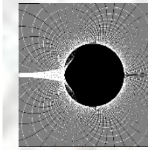
WHAT STARTS HERE CHANGES THE WORLD

THE UNIVERSITY OF TEXAS AT AUSTIN



Dynamic fracture

Krishnaswamy Ravi-Chandar

Lecture 4 presented at the

University of Pierre and Marie Curie

May 21, 2014

Center for Mechanics of Solids, Structures and Materials

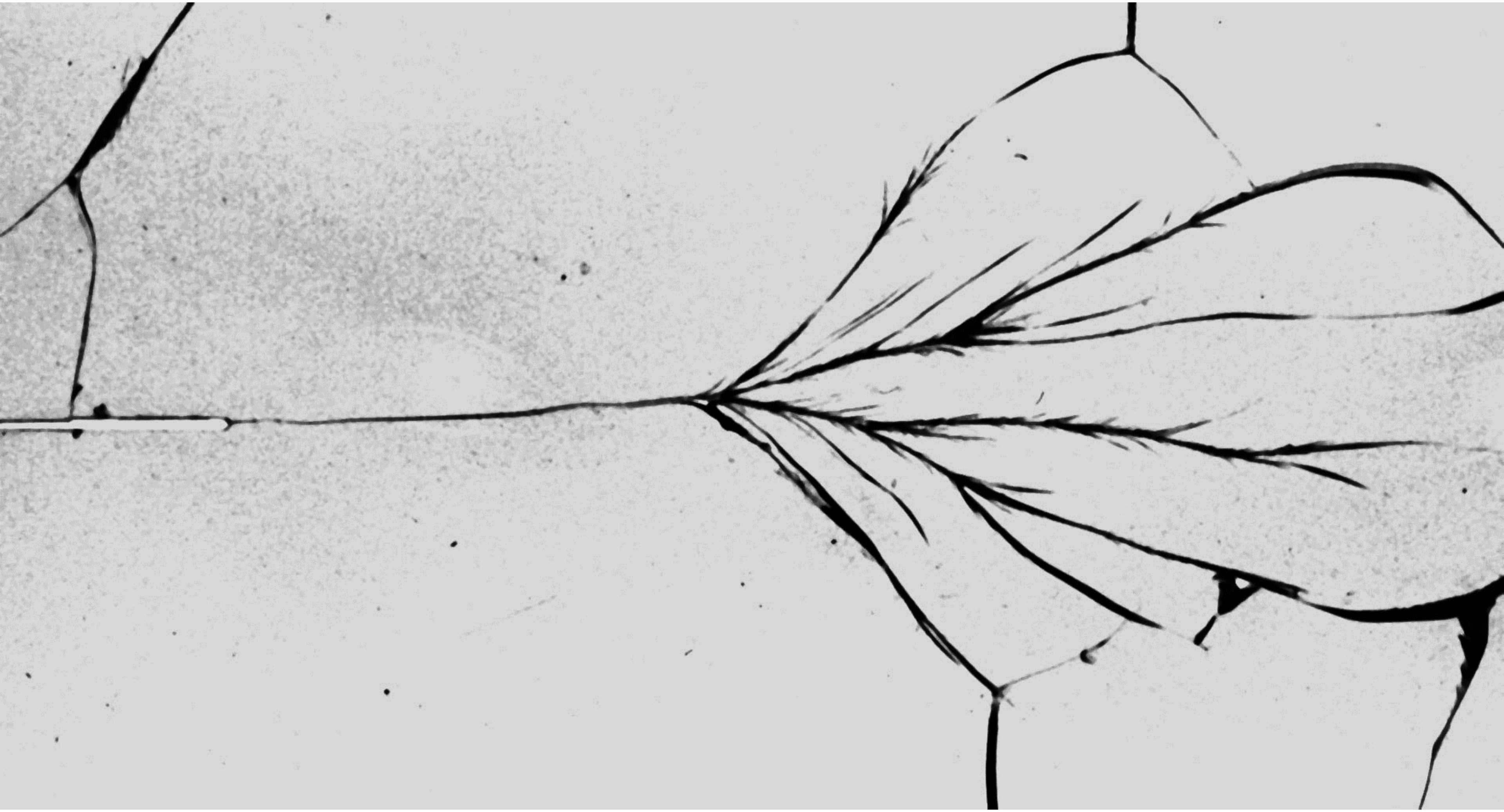
Department of Aerospace Engineering and Engineering Mechanics

THE UNIVERSITY OF TEXAS AT AUSTIN

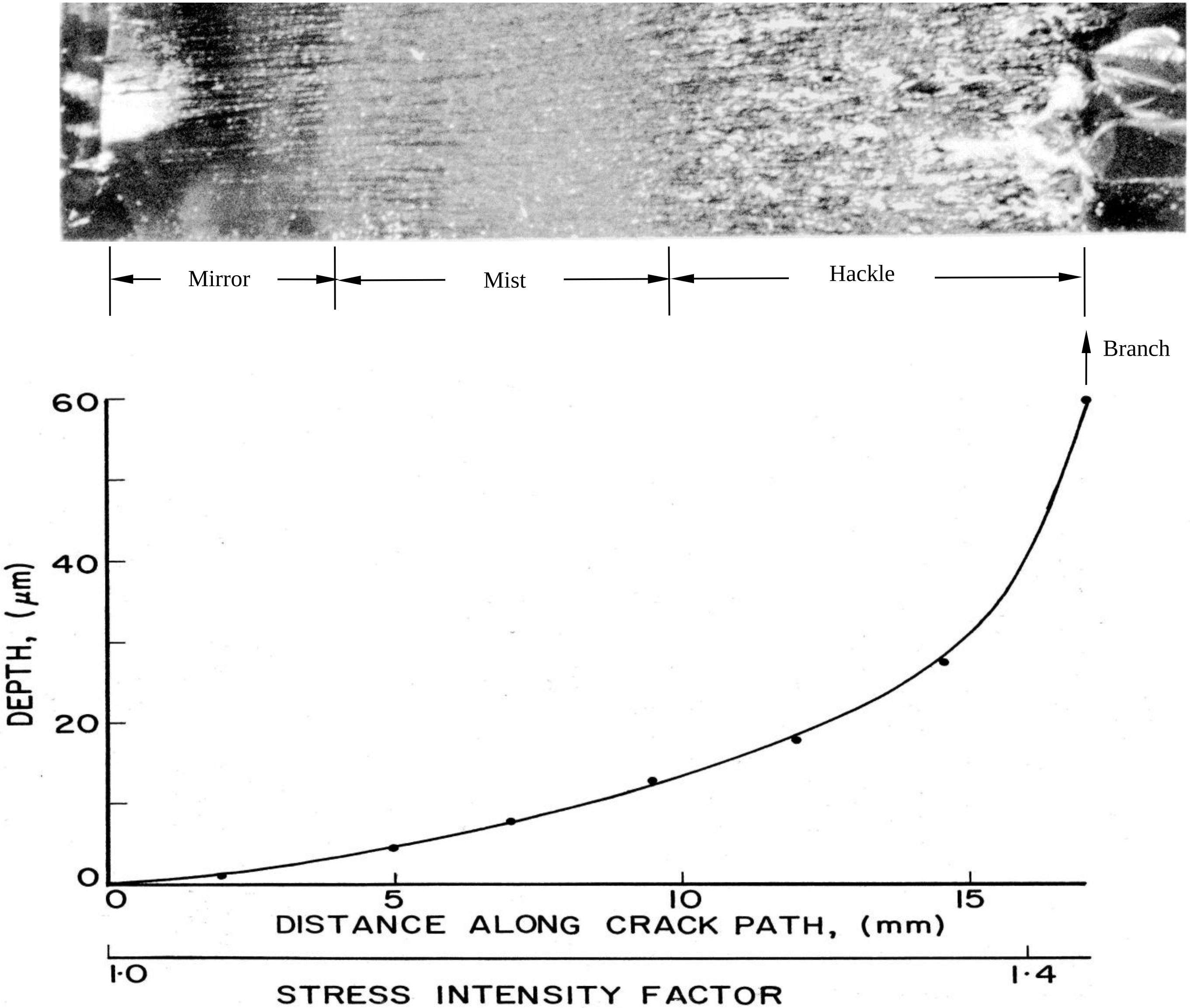
Outline

- Crack surface roughness development
- Crack front waves
- Crack branching
- Tape peeling

