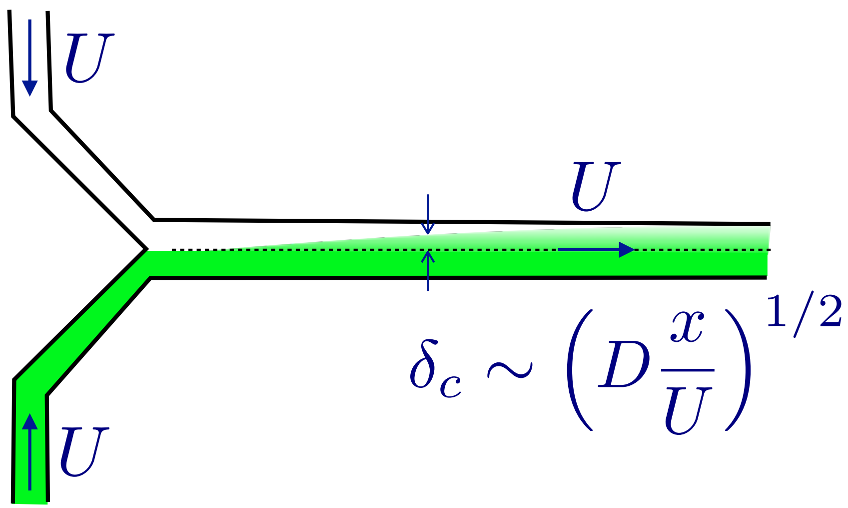
Transport phenomena

4. Forced convection

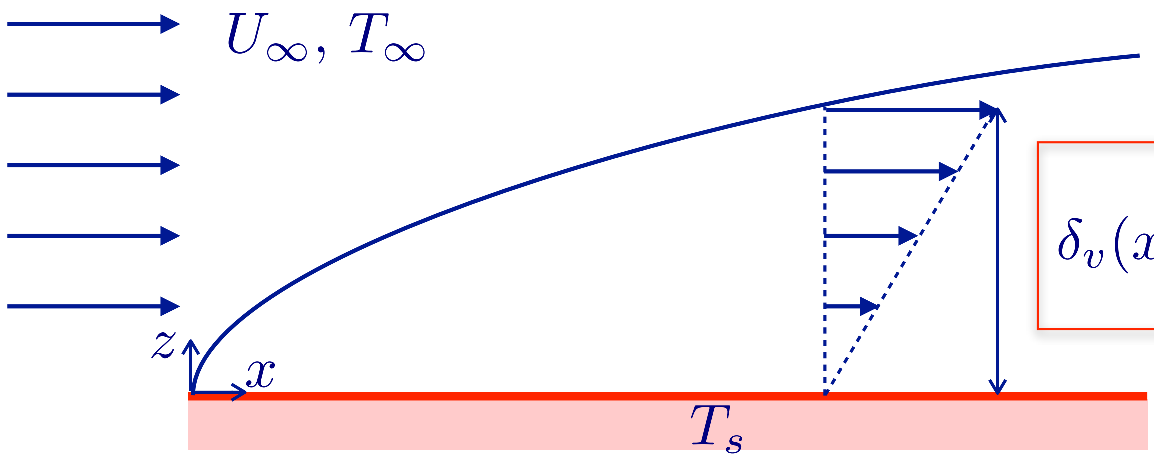
4.0 Coupling flow with heat/mass exchange

