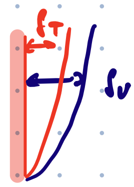
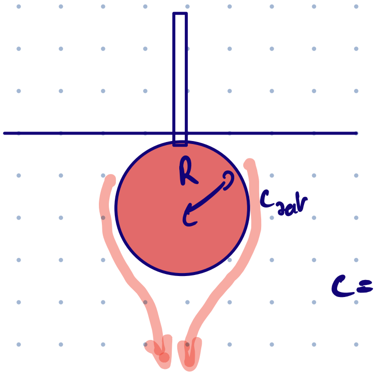
Recap : Free convection



$$Ra = \frac{\beta g \theta L^3}{\nu \kappa} \qquad Nu = \frac{hL}{k} \sim Ra^{1/4}$$

$$\delta_T \sim \left(\frac{\nu\kappa}{\beta g \theta}\, x\right)^{1/4} \qquad U \sim (\beta g \theta x)^{1/2}$$

Dissolution of a lollipop



At the surface of the lollipop $c = c_{sat}$

mass concentration

Liquid density $\rho = \rho_0 + c = \rho_0 (1 + c/\rho_0)$

$\beta\theta$ for thermal flow

$c = 0$

$$\text{"}Ra\text{"} = \frac{g\, c_{sat}/\rho_0\, R^3}{\nu D}$$

$$\Rightarrow J_m \sim \frac{D\, c_{sat}}{R} \left(\frac{g\, c_{sat}/\rho_0\, R^3}{\nu D}\right)^{1/4}$$

mass conservation: $\frac{dm}{dt} = \rho_s\, 4\pi R^2 \frac{dR}{dt} = -4\pi R^2 \frac{D c_{sat}}{R}\left(\frac{g\, c_{sat}/\rho_0\, R^3}{\nu D}\right)^{1/4}$

$$\rightarrow \frac{dR}{dt} \sim -\frac{D\, c_{sat}}{\rho_s}\left(\frac{g\, c_{sat}/\rho_0}{\nu D}\right)^{1/4} \frac{1}{R^{1/4}}$$

$$\frac{dR/R_0}{dt} \sim -\frac{D\, c_{sat}}{\rho_s R_0}\left(\frac{g\, c_{sat}/\rho_0}{\nu D R_0}\right)^{1/4} \frac{1}{(R/R_0)^{1/4}}$$

$$\left(R/R_0\right)^{1/4} \frac{d(R/R_0)}{dt} \sim -\frac{D\, c_{sat}}{\rho_s R_0}\left(\frac{g\, c_{sat}/\rho_0}{\nu D R_0}\right)^{1/4}$$

$$1 - \left(\frac{R}{R_0}\right)^{5/4} = t/\tau \qquad \tau \sim \frac{\rho_s R_0}{D\, c_{sat}}\left(\frac{\nu D R_0}{g\, c_{sat}/\rho_0}\right)^{1/4}$$

$$R = R_0 \left(1 - t/\tau\right)^{4/5}$$

Comparison experiment: $R_0 = 3\,cm$ $\quad c_{sat} = 0{,}3\,g/cm^3$ $\quad \rho_s = 1{,}43\,g/cm^3$

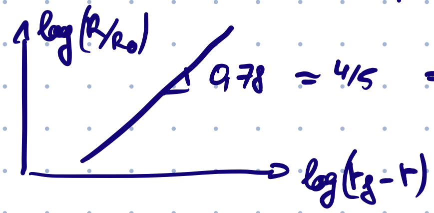
$D \sim 4{,}3\cdot 10^{-10}\,m^2/s$ (sucrose in pure water)

$\nu_{water} = 10^{-6}\,m^2/s$ / $\nu_{saturated\ sucrose} \simeq 8\cdot 10^{-4}\,m^2/s$

↳ for ν_{water} : $\tau = \frac{1{,}43}{0{,}3}\, \frac{3\cdot 10^{-2}}{4{,}3\cdot 10^{-10}} \left(\frac{10^{-6}\cdot 4{,}3\cdot 10^{-10}\cdot 3\cdot 10^{-2}}{10\cdot 0{,}3/1}\right)^{1/4} = 15000\,s = 250\,min$

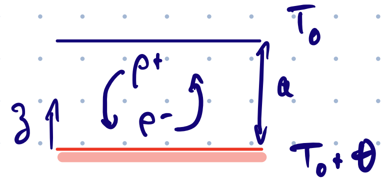
experiment $\tau = 175\,min$

→ Plot



$0{,}78 \simeq 4/5 \Rightarrow$ good agreement!

