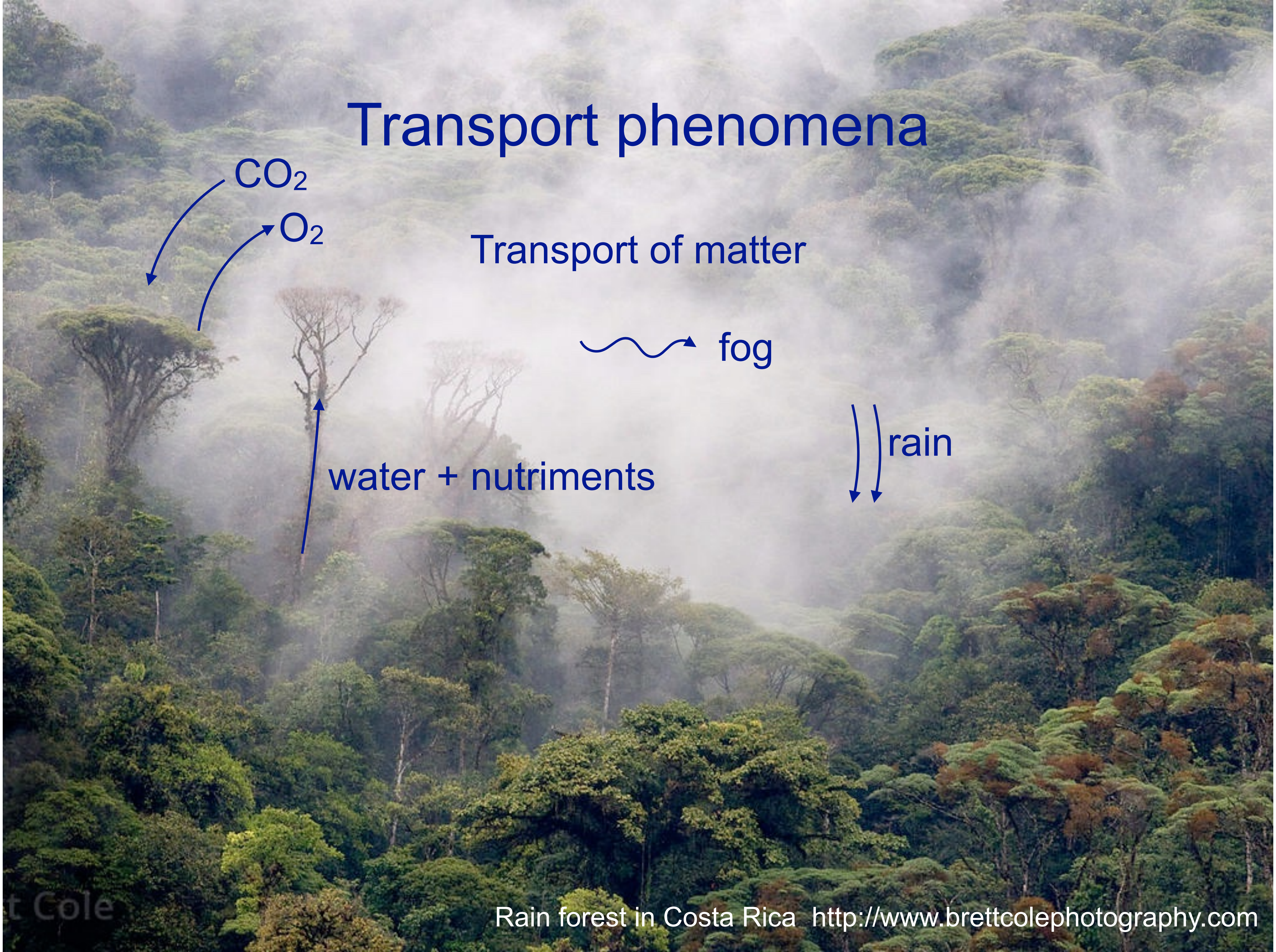
