

Transport phenomena

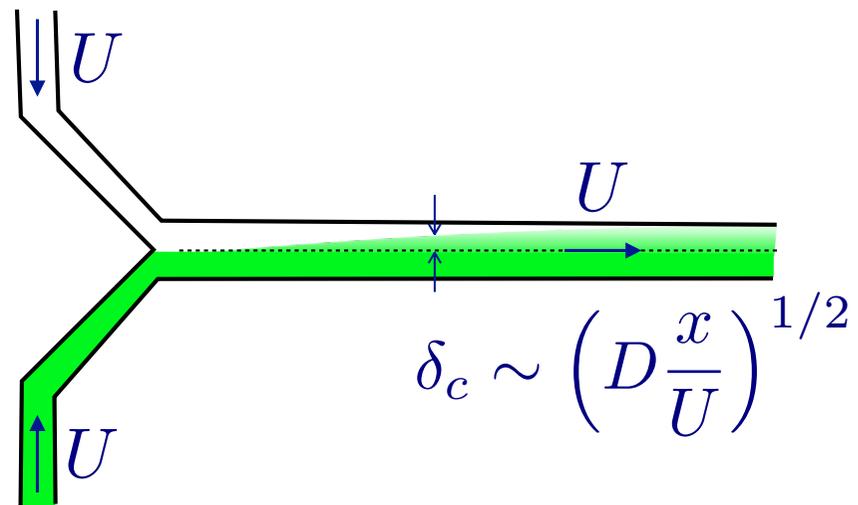
4. Forced convection



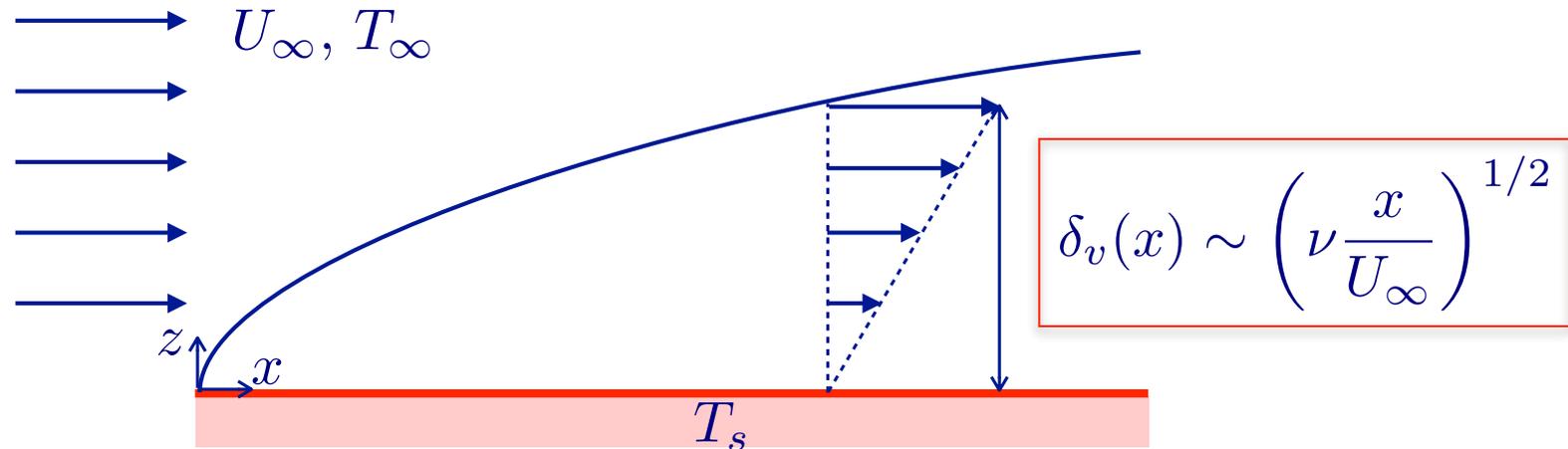
4.0 Coupling flow with heat/mass exchange