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} c = D\,\Delta c + r$$

Simple configuration: U uniform



3.1 Cooling flow over a plate



$$\boxed{\delta_v(x) \sim \left(\nu \frac{x}{U_\infty}\right)^{1/2}}$$

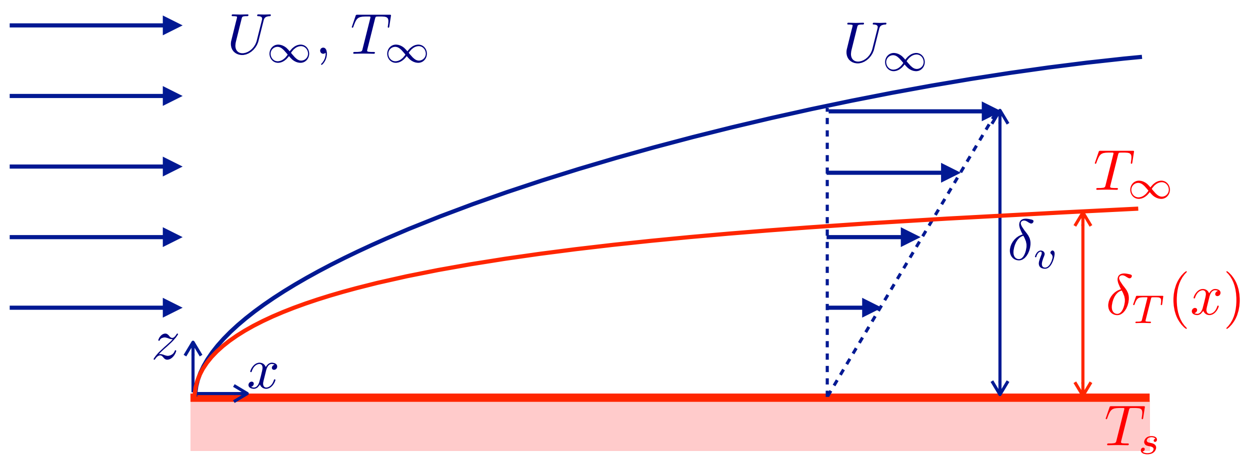
incompressible flow: $\boldsymbol{\nabla} \cdot \mathbf{u} = 0 = \dfrac{\partial u_x}{\partial x} + \dfrac{\partial u_z}{\partial z} \;\Rightarrow\; \dfrac{U_\infty}{x} \sim \dfrac{u_z}{\delta_v}$

$$u_z \sim \frac{\delta_v}{x} U_\infty \ll U_\infty \quad \text{for } x \gg \delta_v$$

velocity profile inside the boundary layer:

$$\boxed{u_x(x,z) \sim \frac{U_\infty}{\delta_v(x)}\, z}$$

3.1.1 Thermal boundary layer (hypothesis: $\delta_T < \delta_v$)



heat equation:

$$\cancel{\frac{\partial T}{\partial t}} + \mathbf{u} \cdot \boldsymbol{\nabla} T = \alpha \Delta T + \cancel{\frac{r}{\rho C_p}}$$

steady

$$\simeq u_x \frac{\partial T}{\partial x}$$

in scaling laws:

$$\frac{U_\infty}{(\nu\, x/U_\infty)^{1/2}}\, \delta_T \frac{T}{x} \sim \alpha\, \frac{T}{\delta_T^2}$$

$$\Rightarrow \boxed{\delta_T(x) \sim \alpha^{1/3}\nu^{1/6}\left(\frac{x}{U_\infty}\right)^{1/2} \sim \delta_v(x)\left(\frac{\alpha}{\nu}\right)^{1/3} = \delta_v(x)\,\mathrm{Pr}^{-1/3}}$$

