Advanced Mechanics: Transport

April 10th, 2024

Duration 1h. Authorized document: lecture notes.

1 Thermal insulation of marine mammals

Like us, marine seals have an internal temperature close to 37°C. An insulating layer of fat protects them from cold water. We propose to make a kick estimate of the thickness of this layer.

Seals have roughly the shape of a cylinder with a length of 1.5 m and a diameter of 0.5 m. Their metabolism releases a heat flux of typically $100\,\mathrm{W/m^2}$. If T_w is the temperature of water, what is roughly the temperature T_s at the surface of a seal? (**2pts**)

What is the thickness of fat required to be comfortable in a water at $T_w = 0°\mathrm{C}$? (**2pts**)

Material constants for water:
$\beta = 4 \cdot 10^{-4}\,\mathrm{K^{-1}}$, $\nu = 1 \cdot 10^{-6}\,\mathrm{m^2s^{-1}}$, $\alpha = 0.14 \cdot 10^{-6}\,\mathrm{m^2s^{-1}}$, $\kappa = 0.6\,\mathrm{Wm^{-1}K^{-1}}$
Fat thermal conductivity:
$\kappa_{fat} \simeq 0.23\,\mathrm{Wm^{-1}K^{-1}}$

Figure 1: Common seal laying on ice.

Solution: We are in a case without any imposed fluid flow. IR radiations are not much transmitted by water. The remaining candidate to eliminate the heat flux from metabolism is natural convection (as in the exemple shown in the course):

$$J_s \sim \frac{\kappa_w}{R}(T_s - T_w)\left(\frac{\beta_w g (T_s - T_w) R^3}{\alpha_w\,\nu_w}\right)^{1/4}$$

$$T_s - T_w \sim \frac{J_s}{\kappa_w^{4/5}}\left(\frac{\alpha_w\,\nu_w\,R}{\beta_w g}\right)^{1/5} \sim 1°\mathrm{C}$$

We have the same heat flux through the layer of fat, which lead to:

$$J_s = \frac{\kappa_{fat}}{e_{fat}}(T_{body} - T_s)$$

$$e_{fat} \simeq 10\,\text{cm}$$

Reasonable order of magnitude.

2 Sniffing

When we want to appreciate the smell of a flower, fragrance molecules are convected through our nostrils and interact with nervous connections located in the olfactory bulb. As a rough approximation, we model this region as a tube of radius R and length L, where air goes trough at a velocity U (Fig. 2). We propose to discuss the optimal way of smelling.

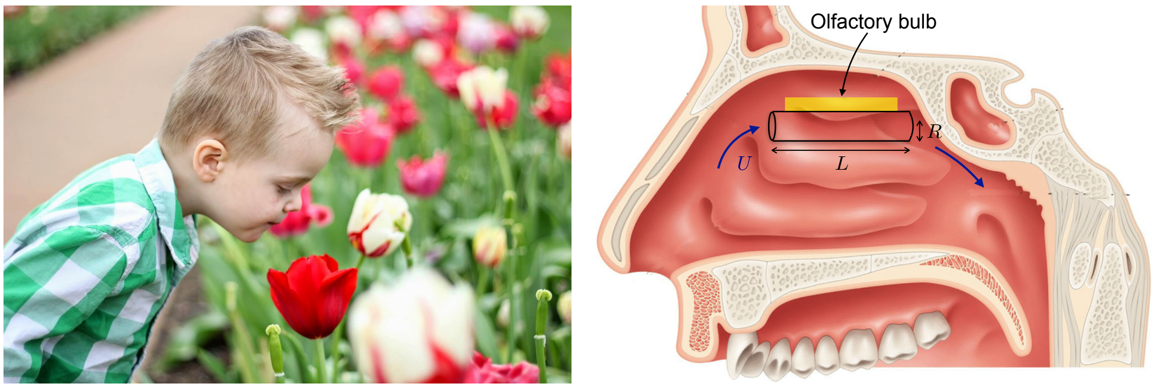

Figure 2: Left, child sniffing a flower. Right, nostril viewed a a tube of length L and radius R in the vicinity of the olfactory bulb.

Give an expression for the typical convection time τ_c of a fragrance molecule along the tube (**1 pt**). If D is the diffusion coefficient of fragrance molecules, what is the typical time τ_d for a fragrance to diffuse across R? (**1 pt**)
What is the condition to ensure that most fragrance molecules will have a chance to interact with the olfactory bulb at the upper surface of the tube? (**1 pt**)
A typical value for a diffusion coefficient of a fragrance molecule in air is $D \sim 10^{-6}\,\text{m}^2/\text{s}$. What is the maximal breathing velocity that ensures the previous condition for $R \sim 5\,\text{mm}$ and $L \sim 50\,\text{mm}$? (**1 pt**)

Solution: Convection along L: $\tau_c \sim L/U$
Diffusion across R: $\tau_d \sim R^2/D$
We want $\tau_d < \tau_c$, in other words $\frac{UR^2}{DL} < 1$ (this is a Peclet number)
$U_{max} \sim \frac{DL}{R^2} \sim 2\,\text{mm/s}$.