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = D \Delta c + r$$

Simple configuration: U uniform



3.1 Cooling flow over a plate



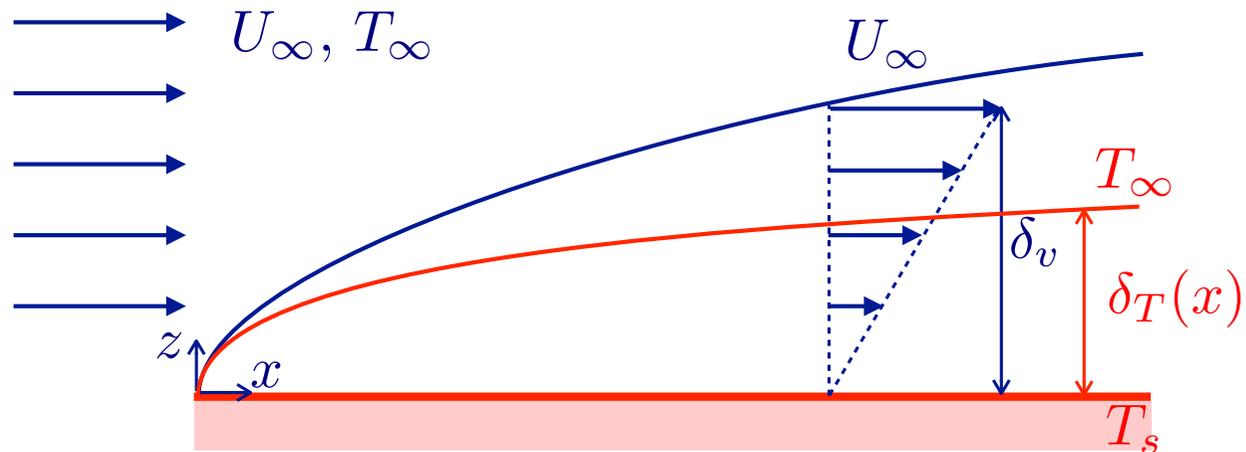
incompressible flow: $\nabla \cdot \mathbf{u} = 0 = \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \Rightarrow \frac{U_\infty}{x} \sim \frac{u_z}{\delta_v}$

$$u_z \sim \frac{\delta_v}{x} U_\infty \ll U_\infty \quad \text{for } x \gg \delta_v$$

velocity profile inside the boundary layer:

$$u_x(x, z) \sim \frac{U_\infty}{\delta_v(x)} z$$

3.1.1 Thermal boundary layer (hypothesis: $\delta_T < \delta_v$)



heat equation:

$$\cancel{\frac{\partial T}{\partial t}} + \mathbf{u} \cdot \nabla T = \alpha \Delta T + \cancel{\frac{r}{\rho C_p}}$$

steady

$$\simeq u_x \frac{\partial T}{\partial x}$$

in scaling laws:

$$\frac{U_\infty}{(\nu x / U_\infty)^{1/2}} \delta_T \frac{T}{x} \sim \alpha \frac{T}{\delta_T^2}$$

$$\Rightarrow \delta_T(x) \sim \alpha^{1/3} \nu^{1/6} \left(\frac{x}{U_\infty} \right)^{1/2} \sim \delta_v(x) \left(\frac{\alpha}{\nu} \right)^{1/3} = \delta_v(x) \text{Pr}^{-1/3}$$