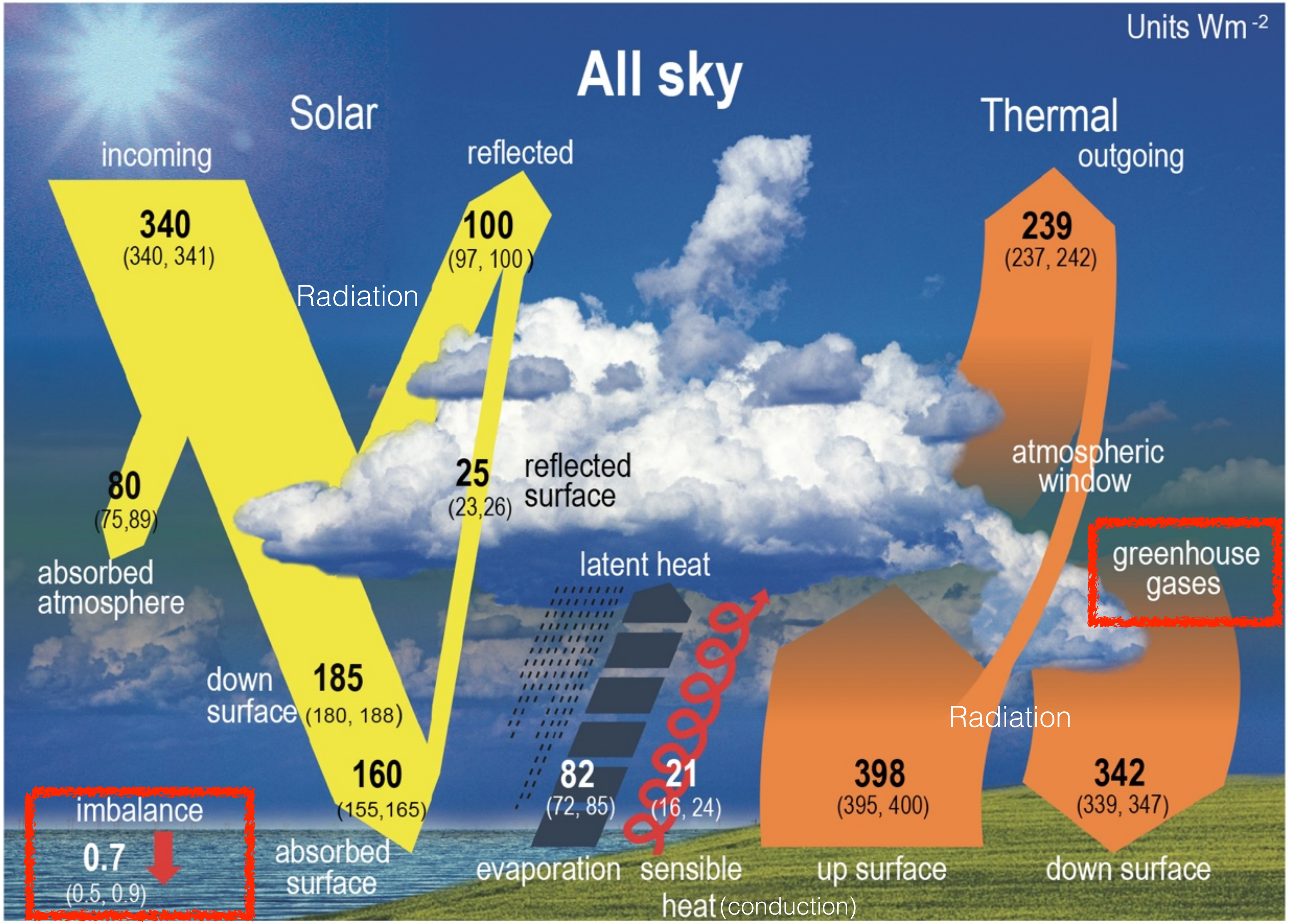
# Transport phenomena




