchbinder, A. Livne and J. Fineberg, Science **327**, 1359–1363 (2010).

The theory also supplies a dynamic scale: l_{nl}

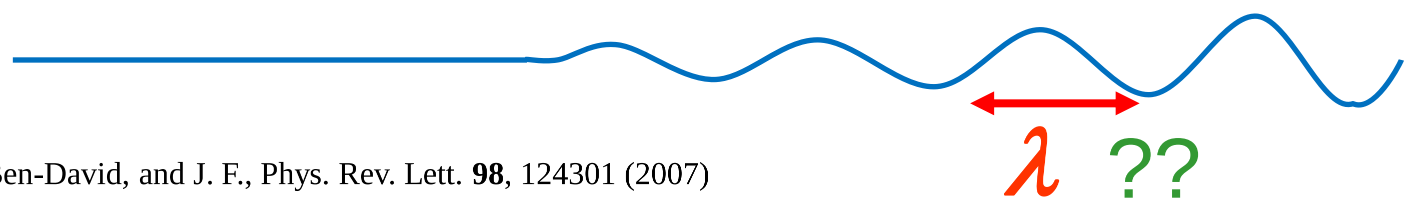
What scales are important for crack instabilities?

A. Livne, G. Cohen and J. F.,
Phys. Rev. Lett. **94,** 224301 (2005)

Micro-branching



Oscillatory instabilities $v>0.9C_S$



A. Livne, O. Ben-David, and J. F., Phys. Rev. Lett. **98**, 124301 (2007)

LEFM has no intrinsic scales…

Instabilities generate dynamic scales

➔*Instabilities might be described with scales ~* l_{nl} *given by the* ***weakly nonlinear theory*** *!??!*

Next Step…

Utilize this new theoretical framework to go *beyond* single-crack dynamics

(e.g. Is this sufficient to understand the origin of crack instabilities..)

Summary:

We now understand "simple" cracks in brittle materials!

√ *Equation of motion ⇔ Energy Balance*
√ *Singular fields near crack tip ⇔* ***nonlinear elasticity*** *matters*

➔ ***Worse*** *singularity*

➔ *New* ***intrinsic*** *nonlinear length scale,* l_{nl}

What's next:

Understanding crack instabilities…using l_{nl} ***. (Next week)***

- The dynamics of crack instabilities and crack branching?
  ⇔ perturb the **correct** "ground state"!

- Beyond brittle materials ⇔ The near-tip stress field structure:
  How does ***microscopic*** structure affect ***macroscopic*** behavior?

Thank You!