Prandtl number $\boxed{\mathrm{Pr} = \frac{\nu}{\alpha}}$

water ~ 7

air ~ 1

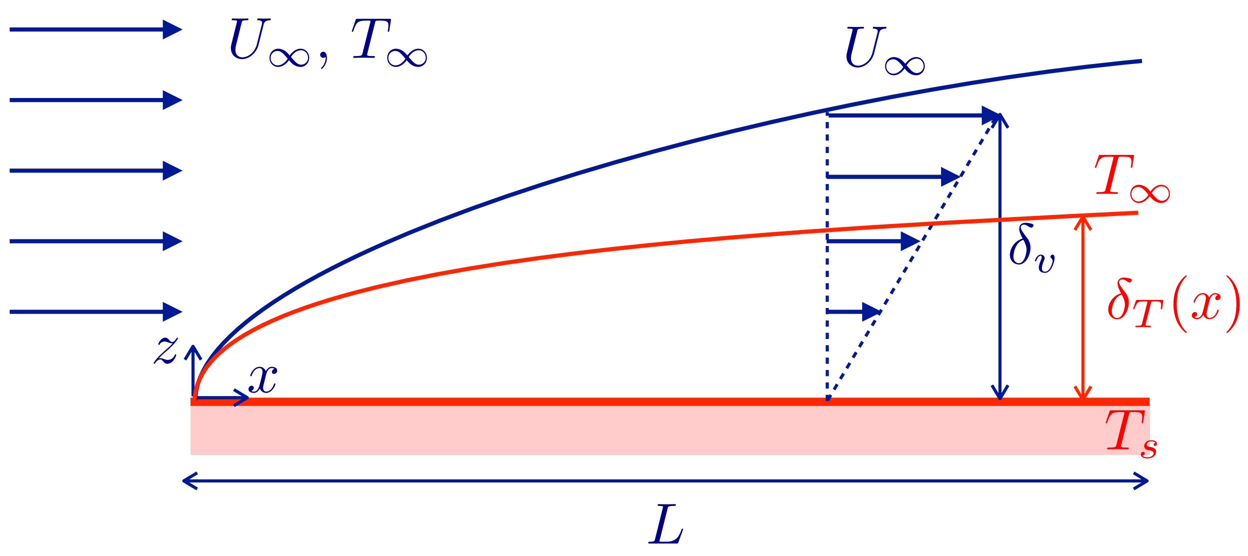
Hg ~ 0.02 ⇒ different behavior

glycerin ~ 1000

$\delta_T < \delta_v$ ~ OK, if $\mathrm{Pr} > 0.6$

Schmidt number for mass transport $\boxed{\mathrm{Sc} = \frac{\nu}{D}}$

3.1.2 Heat transport across the thermal boundary layer



Heat flux $$\mathbf{J}_T = -\kappa \frac{\partial T}{\partial z} \sim \kappa \frac{T_\infty - T_s}{\delta_T}$$

Heat transfer coefficient $\boxed{J_T = -h_T(T_\infty - T_s)}$ $\boxed{h_T \sim \frac{\kappa}{\delta_T}}$

Nusselt number $\boxed{\mathrm{Nu} = \frac{h_T L}{\kappa} \sim \frac{L}{\delta_T}}$

Forced convection over a plate $\boxed{\mathrm{Nu} \simeq 0.33\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}}$

Other expressions for different geometries $\mathrm{Nu} = f(\mathrm{Re},\, \mathrm{Pr})$

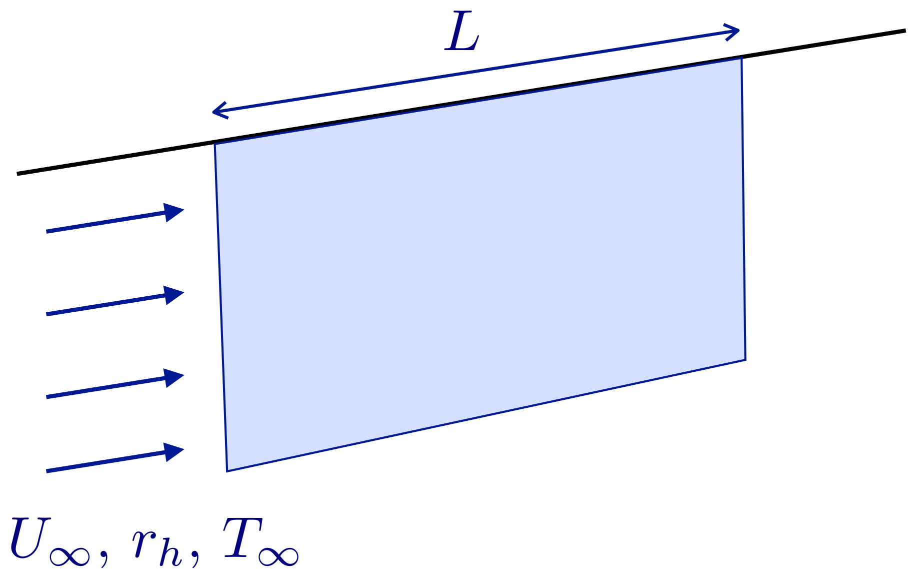
Same formalism for mass transport:

Mass transfer coefficient $\boxed{J_m = -h_m(c_\infty - c_s)}$ $\boxed{h_m \sim \frac{\kappa}{\delta_c}}$