3 Hot wire anenometry

You may have used hot wire anemometry during your lab teaching classes (wind tunnel experiment). The measuring probe is based on a tungsten filament heated up to a fixed temperature T_w with an imposed electric current I (Fig. 3). Blowing air tends to cool down the wire, which changes its electrical resistance. A feedback loop maintains the resistance of the wire to its prescribed value R_w (ie, its prescribed temperature) by adjusting the electric current. The wind temperature can be inferred from monitoring the feedback signal after calibrating the device.
We will assume that the heat transfer laws that we have explored during classes (that assumes a rather large Re number) are still valid in the present situation.
The probe suppliers propose the following law for the evolution of the applied current I with the wind velocity U:

$$R_w I^2 = A + BU^{1/2}$$

where A and B depend on T_w and on the room temperature T_{ext}, on the geometry of the probe (length $L = 1\,\text{mm}$ and tiny radius of $r = 2.5\,\mu\text{m}$) and on material constants (we express U in m/s).

What heat transfer mechanisms may contribute to A and B, respectively (**1 pt**)? What is the origin of the $U^{1/2}$ term (**1 pt**)?
Give a numerical estimate of the different heat fluxes for a prescribed temperature $T_w = 200°\text{C}$ and ambient temperature $T_{ext} = 20°\text{C}$ and a wind velocity of $1\,\text{m/s}$ (**6 pts**). How does this velocity compares with natural convection (**1 pt**)?
Find numerical estimates of both parameters A and B (**2 pt**).
Do you expect this law to hold for low values of the velocity (**1 pt**)?

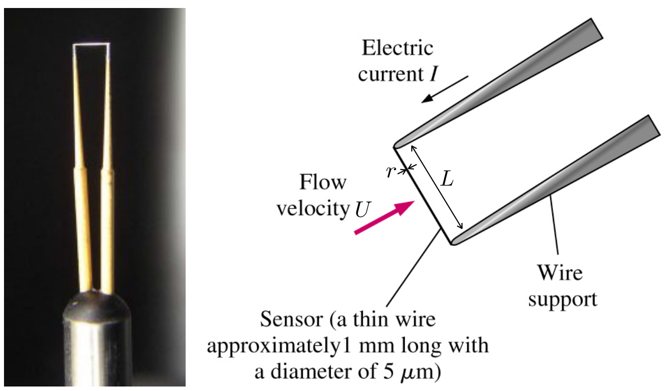

Figure 3: Hot wire anenometry. Left, detail of a probe. Right, principle of measurement: the resistance of the probe is related to its temperature (resistivity increases with the temperature). A feedback loop on the applied electrical intensity sets a prescribed value of the resistance (ie, of the temperature). Blowing air on the probe tends to decrease its temperature, but the electronic system increases the applied current accordingly. Measuring the current thus indicates the wind velocity.

Material constants for air:

$\beta_{air} = 3.4 \cdot 10^{-3}\,\mathrm{K}^{-1}$, $\nu_{air} = 1.5 \cdot 10^{-5}\,\mathrm{m}^2\mathrm{s}^{-1}$, $\alpha_{air} = 2 \cdot 10^{-5}\,\mathrm{m}^2\mathrm{s}^{-1}$, $\kappa_{air} = 0.026\,\mathrm{Wm}^{-1}\mathrm{K}^{-1}$

Material constants for the probe:
$\alpha_w \sim 7 \cdot 10^{-5}\,\mathrm{m}^2\mathrm{s}^{-1}$, emissivity of the probe $\varepsilon_w \simeq 1$, emissivity of the surrounding walls $\varepsilon_{ext} \simeq 1$

Fundamental constants:
Absolute 0: $-273.15°$C, Stefan-Bolztman constant $\sigma = 5.7 \cdot 10^{-8}\,\mathrm{Wm}^{-2}\mathrm{K}^{-4}$

Solution: Obviously, $R_w I^2$ corresponds to the total power emitted by the wire.
A accounts for radiative exchanges and natural convection.
B corresponds to forced convection.
Radiative flux is given by $J_r = \sigma(T_w^4 - T_{ext}^4) = 2400\,\mathrm{W/m}^2$
Natural convection is given by $J_{nc} \sim \frac{\kappa_{air}}{r}(T_w - T_{ext})\left(\frac{\beta_{air} g (T_w - T_{ext}) r^3}{\alpha_{air}\,\nu_{air}}\right)^{1/4} = 45\,\mathrm{kW/m}^2$ (apparently huge value, but the corresponding area is small).
Velocity for natural convection: $U_{nc} \sim (\beta_{air} g (T_w - T_{ext}) r)^{1/2} \sim 4\,\mathrm{mm/s}$.
Forced convection, $Nu = Pr^{1/3} Re^{1/2}$, $Pr = \nu/\alpha$,

$$J_{fc} \simeq \frac{\kappa_{air}}{r}\left(\frac{\nu_{air}}{\alpha_{air}}\right)^{1/3}\left(\frac{r\,U}{\nu_{air}}\right)^{1/2}(T_w - T_{ext}) \sim 740\,\mathrm{kW/m}^2$$

The heat power released is $2\pi r L(J_r + J_{nc} + J_{fc}) = 7.4 \cdot 10^{-4} + 1.2 \cdot 10^{-2} U^{1/2}\,\mathrm{W}$.
At low values of the velocity, Re becomes small and the scaling based on Blasius boundary layer is not valid any more.