Rain forest in Costa Rica  http://www.brettcolephotography.com

# Transport phenomena

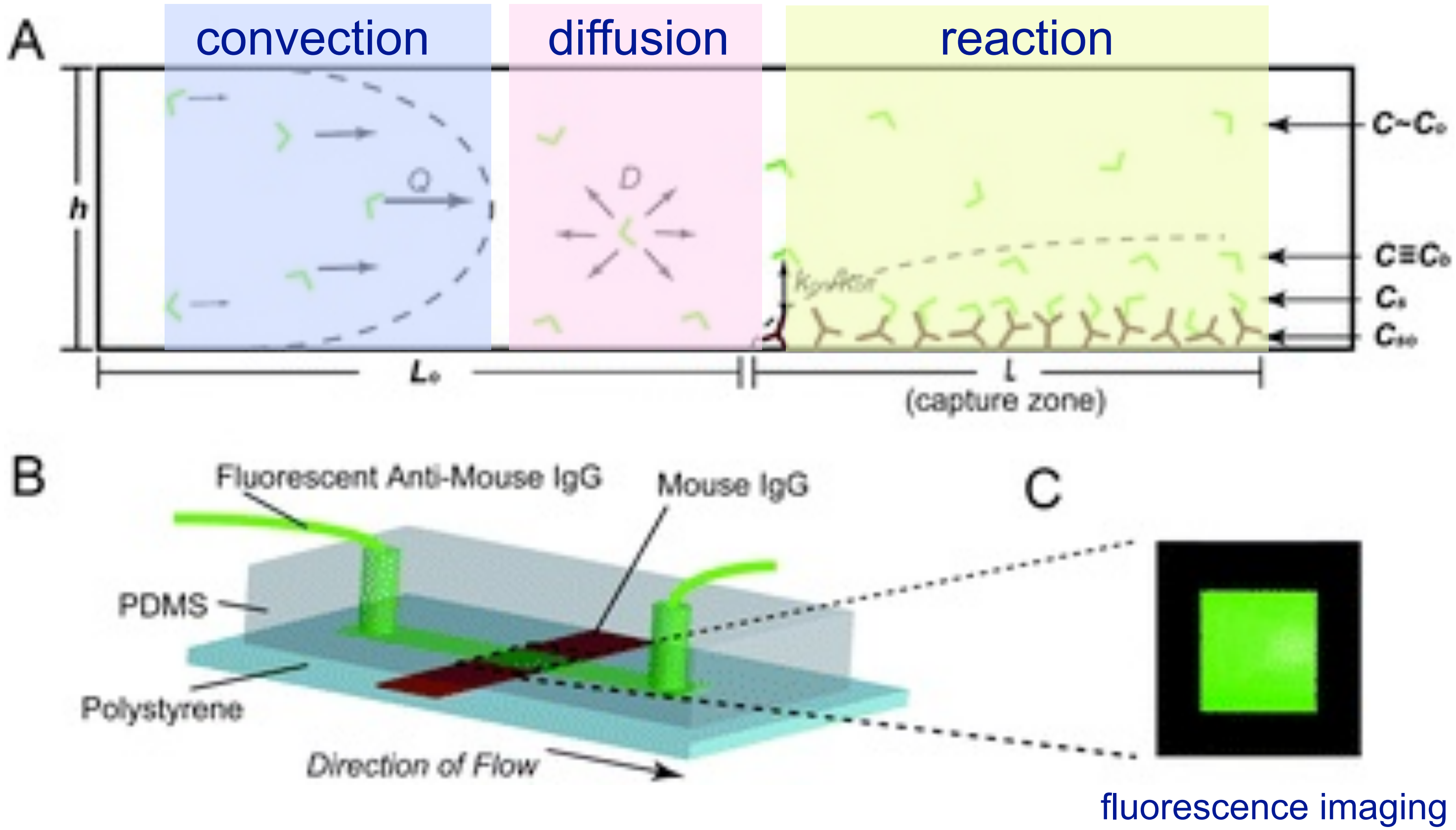
# Earth thermal budget




0.2% of global exchanges

IPCC technical report

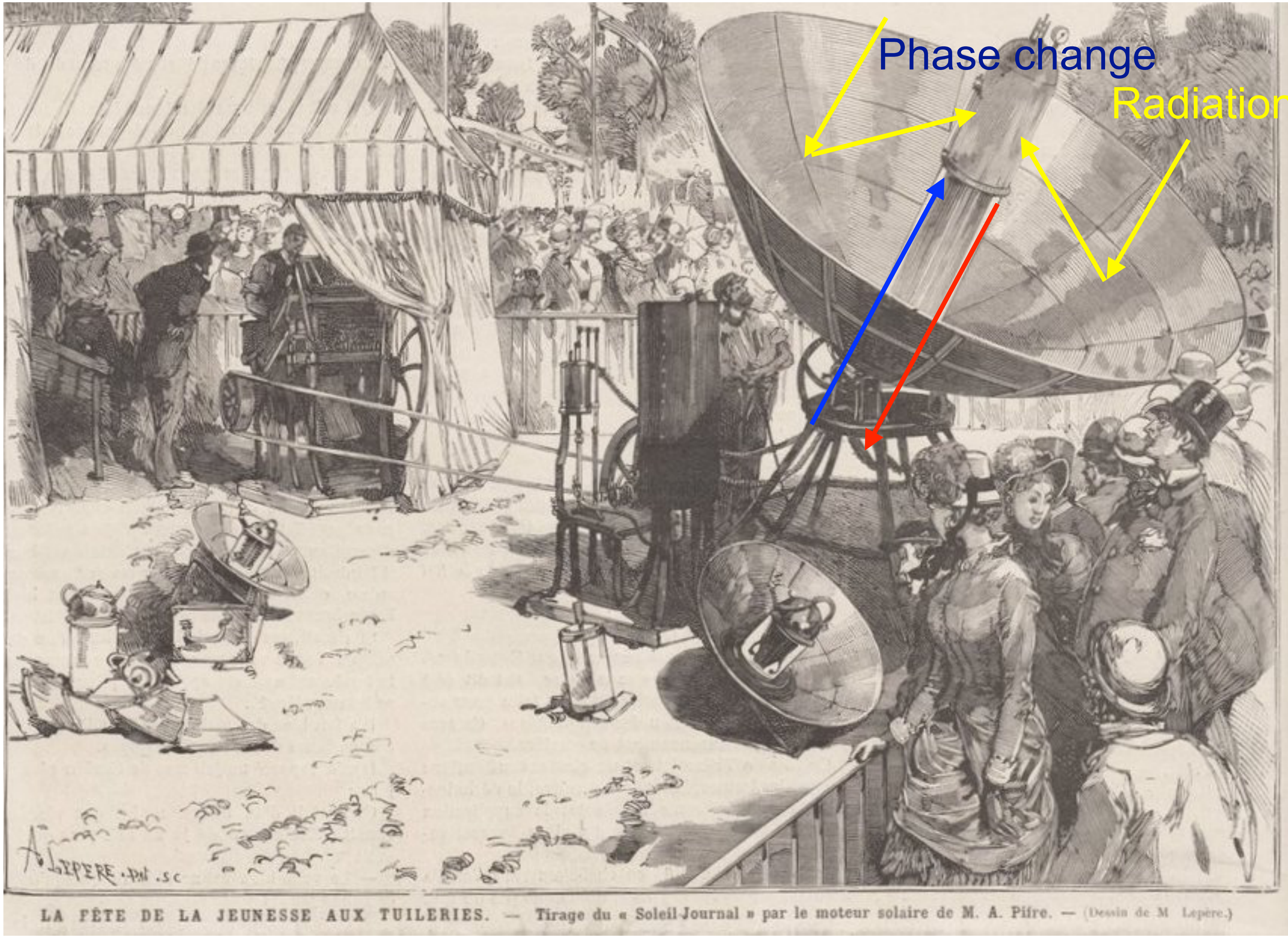