Sherwood number $\boxed{\mathrm{Sh} = \frac{h_m L}{D}}$

5. Forced convection: Practical exemples

5.1 Drying a wet sheet in the wind

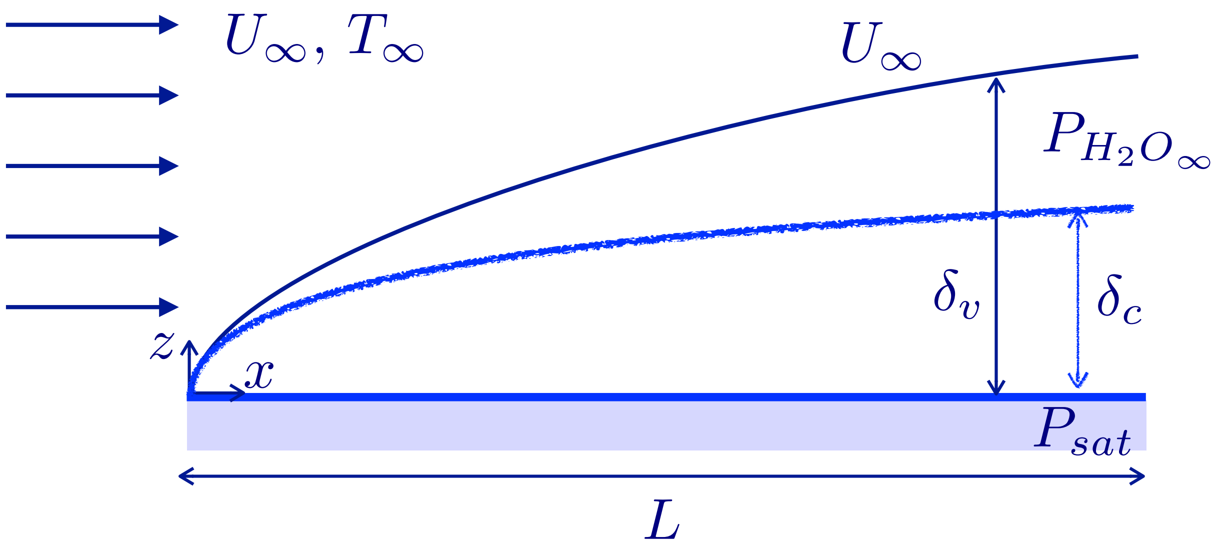


Simplified model

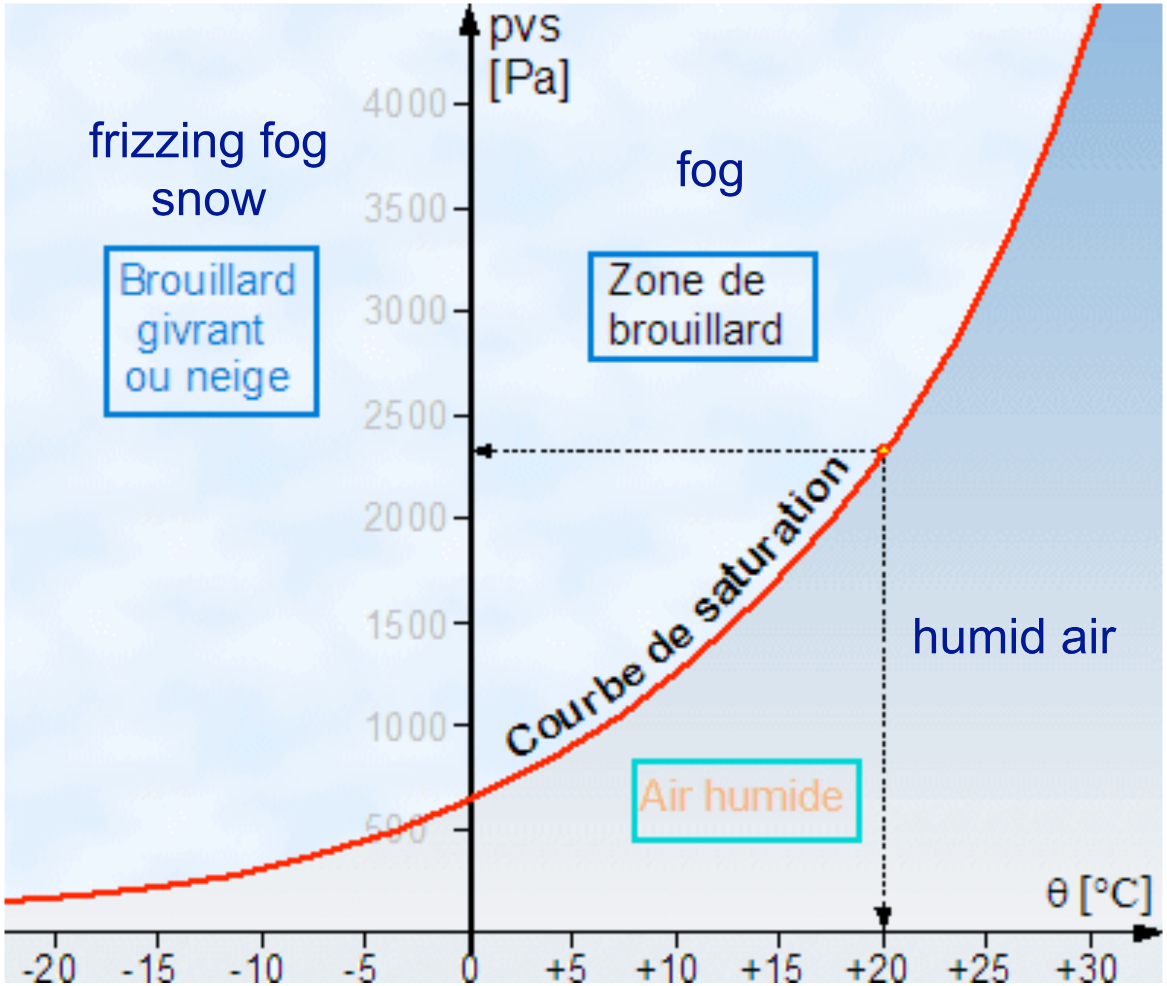
- uniform temperature
- radiative exchanges neglected
- P_{H_2O} at the surface = P_{sat}