Dynamic crack evolution

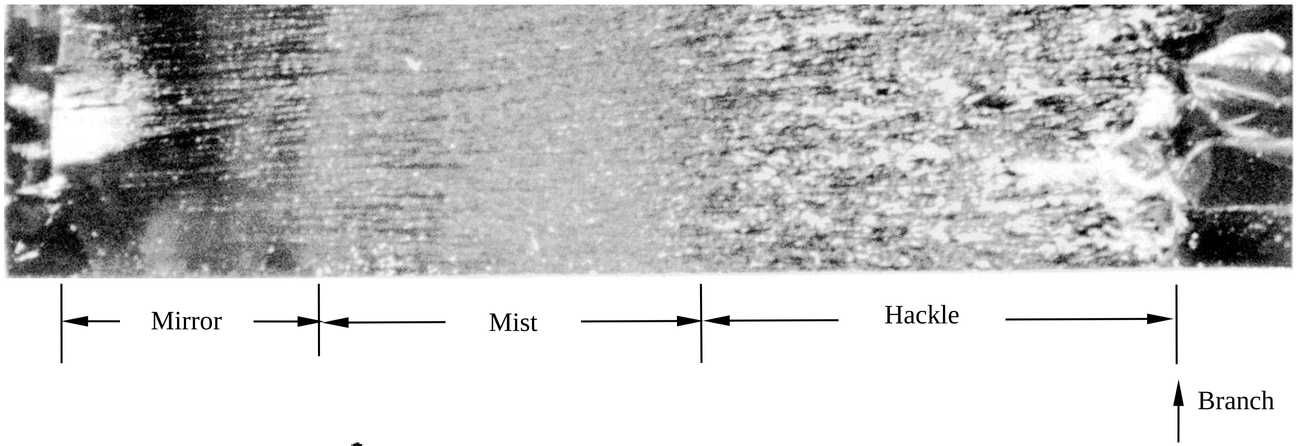


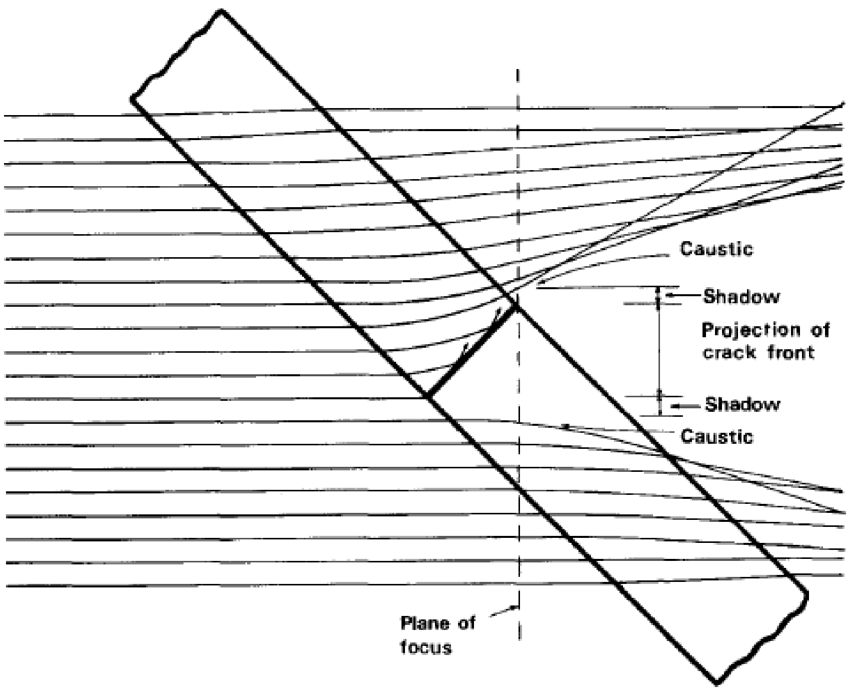
Surface roughening

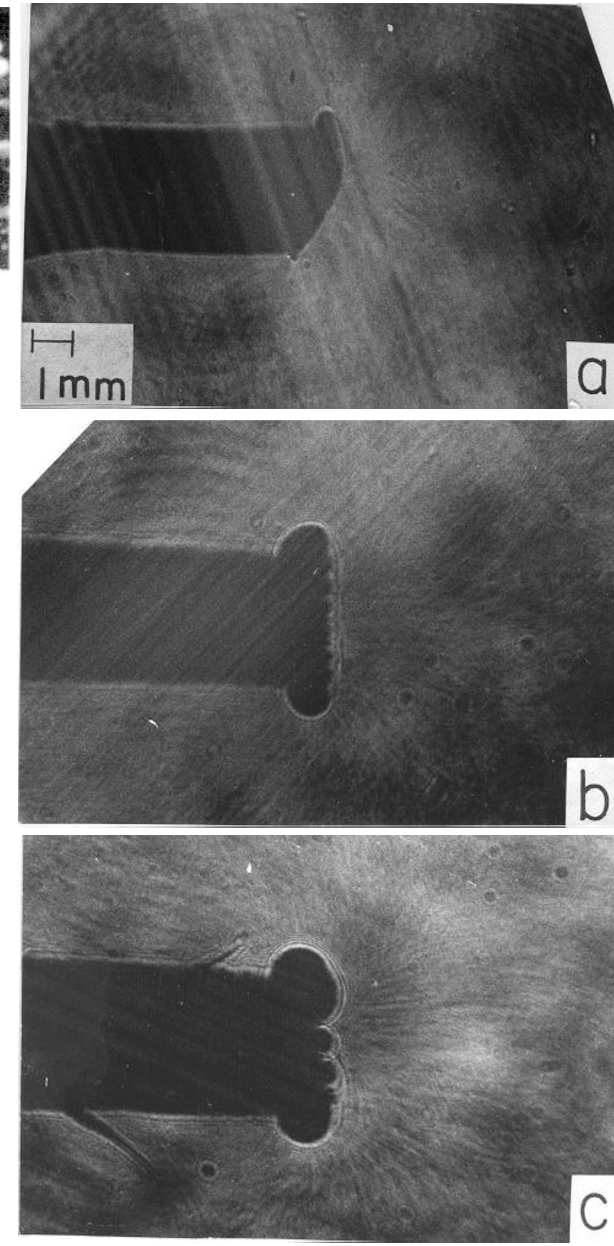


Ravi-Chandar and Knauss, *Int J Fract*, 1984

Dynamic crack front evolution

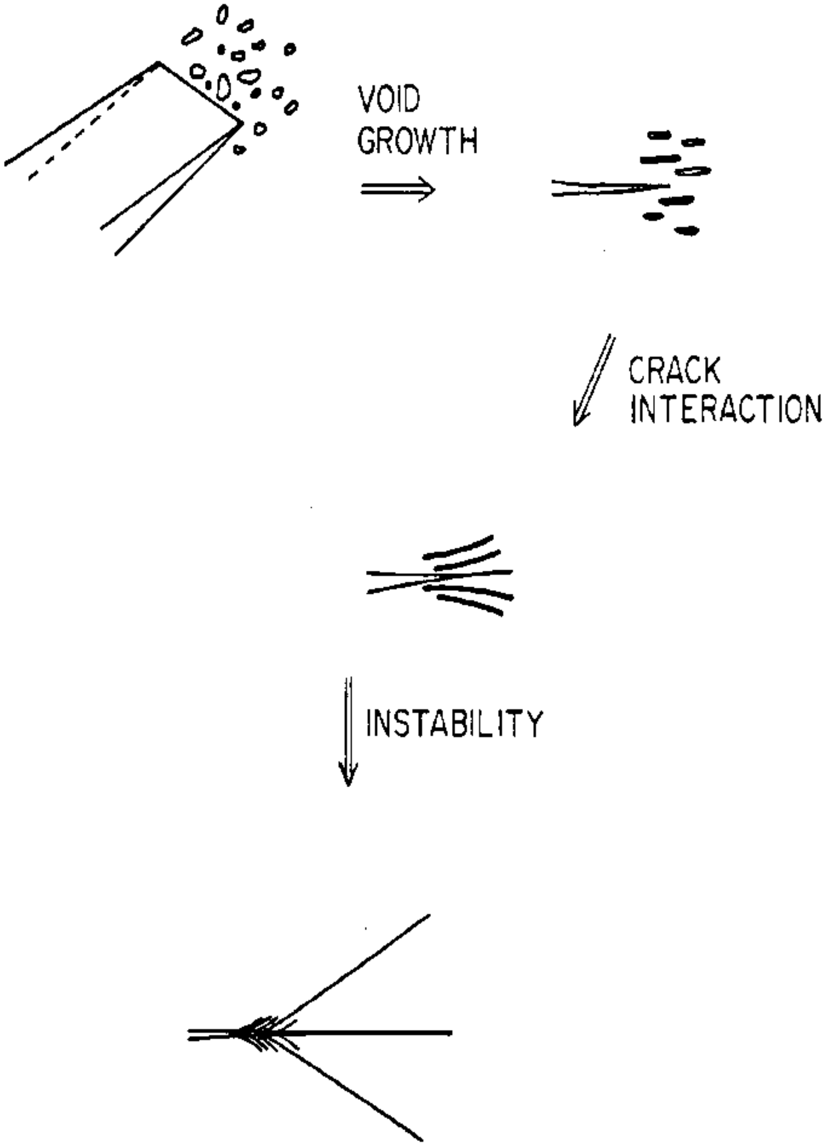


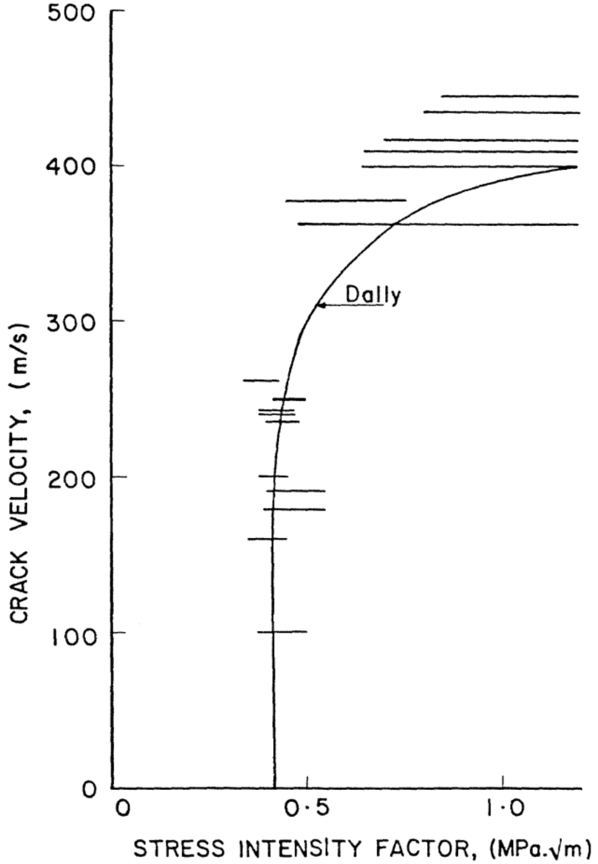




Ravi-Chandar and Knauss, *Int J Fract*, 1984

Microcracking model for dynamic fracture

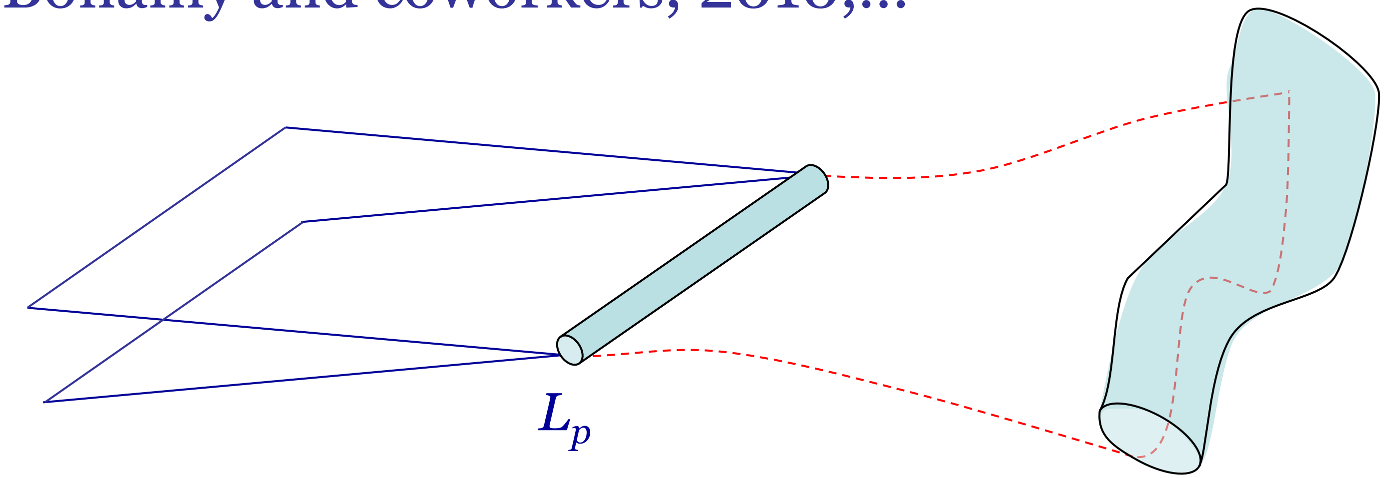




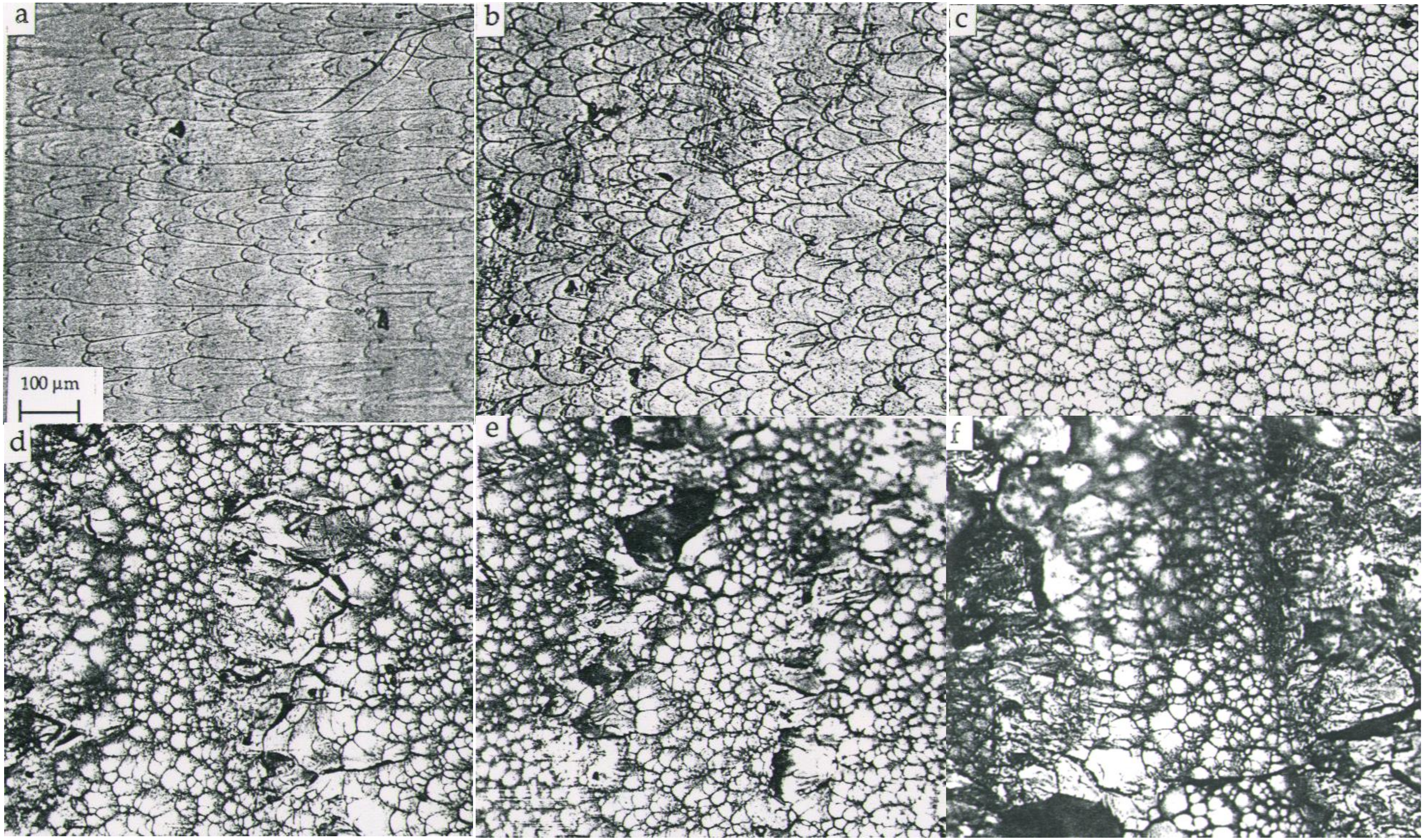
Ravi-Chandar and Knauss, *Int J Fract*, 1984 – Homalite - 100

Surface roughening and limiting speed

- Microcracking, instability
  - R-C and Knauss, *IJF*, 1984, R-C and Yang, JMPS, 1996
- Dynamic instability and microbranching
  - Sharon and Fineberg, *PRL*, 1996
- Crack twisting – mode III perturbations
  - Hull, *J Mat Sci*, 1997
- Microcracking, roughening
  - Bonamy and coworkers, 2010,…

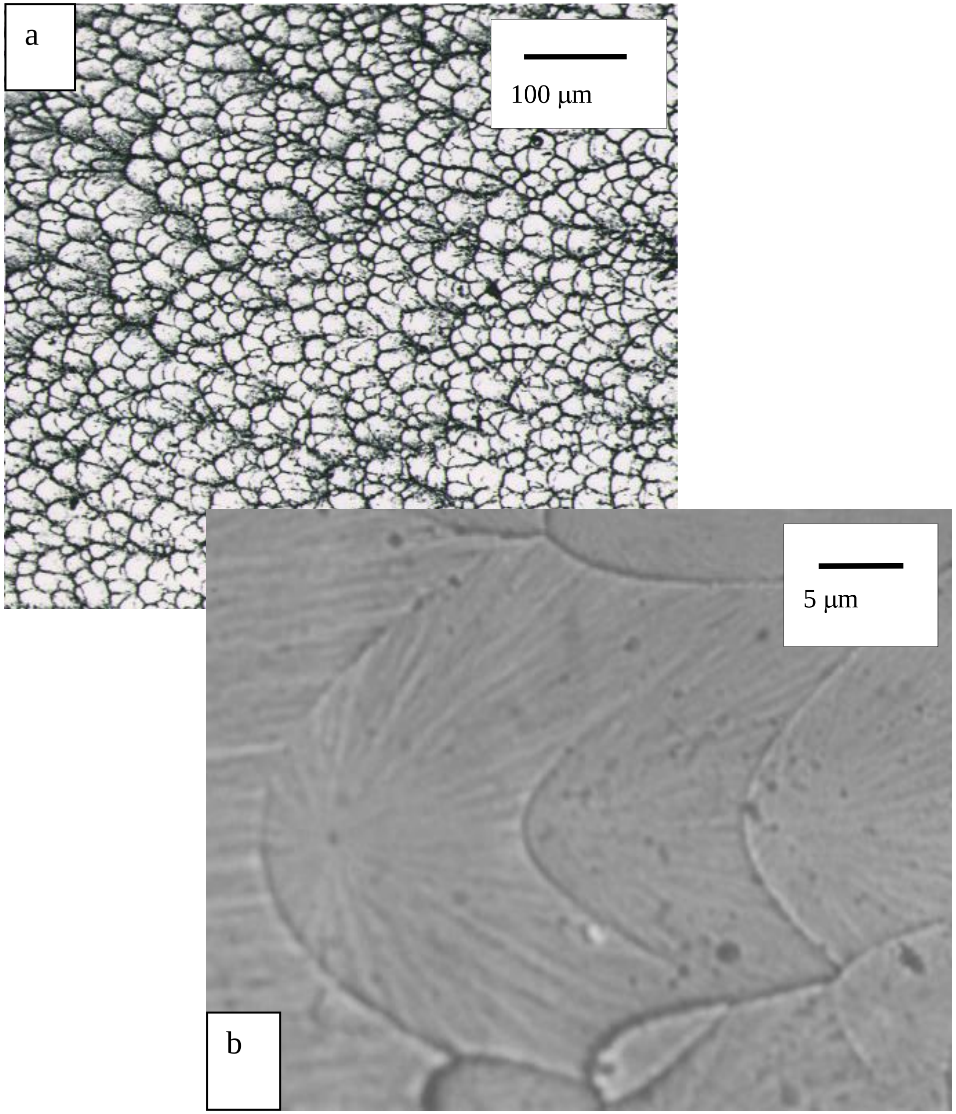


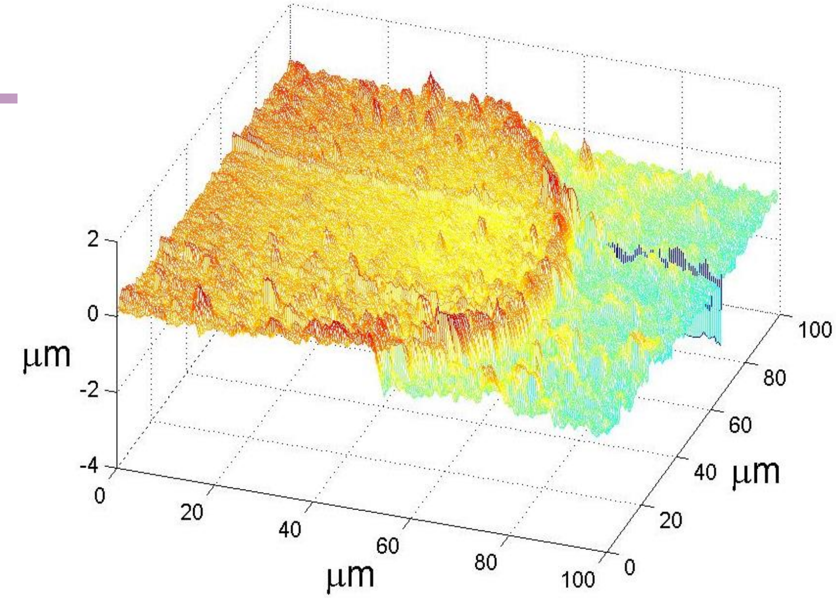
Surface roughening - PMMA

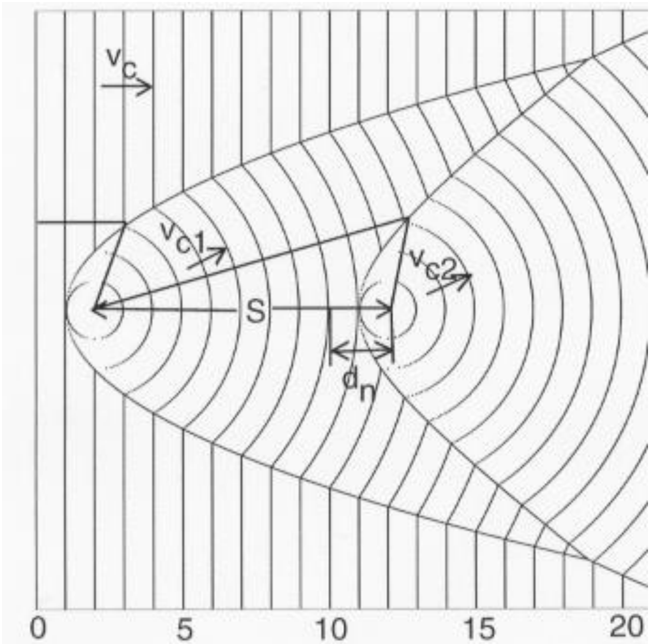


Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997 – PMMA

Formation of conic marks

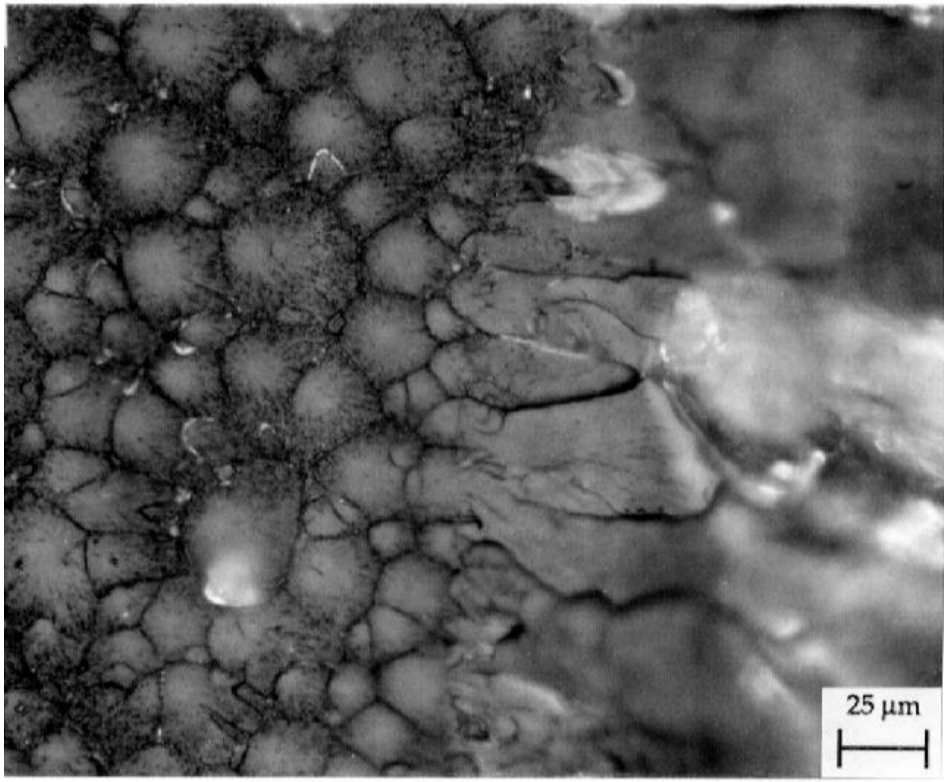


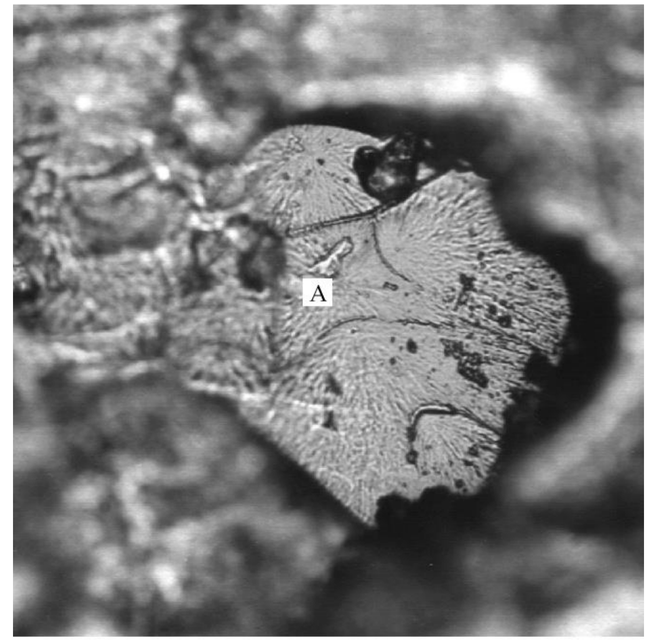




Smekal, 1953
Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997 ; Ravi-Chandar, *Int J Fract*, 1998 – PMMA

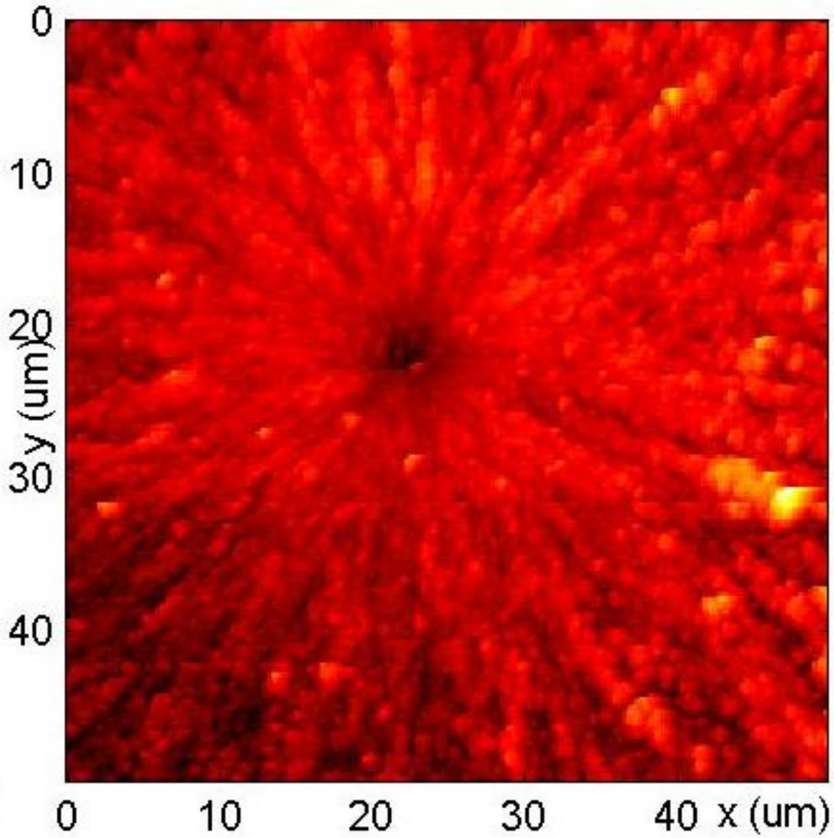
‘Plane’ of the microcracks - microbranches