# Engineering at small scale : designing a microchip ?




→ course on Microfluidics: Nicolas Bremond

*Parsa et al.*, Lab on a Chip. 8, 2062 (2008)

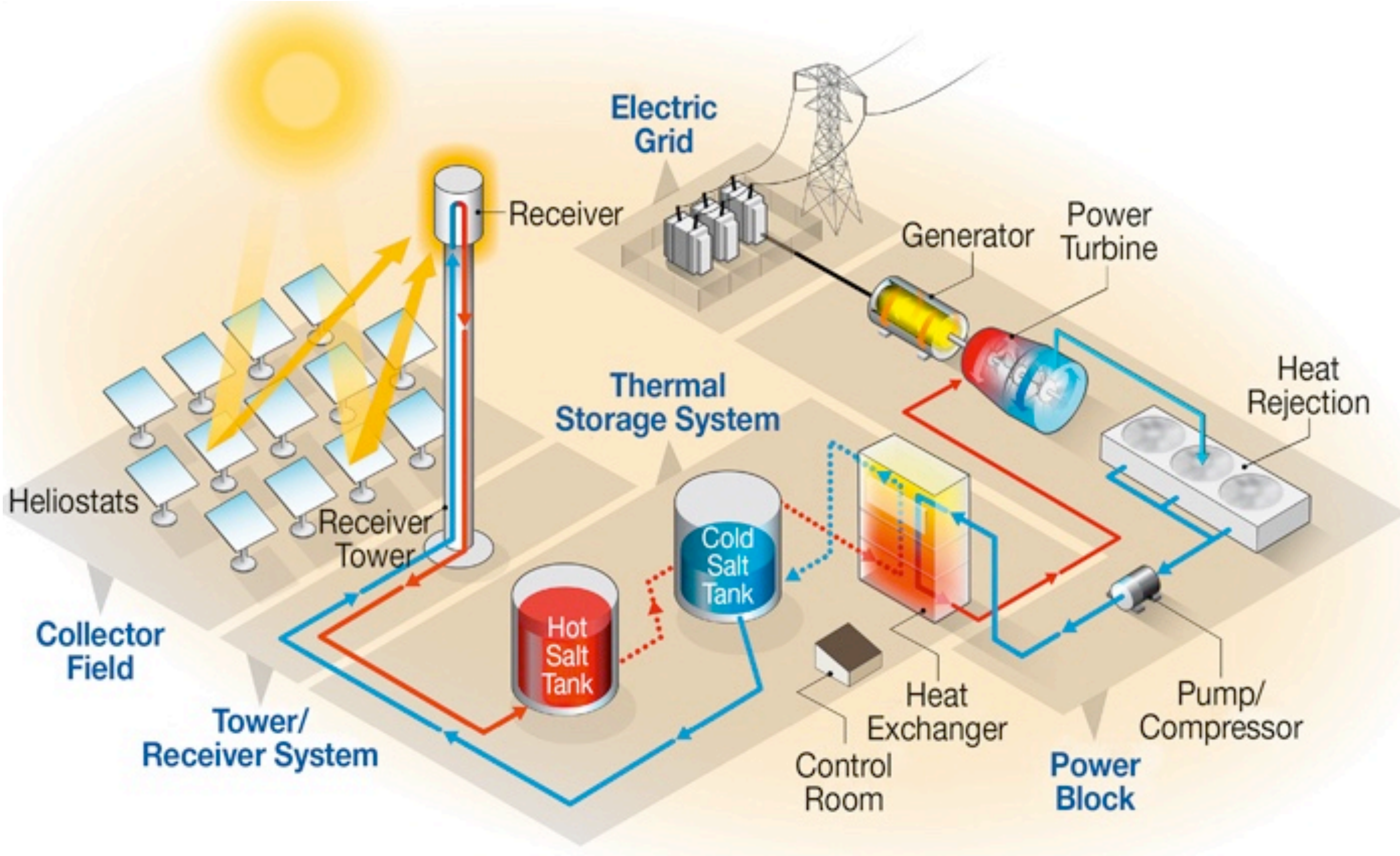
# Engineering at large scale : a solar power plant




