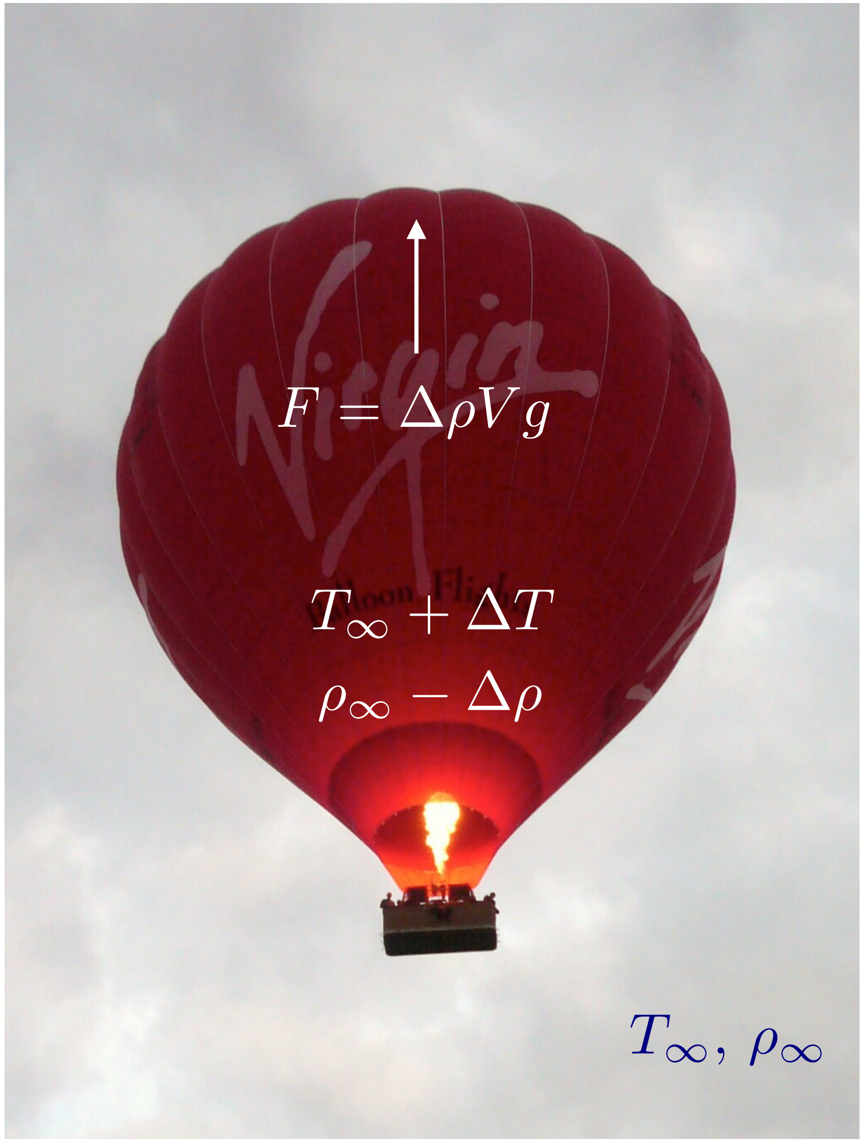
6. Natural convection

4. Natural convection

Principle: hot air balloon (montgolfière)



Thermal expansion coefficient:

$$\rho(T) = \rho_\infty \left(1 - \beta(T - T_\infty)\right)$$

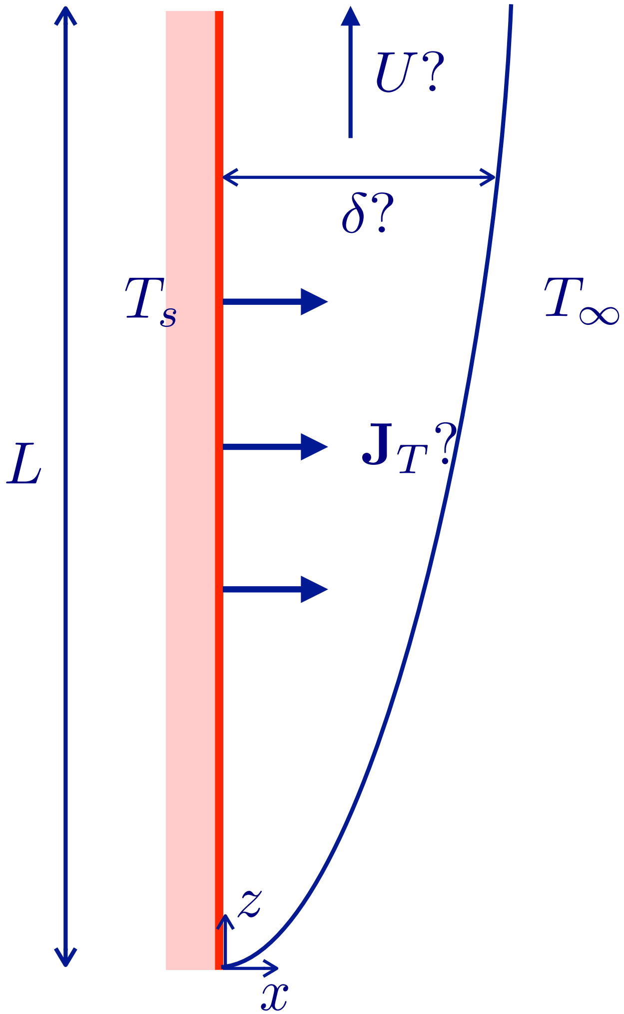
$$\boxed{\beta = -\frac{1}{\rho}\frac{\partial\rho}{\partial T}}$$

hot fluid rises up

⇒ natural convection ↔ gravity

4. Natural convection

Simplified configuration: vertical wall



$$U_\infty = 0$$

Fluid flow induced by temperature contrast
⇒ 1st approximation, single boundary layer

We need to solve 2 coupled equations:

Flow equation:

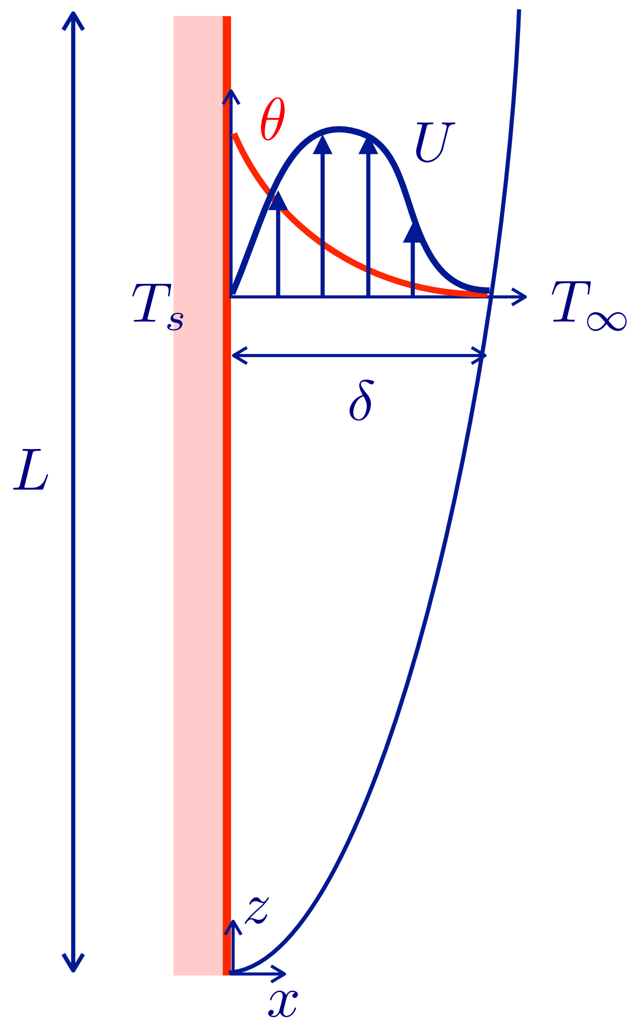
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho(\mathbf{u}.\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla} P + \eta \Delta \mathbf{u} + \rho_\infty (1 - \beta(T - T_\infty))\mathbf{g}$$