Prandtl number $\text{Pr} = \frac{\nu}{\alpha}$

water ~ 7

air ~ 1

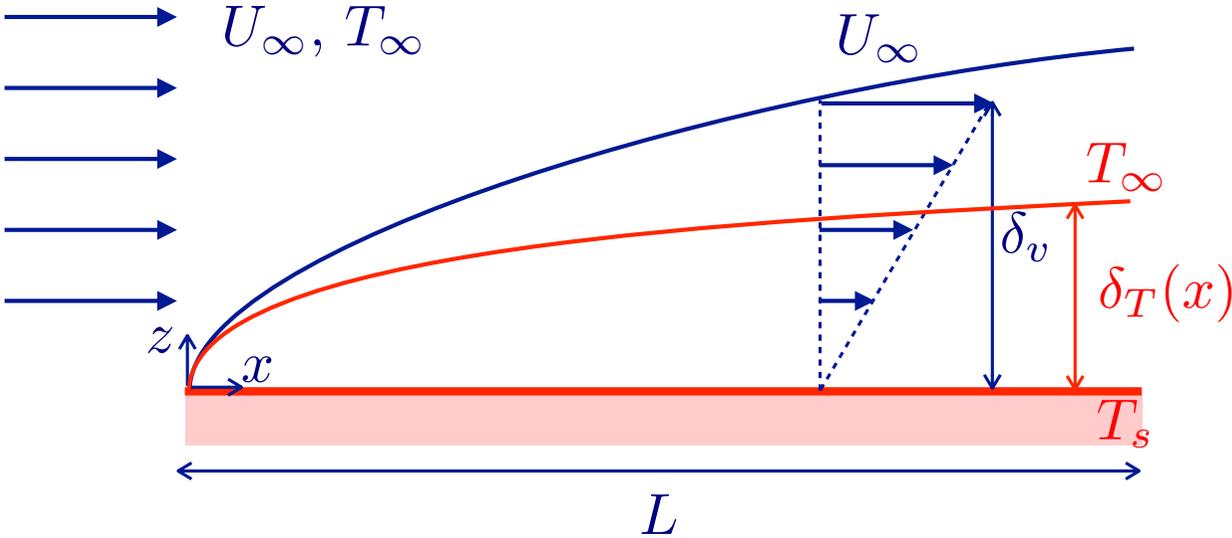
Hg ~ 0.02 \Rightarrow different behavior

glycerin ~ 1000

$$\delta_T < \delta_v \sim \text{OK, if } \text{Pr} > 0.6$$

Schmidt number for mass transport $\text{Sc} = \frac{\nu}{D}$

3.1.2 Heat transport across the thermal boundary layer



Heat flux $\mathbf{J}_T = -\kappa \frac{\partial T}{\partial z} \sim \kappa \frac{T_\infty - T_s}{\delta_T}$

Heat transfer coefficient

$$J_T = -h_T(T_\infty - T_s)$$

$$h_T \sim \frac{\kappa}{\delta_T}$$

Nusselt number

$$\text{Nu} = \frac{h_T L}{\kappa} \sim \frac{L}{\delta_T}$$

Forced convection over a plate

$$\text{Nu} \simeq 0.33 \text{Re}^{1/2} \text{Pr}^{1/3}$$

Other expressions for different geometries $\text{Nu} = f(\text{Re}, \text{Pr})$

Same formalism for mass transport:

Mass transfer coefficient

$$J_m = -h_m(c_\infty - c_s)$$

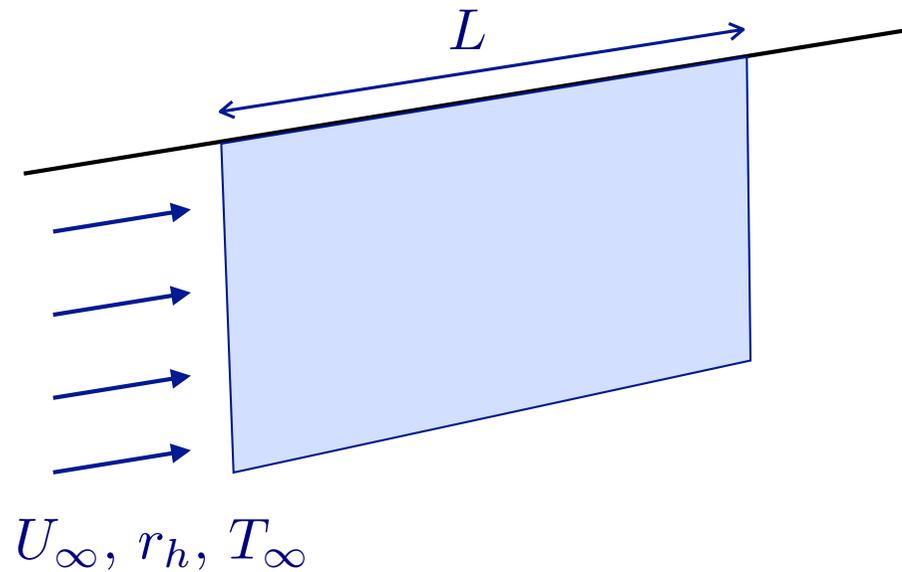
$$h_m \sim \frac{\kappa}{\delta_c}$$

Sherwood number

$$\text{Sh} = \frac{h_m L}{D}$$

5. Forced convection: Practical examples

5.1 Drying a wet sheet in the wind

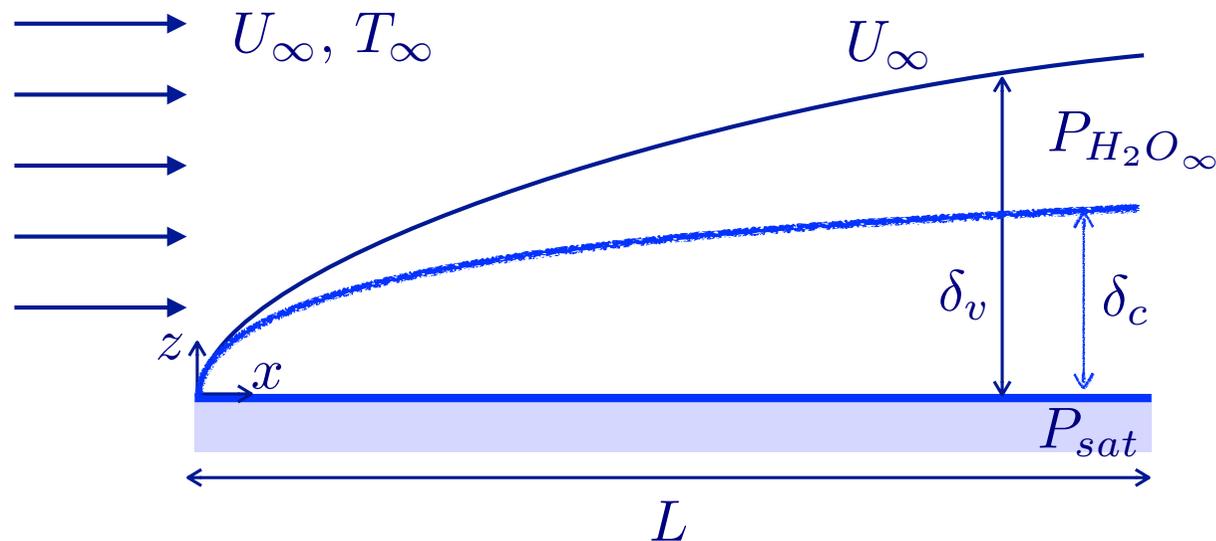


Simplified model

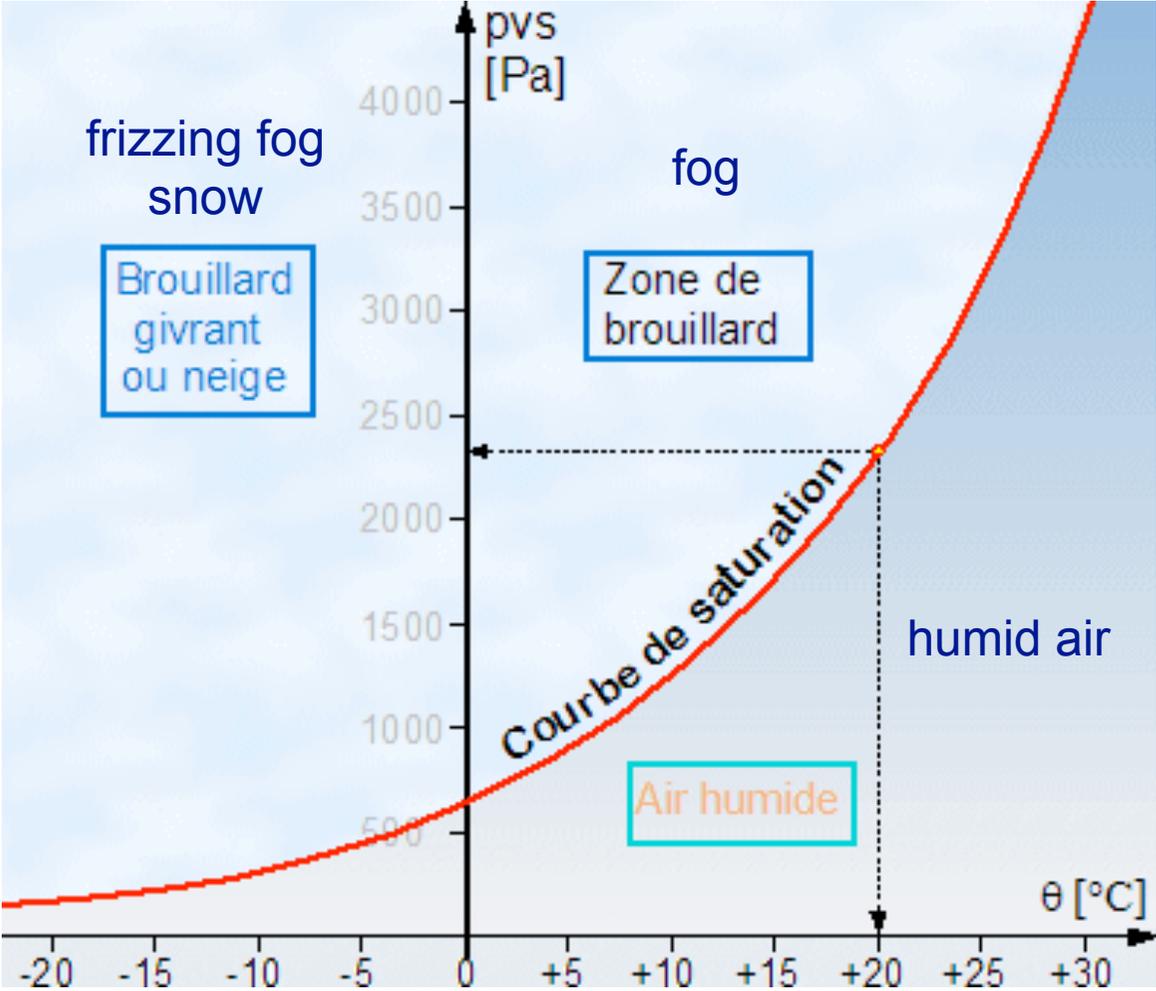
- uniform temperature
- radiative exchanges neglected
- P_{H_2O} at the surface = P_{sat}

relative humidity r_h

$$P_{H_2O_\infty} = r_h P_{sat}$$



Vapor pressure



source: dimclim.fr

$$P_{H_2O} \leftrightarrow c_{H_2O} \quad PV = nRT \Rightarrow c_{H_2O} = \frac{P_{H_2O}}{RT}$$

\downarrow
 mol.m⁻³

H₂O molar flux: $J_{H_2O} = h c_{sat}(1 - r_h)$

$$\downarrow \quad \frac{D}{L} \text{Sh} \sim \frac{D}{L} \left(\frac{U_\infty L}{\nu} \right)^{1/2} \left(\frac{\nu}{D} \right)^{1/3}$$

\downarrow
 fine to call it "Nu"!

$$J_{H_2O} \sim \frac{D}{L} \left(\frac{U_\infty L}{\nu} \right)^{1/2} \left(\frac{\nu}{D} \right)^{1/3} c_{sat}(1 - r_h) \quad [\text{mol.m}^{-2}.\text{s}^{-1}]$$

if r_h fixed, $T \nearrow \Rightarrow c_{sat} \nearrow \Rightarrow$ higher evaporation

mass flux: $J_{m_{H_2O}} = M_{H_2O} J_{H_2O} \quad [\text{kg.m}^{-2}.\text{s}^{-1}]$

volumic flux: $J_{v_{H_2O}} = \frac{1}{\rho_{H_2O}} J_{m_{H_2O}} \quad [\text{m.s}^{-1}]$