relative humidity r_h

$$\boxed{P_{H_2O_\infty} = r_h \, P_{sat}}$$



Vapor pressure



source: dimclim.fr

$P_{H_2O} \;\leftrightarrow\; c_{H_2O}?$ $\qquad PV = nRT \;\Rightarrow\; c_{H_2O} = \dfrac{P_{H_2O}}{RT}$

mol.m^{-3}

H$_2$O molar flux: $\quad J_{H_2O} = h\, c_{sat}(1 - r_h)$

$$\frac{D}{L}\mathrm{Sh} \sim \frac{D}{L}\left(\frac{U_\infty L}{\nu}\right)^{1/2}\left(\frac{\nu}{D}\right)^{1/3}$$

fine to call it "Nu" !

$$J_{H_2O} \sim \frac{D}{L}\left(\frac{U_\infty L}{\nu}\right)^{1/2}\left(\frac{\nu}{D}\right)^{1/3} c_{sat}(1 - r_h) \qquad \text{[mol.m}^{-2}\text{.s}^{-1}\text{]}$$

if r_h fixed, $T \nearrow \;\Rightarrow\; c_{sat} \nearrow \;\Rightarrow$ higher evaporation

mass flux: $\quad J_{m_{H_2O}} = M_{H_2O} J_{H_2O}$ [kg.m^{-2}.s^{-1}]

volumic flux: $\quad J_{v_{H_2O}} = \dfrac{1}{\rho_{H_2O}} J_{m_{H_2O}}$ [m.s^{-1}]

Numerical estimate: L = 1m U_∞~ 5 m/s

$D_{H_2O/air}$ = 2,6.10^{-5} m^2/s ν_{air} = 1,5.10^{-5} m^2/s

P_{sat} = 2.3 kPa $\Rightarrow$ c_{sat} = 0.9 mol/m^3

$$J_{H_2O} \sim 0.03\,(1 - r_h)$$ [mol.m^{-2}.s^{-1}]

(with prefactor 0.33 for Sh)

$$J_{m_{H_2O}} \sim 7.10^{-5}(1 - r_h)$$ [kg.m^{-2}.s^{-1}]

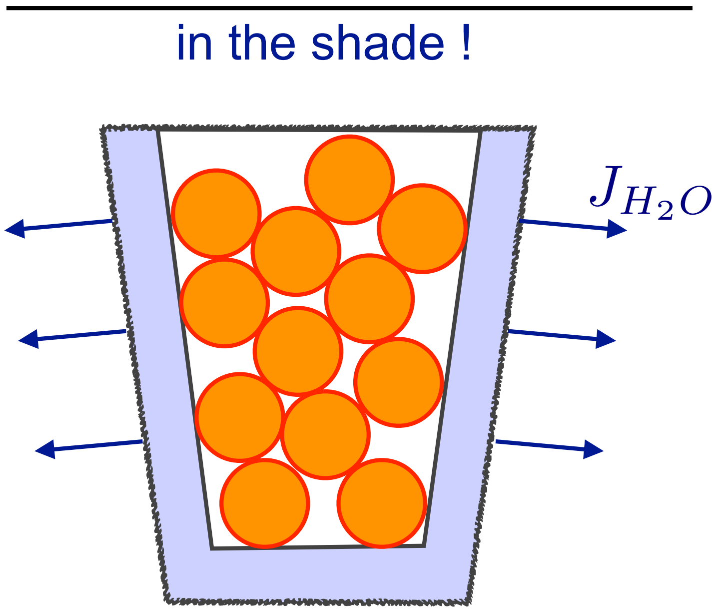
$$J_{v_{H_2O}} \sim 0.07\,(1 - r_h)$$ [µm/s]

to evaporate 0.5 kg/m^2 of water (i.e. 0.5 mm) with r_h = 50%

$$\tau_{evap} \sim \frac{0.5 \cdot 10^{-3}}{\underbrace{2}_{\text{2 faces}} \times 0.07 \cdot 10^{-6} \times 0.5} \sim 2\text{h}$$

probably underestimated: evaporation ⇒ cooling down ⇒ $c_{sat} \searrow \Rightarrow J_{H_2O} \searrow$