Heat equation:

$$\frac{\partial T}{\partial t} + (\mathbf{u}.\boldsymbol{\nabla})T = \alpha \Delta T$$

4. Natural convection

Profiles in the boundary layer



$$U_\infty = 0$$

$$\theta = T - T_\infty$$

4. Natural convection

Doing our best to simplify !

$$\cancel{\rho\frac{\partial \mathbf{u}}{\partial t}} + \underset{\rho_\infty}{\rho}(\mathbf{u}.\boldsymbol{\nabla})\mathbf{u} = -\boldsymbol{\nabla}P + \eta\Delta\mathbf{u} + \rho_\infty(1-\beta(T-T_\infty))\mathbf{g}$$

steady

Boussinesq approximation: $\rho \simeq \rho_\infty$
except in $\rho\mathbf{g}$

for a uniform temperature $\mathbf{u}=0$ and $T=T_\infty \Rightarrow -\boldsymbol{\nabla}P + \rho_\infty\mathbf{g} = 0$

We assume to get the same $\boldsymbol{\nabla}P$ in the presence of convection (1st order approximation)

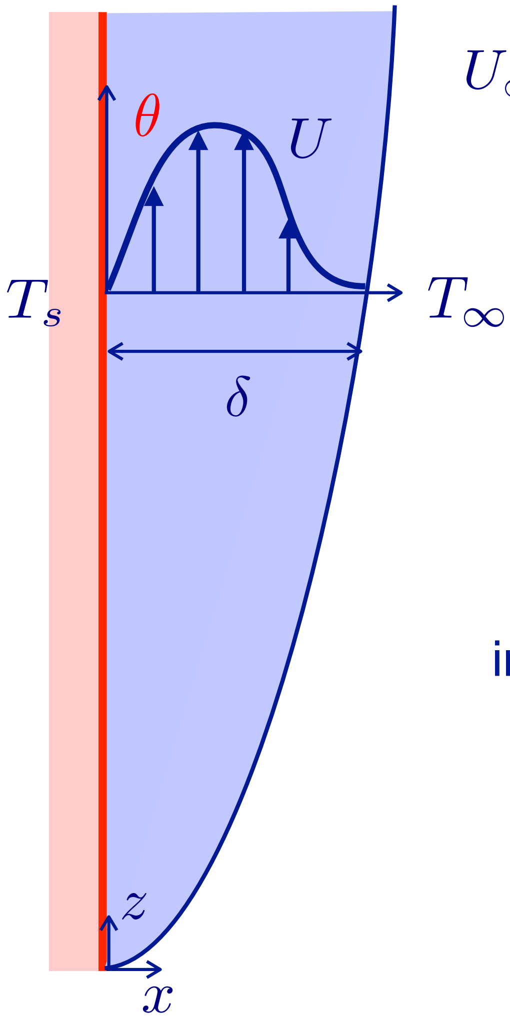
Remains $\rho_\infty(\mathbf{u}.\boldsymbol{\nabla})\mathbf{u} \simeq \eta\Delta\mathbf{u} + \rho_\infty\beta\theta\mathbf{g}$ with $\theta = T - T_\infty$

$$\cancel{\frac{\partial \theta}{\partial t}} + (\mathbf{u}.\boldsymbol{\nabla})\theta = \alpha\Delta\theta$$

steady

4. Natural convection

Balance shear force on the wall / buoyancy force



$$U_\infty = 0$$

shear stress $\sigma = \eta \left. \frac{\partial u_z}{\partial x} \right|_{x=0}$

