Transport Phenomena:

8. Radiative transfer

Take home message:

Black body emission

$$E_{bb} = \sigma T^4 \qquad \sigma = 5.76 \cdot 10^{-8}\,\text{W.m}^2.\text{K}^{-4}$$

Wien's displacement law

$$\lambda_{max}\,[\mu\text{m}] = \frac{2898}{T\,[\text{K}]}$$

Real surface
emissivity: $\varepsilon(\lambda,\theta) = E(\lambda,\theta)/E_{bb}(\lambda)$
absorptivity: $\alpha(\lambda,\theta)$

Kirchhoff's law: $\alpha(\lambda,\theta) = \varepsilon(\lambda,\theta)$

Grey surface approximation: $\varepsilon = \alpha$ independent from λ, T or θ.

Glass window $\simeq$ transparent to visible light / opaque to IR

Heat and mass can be transported through diffusion and convection processes as described in the previous lectures. However heat can also be transported by electromagnetic waves as the radiation from the sun that travels through $1.5 \cdot 10^{11}$ m of sidereal vacuum before reaching us. After a brief recap on black body emission from statistical mechanics, we will consider some illustrative exemples.

1 Recap from statistical physics

1.1 Black body emission

In 1900 Max Planck described the electromagnetic radiations exchanged between a perfectly opaque body and its environment at the equilibrium temperature T. Such *black body* ideally absorbs any incoming radiation and is a diffuse emitter (the same intensity is emitted in any direction). Within this hypothesis, the radiative flux emitted in the range of wavelength $[\lambda, \lambda + d\lambda]$ is dictated by the celebrated Plank-Boltzmann law:

$$E(\lambda,T)d\lambda = \frac{2\pi hc^2}{\lambda^5}\frac{1}{\exp(hc/\lambda k_B T) - 1}\,d\lambda$$

with the Planck constant $h = 6.6 \cdot 10^{-34}$ J.s, the light velocity $c = 3 \cdot 10^8$ m/s, the Boltzmann constant $k_B = 1.38 \cdot 10^{-23}$ J/K.
The distribution is broad and typically spans from the infrared to the visible range (Fig. 1). Following Wien's law, the maximum flux corresponds to a wavelength λ_{max} inversely proportional to the absolute temperature:

$$\lambda_{max}\,[\mu\text{m}] = \frac{2898}{T\,[\text{K}]}$$

For instance, we obtain at room temperature $\lambda_{max}(300°C) \simeq 10\,\mu m$ which corresponds to infrared. Conversely, the light emitted by the sun $\lambda_{max}(5800°C) \simeq 0.5\,\mu m$ is in the visible range.

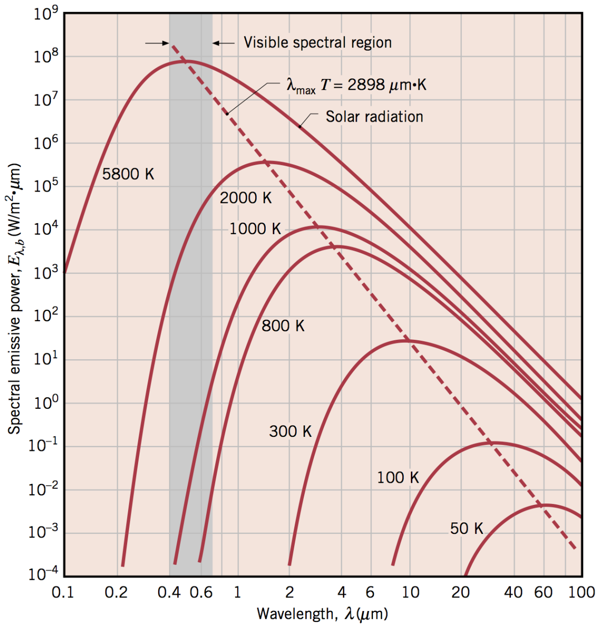

Figure 1: Spectral emissive power from a black body (from Incropera *et al.*, *Fundamentals of Heat and mass transfer*, Ed. John Wiley & Sons).

The global flux is readily obtained by integrating E over the possible values of λ:

$$E_{bb} = \int_0^{\infty} \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}\, d\lambda = \sigma T^4$$

where $\sigma = \frac{2\pi^5 k_B^4}{15h^3c^2} = 5.67 \cdot 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$ is the Stefan-Boltzmann constant

1.2 Energy loss by radiation

In the previous lectures, we have totally ignored radiation in our estimates of energy fluxes. Consider a naked body with a skin temperature $T_s = 35°C$ in a room at $T_{ext} = 20°C$. What is the corresponding radiative flux?
If we wear clothes, we have seen that the surface temperature of clothes is significantly lower than body temperature. What is the radiative flux if we now consider a surface temperature $T_s = 30°C$. In any case, are radiative fluxes negligible?