5.2 Clay pot cooling



evaporation ⇒ latent heat

heat flux at the surface

$$\boxed{J_{latent} = L_{vap}\, J_{m_{H_2O}}}$$

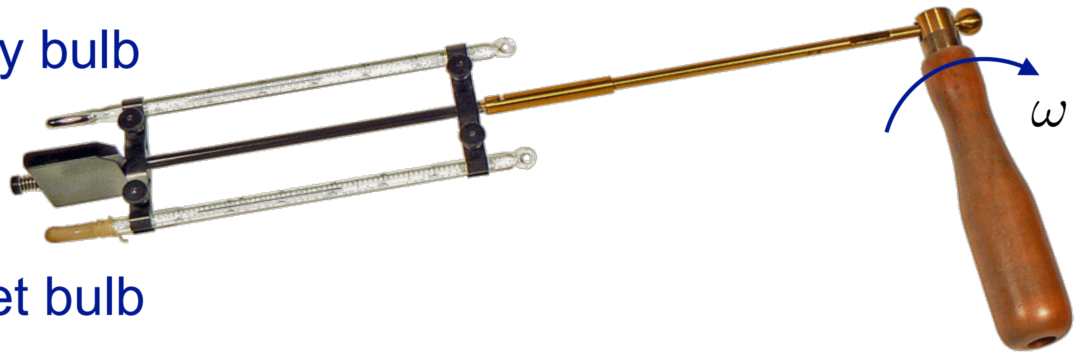
$$J_{latent} \sim 150\,(1 - r_h)\,\mathrm{W/m^2}$$

standard fridge ~ 30 to 50 W

5.3 Measuring relative humidity

psychrometer

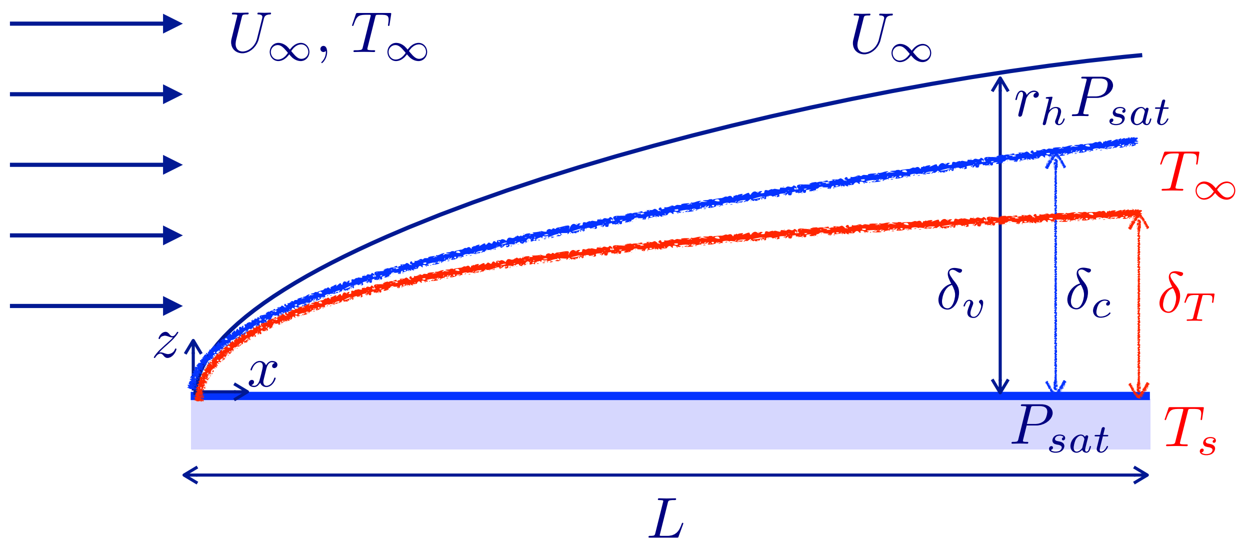
dry bulb



wet bulb

measure $\Delta T \Rightarrow r_h$

3.2.3 Thermal balance



heat flux from the fluid to the surface

$$J_T \sim \frac{\kappa}{L}\left(\frac{U_\infty L}{\nu}\right)^{1/2}\left(\frac{\nu}{\alpha}\right)^{1/3}(T_\infty - T_s)$$

heat flux from latent heat

$$J_{latent} \sim L_{vap} M_{H_2O}\,\frac{D}{L}\left(\frac{U_\infty L}{\nu}\right)^{1/2}\left(\frac{\nu}{\alpha}\right)^{1/3} c_{sat}(1-r_h)$$