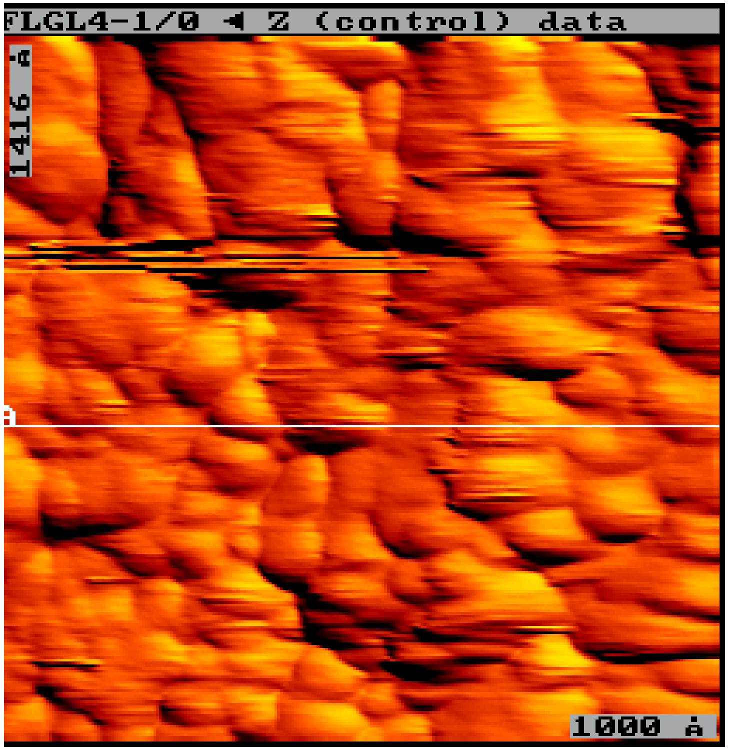




Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997 – PMMA

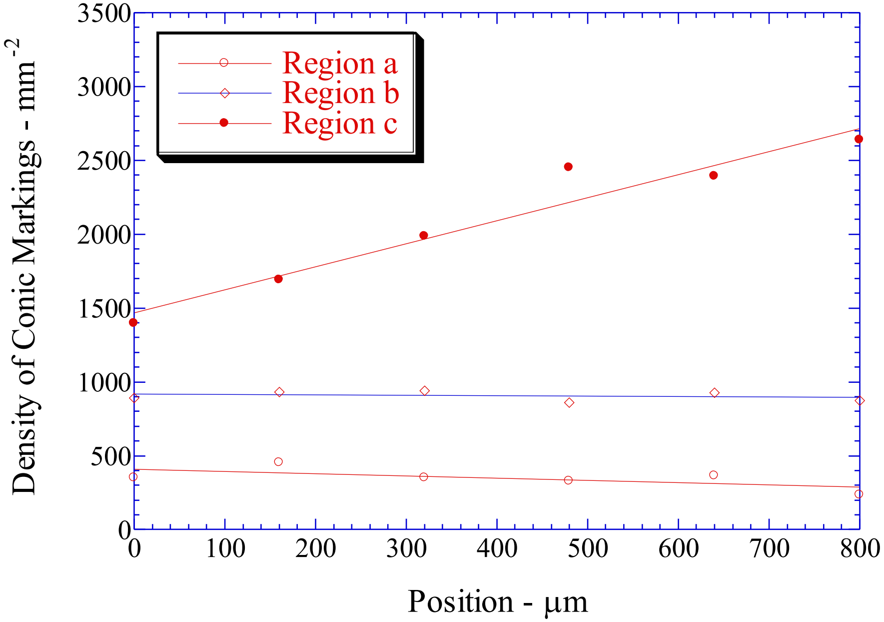
Nucleus of the conic

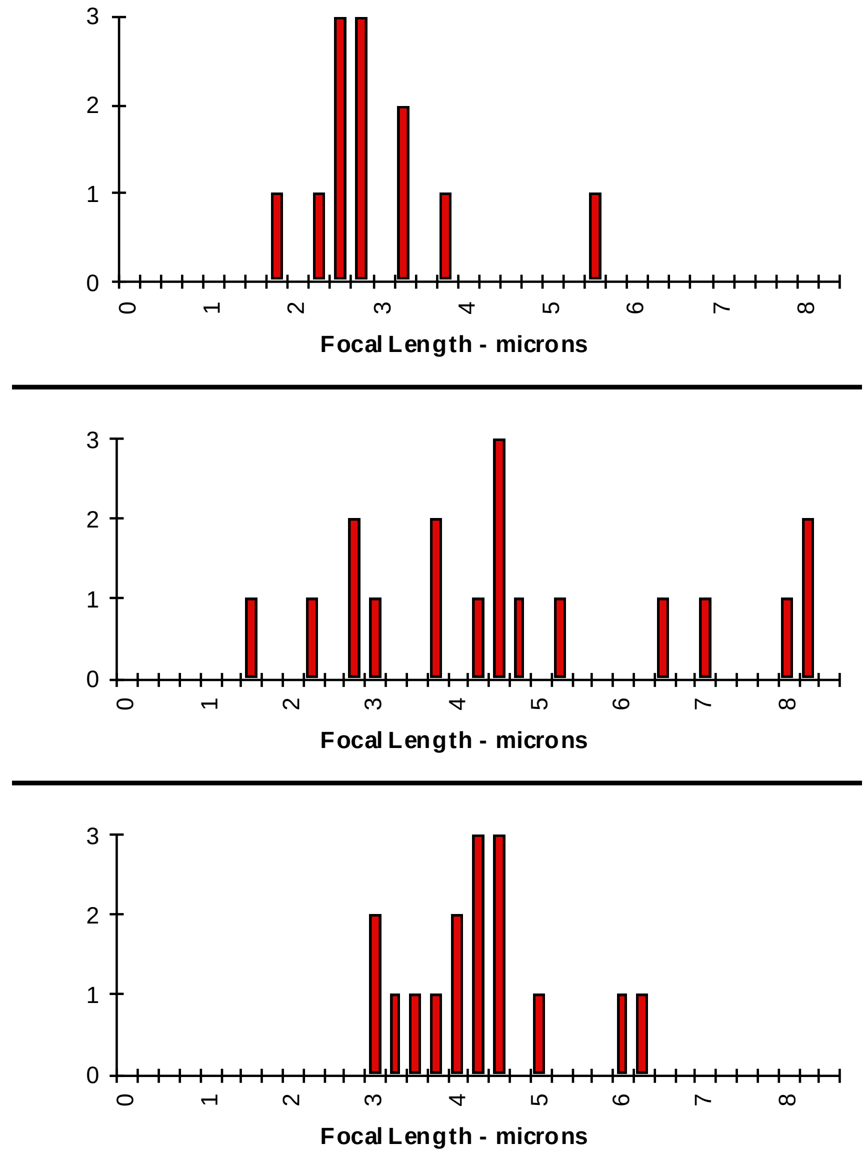




Ravi-Chandar, *Int J Fract*, 1998 – PMMA

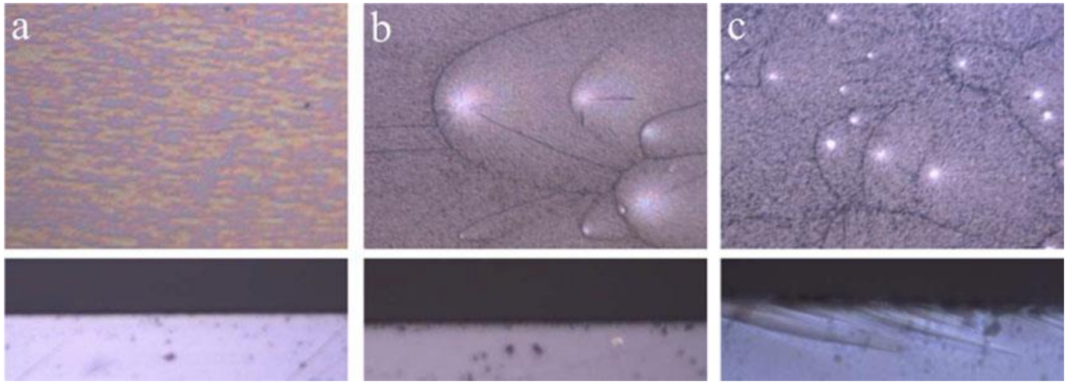
Statistics of conic marks - 1

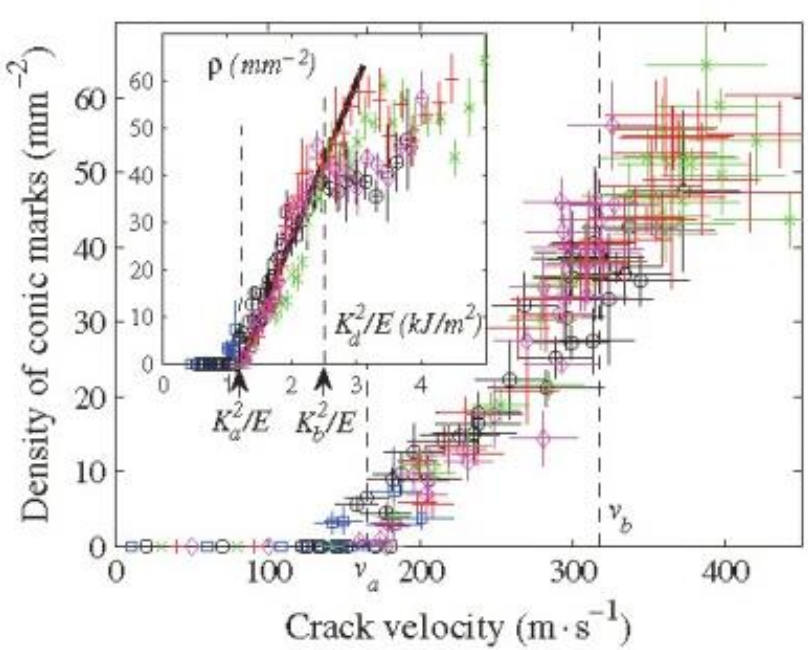


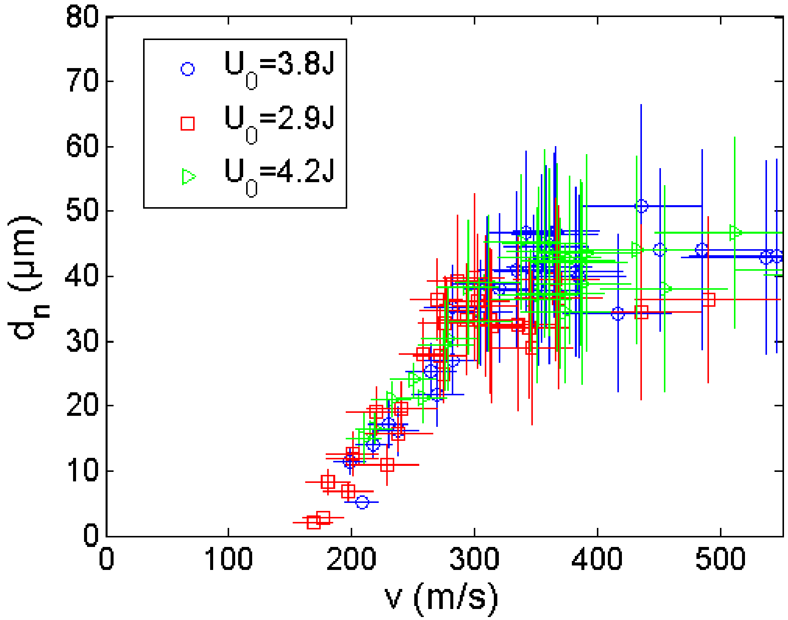


Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997 – PMMA

Statistics of conic marks - 2







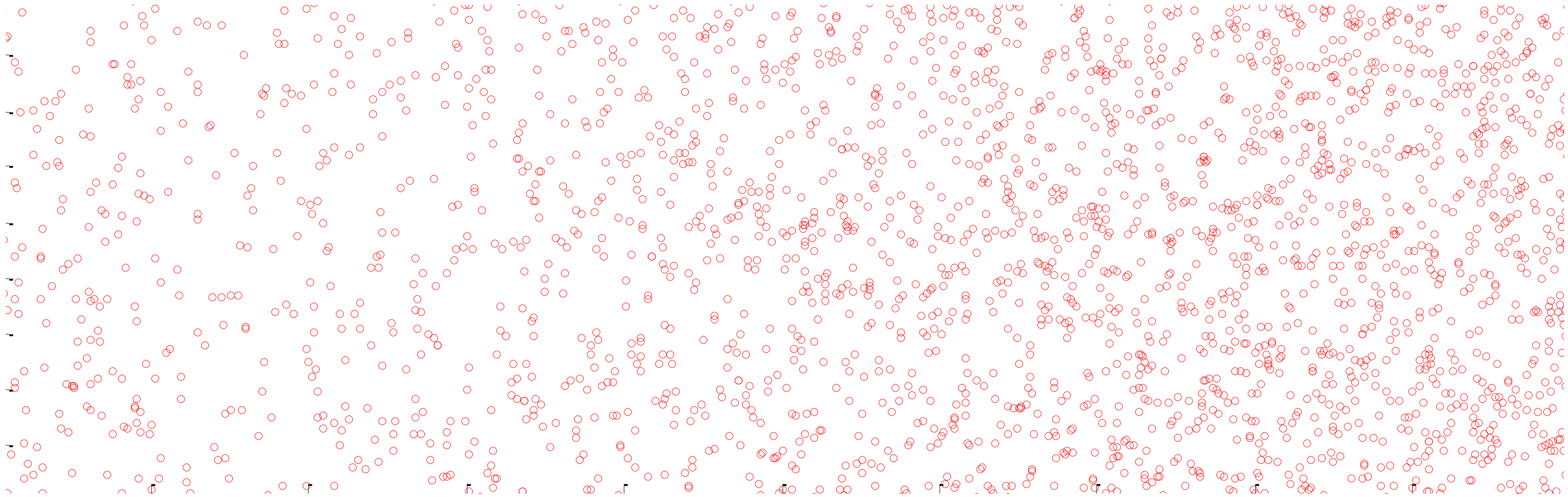
Scheibert, **Guerra**, Celarie, Dalmas and Bonamy, *PRL*, 2010

Speed of microcracks?

- Assume that microcracks grow with the same speed
  - If microcrack speeds are of equal, then the shape of the conic is independent of the speed (R-C and Yang, 1997)
  - Dalmas et al (IJF, 2013) show this by examining the detailed shapes of conics
- How fast do the microcracks grow?
  - It is not really possible to examine this experimentally. To be determined by other means!
  - This is still an open issue
- Can we simulate this? – just the kinematics!

Nucleation and growth model: Flaw Nuclei

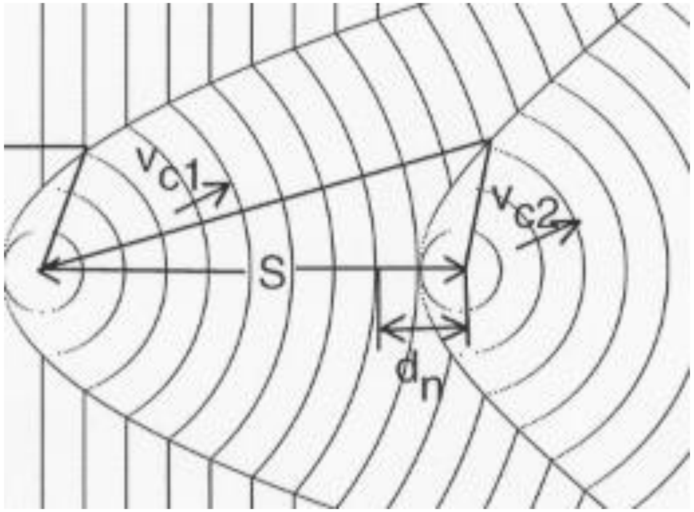
- Nucleation is by cavity growth
- Spacing and density are governed by the stress level
- Measured densities are in the range of 500 to 2500 nuclei per mm^2.

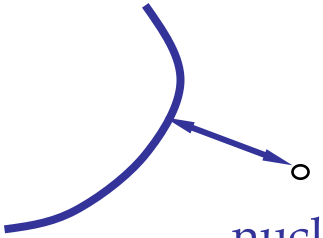


Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997

Nucleation and growth model: Nucleation

- When the stress at a flaw nucleus reaches a critical value, the nucleus becomes an active microcrack; calculation of this requires detailed knowledge of the flaw dimensions and the detailed stress field in its vicinity.
- Here we assume that when an active crack front approaches a nucleus to within a critical nucleation distance, d_n the flaw is nucleated into a microcrack.





d_n, nucleation distance

s, spacing between nuclei

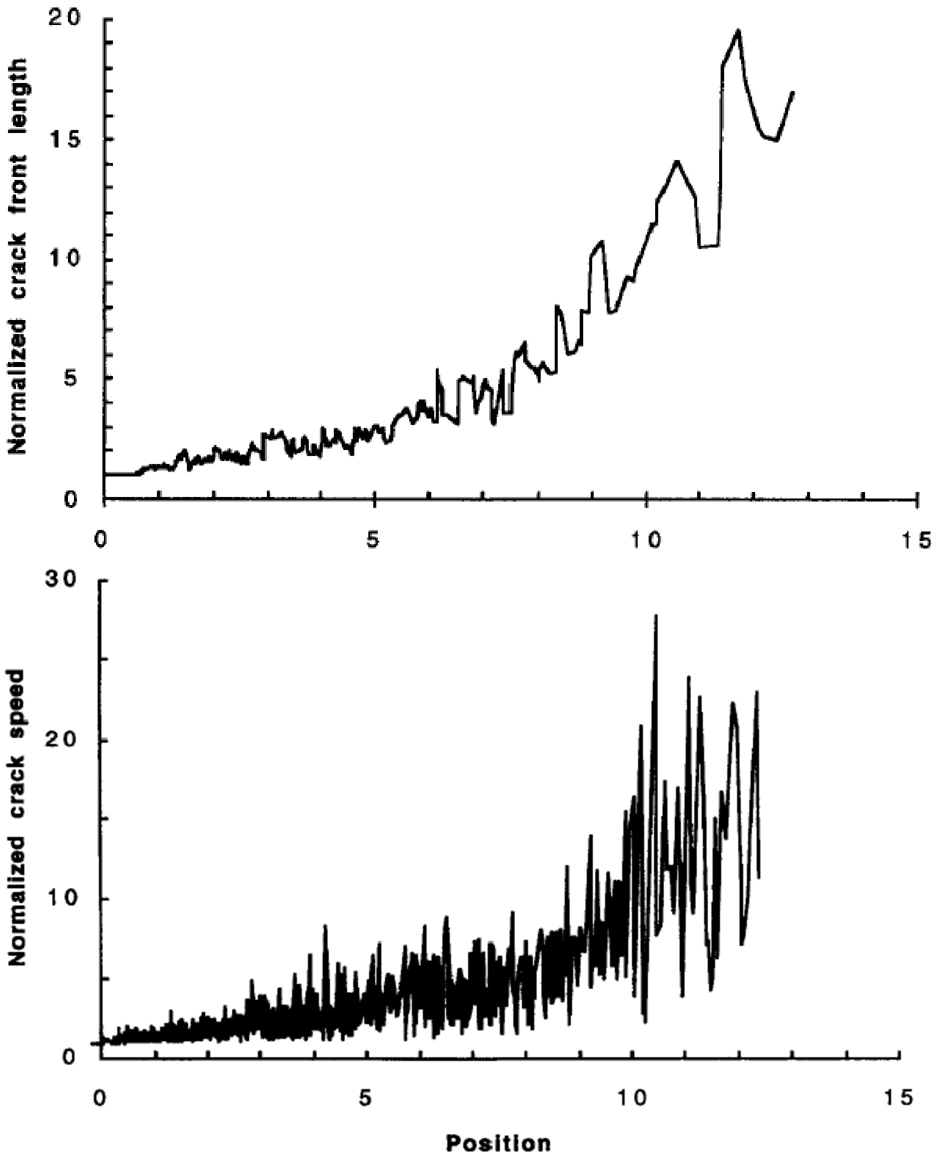
Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997