Rayleigh - Bénard Instability



$$T(z) = T_0 + \theta - \frac{\theta}{a} z$$

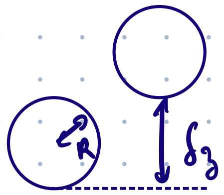
$$\rho = \rho_0 \left(1 - \beta \left(T(z) - T_0\right)\right)$$

$$\rho(z) = \rho_0 \left(1 - \beta \left(\theta - \frac{\theta}{a} z\right)\right)$$

↳ hot in the bottom ⇒ unstable?

fluid bubble of size R → moves perturbation of δz

$$\Rightarrow \quad \delta\rho = -\rho_0 \beta \frac{\Delta T}{a} \delta z$$



buoyancy force: $\frac{4}{3}\pi R^3 \delta\rho\, g = \frac{4}{3}\pi R^3 \rho_0 g \beta \frac{\theta}{a} \delta z$

if inertia is neglected ($Re \ll 1$)

buoyancy balanced by viscous drag: $6\pi\eta R U$

$$\frac{4}{3}\pi R^3 \rho_0 g \beta \theta/a\, \delta z = 6\pi\eta R U$$

$$\Rightarrow \quad U = \frac{g}{2} \frac{\rho_0 g R^2}{\eta} \beta \frac{\theta}{a} \delta z = \frac{d\delta z}{dt}$$

$$\Rightarrow \quad \delta z(t) = \delta z_0 \exp\left(t/\tau_{conv}\right) \qquad \tau_{conv} \sim \frac{\eta}{\rho_0 g R^2} \frac{a}{\beta \Delta T}$$

However, thermal diffusion attenuates θ

→ diffusion time $\tau_{diff} \sim \frac{R^2}{\alpha}$

⇒ unstable if $\tau_{diff} \gg \tau_{conv}$

$$\rightarrow \frac{R^2}{\alpha} \gg \frac{\eta}{\rho_0 g R^2} \frac{a}{\beta\theta} \rightarrow \frac{g R^4 \beta\theta}{\nu \alpha a} \gg 1$$

→ the bigger the bubble the more unstable

most unstable configuration: $R \sim a$

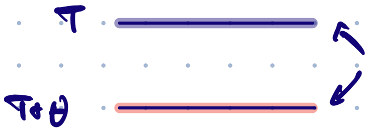
→ unstable if $\boxed{Ra = \frac{g a^3 \beta\theta}{\nu\alpha} > Ra_c}$

↳ lectures L. Duchemin → $Ra_c \sim 1000$

Numerical estimate : ex : $\theta = 50°C$

	$\beta (K^{-1})$	$\nu (m^2/s)$	$\alpha (m^2/s)$	$B/\nu\alpha (J.m^{-2}.K^{-1})$	a_c
air	$3,4\ 10^{-3}$	$1,5.10^{-5}$	2.10^{-5}	10^7	6 mm
water	2.10^{-4}	10^{-6}	$1,4.10^{-7}$	$1,4.10^{9}$	1 mm
glycerin	$4,8.10^{-4}$	10^{-3}	10^{-7}	5.10^{6}	7 mm

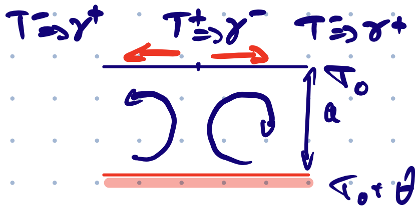
→ Comparison theory / experiments OK for closed cells

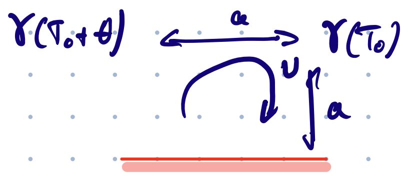


walls ⇒ no interface

→ <u>Free interfaces</u> : mismatch → controversy until ~ 1960

$$\gamma(T) = \gamma(T_0)\left(1 - \beta(T - T_0)\right)$$





⇒ surface stress $\frac{\Delta\gamma}{a} = \beta_T \frac{\gamma_0}{a}\theta$

→ viscous stress $\frac{\eta U}{a}$

→ $U \sim \frac{\gamma_0}{\eta}\beta_T\theta$ → $\tau_{conv} \sim \frac{a}{U} \sim \frac{\eta a}{\gamma_0 \beta_T \theta}$

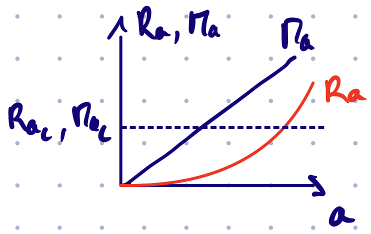
diffusion over a length a $\tau_{dif} \sim a^2/\alpha$

→ unstable if $\tau_{dif} \gg \tau_{conv}$

$$\frac{a^2}{\alpha} \gg \frac{\eta a}{\gamma_0 \beta_T \theta}$$

$$Ma = \frac{\gamma_0 \beta_T \theta a}{\eta \alpha} > M_c$$

↳ Marangoni number



→ Marangoni may appear before Rayleigh