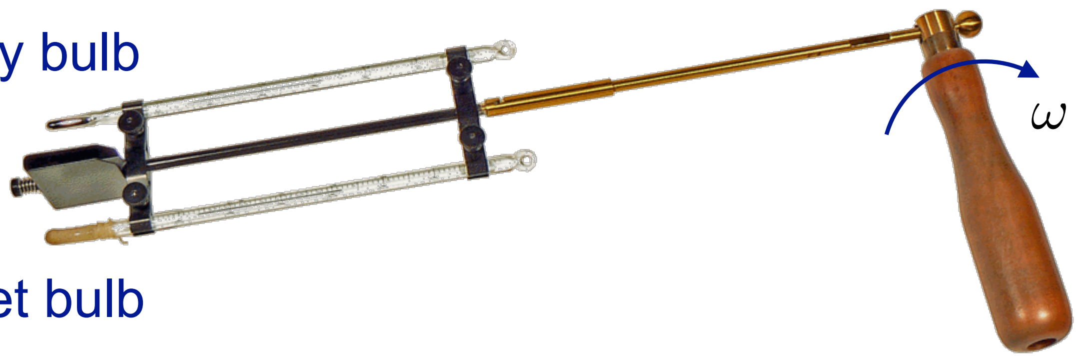
$$\boxed{\Delta T \sim L_{vap} M_{H_2O}\,\frac{D}{\kappa}\left(\frac{\alpha}{D}\right)^{1/3} c_{sat}(1-r_h)}$$ independent of U_∞

Measuring relative humidity

Numerical estimate: κ_{air} = 0.03 J.m^{-1}.K^{-1} α_{air} = 1.9 10^{-5} m^2/s

psychrometer

dry bulb

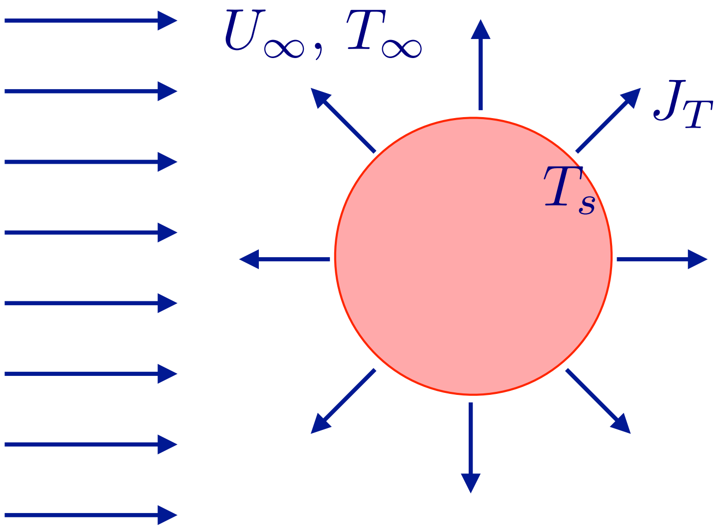


wet bulb

measure $\Delta T \Rightarrow r_h$

$$\Delta T \simeq 27^\circ\mathrm{C}(1 - r_h)$$

5.4 Blowing to cool down



$$J_T \sim \frac{\kappa_{air}}{R}\left(\frac{U_\infty R}{\nu_{air}}\right)^{1/2}\left(\frac{\nu_{air}}{\alpha_{air}}\right)^{1/3}(T_s - T_\infty)$$

hypothesis: T uniform in the solid

heat loss from the solid:

$$\frac{dQ}{dt} = -\frac{4}{3}\pi R^3 \rho_s C_{p_s}\frac{dT_s}{dt}$$

$$J_T \sim -R\,\rho_s C_{p_s}\frac{dT_s}{dt}$$

$$\frac{d(T_s - T_\infty)}{dt} \sim \frac{\kappa_{air}}{R}\left(\frac{U_\infty R}{\nu_{air}}\right)^{1/2}\left(\frac{\nu_{air}}{\alpha_{air}}\right)^{1/3}\frac{(T_s - T_\infty)}{\rho_s C_{p_s} R}$$