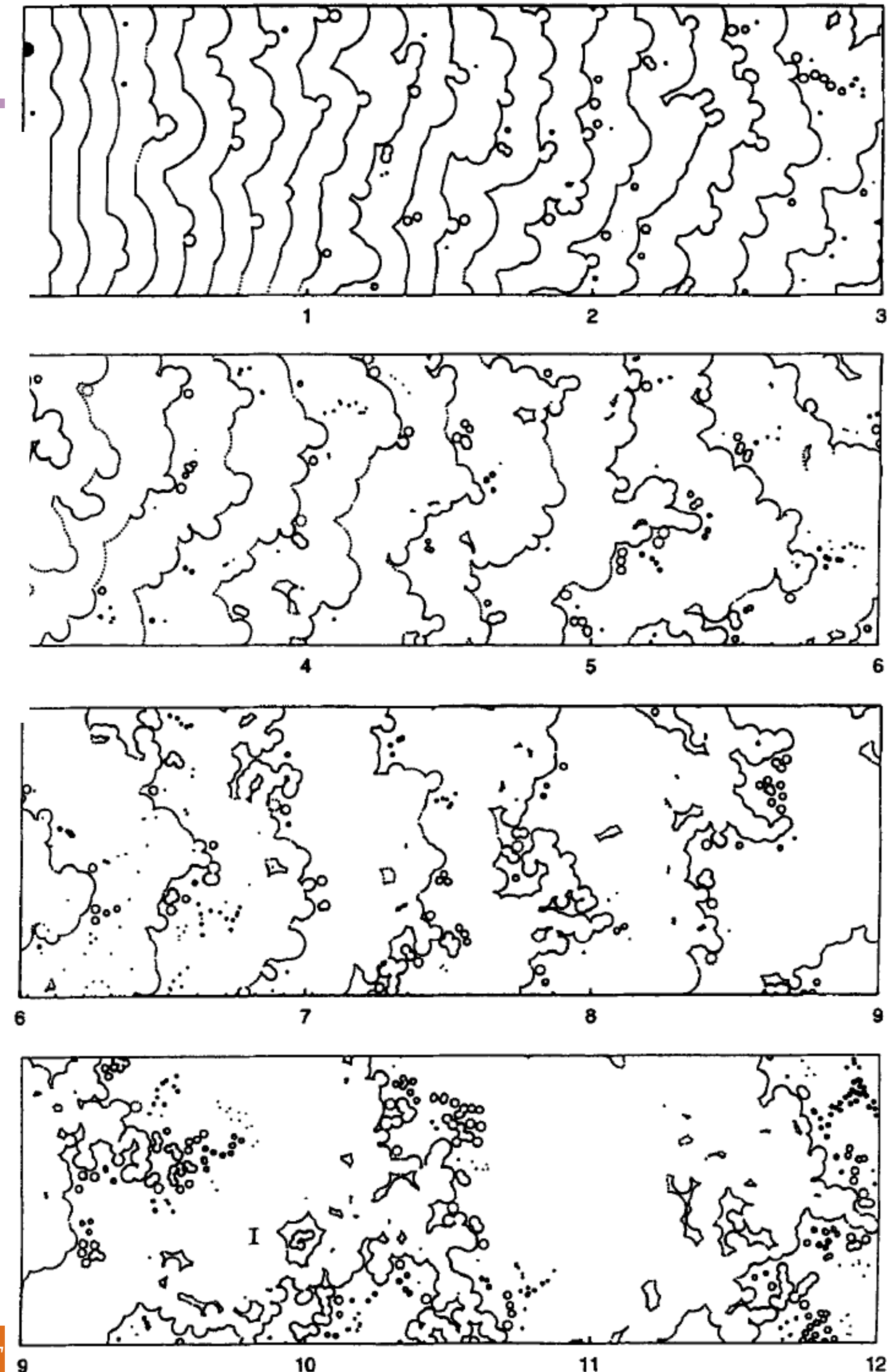
Nucleation and growth model: Growth

- Constant microcrack speed
  - all active microcracks grow with the same speed regardless of the density of microcracks
- Constant energy flux
  - microcrack speed depends on the number of active microcracks and the available energy

Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997

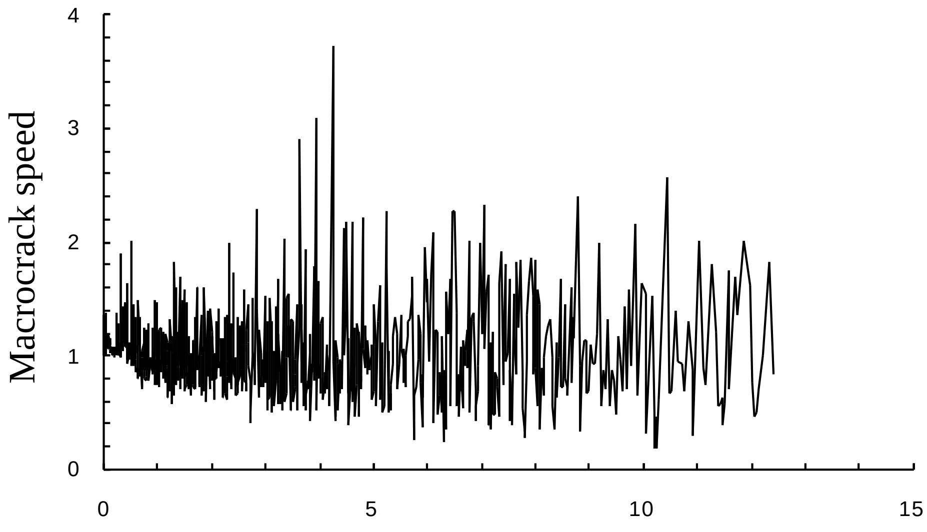
Constant microcrack speed

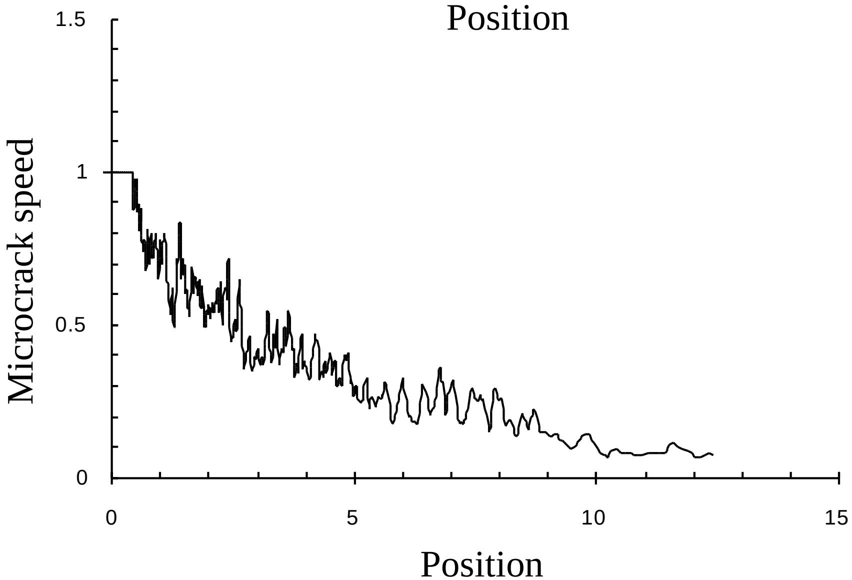


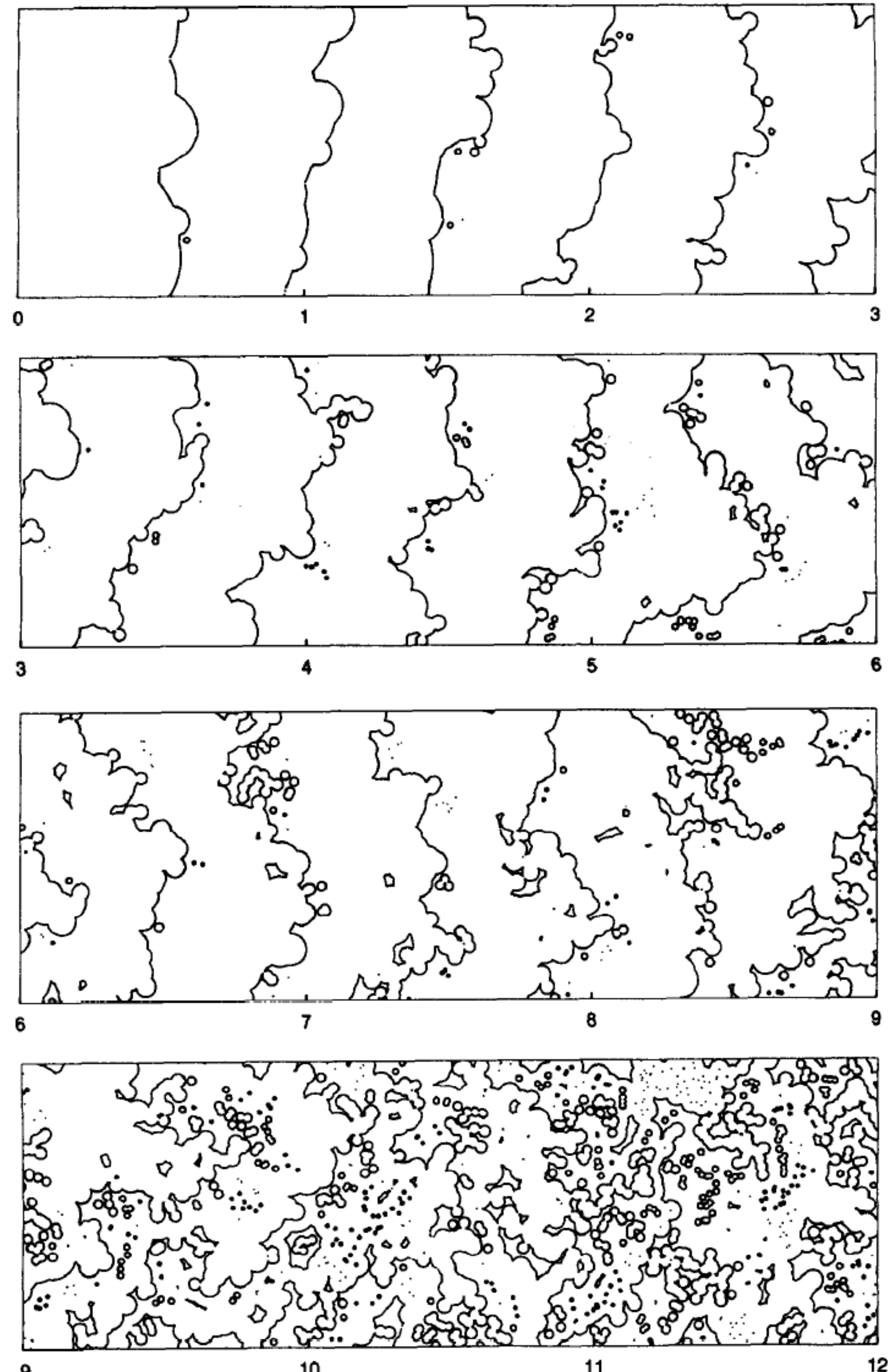


Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997

Constant power input







Ravi-Chandar and Yang, *J Mech Phys Solids*, 1997

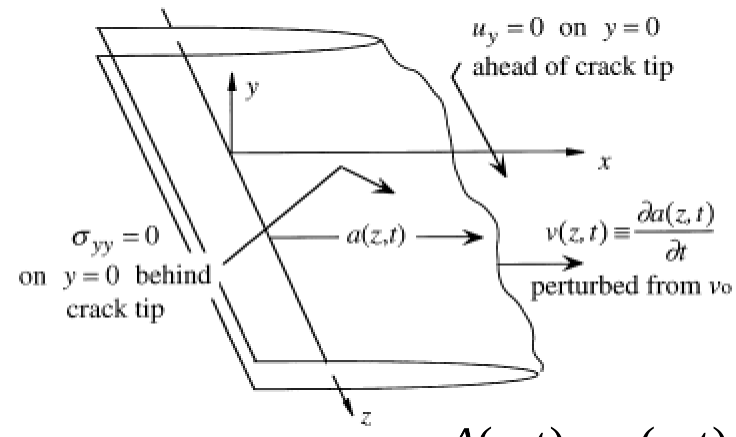
Comments…

- Differences in density of conic marks between different types of PMMA need to be reconciled
  - Bonamy et al have one to two orders of magnitude lower conic marks per unit area
- Quantitative modeling of the deformation within the fracture process zone – cavitation, crack initiation, growth, coalescence,…
- Continuum modeling based on heterogeneity?
  - Line models and quantitative calibration/comparison
  - Crack front waves

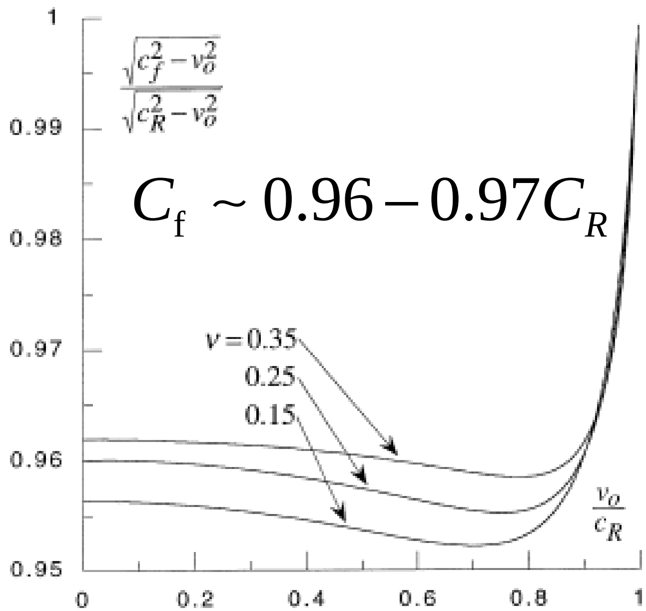
Crack front waves – a quick summary

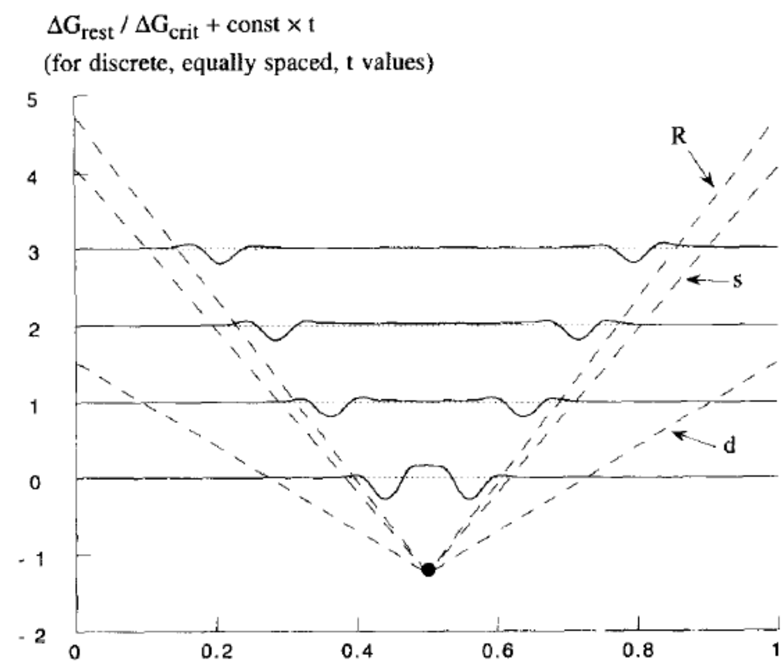
$$\frac{\Delta\hat{G}(k,\omega)}{G_0} = -\hat{P}(k,\omega)\hat{A}(k,\omega)$$

$\hat{P}(k,\omega)$… obtained from perturbation solution of Willis and Movchan; has a simple zero corresponding to a propagating mode with speed C_f ; this is the *crack front wave.*