Augustin Mouchot, a pioneer

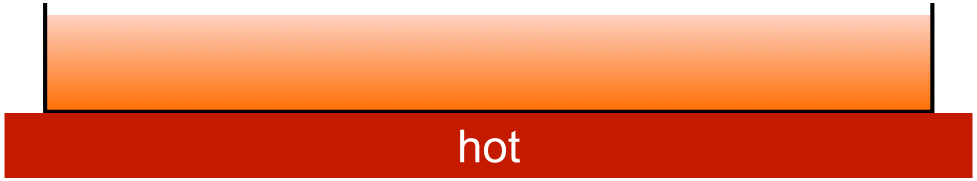
*Le Monde Illustré* (1882)

# Engineering at large scale : a solar power plant




*Karatairi,* MRS Bull. (2018)

# And some instabilities



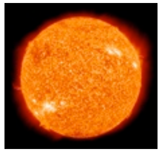

https://youtu.be/nQUH9nGTZTY

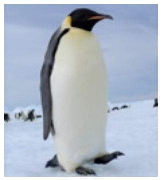
→ course on Instabilities: Laurent Duchemin

# Outline: learning from problems







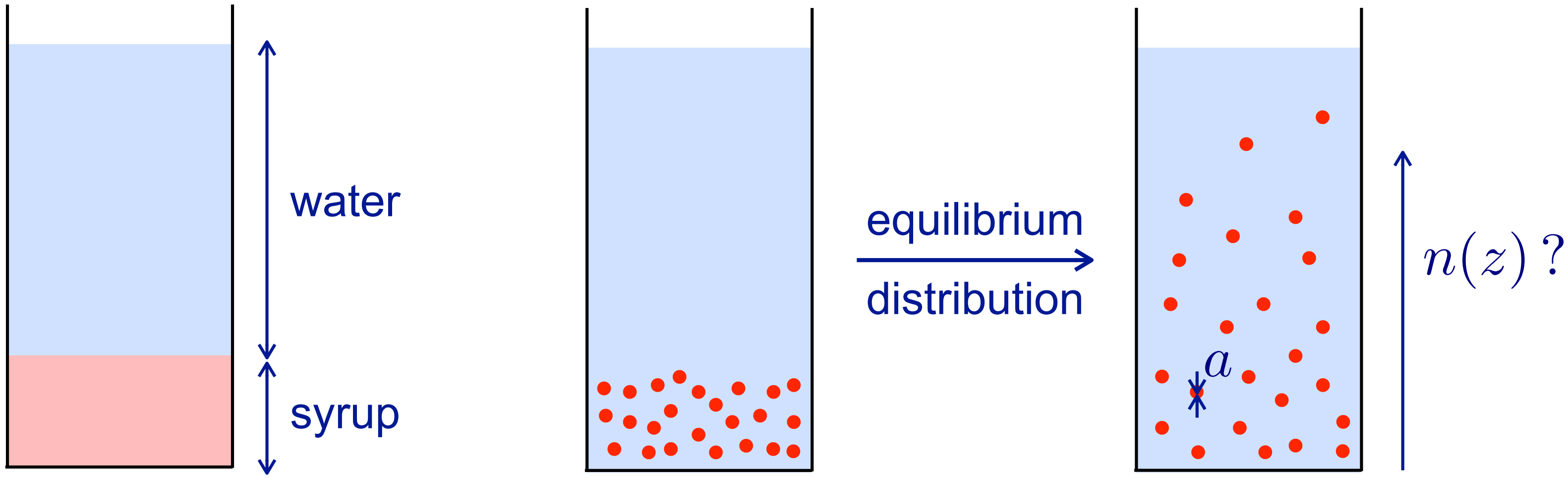
1. Diffusion: mixing time of a cocktail, Adding convection
2. Simple transport problems #1: measuring a diffusion coefficient
3. Simple transport problems #2: contact temperature, melting time of an ice cube, temperature inside a cave
4. Forced convection & boundary layers: wind powered fridge
5. Natural convection: designing an igloo
6. Convection-induced instabilities: convection cells
7. Radiative transfer
8. Thermal imaging
9. Recap: Solidifying droplet, Penguins feet.