2 Real surfaces

Real materials do not behave has ideal black bodies. For instance, the flux may not be diffuse and the intensity may depend on the emission angle θ (Fig. 2a) and the dependance with temperature may differ from Planck theory. The ratio of the actual flux with the ideal black body prediction is defined as *emissivity*:

$$\varepsilon(\lambda, T, \theta) = \frac{E(\lambda, T, \theta)}{E_{bb}(\lambda, T)}$$

Similarly, a real body will not perfectly absorb all the incoming radiation. Part of the radiation can be absorbed, reflected or transmitted (Fig. 2b). Energy conservation imposes that the sum of the relative contributions is equal to 1.

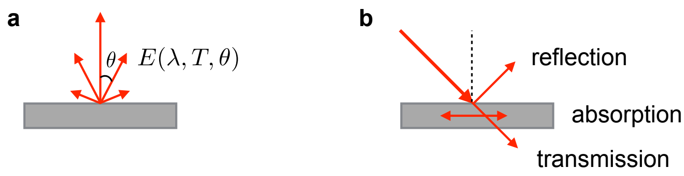

Figure 2: Real surface: (a) emission may depend on the orientation; (b) part of the incoming radiation can be absorbed as for a black body but also reflected of transmitted.

Absorption and emission are nevertheless related. Kirchhoff law states that *absorptivity* equals *emissivity* for a given wavelength:

$$\alpha(\lambda) = \varepsilon(\lambda)$$

However numerous materials exhibit $\varepsilon(\lambda_1) \neq \varepsilon(\lambda_2)$. For instance white or black paint display almost the same emissivity in infrared, but obviously not the same absortivity for visible light.

3 Radiative balance for the Earth and greenhouse effect

3.1 Earth deprived of atmosphere

The Sun can be approximated as a black body (strange definition for an object feeding us with light!). It surface temperature is $T_S = 5800°\text{K}$. What is the corresponding radiation flux? The radius of the Sun is $R_S = 1.4 \cdot 10^9$ (Fig. 3). What is the radiative flux reaching the Earth at a distance $d = 1.5 \cdot 10^{11}$ m from the Sun? Obviously, not all the regions of the Earth get the same intensity. What is the average flux reaching the surface in a Earth deprived of atmosphere?

If we assume that the Earth behaves as a perfect black body ($\varepsilon = \alpha = 1$), what should be its equilibrium average temperature (Fig. 4)?

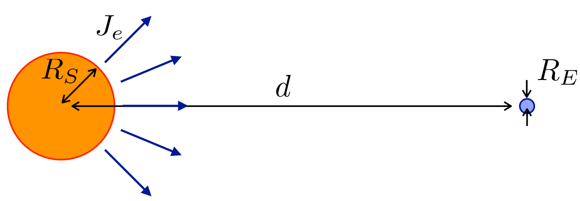

Figure 3: Radiative flux from the Sun ($T_S = 5800°\text{K}$, $R_S = 1.4 \cdot 10^9$ m, $d = 1.5 \cdot 10^{11}$ m).

Now consider a Earth painted in white (or covered with snow), such that $\alpha(\text{visible}) = 0.2$ and $\varepsilon(IR) = 0.9$. What would then be the average equilibrium temperature? This effect is referred to as *albedo effect* and plays an important role in climate models.

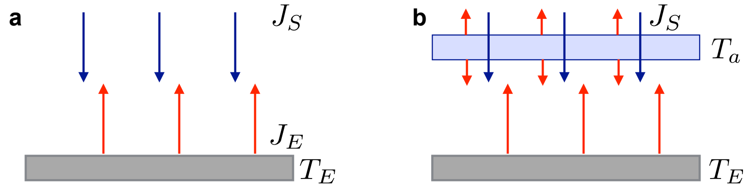

Figure 4: Radiative balance without atmosphere (a) or with a very simplified model for the atmosphere (b).

3.2 Greenhouse effect

In reality, atmosphere plays an important role as it reflects about 30% of the incoming radiations and absorbs about 20% of them, leaving only 50% of the radiations from the Sun reaching the surface. We consider here a very simplified model for the atmosphere that applies for greenhouses used in agriculture to grow vegetables in cold seasons. Imagine the atmosphere could be modeled as a glass panel sitting above the surface. As a first approximation, we assume that this panel is perfectly transparent to light ($\alpha(\text{visible}) = \varepsilon(\text{visible}) = 0$), but perfectly opaque to infrared $\alpha(IR) = \varepsilon(IR) = 1$. What are the balances of fluxes for the surface of the Earth and for the glass panel? What is now the surface temperature?