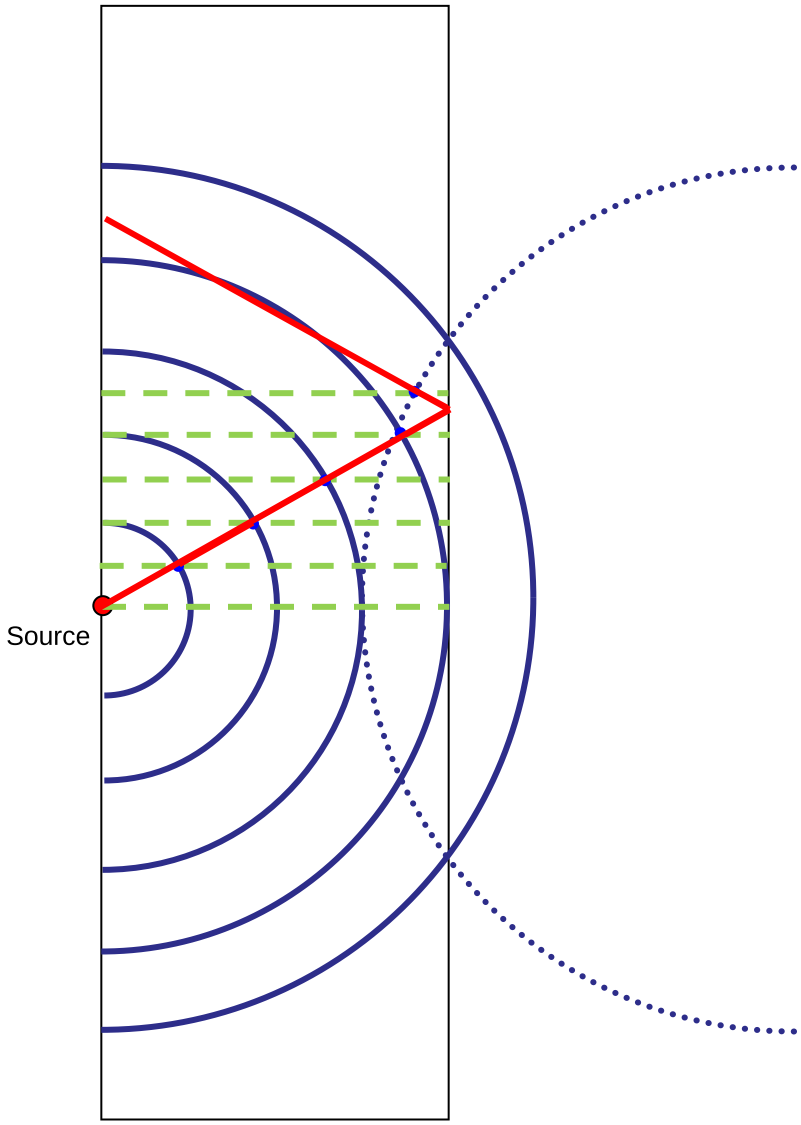
$$A(z,t) = a(z,t) - v_0 t$$

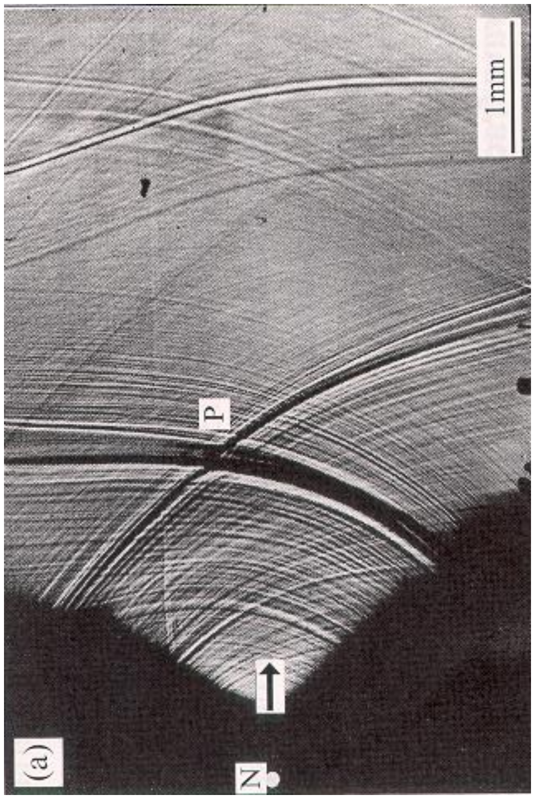




Morrissey and Rice, *JMPS*, 2000

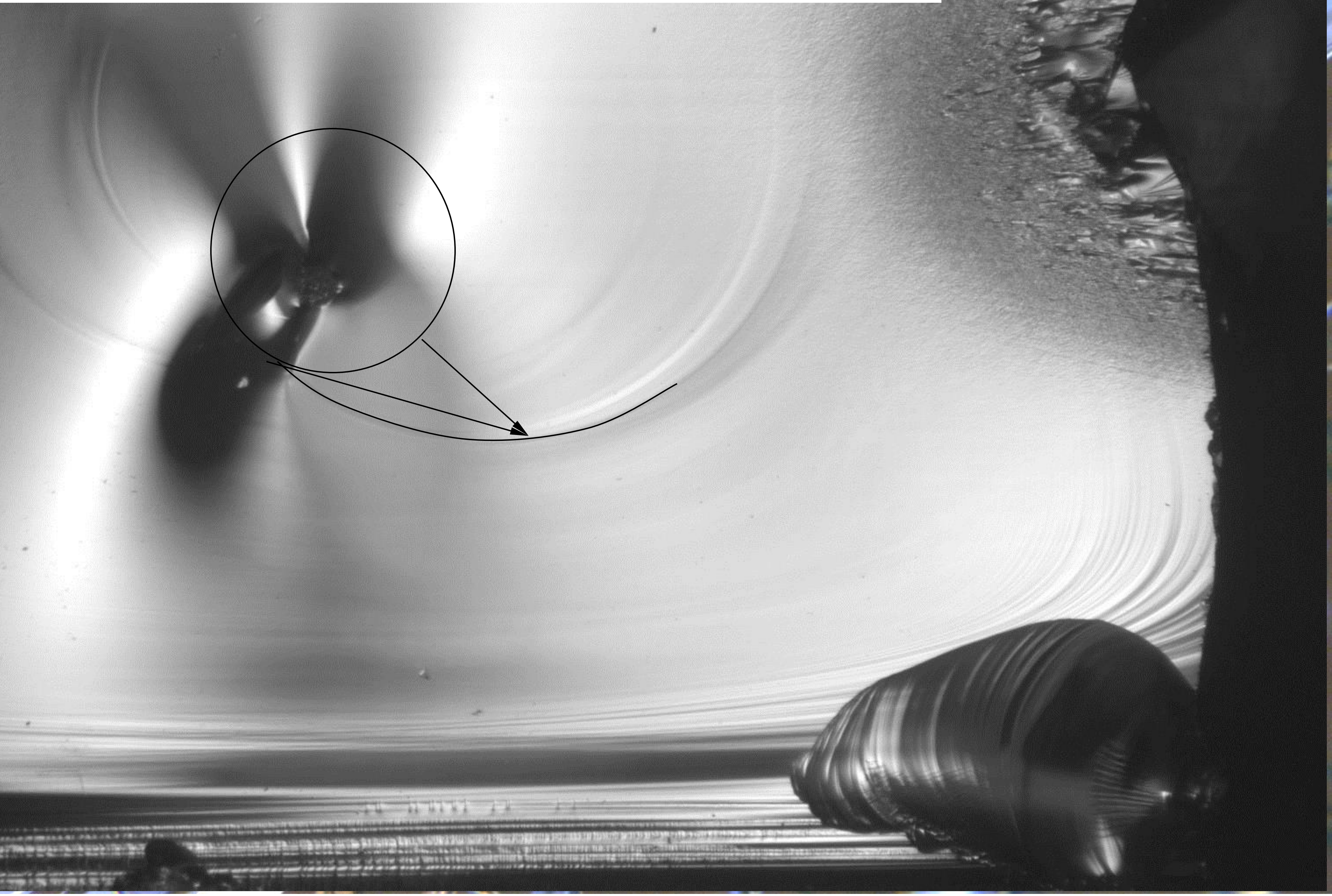
Wallner lines



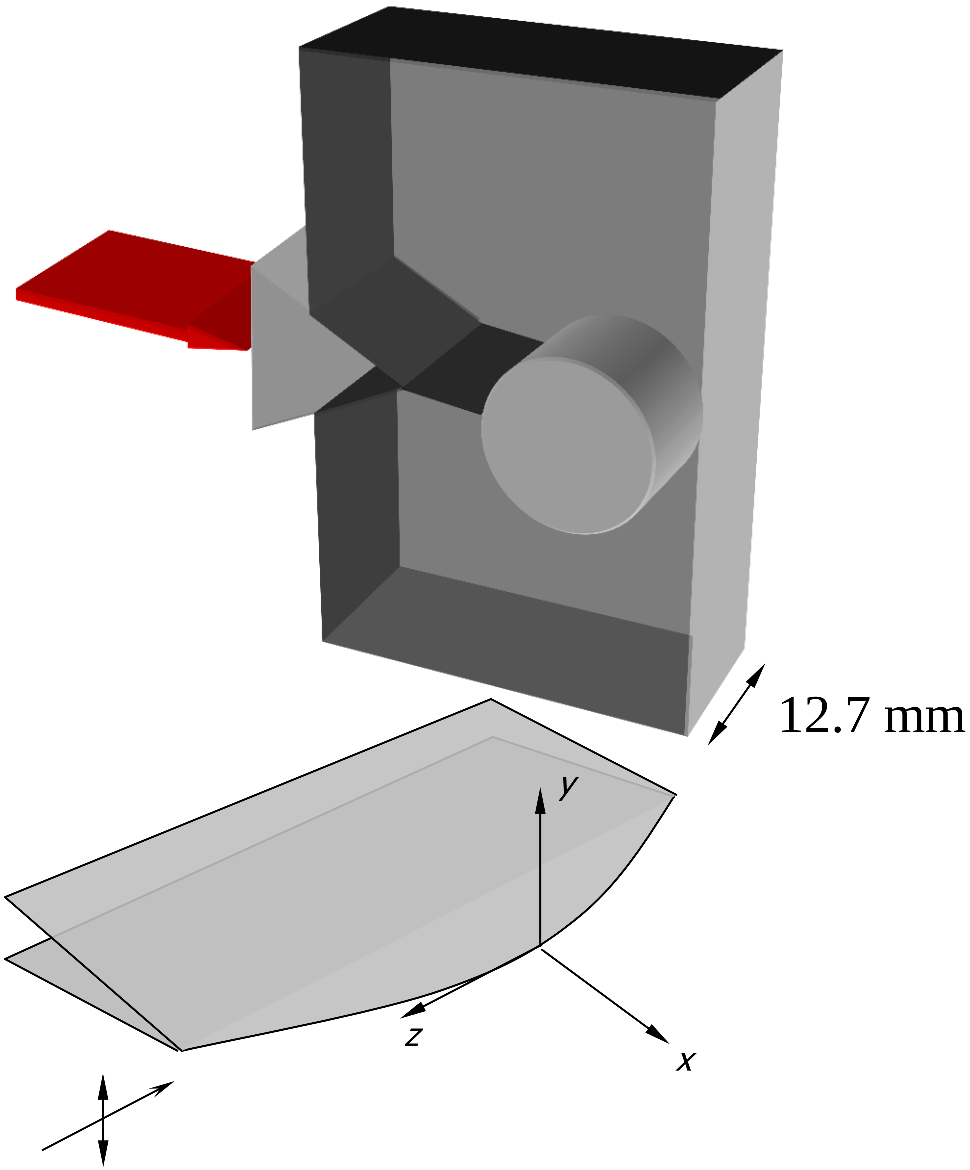


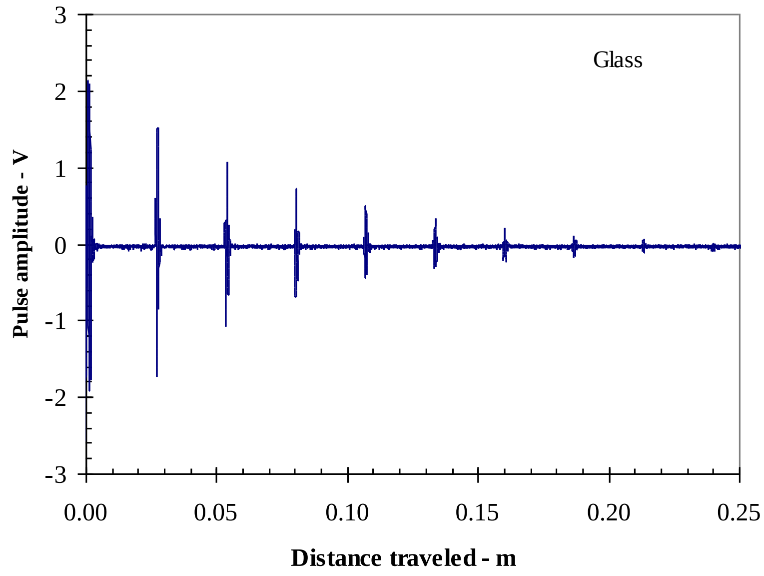
Field, *J Contemporary Physics*, 1971

Photographs: Jill Glass, Sandia National Laboratories



Mode III perturbations

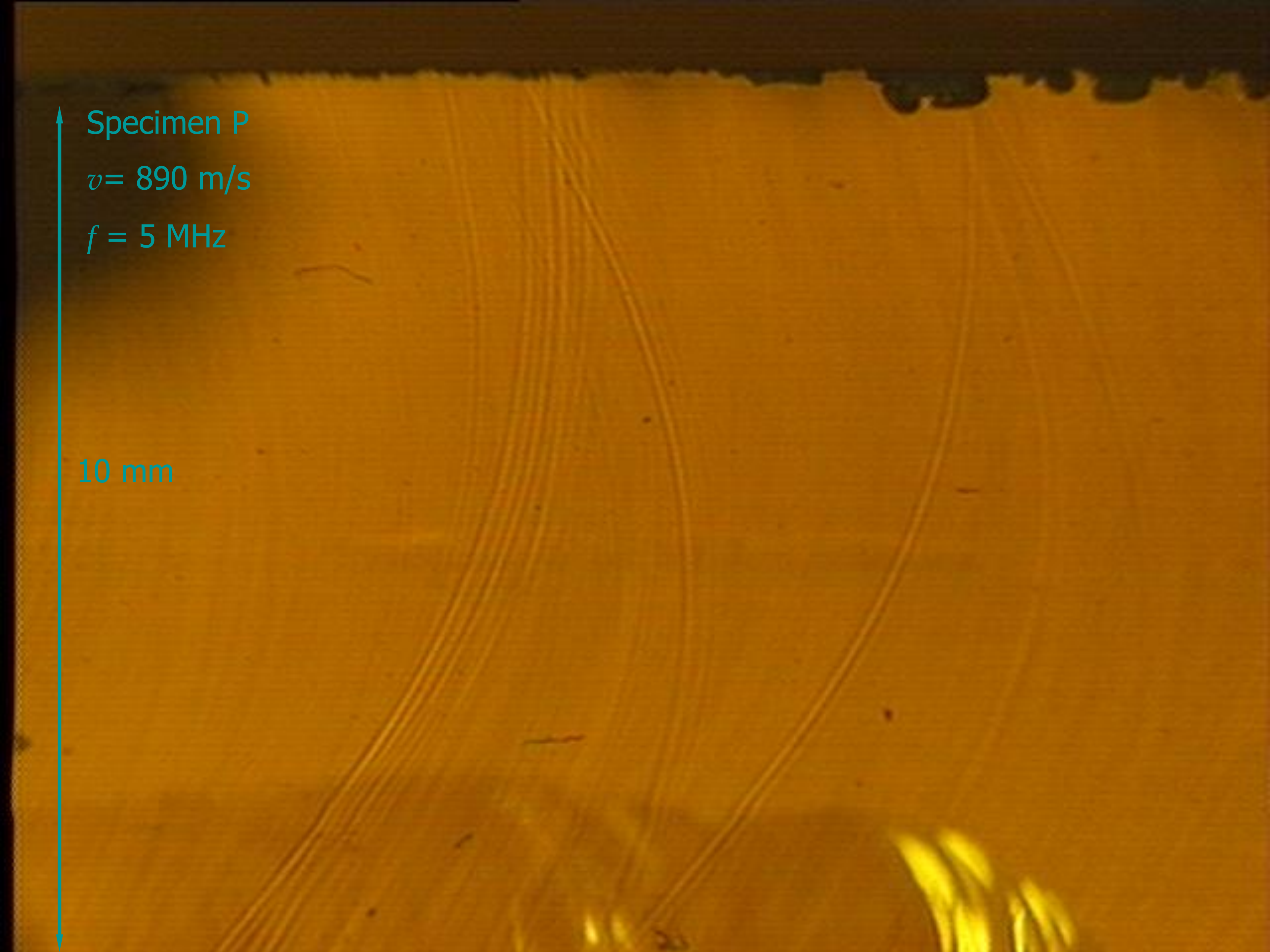




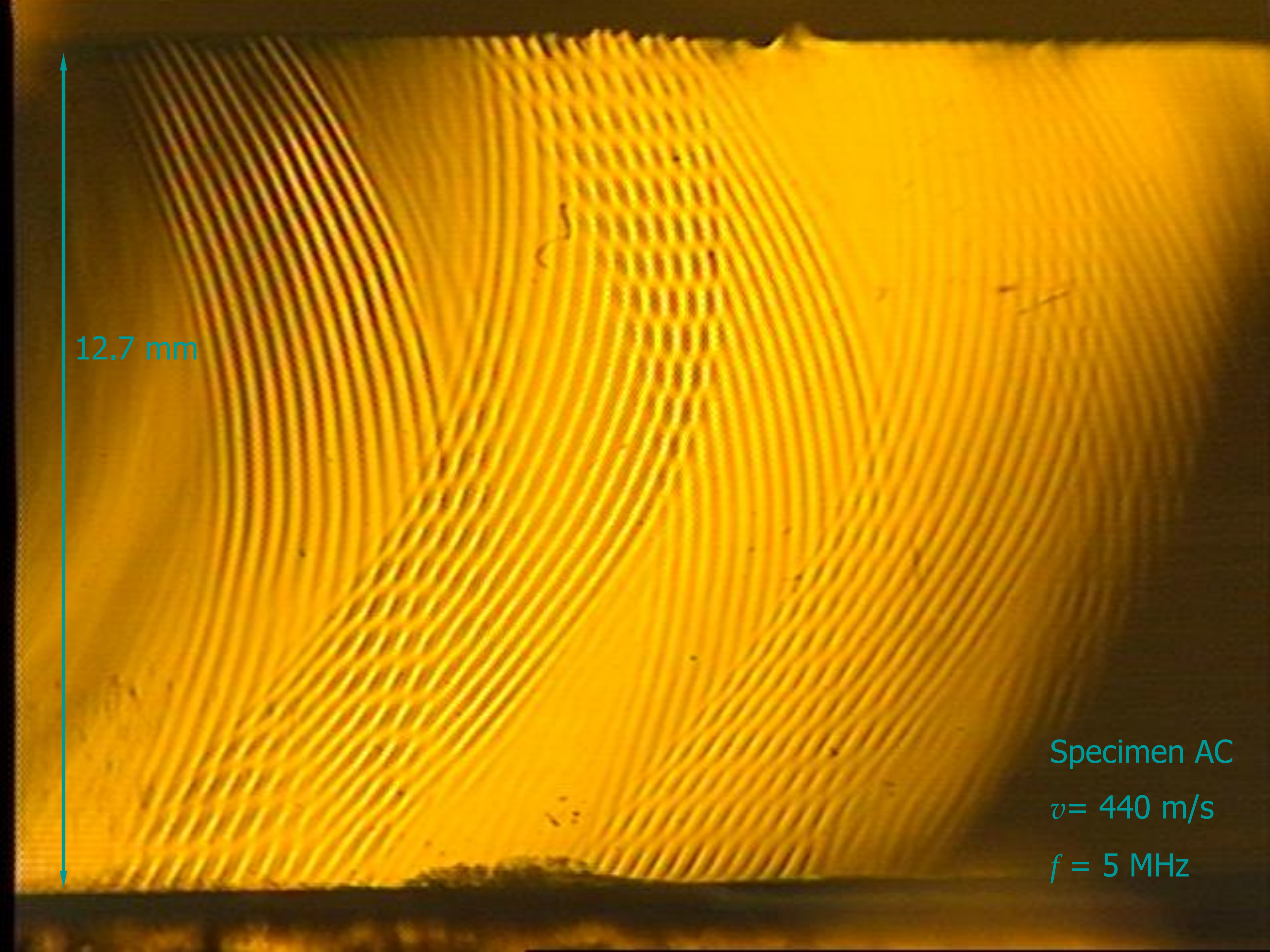
Wavelength and duration for glass

f MHz	λ μm	τ ns
0.5	6880	2000
5	688	200
20	172	50

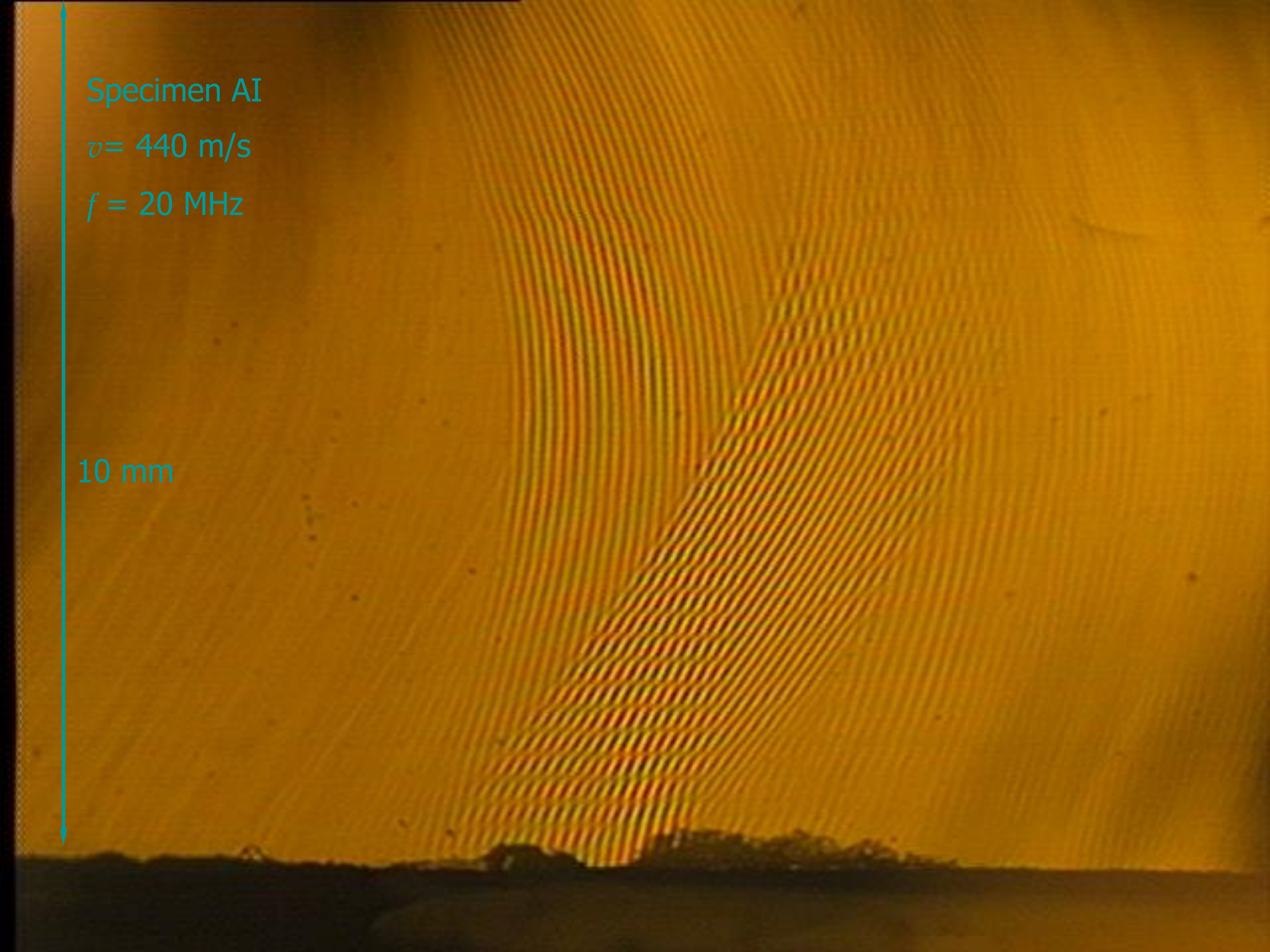
Bonamy and Ravi-Chandar, 2003



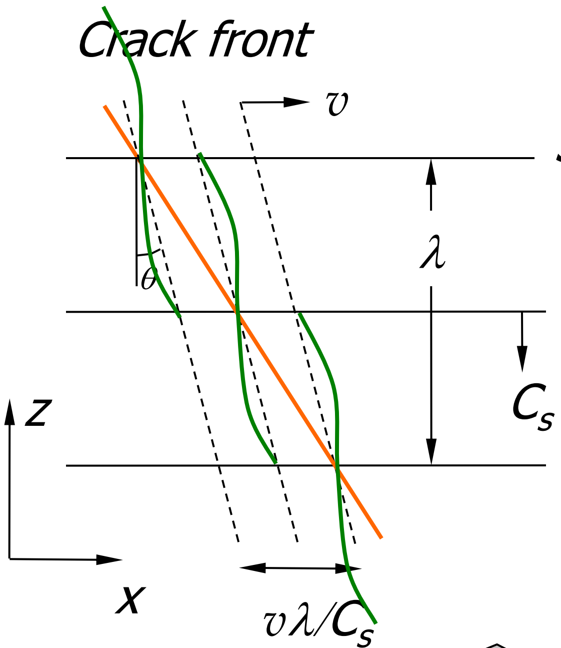
Specimen P
v= 890 m/s
f = 5 MHz
10 mm

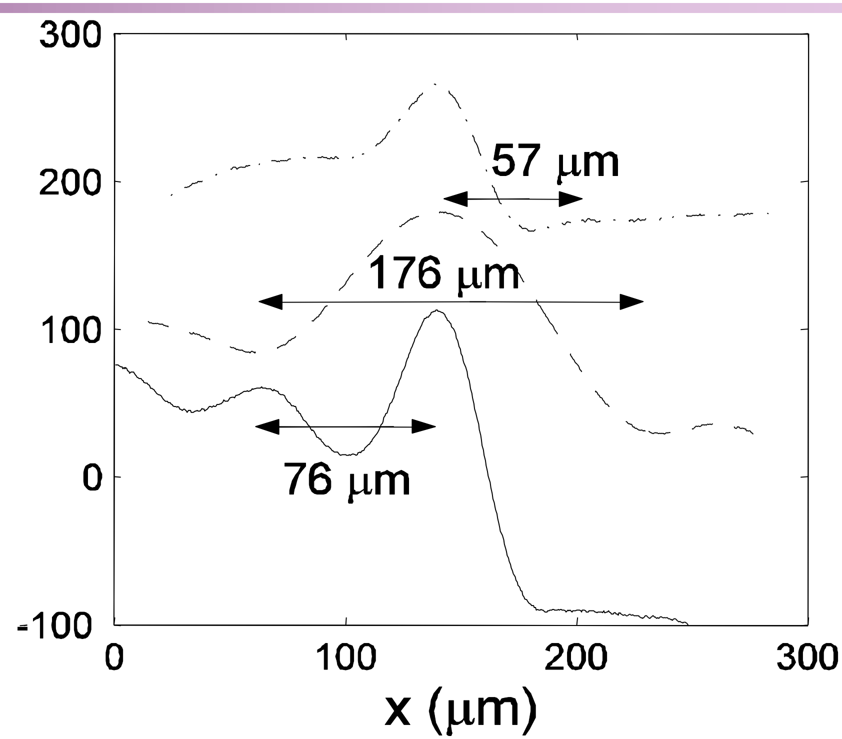


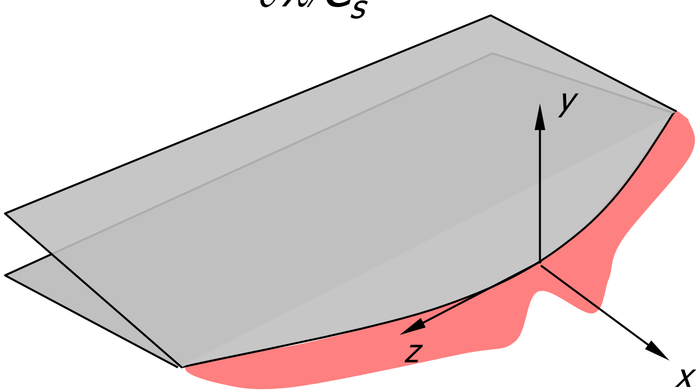
12.7 mm
Specimen AC
v= 440 m/s
f = 5 MHz


Specimen AI
v= 440 m/s
f = 20 MHz
10 mm

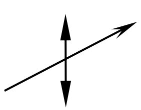