# Diffusion in a layered cocktail

Is the configuration stable ?

How long will it take to vanish ?

# Diffusion in a layered cocktail




Statistical Physics

$$\mathcal{U}_p = \frac{4}{3}\pi a^3 \Delta\rho g z \qquad n(z) = n_0 \exp\left(-\frac{\mathcal{U}_p}{k_B T}\right) = n_0 \exp(-z/\lambda)$$

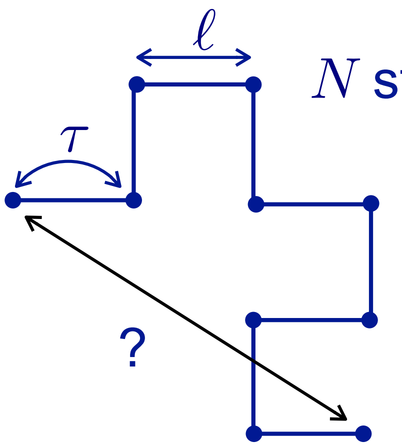
with $$\boxed{\lambda = \frac{k_B T}{\frac{4}{3}\pi a^3 \Delta\rho g}}$$

numerical estimate:

$k_B T = 4.10^{-21}$ J $\quad \Delta\rho \sim 600$ kg/m$^3$ $\quad a \sim 10^{-9}$ m $\quad \Rightarrow \quad \lambda \sim 100$ m !

# Random walks

Very simplified model




$N$ steps

$$x = \sum^{N} \ell \delta_i \qquad \delta_i = \pm 1 \quad \Rightarrow \quad < x >= 0$$

$$x^2 = \ell^2 \left( \sum^{N} \delta_i^2 + \sum^{N,\,N} \delta_i \delta_j \right)$$

$$< \delta_i \delta_j >= 0$$

$$< x^2 >= N\ell^2$$

$$t = N\tau \quad \Rightarrow \quad < x^2 >= \frac{\ell^2}{\tau} t$$

Dissipation for each step

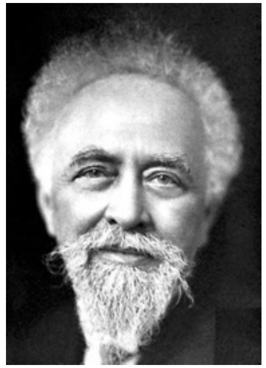
viscous drag $6\pi\eta a U \ell \sim k_B T \qquad U \sim \frac{\ell}{\tau} \quad \Rightarrow \quad \frac{\ell^2}{\tau} = \boxed{D = \frac{k_B T}{6\pi\eta a}}$

numerical estimate:

$k_B T =$ 4.$10^{-21}$ J $\quad \eta =$ $10^{-3}$ Pa.s $\quad a \sim$ $10^{-9}$ m $\quad D \sim$ 2.$10^{-10}$ m$^2$/s

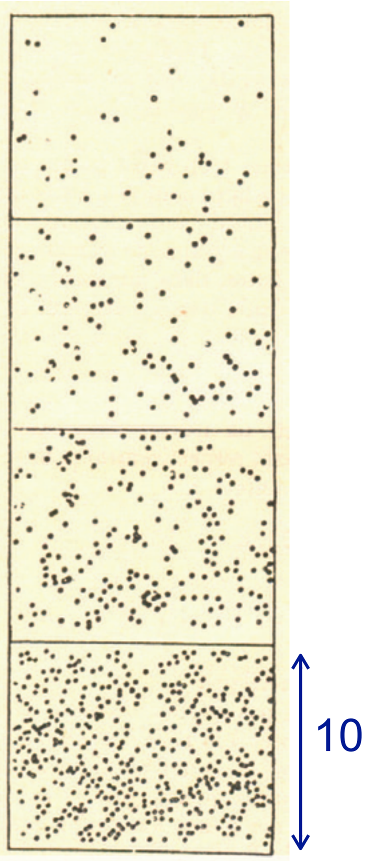
homogenization over 10 cm: 5.$10^{7}$ s ~ 1.5 year !