Numerical estimate: $L = 1\text{m}$ $U_\infty \sim 5\text{ m/s}$

$$D_{H_2O/air} = 2,6 \cdot 10^{-5} \text{ m}^2/\text{s} \quad \nu_{air} = 1,5 \cdot 10^{-5} \text{ m}^2/\text{s}$$

$$P_{sat} = 2.3 \text{ kPa} \Rightarrow c_{sat} = 0.9 \text{ mol/m}^3$$

$$J_{H_2O} \sim 0.03 (1 - r_h) \text{ [mol.m}^{-2}\text{.s}^{-1}\text{]}$$

(with prefactor 0.33 for Sh)

$$J_{m_{H_2O}} \sim 7 \cdot 10^{-5} (1 - r_h) \text{ [kg.m}^{-2}\text{.s}^{-1}\text{]}$$

$$J_{v_{H_2O}} \sim 0.07 (1 - r_h) \text{ [}\mu\text{m/s]}$$

to evaporate 0.5 kg/m^2 of water (i.e. 0.5 mm) with $r_h = 50\%$

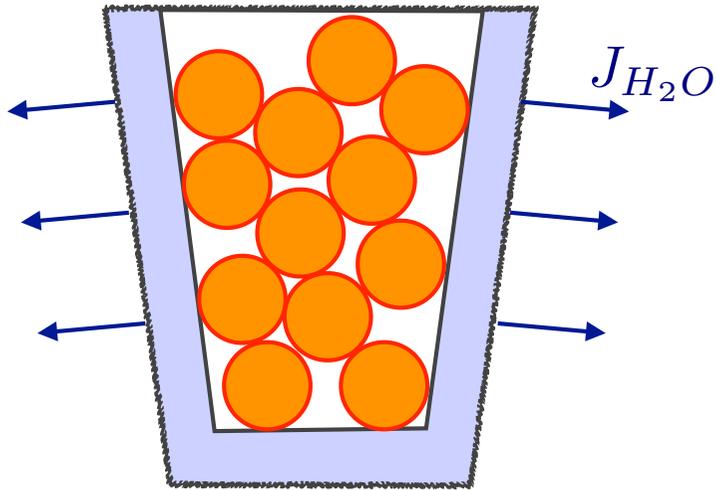
$$\tau_{evap} \sim \frac{0.5 \cdot 10^{-3}}{2 \times 0.07 \cdot 10^{-6} \times 0.5} \sim 2\text{h}$$

↙
2 faces

probably underestimated: evaporation \Rightarrow cooling down $\Rightarrow c_{sat} \searrow \Rightarrow J_{H_2O} \searrow$

5.2 Clay pot cooling

in the shade !