9. Thermal imaging

We have seen how warm surfaces emit radiations with a spectral distribution that depends on temperature. In the case of room temperatures ($\sim 300°$K), the peak in emissivity is of the order of $10\,\mu$m. Luckily, air is transparent in this range of radiation spectrum (Fig. 7) so that thermal imaging can be envisioned. Did you know that rattle snakes, cobras or pythons, as well as vampire bats, could "see" infrared light? We will briefly describe the detection strategy they developed through millions of years of evolution. Can these mechanisms be a source of inspiration for engineered thermal imaging?

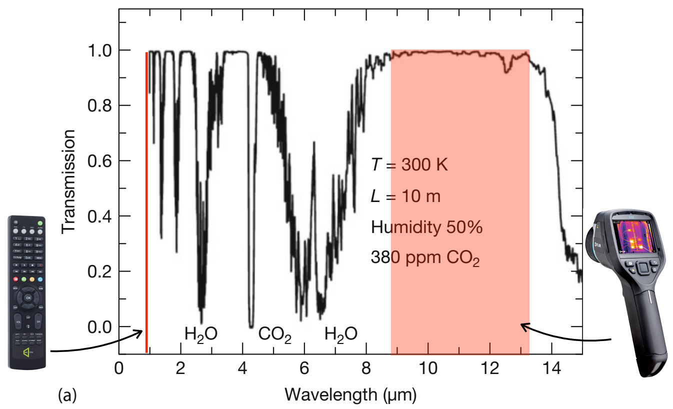

Figure 5: Transmission spectrum for a 10 m thick layer of air (from Vollmer & Möllmann, *Infrared thermal imaging*, Ed. John Wiley). The wavelength of the LEDs used in remote controls is typically 940 nm in the near infrared.

4 Seeing like a rattle snake

Rattles snakes posses loreal pit organs located between their eyes and nostrils (Fig. 8). The cavity presents an aperture about 1 mm in diameter. A membrane of typical thickness $10\,\mu$m is located 2 mm behind the aperture and separates an outer cavity from an inner side. Innervation is dense through this membrane, which make this organ very sensitive to temperature changes.

The purpose of this peculiar organ is to locate preys from their heat, even in the dark. Considering the range of wavelength involved, can these radiations be detected by photoreceptors as for visible light?

Consider a rodent modeled as a sphere of diameter $D = 10$ cm located 1 m away from the snake. Its surface temperature is about 300°K.

What is the radiated flux flux at the surface of the rodent if we consider it as a perfect black body? What is the corresponding power?

What is the flux the snake receipts 1 m away? What is the power of the thermal radiation penetrating inside the pit?

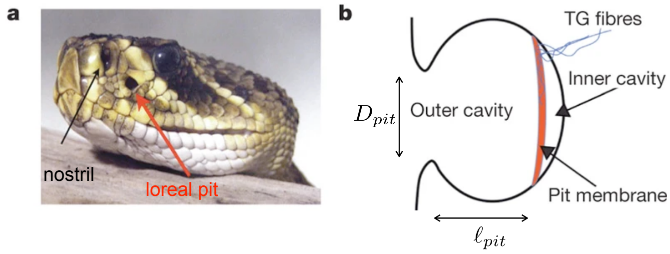

Figure 6: (a) Rattle snake head showing the left loreal pit organ between the nostril and the eye. (b) Sketch of the pit cavity with a sensitive membrane separating a closed inner cavity from an outer open cavity. The diameter of the aperture is typically of 1 mm and the membrane is located 2 mm behind (source: Gracheva *et al.*, Molecular basis of infrared detection by snakes, *Nature* 464, 1006 (2010)).

If we assume that the membrane is perfectly insulated and has the same thermal properties as water, what would be the increase of temperature of the irradiated region after an exposure of 100 ms?
This integration times results from thermal exchanges with the environment of the membrane as we will now discuss in the case of detectors for IR cameras.

5 Microbolometers

The detection mechanism used in thermal cameras operating at room temperature is actually the same as developed by snakes. The detector is composed of an array of *microbolometers* obtained through microlithography techniques. In the Each microbolometer consists of a thin platelet separated from its base by electrically (and thermally) conductive arms (Fig. 9). When the platelet is heated by radiation, its electric resistance is slightly modified. This minute variation is measured by differential detection with a microtransitor located at the base of each platelet.
To get numbers, we will consider platelets of size $S = 40 \times 40\,\mu$m, of thickness $t = 0.2\,\mu$m with arms of length $L_a = 30\,\mu$m and width $w = 2\,\mu$m, made of Si_3N_4, of thermal conductivity $\kappa = 2\,\mathrm{Wm^{-1}K^{-1}}$, thermal capacity $C_p = 500\,\mathrm{Jkg^{-1}K^{-1}}$ and density $\rho = 3200\,\mathrm{kg/m^3}$.