Crack-ultrasonic pulse interaction





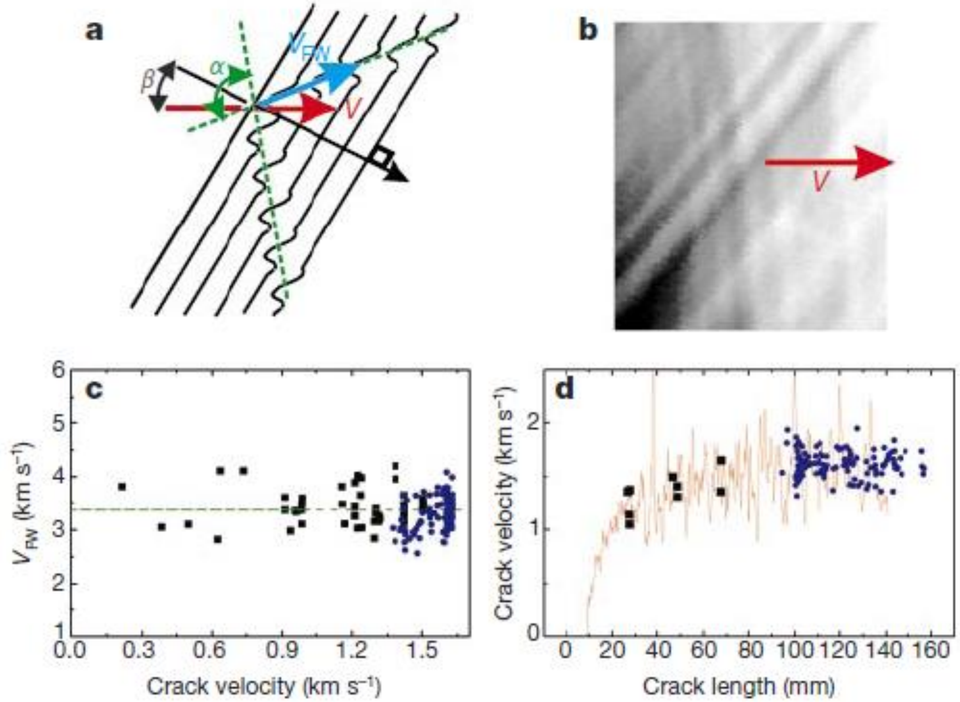


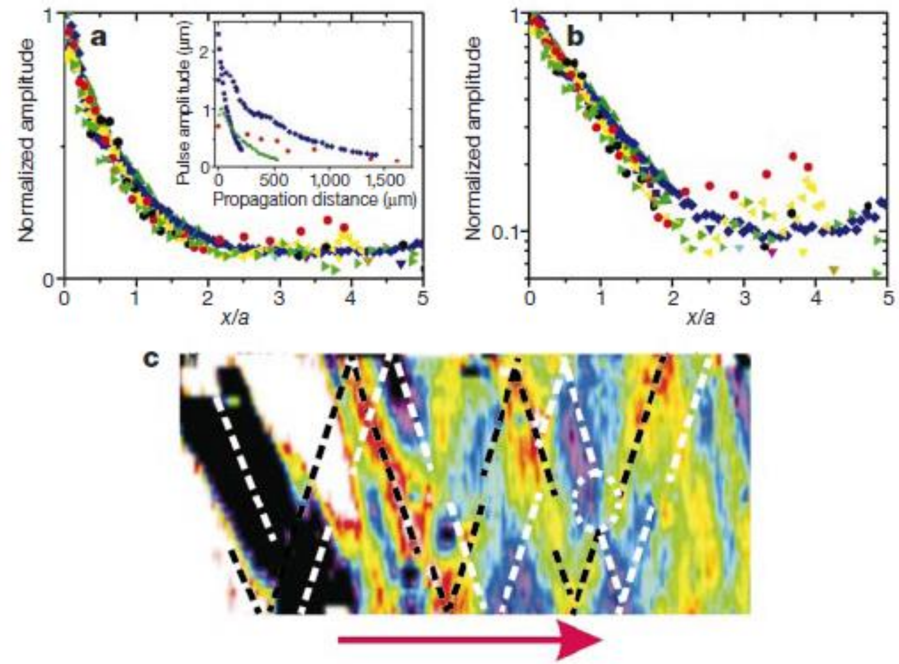
v m/s	f MHz	$\lambda_x = \lambda v/C_s$ μm
480	5	96
867	5	174
895	20	45



Attenuation is similar to that of bulk glass

Results of Fineberg et al

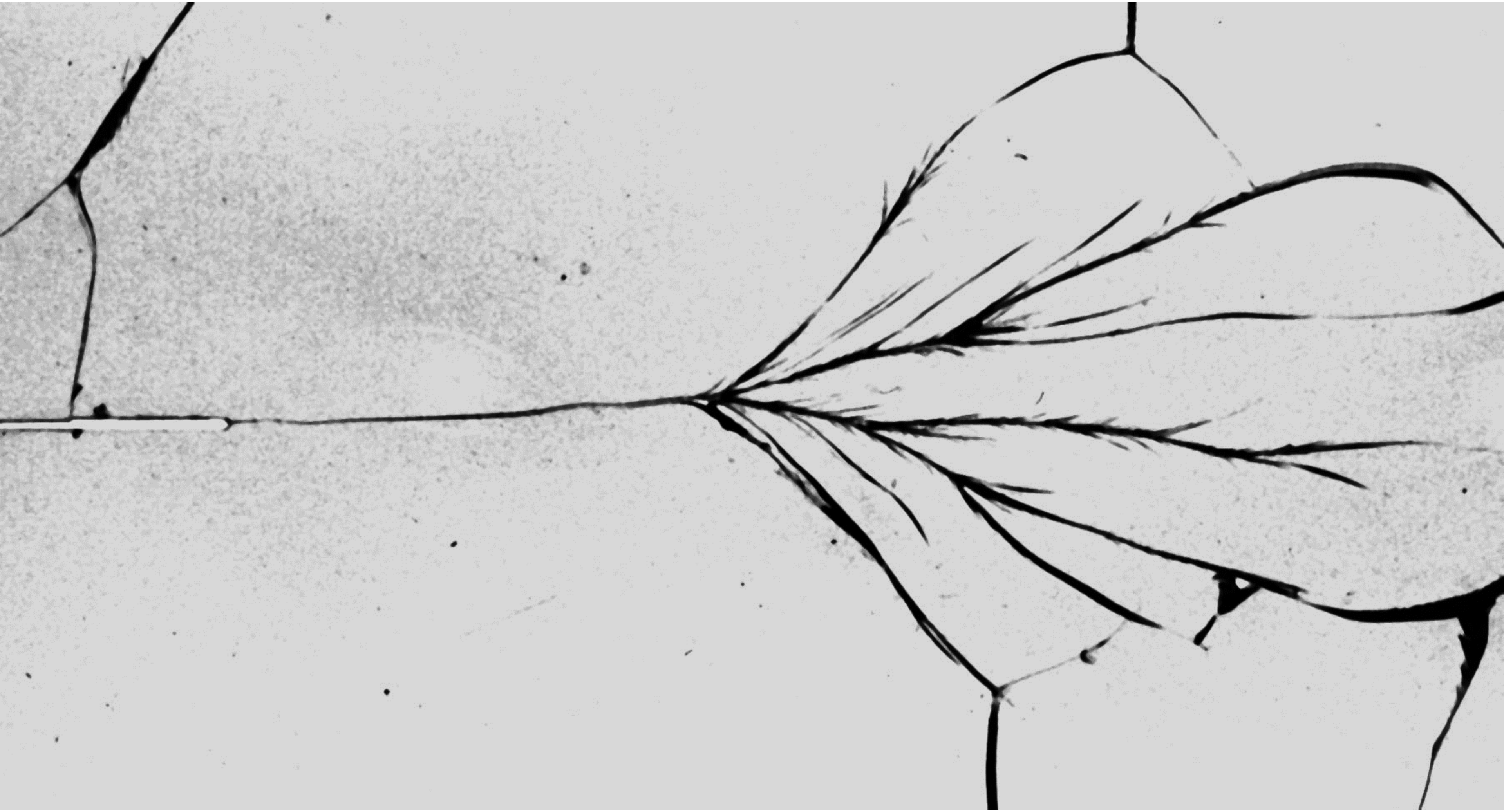


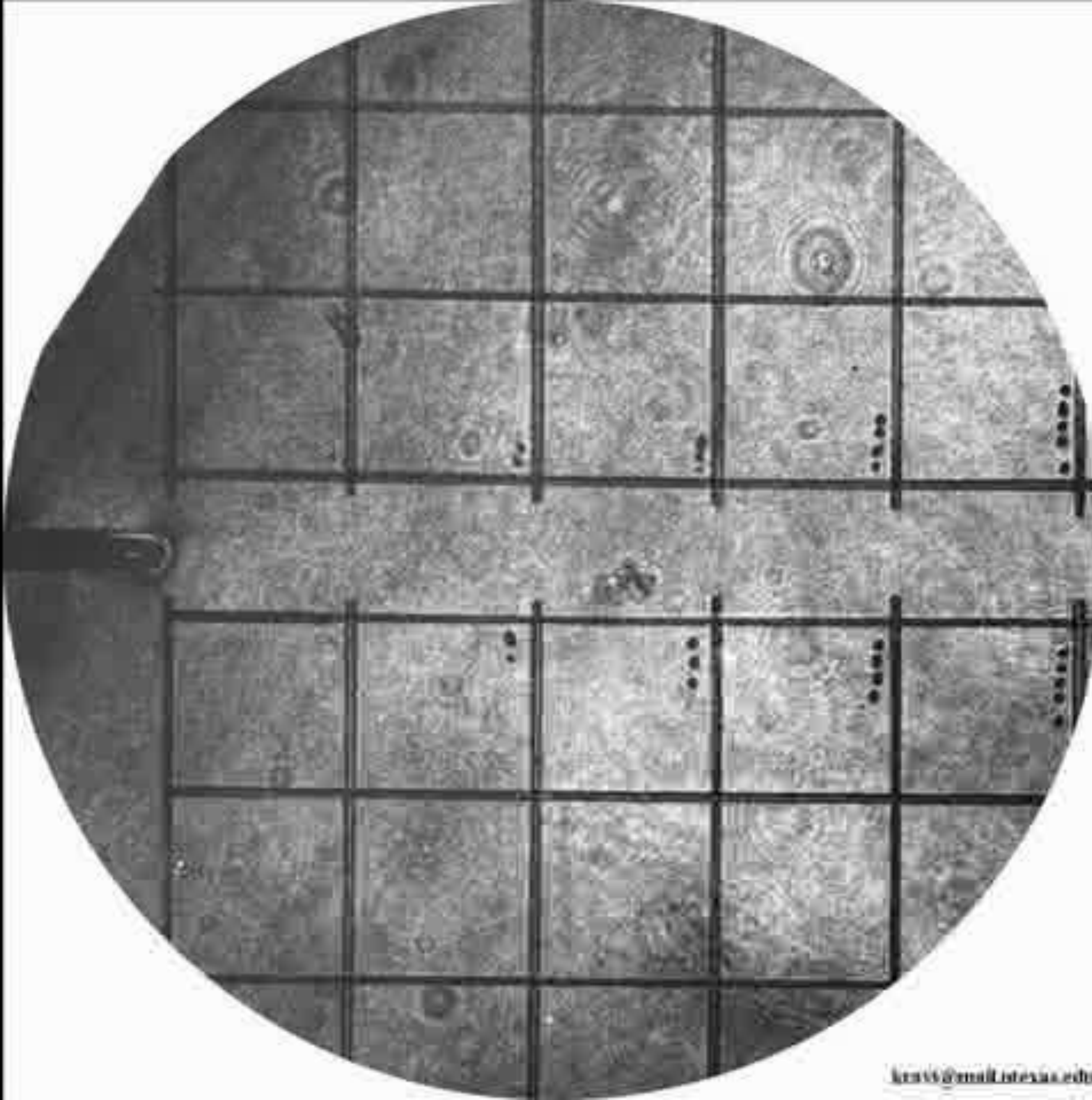


Summary on crack fronts

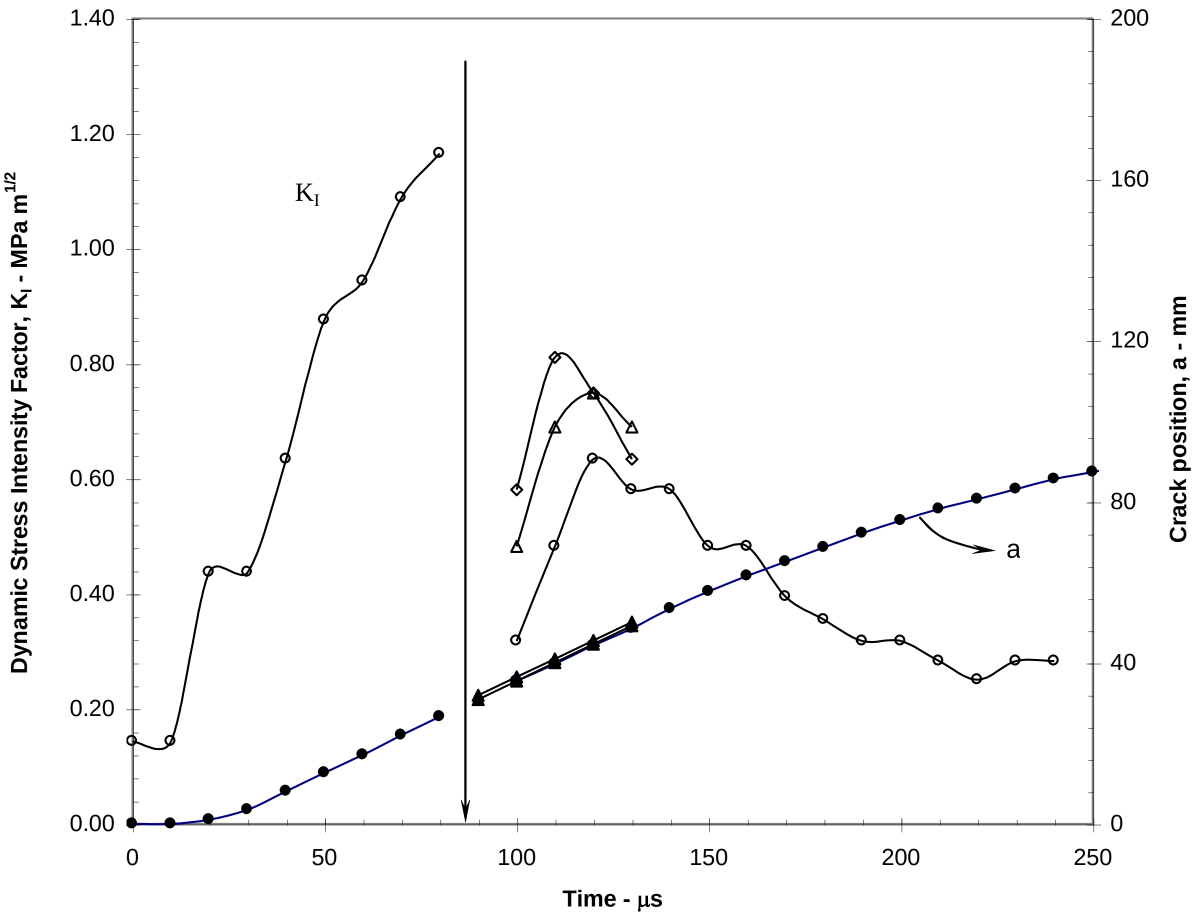
- Theory predicts that they exist
  - Important in roughness generation; bulk waves can also deliver energy to the cracks and cause roughness, but these CF waves are more effective.
  - Difficult to distinguish from Wallner lines
- Small amplitude plane wave perturbation
  - Perfectly linear response to mode III perturbations
  - Responds to wavelength of input, speed of crack – no inherent characteristic length!
  - No persistent interaction between different pulses
  - No persistence when perturbation is removed
- Need larger amplitude perturbations?