$$T_s - T_\infty = (T_{s_0} - T_\infty)\exp(-t/\tau)$$

with $$\boxed{\tau \sim \frac{\rho_s C_{p_s} R^2}{\kappa_{air}} \left(\frac{\alpha_{air}}{\nu_{air}}\right)^{1/3} \left(\frac{\nu_{air}}{U_\infty R}\right)^{1/2}}$$

numerical application: $\tau \sim 15\,\mathrm{min}$ for $R = 2\,\mathrm{cm}$

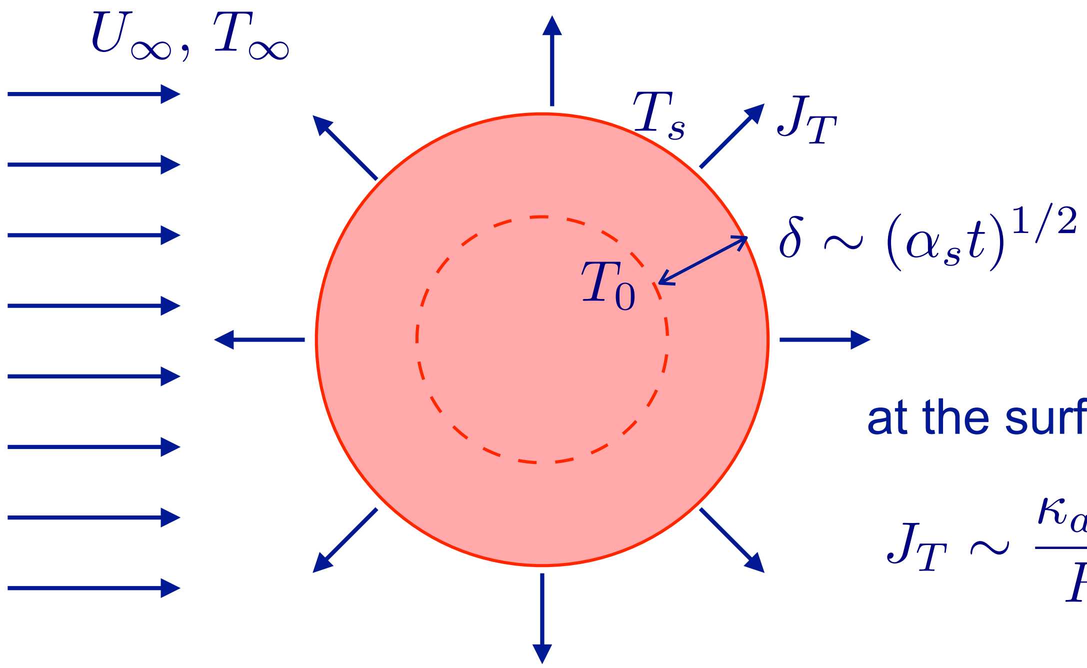
T uniform in the solid ?

$\Rightarrow$ comparison with diffusion time through the solid $\tau \gg \tau_{diff} \sim \dfrac{R^2}{\kappa_s}$?

$$\Rightarrow \quad R \ll \frac{\nu_{air}}{U_\infty} \left(\frac{\kappa_s}{\kappa_{air}}\right)^2 \left(\frac{\alpha_{air}}{\nu_{air}}\right)^{2/3}$$

numerical application: $R \ll 2\,\mathrm{cm}$

and if T not uniform?



at the surface:

$$J_T \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} (T_s - T_\infty)$$

across the boundary layer same flux:

$$J_T \sim \frac{\kappa_s}{\delta} (T_0 - T_s)$$

$$\frac{\kappa_s}{(\alpha_s t)^{1/2}} (T_0 - T_s) \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} (T_s - T_\infty)$$

$$T_s \sim \frac{\frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} T_\infty + \frac{\kappa_s}{(\alpha_s t)^{1/2}} T_0}{\frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3} + \frac{\kappa_s}{(\alpha_s t)^{1/2}}}$$

We recover:

$$t \to 0 \;\Rightarrow\; T_s \to T_0$$

$$t \to \infty \;\Rightarrow\; T_s \to T_\infty$$

characteristic cooling time:

$$\frac{\kappa_s}{(\alpha_s \tau)^{1/2}} \sim \frac{\kappa_{air}}{R} \left(\frac{U_\infty R}{\nu_{air}} \right)^{1/2} \left(\frac{\nu_{air}}{\alpha_{air}} \right)^{1/3}$$

$$\tau \sim \left(\frac{\kappa_s}{\kappa_{air}} \right)^2 \left(\frac{\alpha_{air}}{\nu_{air}} \right)^{2/3} \frac{\nu_{air}}{\alpha_s} \frac{R}{U_\infty}$$