Behind the platelet, the base of the detector is covered with a reflective layer. The distance from the platelet to the base is $d = 2.5\,\mu$m. Why has this specific distance been selected?

In order to get estimates, we assume that the radiation flux calculated for the snake is the same that each bolometers gets. What would be the increase in temperature for an acquisition time of 10 ms if we assume that the platelet is perfectly thermally isolated?

We now try to describe the origin of this integration time. In reality, the platelet is connected to the base (at room temperature) by the thin thermally conductive arms and heat may also be lost by radiative exchanges with the surrounding environment. As microbolometers operate under vacuum, conduction through air is not considered. What equation describes the evolution of ΔT the difference in temperature of the platelet with the base as a function of time and the incoming radiative power?
What is the response to a step function?

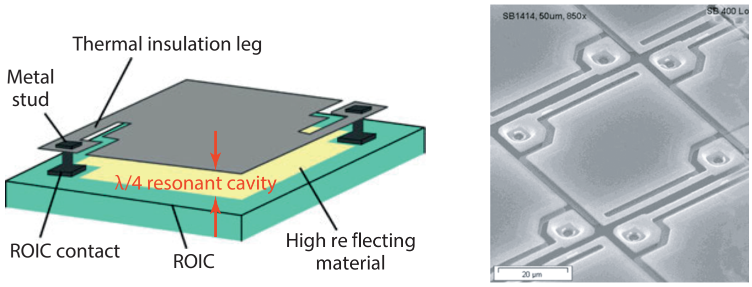

Figure 7: Sketch of a unit microbolometer (source: Vollmer & Möllmann, *Infrared thermal imaging*, Ed. John Wiley). Close-up SEM image of an array of bolometers (source: Lohrmann *et al.*, Uncooled long-wave infrared small pixel focal plane array and system challenges, *Optical Engineering*, 52, 061305 (2013)).

What is the saturating value of ΔT? What limits the acquisition time?
Why is it important to operate microbolometers under vacuum ($\kappa_{air} = 0.026\,\mathrm{Wm^{-1}K^{-1}}$)?

6 Playing with a thermal camera

6.1 Magical tricks

Visualize a warm object (a hand for instance) with a thermal camera and "hide" it under a transparent (for visible light) glass beaker. Surprising, no? This is why optical lenses suitable for thermal cameras are usually made from Germanium (and therefore quite expensive).
Even more surprising: put your hand inside a black plastic bag. What does the thermal camera display?

An unpolished copper plate is usually a mediocre mirror as micron-sized roughness diffuses visible light. What about for "thermal" light?

6.2 Diffusion in paper

Put your hand on the cover of a notebook for 10 s. Then visualize the papes across the notebook with the thermal camera

6.3 Friction

Thermal imaging is an excellent tool to assess friction effects. As a good demonstration, try to visualize the effect of rubbing a shoe on the floor.

Consider a more controlled experiment where a piece of wood of size $1.5 \times 10\,\mathrm{cm}$ is rubbed on another wood plate at a typical velocity of 1 cm/s with a normal load of 50 N (Fig. 10).
What is the order of magnitude of the heat flux due to friction for a typical friction coefficient of 0.4?
What is the time evolution of the typical thickness involved in the temperature distribution?
What is the expected surface temperature after 10 s of rubbing? For information, the temperature for spontaneous ignition of wood is about 500°C.

Wood material properties: $\kappa = 0.15\,\mathrm{W m^{-1} K^{-1}}$, $\rho = 600\,\mathrm{kg/m^3}$, $C_p = 1700\,\mathrm{J\,kg^{-1} K^{-1}}$.

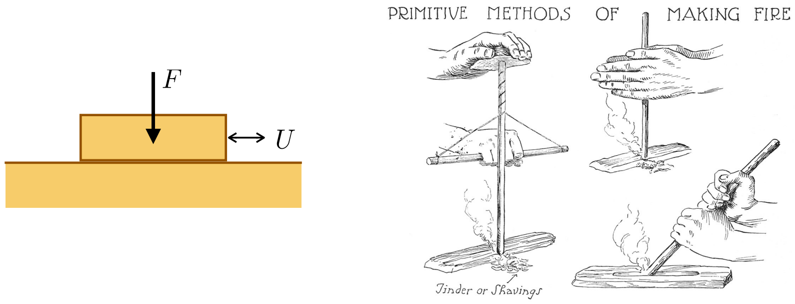

Figure 8: Rubbing a piece or wood on another one. Traditional ways of making fire (source: C.W. Jefferys, *The Picture Gallery of Canadian History*, 1942).