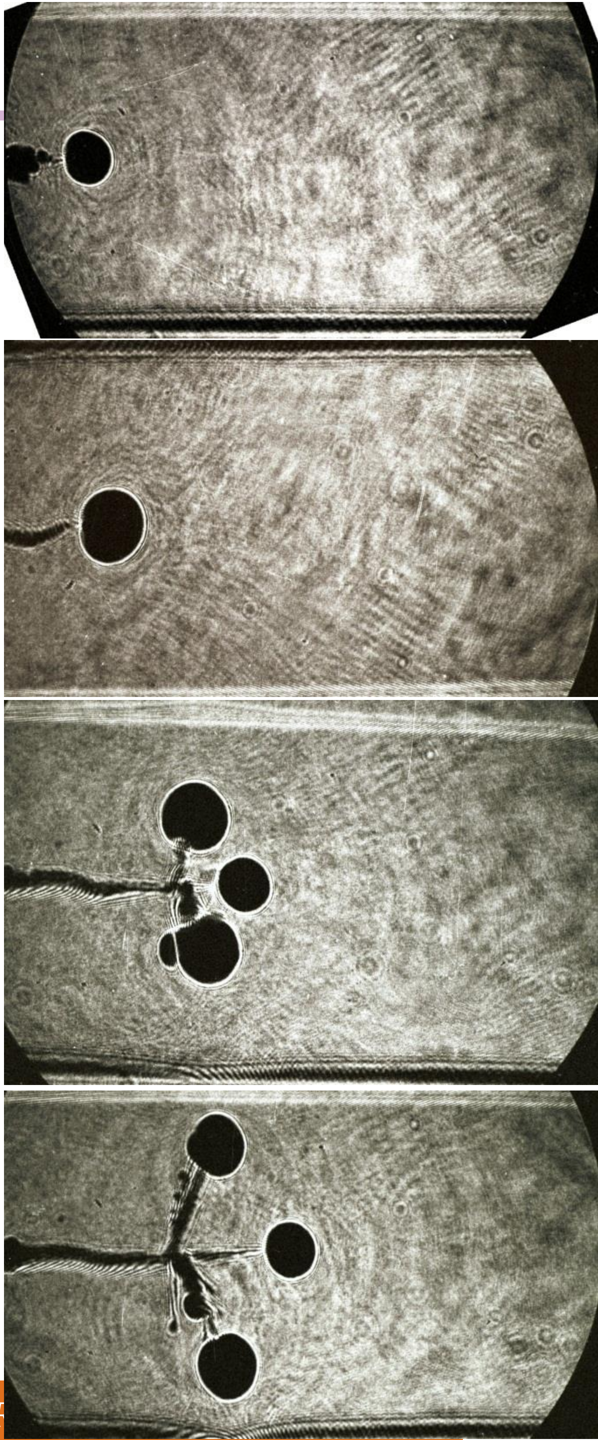
Dynamic crack branching



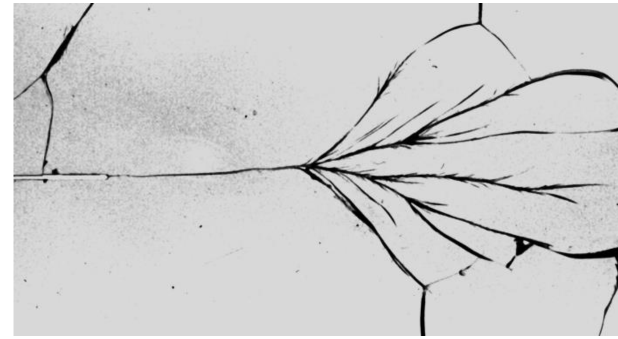


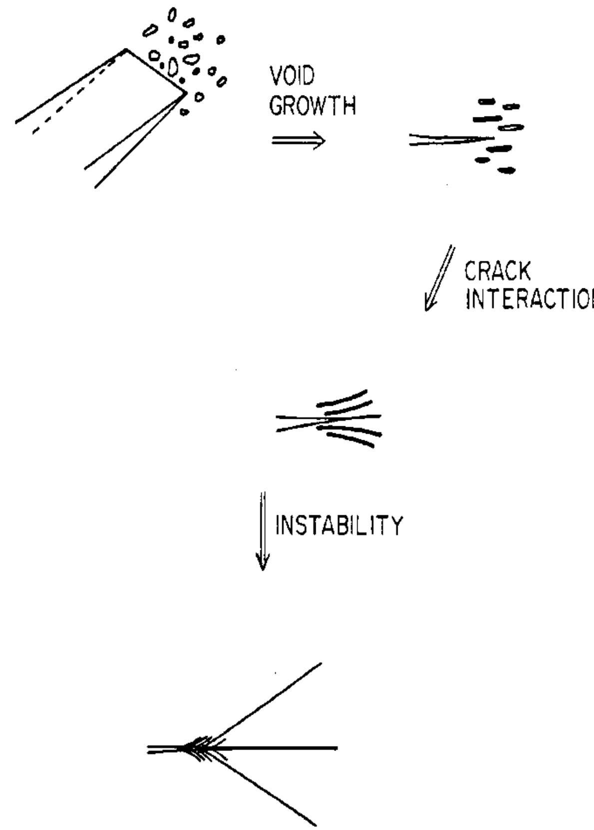
Crack branching





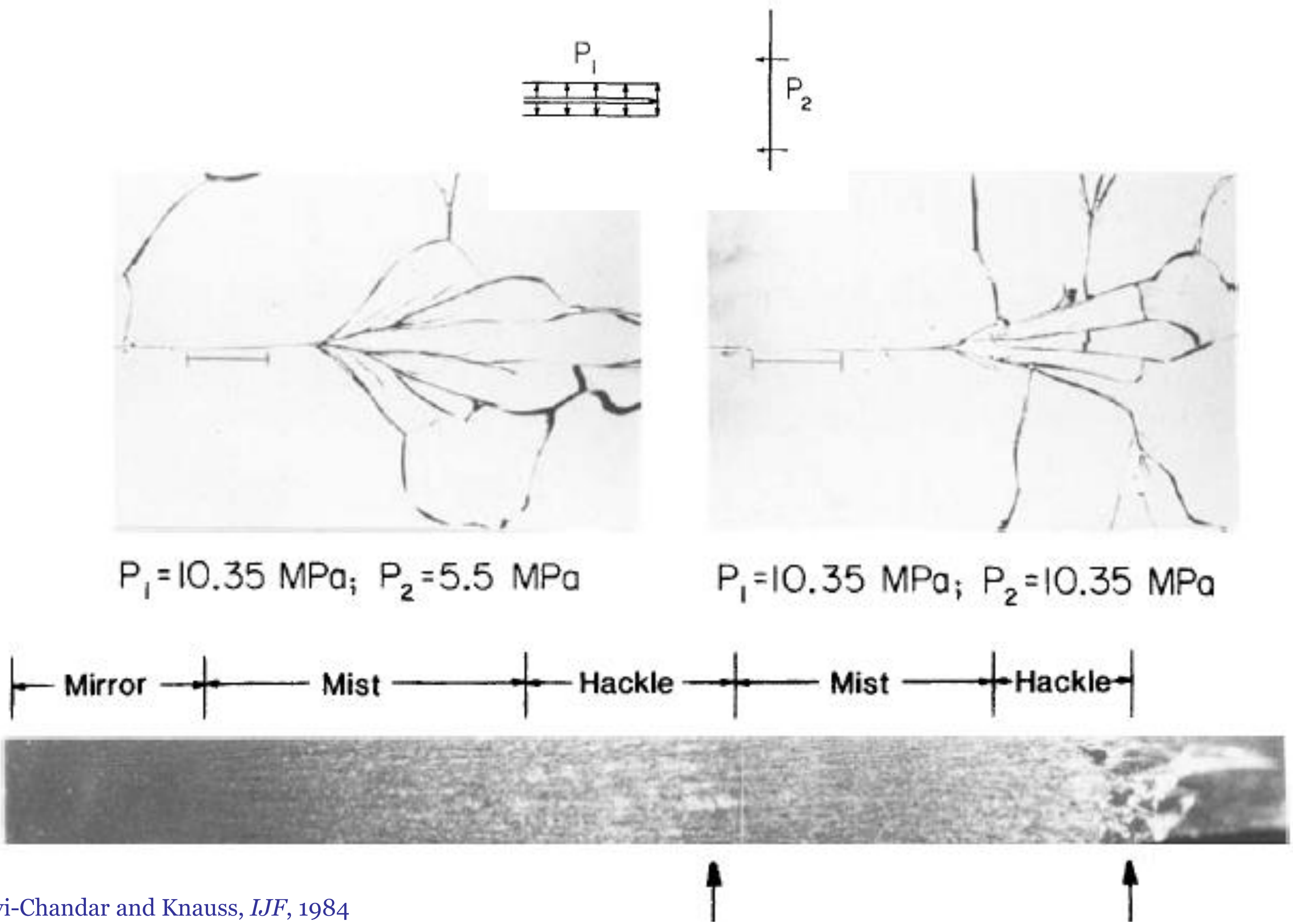
Microcracking model for dynamic fracture





Ravi-Chandar and Knauss, *Int J Fract*, 1984 – Homalite - 100

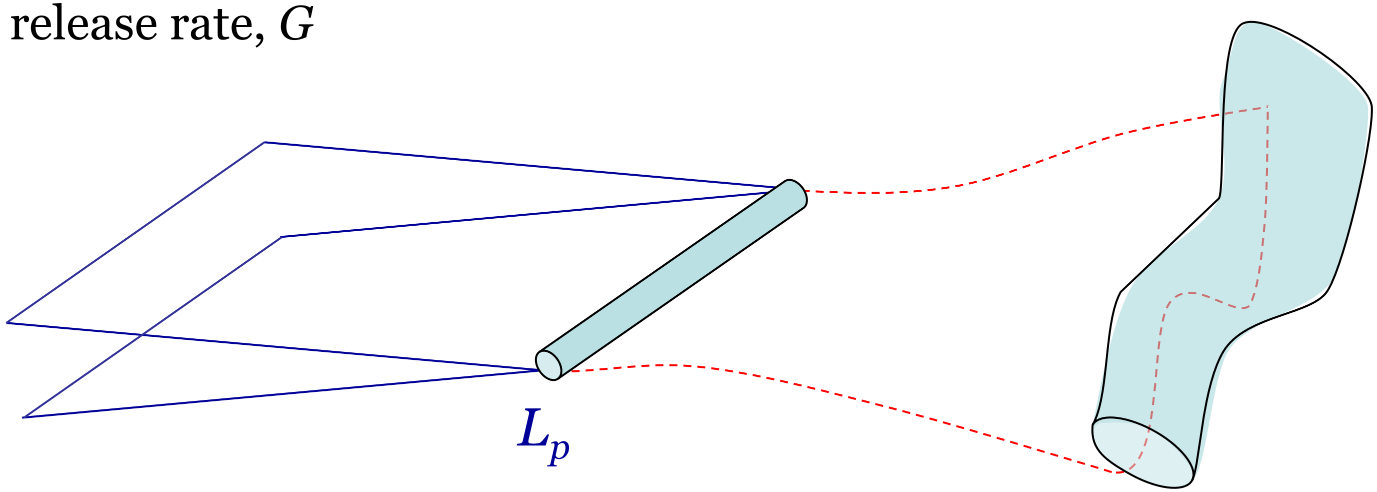
Control of surface roughening and branching



Ravi-Chandar and Knauss, *IJF*, 1984

Fracture mechanics

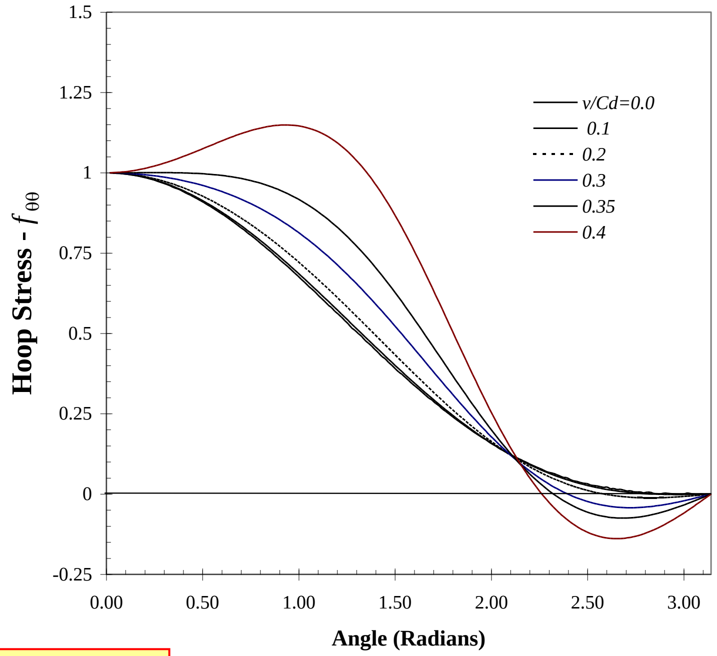
Outer Problem: Knowledge of constitutive laws and balance equations is adequate to calculate the energy release rate, *G*



Inner Problem: Are the details of the failure mechanisms and fracture processes important ***only*** in determining the fracture energy, Γ?

Crack branching analysis

- Stress field based
  - Yoffe (1951): v~0.65 C_R
- Energy-based
  - Eshelby (1970)
  - Freund (1972): v~0.53 C_R
  - Adda-Bedia et al. (2005, 2007)



$$G = \frac{1-\nu^2}{E} A_I(v) K_I^2 \cdots \text{dynamic energy release rate}$$

Energy balance calculations of Adda-Bedia et al

$$K_p(t,v) = k_p(v) K_p^0(t,0),\ p = I, II, III$$

$$K'_p(t,v) = \sum_l k_l(v) H_{pl}\left(\lambda, v', v\right) K_l^0(t,0)$$

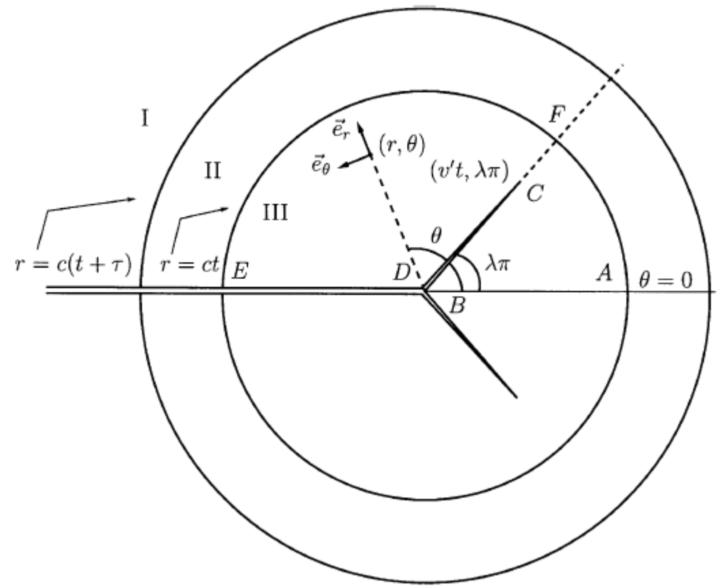
Not all the H_{pl} are available, but H_{33} was calculated by Adda-Bedia et al, corresponding to the mode III problem.

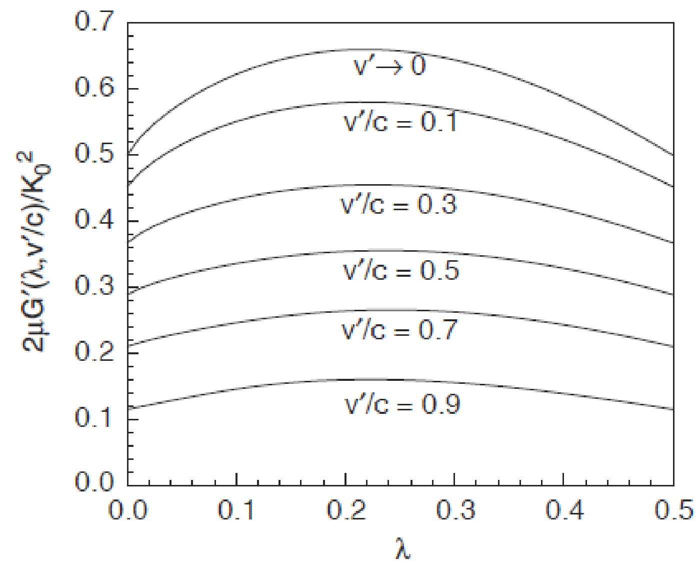
$$K'_{III}(t,v) = k_{III}(v) H_{33}\left(\lambda, v', v\right) K_{III}^0(t,0)$$

$$G' = \frac{1}{2\mu} g(v')\left(H_{33}\left(\lambda, v'\right) K_{III}^0(t,0)\right)^2$$

$$G' = \Gamma\left(v'\right); G = \Gamma\left(v\right)$$

$$\boxed{v = 0.39 C_s; \lambda = 0.22(39.6^\circ); v' = 0}$$





Adda-Bedia et al. 2005, 2007,...

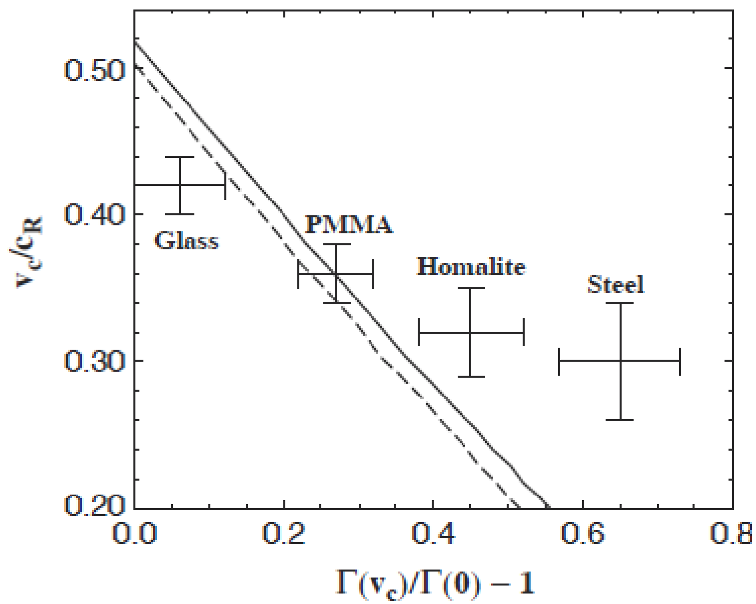
In-plane modes

$$G' = \frac{1}{2\mu}\left[g_I(v')\left(H_{11}\left(\lambda, v'\right)K_I^0(t,0)\right)^2 + g_{II}(v')\left(H_{21}\left(\lambda, v'\right)K_{II}^0(t,0)\right)^2 \right]$$

$$G' = \Gamma\left(v'\right); G = \Gamma\left(v\right)$$

H_{11} and H_{21} are not available, but Adda-Bedia argues that in the limit of zero crack speed, these should behave similarly to H_{33}, corresponding to the mode III problem.

$$\boxed{v = 0.518C_R; \lambda = 0.13(23.4^\circ); v' = 0}$$

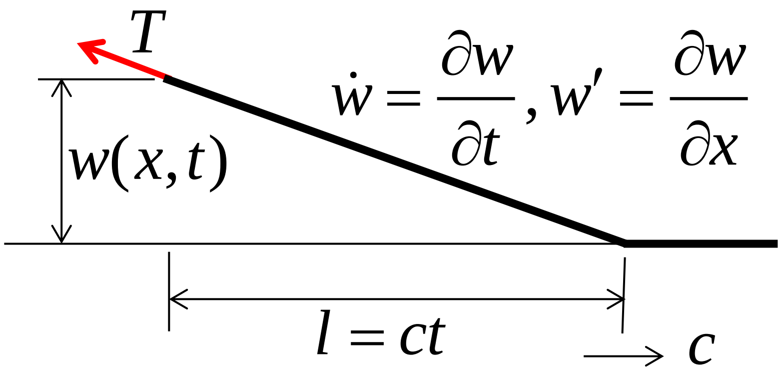


$\Gamma\left(v\right)$ influences the critical speed, but not the branching angle

Adda-Bedia et al. 2005, 2007,...