evaporation \Rightarrow latent heat

heat flux at the surface

$$J_{latent} = L_{vap} J_{m_{H_2O}}$$

$$J_{latent} \sim 150 (1 - r_h) \text{ W/m}^2$$

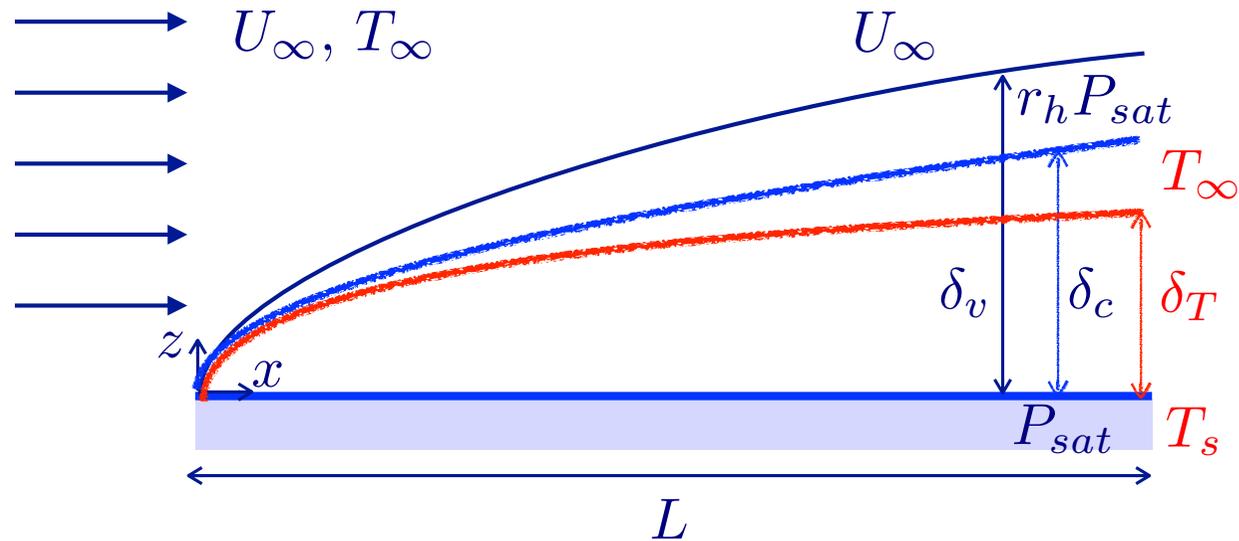
standard fridge \sim 30 to 50 W

5.3 Measuring relative humidity



measure $\Delta T \Rightarrow r_h$

3.2.3 Thermal balance



heat flux from the fluid to the surface

$$J_T \sim \frac{\kappa}{L} \left(\frac{U_\infty L}{\nu} \right)^{1/2} \left(\frac{\nu}{\alpha} \right)^{1/3} (T_\infty - T_s)$$

heat flux from latent heat

$$J_{latent} \sim L_{vap} M_{H_2O} \frac{D}{L} \left(\frac{U_\infty L}{\nu} \right)^{1/2} \left(\frac{\nu}{\alpha} \right)^{1/3} c_{sat} (1 - r_h)$$

$$\Delta T \sim L_{vap} M_{H_2O} \frac{D}{\kappa} \left(\frac{\alpha}{D} \right)^{1/3} c_{sat} (1 - r_h) \quad \text{independent of } U_\infty$$

Measuring relative humidity

Numerical estimate: $\kappa_{air} = 0.03 \text{ J.m}^{-1}.\text{K}^{-1}$ $\alpha_{air} = 1.9 \cdot 10^{-5} \text{ m}^2/\text{s}$

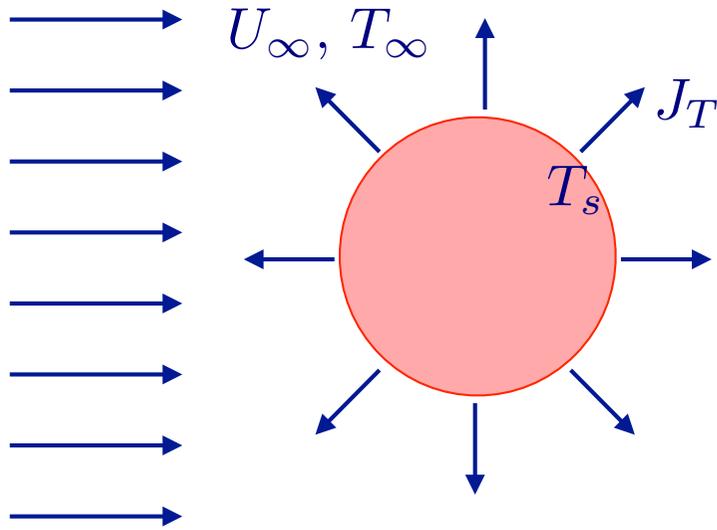


measure $\Delta T \Rightarrow r_h$

$$\Delta T \simeq 27^\circ\text{C}(1 - r_h)$$

5.4 Blowing to cool down





$$J_T \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} (T_s - T_\infty)$$

hypothesis: T uniform in the solid

heat loss from the solid:

$$\frac{dQ}{dt} = -\frac{4}{3}\pi R^3 \rho_s C_{p_s} \frac{dT_s}{dt}$$

$$J_T \sim -R \rho_s C_{p_s} \frac{dT_s}{dt}$$

$$\frac{d(T_s - T_\infty)}{dt} \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} \frac{(T_s - T_\infty)}{\rho_s C_{p_s} R}$$

$$T_s - T_\infty = (T_{s_0} - T_\infty) \exp(-t/\tau)$$

with

$$\tau \sim \frac{\rho_s C_{p_s} R^2}{\kappa_{air}} \left(\frac{\alpha_{air}}{\nu_{air}} \right)^{1/3} \left(\frac{\nu_{air}}{U_\infty R} \right)^{1/2}$$

