Advanced Mechanics: Transport

April 9th, 2025

Duration 1h. Authorized document: lecture notes.

1 Insulating an oven [8 pts]

We consider an oven of approximative cube shape of $1 \times 1 \times 1\,\mathrm{m}$ (Figure 1). The inside temperature can reach 225° and the ambiant temperature in the kitchen is 25°. The walls of the oven are insulated so that the surface temperature does not exceed 45°.

What thickness of glass wool should be used to insulate the oven?
Hint: do not forget to consider the different heat transfer processes from the oven to the atmosphere.
What electrical power will the oven use in these conditions?

Thermal conductivity of glass wool: $\kappa_{wool} = 0.03\,\mathrm{Wm^{-1}K^{-1}}$
Stephan constant: $\sigma = 5.76 \cdot 10^{-8}\,\mathrm{Wm^2K^{-4}}$. *Emissivity of the surface of the oven:* $\varepsilon \simeq 1$
Material constants for air:
$\beta = 3.4 \cdot 10^{-3}\,\mathrm{K^{-1}}$, $\nu = 1.5 \cdot 10^{-5}\,\mathrm{m^2s^{-1}}$, $\alpha = 2 \cdot 10^{-5}\,\mathrm{m^2s^{-1}}$, $\kappa = 0.026\,\mathrm{Wm^{-1}K^{-1}}$

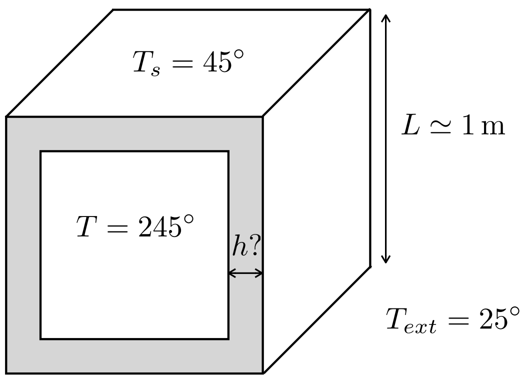

Figure 1: Determining the thickness of the insulating layer in an oven.

Solution: Heat can be transferred from the surface to the atmosphere through radiation or natural convection.
For radiation:
$J_{rad} = \varepsilon\sigma(T_s^4 - T_{ext}^4) \simeq 130\,\mathrm{W/m^2}$
For natural convection:
$J_{nc} = \frac{\kappa_{air}}{L}(T_s - T_{ext})Nu$, with $Nu \sim Ra^{1/4} \sim \left(\frac{\beta g (T_{ext} - T_s) L^3}{\nu_{air}\alpha_{air}}\right)^{1/4} \sim 220$
$J_{nc} = 110\,\mathrm{W/m^2}$

We obtain the total flux: $J_{total} \simeq 240\,\mathrm{W/m^2}$
This flux is the same as through the insulating layer: $J_{total} = \frac{\kappa_{wool}}{h}(T_{in} - T_s)$
which gives: $h = \frac{\kappa_{wool}(T_{in}-T_s)}{J_{total}} \simeq 2.5\,\mathrm{cm}$
Electrical power: $P \sim 6 \times J_s L^2 \sim 1440\,\mathrm{W}$ (maybe we should use 5 as the face in contact with the floor may not dissipate much).

2 Evaporation from a capillary tube [12 pts]

We want to describe the evaporation of a liquid column from a capillary tube of radius $R \sim 600\mu\mathrm{m}$ closed at the bottom end and open at the upper end (Figure 2). The tube is initially filled with isopropanol (IPA) and a gentle air flow is blown to impose $P_{IPA} = 0$ at the extremity of the tube. We assume that this air flow does not induce any convection inside the capillary tube (we will comment later the validity of this assumption). If we neglect short times, we observe a linear evolution of the square of the evaporation length ℓ^2 as a function of time. We propose to model this evolution.

Assuming that IPA behaves as a perfect gas, what is the molar concentration of gaseous IPA in the close vicinity of the liquid meniscus and at the exit of the capillary tube ? What is the concentration gradient along ℓ?
What is the molar flux across the tube section?
And the mass flux?
And finally, the volume flux?
Using flux conservation, deduce the time evolution of $\ell(t)$. Is the predicted law in agreement with the observed result?
Estimate a value for the diffusion coefficient of IPA in the air from the experimental data.
Comment the evolution of ℓ at the beginning of the experiments. Can you estimate the initial rate of evaporation for a wind velocity $U \sim 0.1\,\mathrm{m/s}$?