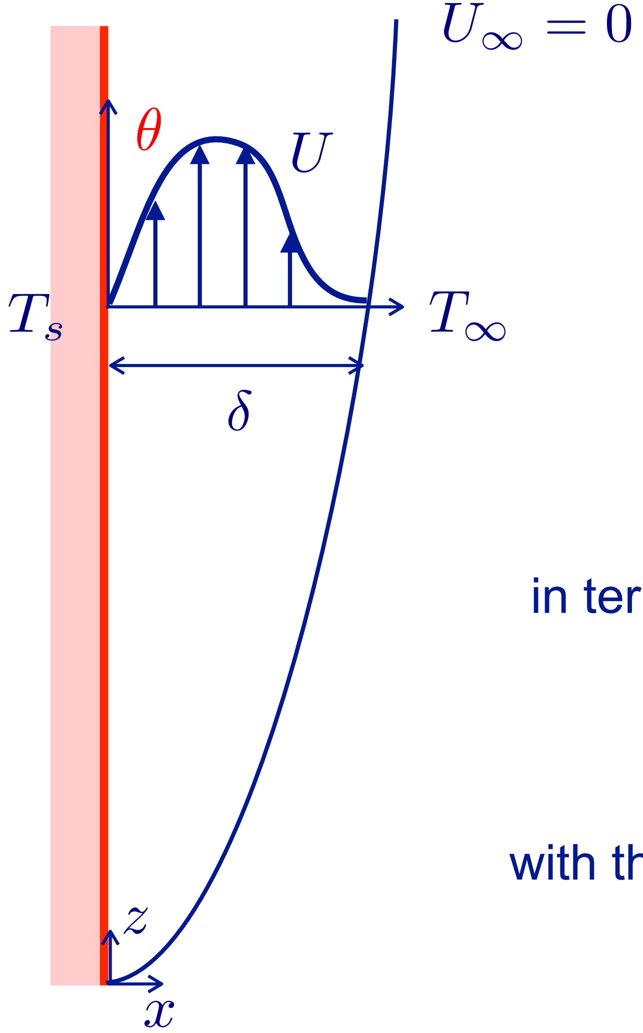
$$W \int_0^z \eta \left. \frac{\partial u_z}{\partial x} \right|_{x=0} dz = W \int_0^z \rho_\infty g \beta \theta \delta(z)\, dz$$

in terms of scaling: $\eta \frac{U}{\delta} \sim \rho_\infty g \beta \theta \delta$

$$U \sim \frac{g \beta \theta \delta^2}{\nu}$$

4. Natural convection

Heat equation



$$\cancel{\frac{\partial \theta}{\partial t}} + (\mathbf{u}.\boldsymbol{\nabla})\theta = \alpha \Delta \theta$$

we consider $u_z \gg u_x$

and $z \gg \delta$

$$u_z \frac{\partial \theta}{\partial z} \simeq \alpha \frac{\partial^2 \theta}{\partial x^2}$$

in terms of scaling:

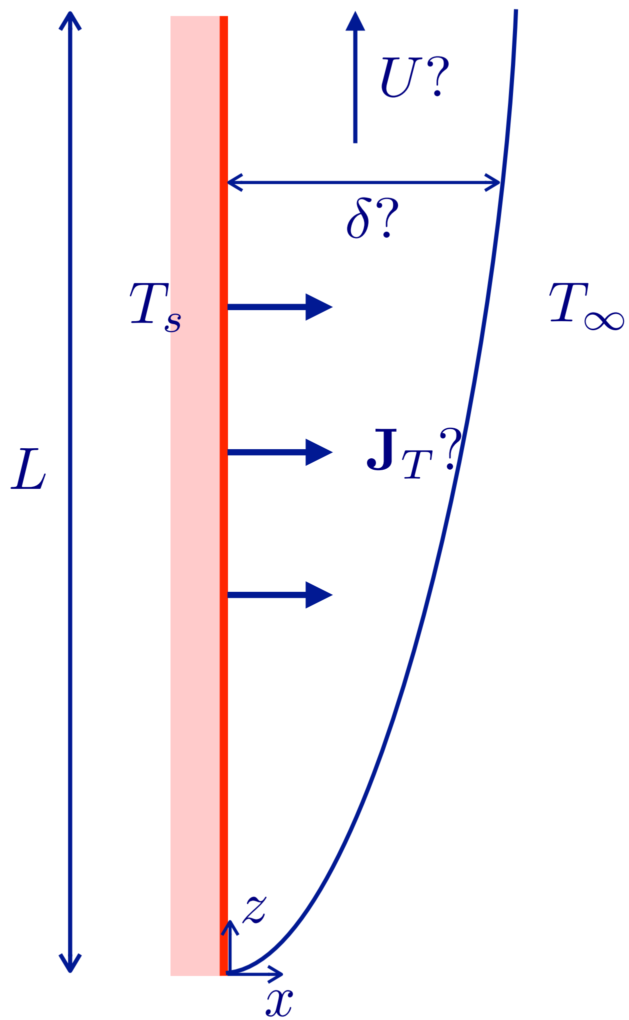
$$\frac{U}{z} \sim \frac{\alpha}{\delta^2}$$