numerical application: $\tau \sim 15$ min for $R = 2$ cm

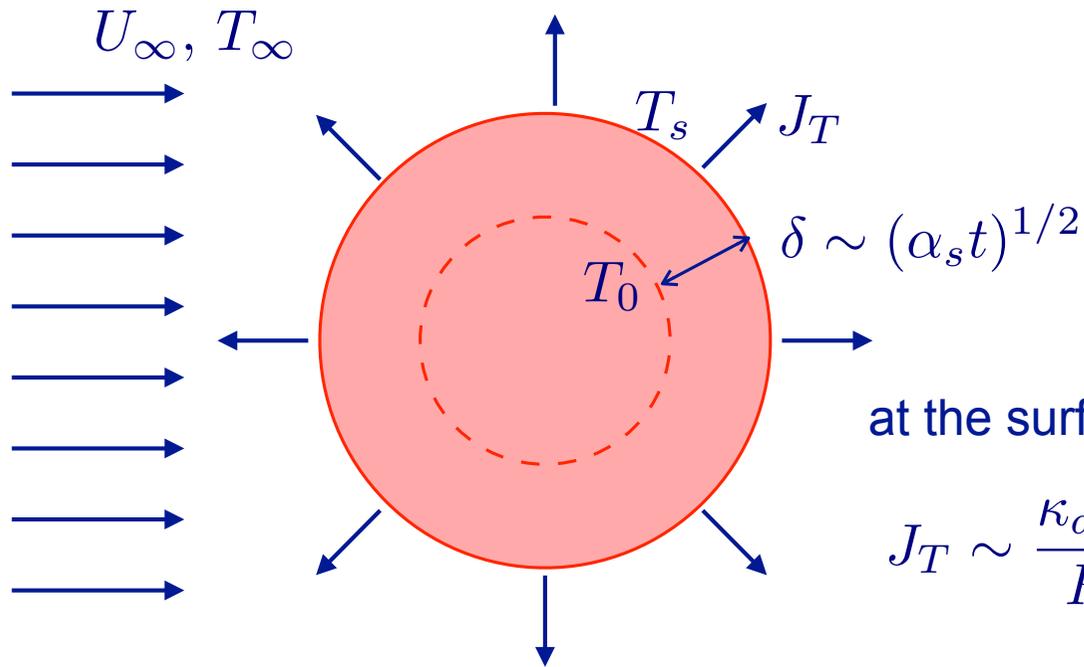
T uniform in the solid ?

\Rightarrow comparison with diffusion time through the solid $\tau \gg \tau_{diff} \sim \frac{R^2}{\alpha_s}$?

$$\Rightarrow R \ll \frac{\nu_{air}}{U_\infty} \left(\frac{\kappa_s}{\kappa_{air}} \right)^2 \left(\frac{\alpha_{air}}{\nu_{air}} \right)^{2/3}$$

numerical application: $R \ll 2$ cm

and if T not uniform?



at the surface:

$$J_T \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} (T_s - T_\infty)$$

across the boundary layer same flux:

$$J_T \sim \frac{\kappa_s}{\delta} (T_0 - T_s)$$

$$\frac{\kappa_s}{(\alpha_s t)^{1/2}} (T_0 - T_s) \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} (T_s - T_\infty)$$

$$T_s \sim \frac{\frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} T_\infty + \frac{\kappa_s}{(\alpha_s t)^{1/2}} T_0}{\frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} + \frac{\kappa_s}{(\alpha_s t)^{1/2}}}$$

We recover:

$$t \rightarrow 0 \Rightarrow T_s \rightarrow T_0$$

$$t \rightarrow \infty \Rightarrow T_s \rightarrow T_\infty$$

characteristic cooling time:

$$\frac{\kappa_s}{(\alpha_s \tau)^{1/2}} \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3}$$

$$\tau \sim \left(\frac{\kappa_s}{\kappa_{air}} \right)^2 \left(\frac{\alpha_{air}}{\nu_{air}} \right)^{2/3} \frac{\nu_{air}}{\alpha_s} \frac{R}{U_\infty} \sim \frac{\tau_{unif}^2}{\tau_{diff}}$$