Dynamic lifting of a tape

- Inextensible, flexible tape with tension T, mass per unit length ρ
- $w(x,t)$ ... transverse deflection
- $\dot{w}(x,t)$ ... transverse velocity
- $w'(x,t)$ ... slope (assumed small)



$$L = \frac{1}{2}\left(\rho \dot{w}^2 - T w'^2\right)$$

$$\Rightarrow c^2 \frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial t^2} = 0 \qquad c = \sqrt{T/\rho} \cdots \text{speed of transverse waves}$$

Expect piecewise linear solutions:

$$\dot{s} = \frac{ds}{dt}$$

$$\frac{\partial w'}{\partial t} = \frac{\partial \dot{w}}{\partial x} \Rightarrow \dot{s} \;\; w' \;\; + \;\; \dot{w} \;\; = 0$$

$$f \;\; = f\left(s^+\right) - f\left(s^-\right)$$

In the absence of adhesion in the string, you can show that $\dot{s} = \pm c$

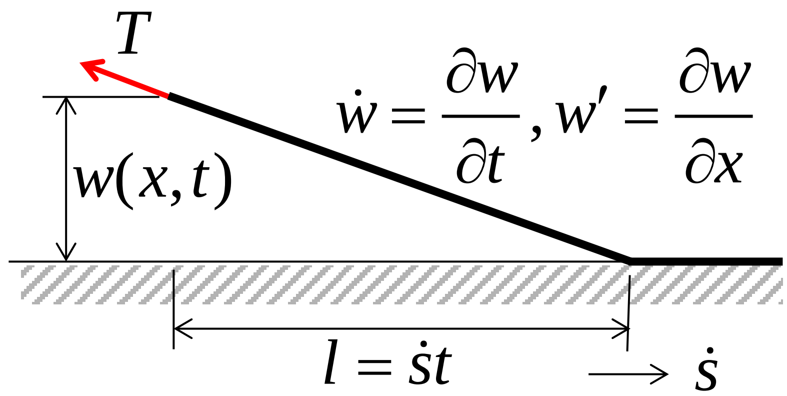
If $\dot{w}$ is prescribed, then $w' = \mp \dot{w}/c$

Burridge and Keller, 1978, Freund, 1990, Duomochel et al, 2008,...

Dynamic peeling of a tape with adhesion

Now let us consider adhesion of the tape to the substrate with an adhesive energy Γ

Consider pulling one end of a semi-infinite tape at a constant speed, $\dot{w}$. As a result, a peel front moves with a speed $\dot{s}$



Expect piecewise linear solutions:

$$\frac{\partial w'}{\partial t} = \frac{\partial \dot{w}}{\partial x} \Rightarrow \dot{s} \quad w' \quad + \quad \dot{w} \quad = 0$$

But another equation is needed to determine $\dot{s}$

This is derived from energy balance: **dynamic energy release rate**

$$G = L\left(s^-\right) - L\left(s^+\right) = \frac{T}{2}\left(\llbracket w'^2 \rrbracket - \llbracket \dot{w}^2 \rrbracket / c^2\right)$$

$$G = \frac{T}{2} w'^2 \left(1 - \frac{\dot{s}^2}{c^2}\right)$$

Dynamic peeling of a tape with adhesion

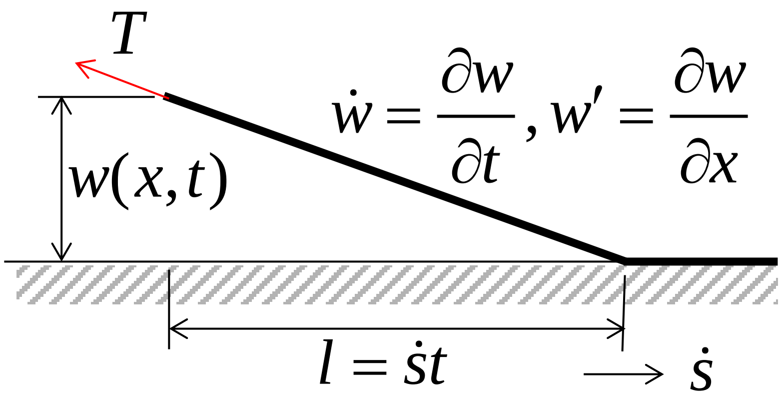
Impose Griffith condition: $G = \Gamma$

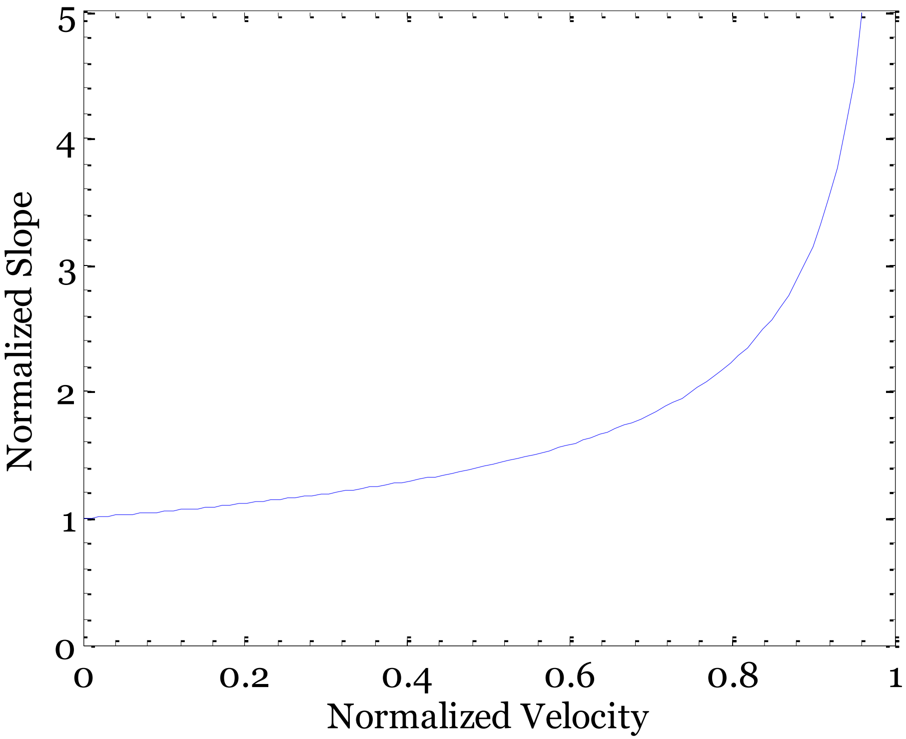
$$G = \frac{T}{2} w'^2 \left(1 - \frac{\dot{s}^2}{c^2}\right) = \Gamma$$

$$\boxed{w' = \left(1 - \frac{\dot{s}^2}{c^2}\right)^{-1/2} w'_{qs}}$$

$$\boxed{w'_{qs} = -\sqrt{2\Gamma / T}}$$

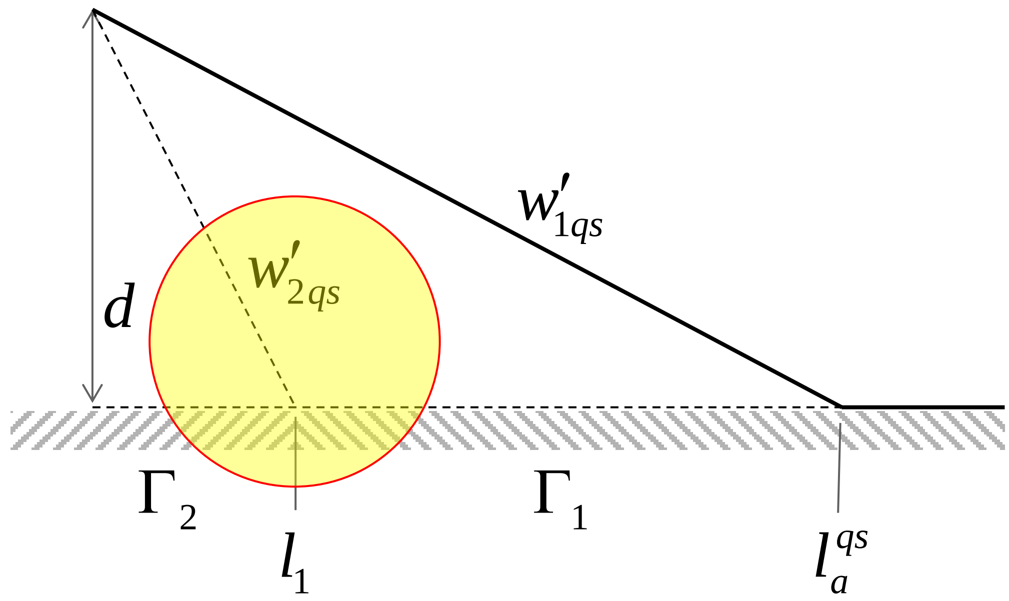
$$\boxed{\frac{\dot{s}^2}{c^2} = \frac{\dot{w}^2 / c^2}{\sqrt{(2\Gamma / T) + \dot{w}^2 / c^2}}}$$





Peeling front encountering a weak patch

$$\Gamma = \begin{cases} \Gamma_2 & 0 < x < l_1 \\ \Gamma_1 < \Gamma_2 & l_1 < x < \infty \end{cases}$$



Let us peel quasi-statically:

$$w'_{2qs} = -\sqrt{2\Gamma_2 / T}; \quad \dot{w}_2 = 0$$

$$d = l_1 \sqrt{2\Gamma_2 / T}$$

At this point, the peel front encounters a weak patch; let us fix d

$$\left| w'_{2qs} \right| > \left| w'_{1qs} \right| = \sqrt{2\Gamma_1 / T}$$

Quasi-statically, crack will extend until

$$w'_{1qs} = -\sqrt{2\Gamma_1 / T}$$

$$\Rightarrow \frac{l_a^{qs}}{l_1} = \sqrt{\frac{\Gamma_2}{\Gamma_1}}$$

$\cdots$equilibrium crack length

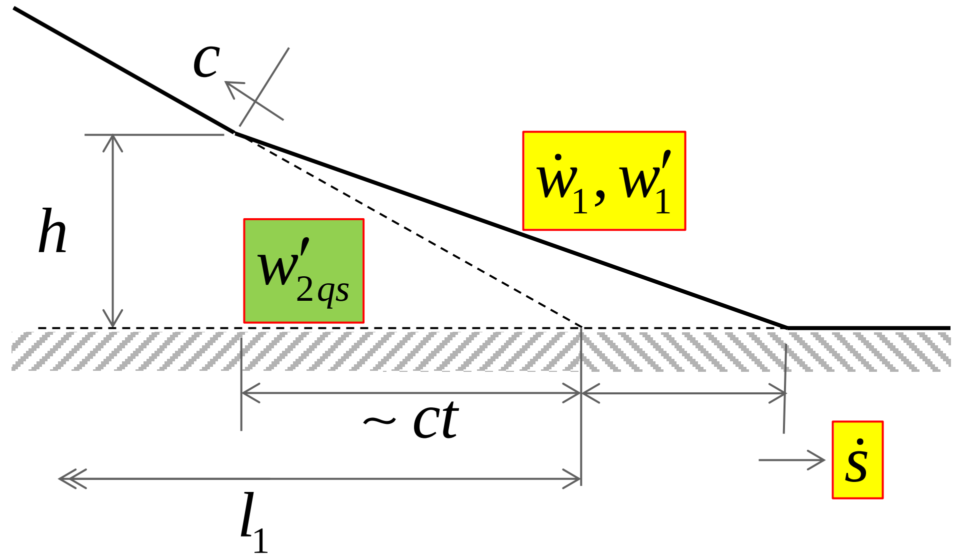
However, the crack will propagate dynamically

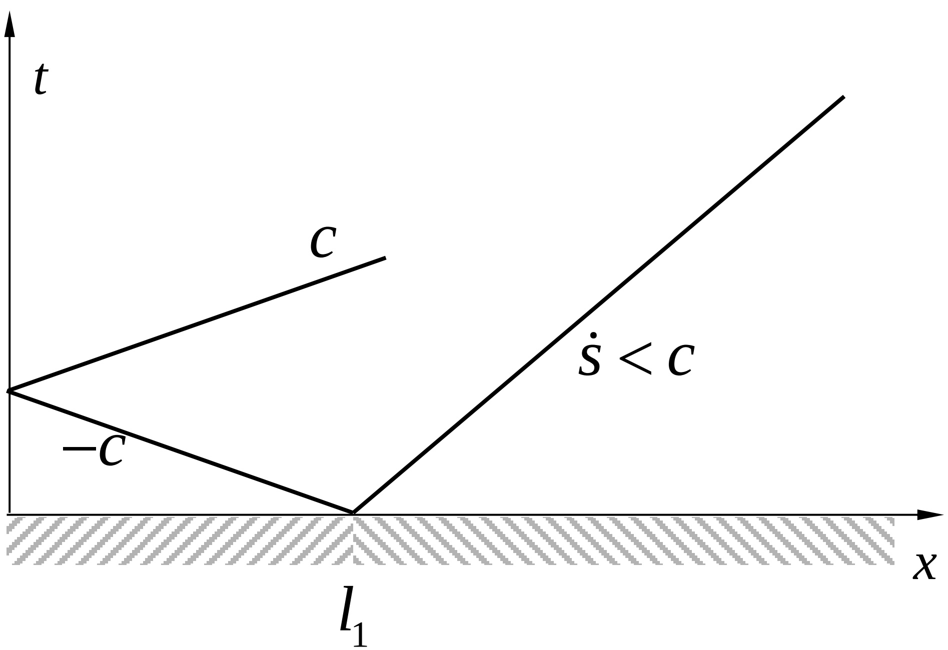
Dynamic peeling front encountering a weak patch

$$\Gamma = \begin{cases} \Gamma_2 & 0 < x < l_1 \\ \Gamma_1 < \Gamma_2 & l_1 < x < \infty \end{cases}$$

But, the crack will advance dynamically, at a speed $\dot{s}$

A kink wave will propagate towards the fixed end in the peeled portion of the tape





crack tip: $\dot{s}w_1' + \dot{w}_1 = 0$

kink: $\left(\dot{w}_1 - \dot{w}_2\right) - c\left(w_1' - w_2'\right) = 0$

dynamic energy release rate

$$cw_2'\left(w_2' - w_1'\right)\left(2 - w_2'/w_1'\right) = \dot{s}2\Gamma_1/T$$

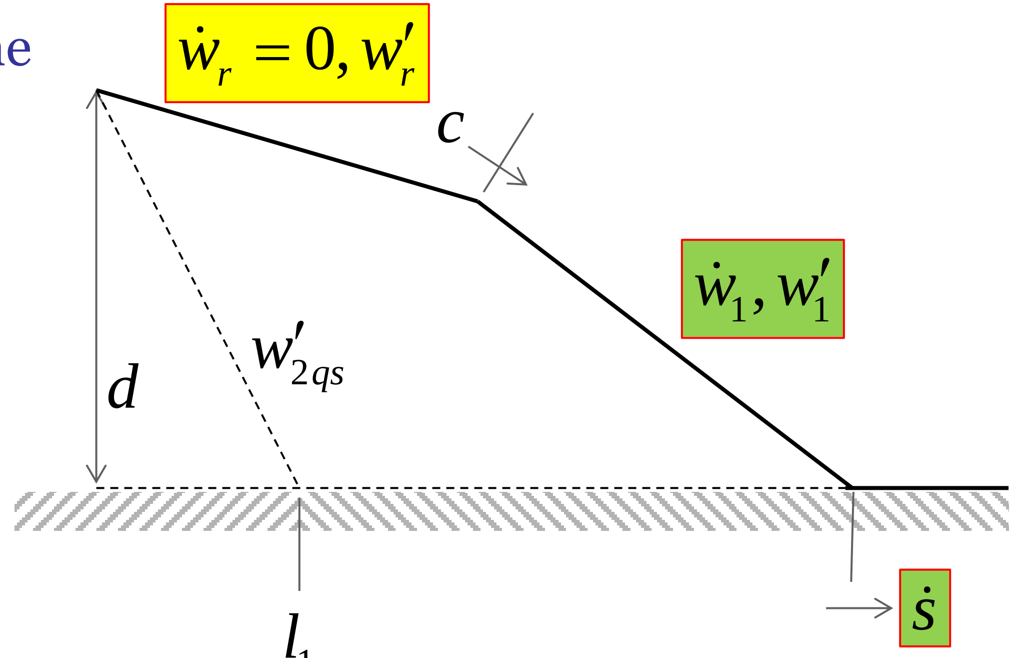
$$\boxed{\frac{w_1'}{w_{1qs}'} = \frac{1+\Gamma_2/\Gamma_1}{2\sqrt{\Gamma_2/\Gamma_1}}} \qquad \boxed{\frac{\dot{s}}{c} = \frac{\Gamma_2/\Gamma_1 - 1}{\Gamma_2/\Gamma_1 + 1}}$$

Dynamic peeling front encountering a weak patch

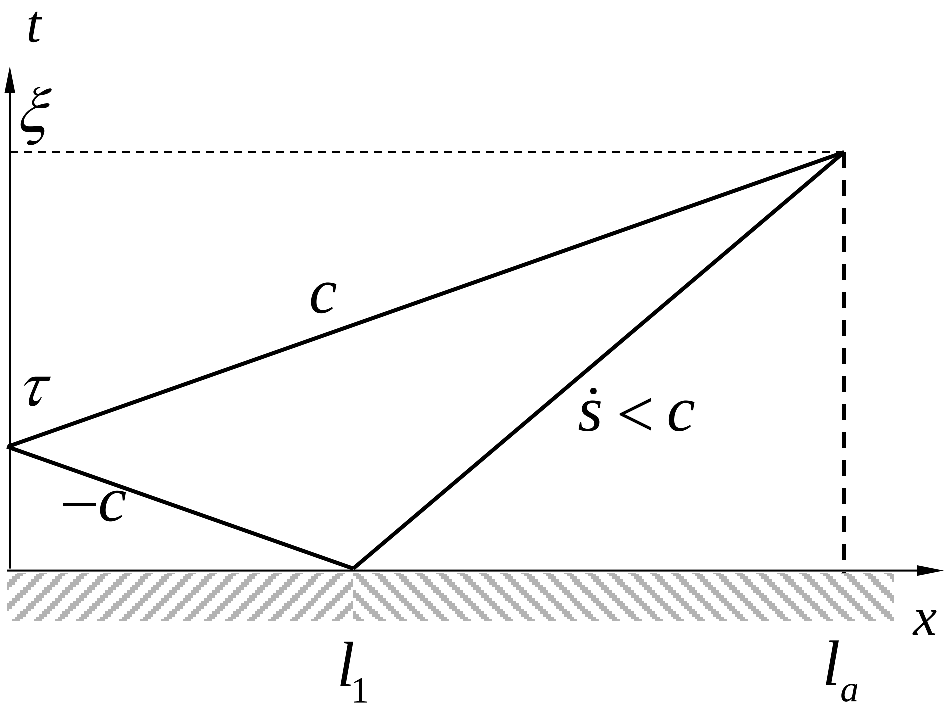
Next, we consider reflection from the fixed end of the kink wave

$$\text{kink: } \left(\dot{w}_1 - \dot{w}_r\right) + c\left(w_1' - w_r'\right) = 0$$

This will arrive at the location $x = l_a$ at $t = \xi = \left(l_a - l_1\right)/\dot{s}$



$$w_r' = w_1'\left(1 - \frac{\dot{s}}{c}\right) = w_{1qs}'\sqrt{\frac{\Gamma_1}{\Gamma_2}} < w_{1qs}'$$



The peel front arrests and everything becomes quiescent!

$$l_a = l_1\frac{\Gamma_2}{\Gamma_1} > l_a^{qs}$$

Extensions to this one-d problem

- Continue loading at $x = 0$; must consider multiple reflections
- Introduce speed dependence to the energy $\Gamma(v)$
- Consider flexibility – straightforward extension to the bending problem
- Consider extensible strings – more complex due to the presence of longitudinal and transverse waves