with the previous relation: $\dfrac{g\beta\theta\delta^2}{\nu z} \sim \dfrac{\alpha}{\delta^2}$

$$\boxed{\delta \sim \left(\frac{\nu\alpha}{\beta\theta g} z\right)^{1/4}}$$

4. Natural convection

Getting the heat flux



$$U_\infty = 0$$

$$J_T = \kappa \frac{\partial T}{\partial x} \sim \kappa \frac{\theta}{\delta} \sim \kappa \frac{\theta}{L} \mathrm{Nu}$$

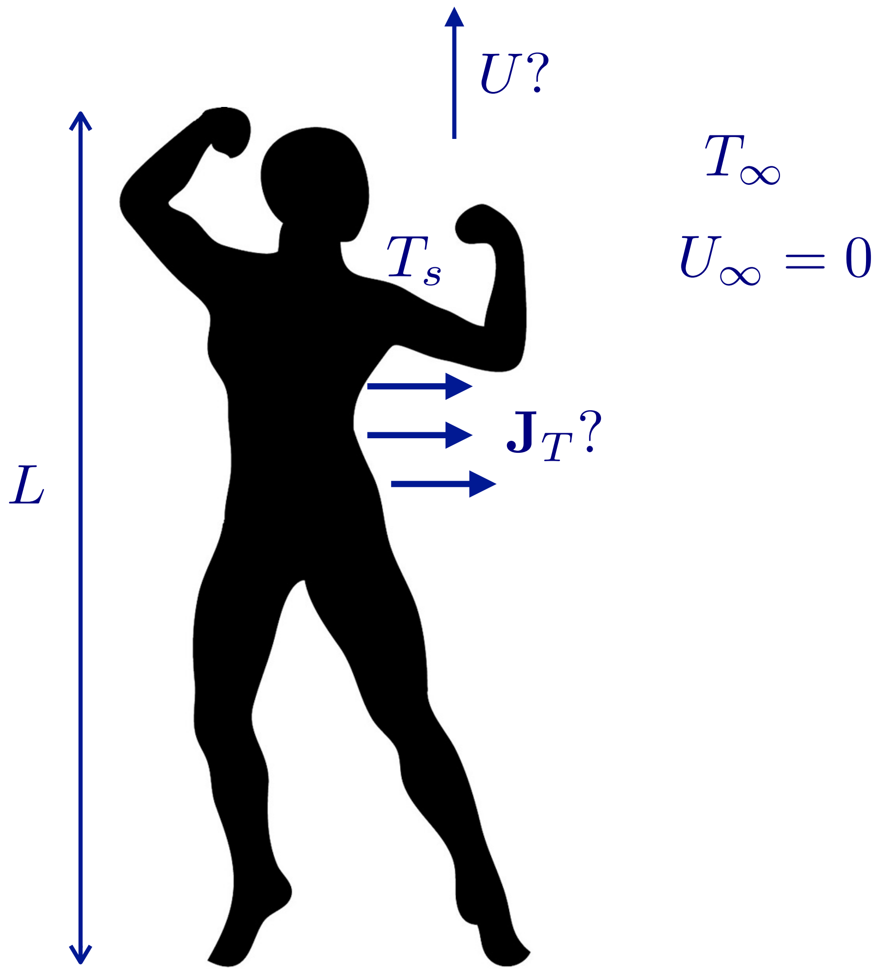
$$\boxed{\mathrm{Nu} \sim \mathrm{Ra}^{1/4}}$$