# Jean Perrin



1870-1942

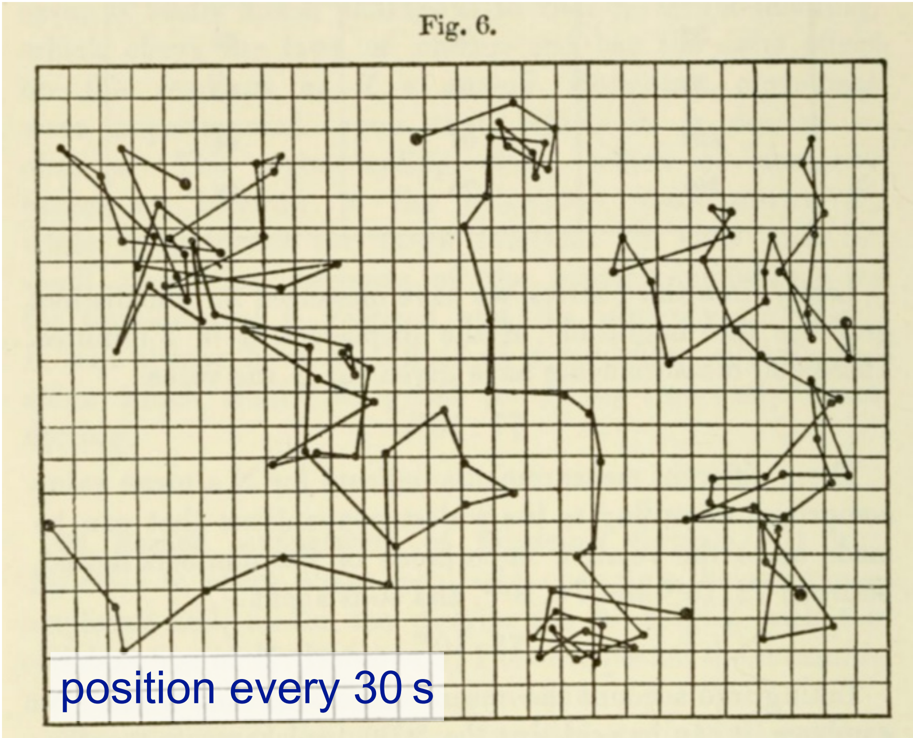
## Sedimentation




resin droplets, *a* ~ 0.3 µm

$$\lambda = \frac{k_B T}{\frac{4}{3}\pi a^3 \Delta\rho g}$$

## Diffusion




$$D = \frac{k_B T}{6\pi\eta a}$$

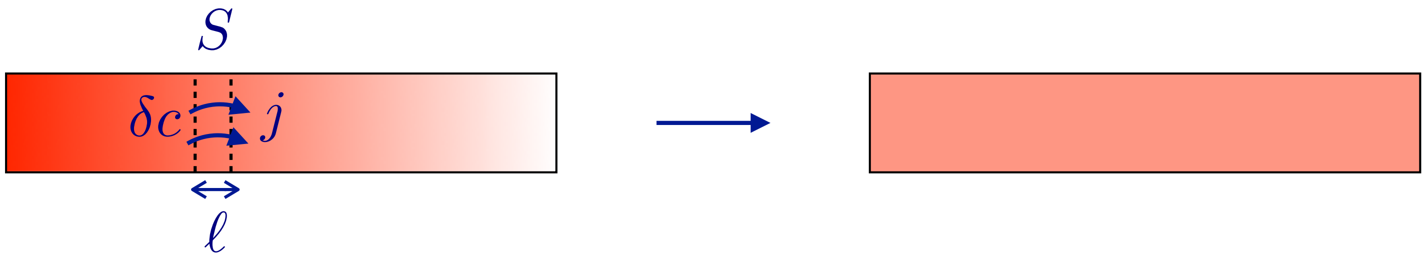
Estimates of $k_B T \Rightarrow \mathcal{N}_A = \frac{R}{k_B}$

Perfect gas constant (already known)

*Les Atomes* (1913)