Material constants for IPA:
vapor pressure at 20°C $P_{sat} = 4400\,\mathrm{kPa}$, diffusion coefficient in air D_{IPA} to be estimated, molecular mass $M_{IPA} = 60\,\mathrm{g/mol}$, density $\rho_{IPA} = 0.786\,\mathrm{g/cm^3}$.
Perfect gas constant: $RT = 8.3\,\mathrm{J.K^{-1}mol^{-1}}$

Solution: We use $p_{IPA} = c_{IPA}RT$, which gives at the meniscus: $c_{IPA} = p_{sat}/RT$. Obviously at the exit, we have $c_{IPA} = 0$.
The gradient in concentration is thus given by: $\nabla c_{sat} = -\frac{p_{sat}}{RT\,\ell}$.
Using Fick's law, we get $J_m = -D_{IPA}\nabla c_{IPA} = \frac{D_{IPA}\,p_{sat}}{RT\,\ell}$ for the molar flux.
We get for the mass flux: $J_M = M_{IPA}J_m = \frac{M_{IPA}D_{IPA}\,p_{sat}}{RT\,\ell}$, and for the volume flux: $J_v = J_M/\rho_{IPA} = \frac{M_{IPA}D_{IPA}\,p_{sat}}{\rho_{IPA}RT\,\ell}$ From flux conservation, we obtain: $\frac{d\ell}{dt} = J_v$. After integration, we get:

$$\ell^2(t) = 2\frac{M_{IPA}D_{IPA}\,p_{sat}}{\rho_{IPA}RT}t$$

Using the slope $4 \cdot 10^{-9}\,\mathrm{m^2/s}$, we deduce $D_{IPA} = 1.4 \cdot 10^{-5}\mathrm{m^2/s}$

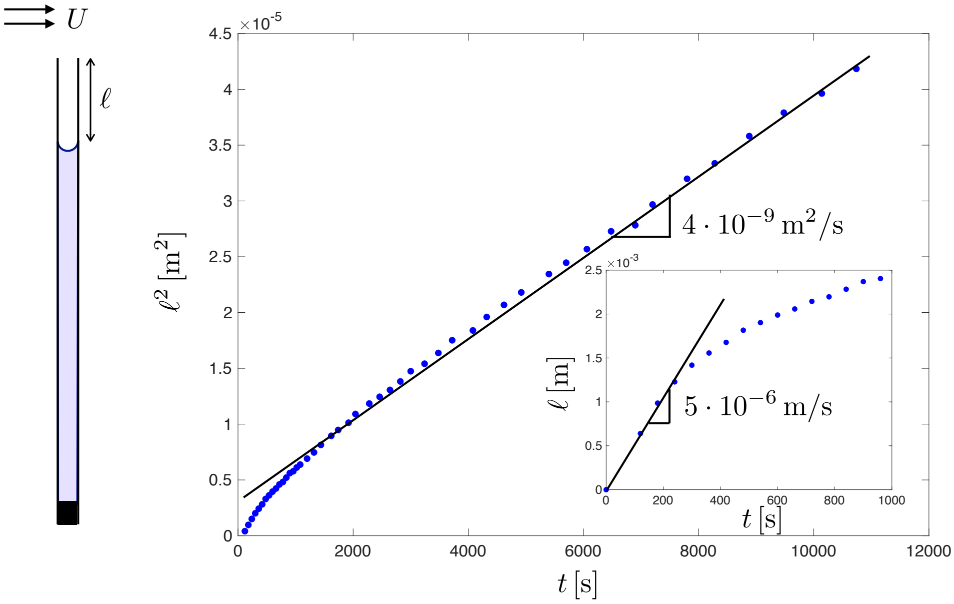

Figure 2: Evaporation of isopropanol (IPA) from a narrow capillary tube (of radius $R \sim 600\,\mu\mathrm{m}$) closed at it bottom end. A gentle air flow is blown at the upper end to ensure $P_{IPA} = 0$ at the exit of the tube. We assume that this flow does not induce any convection inside the tube. The slope of the straight line in the $\ell^2(t)$ plot is of $4 \cdot 10^{-9}\,\mathrm{m}^2/\mathrm{s}$. The inset corresponds to $\ell(t)$ at the beginning of the experiment exhibiting an initial front velocity of $5 \cdot 10^{-6}\mathrm{m/s}$.

The evaporation rate is more important at the beginning of the experiment. We can directly adapt what we have calculated for the drying of a wet piece of fabric. $J_m = D_{IPA}\frac{c_sat-0}{R}Nu$ with $Nu \sim (\nu/D)^{1/3}(UR/\nu)^{1/2}$.
So we obtain $J_v \sim \frac{M_{IPA}D_{IPA}\,p_{sat}}{\rho_{IPA}RT\,R}(\nu/D)^{1/3}(UR/\nu)^{1/2}$, putting numbers, we get $Nu \sim 2$ and $J_v \sim 6\,\mu$m/s, which is compatible with experimental observations.