Rayleigh number: $\boxed{\mathrm{Ra} = \frac{\beta g L^3 \theta}{\nu \alpha}}$

convection velocity: $\boxed{U \sim (\beta g \theta z)^{1/2}}$

4.1 Human plume

Getting the heat flux



Naked human: confort for

$$T_\infty \sim 27°\mathrm{C} \;\Rightarrow\; \theta \sim 10°\mathrm{C}$$

$$\mathrm{Ra} \sim 3.10^{10} \;\Rightarrow\; \mathrm{Nu} \sim 400$$

$$h_T = \frac{\kappa}{L}\mathrm{Nu} \sim 6.5\,\mathrm{W.m^{-2}.K^{-1}}$$

$$J_T = h_T(T_s - T_\infty) \sim 65\,\mathrm{W/m^2}$$

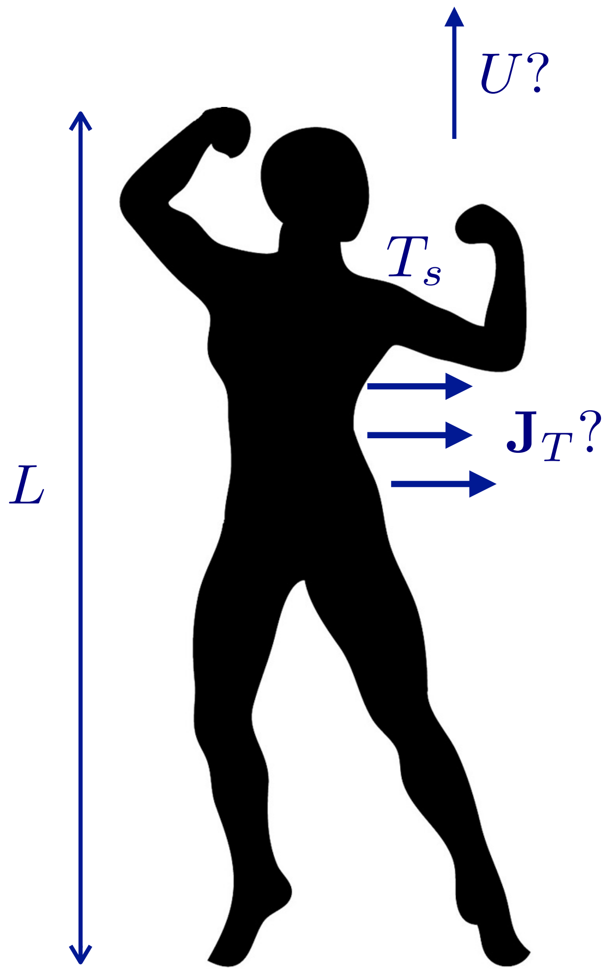
$$S \simeq 1.6\,\mathrm{m}^2$$

$$P = J_T S \simeq 100\,\mathrm{W}$$

good order of magnitude

4.1 Human plume

with a sweater: same $T_s - T_\infty \sim 10°\mathrm{C}$



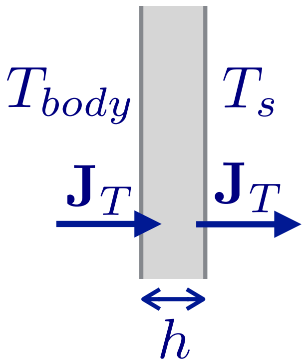
could be $T_\infty \sim 17°\mathrm{C}$ and $T_s \sim 27°\mathrm{C}$

T_∞

$U_\infty = 0$

compatible IR imaging

Thickness of the sweater?