# Fluxes

## Fick's law



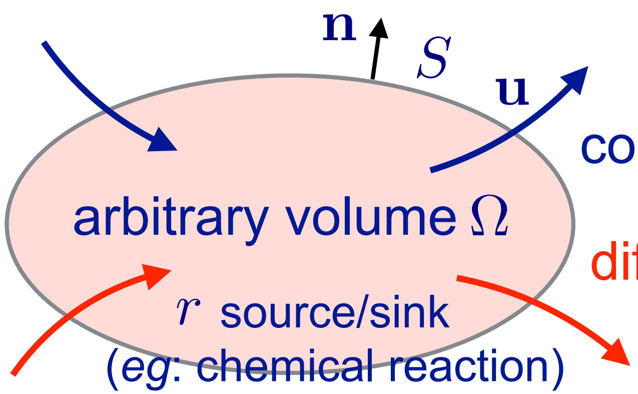

$$Sj\tau = S\ell\,\delta c \quad \Rightarrow \quad j = \frac{\delta c}{\tau}\ell = \frac{\delta c}{\ell}\frac{\ell^2}{\tau}$$

$$\frac{\delta c}{\ell} \;\leftrightarrow\; -\frac{\partial c}{\partial x} \quad \Rightarrow \quad j = -\frac{\ell^2}{\tau}\frac{\partial c}{\partial x} = -D\frac{\partial c}{\partial x}$$

## Generalization

$$\boxed{j = -D\nabla c}$$

# Adding convection




convective flux $\quad \mathbf{j}_c = c\,\mathbf{u}$

diffusive flux $\quad \mathbf{j}_d = -D\boldsymbol{\nabla} c$

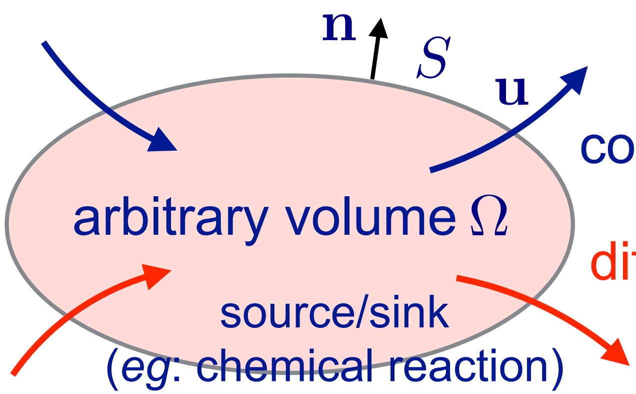
quantity inside the volume (moles, particules, mass): $\quad N = \iiint_\Omega c\, d\Omega$

evolution in time: $\quad \dfrac{dN}{dt} = \iiint_\Omega \dfrac{\partial c}{\partial t}\, d\Omega = -\iint_S (\mathbf{j}_c + \mathbf{j}_d)\cdot \mathbf{n}\, dS + \iiint_\Omega r\, d\Omega$

divergence theorem: $\quad \iiint_\Omega \dfrac{\partial c}{\partial t}\, d\Omega = -\iiint_\Omega \boldsymbol{\nabla}\cdot(\mathbf{j}_c + \mathbf{j}_d)\, d\Omega + \iiint_\Omega r\, d\Omega$

since $\Omega$ is arbitrary: $\quad \dfrac{\partial c}{\partial t} = -\boldsymbol{\nabla}\cdot(\mathbf{j}_c + \mathbf{j}_d) + r$

# Adding convection




convective flux $\mathbf{j}_c = c\,\mathbf{u}$

diffusive flux $\mathbf{j}_d = -D\boldsymbol{\nabla} c$

$$\frac{\partial c}{\partial t} = -\boldsymbol{\nabla}\cdot(c\mathbf{u}) + \boldsymbol{\nabla}(D\boldsymbol{\nabla} c) + r$$

$$\frac{\partial c}{\partial t} = -c\,\cancel{\boldsymbol{\nabla}\cdot\mathbf{u}} - \mathbf{u}\cdot\boldsymbol{\nabla} c + D\Delta c + \boldsymbol{\nabla}\cancel{D\cdot\boldsymbol{\nabla}} c) + r$$

incompressible flow assume *D* uniform

Diffusion-convection equation

$$\boxed{\frac{\partial c}{\partial t} + \mathbf{u}\cdot\boldsymbol{\nabla} c = D\,\Delta c + r}$$

+ Navier & Stokes

$$\boxed{\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\boldsymbol{\nabla})\mathbf{u} = \nu\,\Delta\mathbf{u} - \frac{1}{\rho}\boldsymbol{\nabla} p + \mathbf{g}}$$