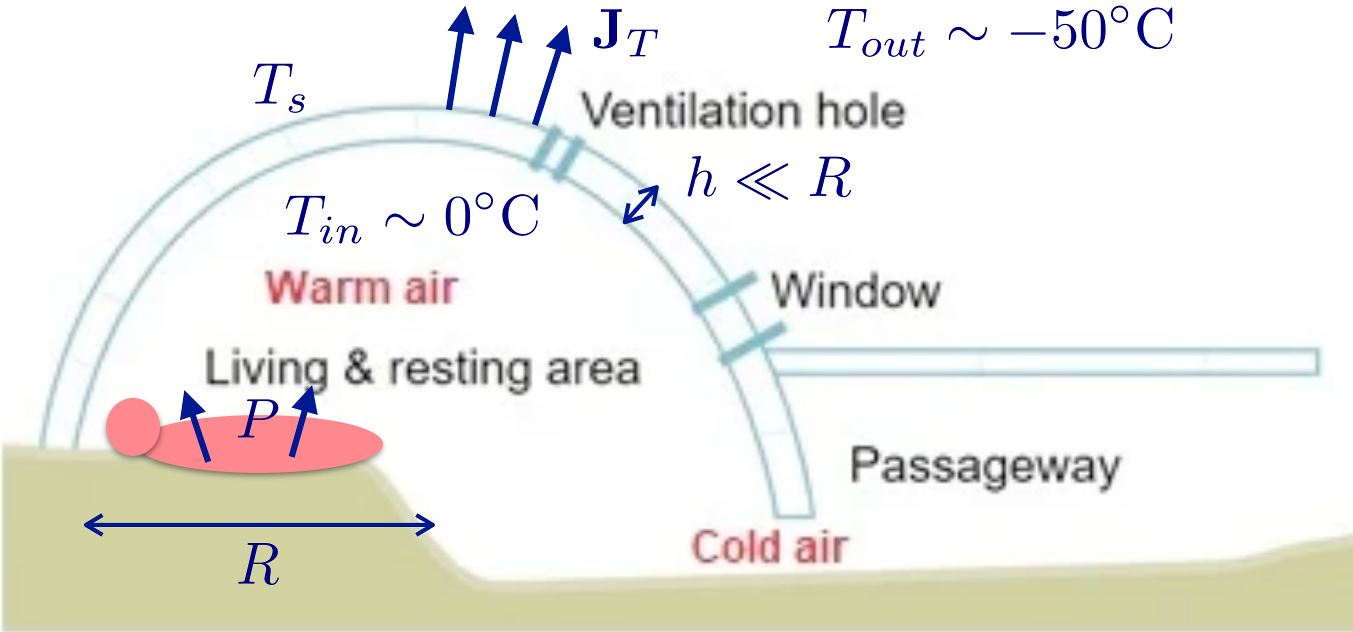


$$J_T = \frac{\kappa_{wool}}{h}(T_{body} - T_s)$$

$$\kappa_{wool} \sim 0.05\,\mathrm{W.m^{-1}.K^{-1}}$$

$$h \sim 8\,\mathrm{mm}$$

4.2 Building an igloo



$$2\pi R^2 J_T = P = 200\,\mathrm{W}$$

$$J_T \sim 30\,\mathrm{W/m^2}$$

If no wind: $J_T = \dfrac{\kappa_{ice}}{h}(T_{in} - T_s) \simeq \kappa_{air}\left(\dfrac{\beta g}{\nu\alpha R}\right)^{1/4}(T_s - T_{out})^{5/4}$

$$T_s - T_{out} = 7°\mathrm{C} \qquad T_s = -43°\mathrm{C} \qquad h \sim 3\,\mathrm{m}$$

Better to make a small igloo!

4.2 Building an igloo

If no external wind, convection flow velocity

$$U \sim \left(\beta g (T_s - T_{out}) R\right)^{1/2} = 0.5\,\mathrm{m/s}$$

Under strong wind (U = 10 m/s >> 0.5 m/s)

$$\mathrm{Nu} \simeq 0.33 \mathrm{Pr}^{1/3} \mathrm{Re}^{1/2} \simeq 240$$

$$J_T = \frac{\kappa}{R} \mathrm{Nu}\,(T_s - T_{out})$$

$$T_s - T_{out} \simeq 5^\circ\mathrm{C}$$

$$h \sim 3\,\mathrm{m}$$