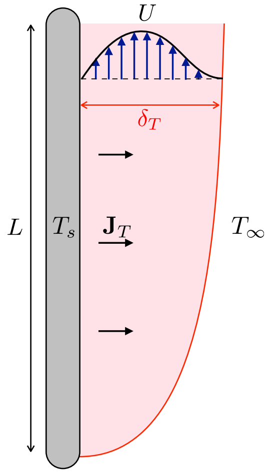
Transport Phenomena:

6. Natural convection

Take home message:



Thermal expansion coefficient:

$$\rho = \rho_0\left(1 - \beta(T - T_0)\right)$$

Rayleigh number:

$$\mathrm{Ra} = \frac{\beta g L^3 \Delta T}{\nu\alpha}$$

Heat transfer coefficient:

$$\mathbf{J}_T = -h_T(T_\infty - T_s)$$

Nusselt number:

$$\mathrm{Nu} = \frac{h_T L}{\kappa} \sim \mathrm{Ra}^{1/4} \quad \text{(vertical plate, Re}_x\text{>>1)}$$

different expressions for other configurations

same formalism for mass transport

1 Self-induced flow along a vertical wall

We consider a vertical hot plate immersed in a colder fluid (Fig. 1). In the vicinity of the plate, the fluid becomes warmer and therefore less dense. As a first approximation, the variation of density in proportional to the temperature difference, $\rho = \rho_0\left(1 - \beta(T - T_\infty)\right)$, where β is the (volume) thermal expansion coefficient. We define the temperature mismatch as $\theta = T - T_\infty$. One usual simplification consist in considering the temperature dependance of ρ solely in the gravity term (Boussinesq approximation). We propose to describe the characteristics of the rising boundary layer in terms of velocity flow and heat flux.

In contrast with forced convection, fluid flow and heat transfer are deeply intricate. The flow in the self-induced boundary layer differs from the case of forced convection. Describe qualitatively the distribution of velocity in this boundary layer.
The coupling between viscous flow and buoyancy forces can be interpreted in terms of force balance. If $\delta(x)$ is the width of the boundary layer, what is the typical viscous shear force acting along the wall? What is the buoyancy force acting on the same boundary layer? Deduce a first relation between U, θ and δ.
Using the heat equation deduce independent scalings for δ and U.

We now need to estimate the heat exchange between the plate and the surrounding air. Express the heat flux J_T, as a scaling law. An important non-dimensional number for natural convection is the Rayleigh number defined as:

$$Ra = \frac{\beta g \theta L^3}{\nu\alpha}$$

How does Nusselt number scales with Ra in the present configuration?

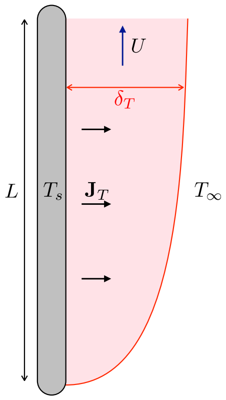


Figure 1: Convection flow along a vertical wall characterized by a boundary layer self-induced by buoyancy forces.

2 Application: Human plumes and igloos

2.1 Human plumes

Figure 2a illustrates the thermal plume emanating from a human body ("schlieren" optical technique). A quick estimate of the velocity of the plume with ImageJ indicates 0.7 m/s. We would like to verify if the previous scaling laws are consistent with standard physiological data (a person at rest typically produces 100 W of heat for a body area on the order of 1.6 m^2).

Can we obtain this heat flux from personal experience? Without clothes, we feel thermally comfortable for an exterior temperature of about 27°C (under moderate humidity). If our skin has a temperature of 37°C, what are the corresponding values for the Rayleigh and Nusselt numbers? Do we recover a heat flux consistent with physiological data?
If the heat flux remains the same, what temperature would you recommend for the bath of a baby?

In the illustrated exemple, the person wears a rather thick sweater, a reasonable estimate of the exterior temperature may be 16°C. Using the same value for the heat flux, is it possible to estimate the temperature at the surface of the clothes? Is the value consistent with IR imaging (Fig 2b)? Is the plume velocity in agreement with the prediction?

Material constants for air:
$\beta = 3.4 \cdot 10^{-3}\,\mathrm{K}^{-1}$, $\nu = 1.5 \cdot 10^{-5}\,\mathrm{m}^2\mathrm{s}^{-1}$, $\alpha = 2 \cdot 10^{-5}\,\mathrm{m}^2\mathrm{s}^{-1}$, $\kappa = 0.026\,\mathrm{Wm}^{-1}\mathrm{K}^{-1}$
for water:
$\beta = 4 \cdot 10^{-4}\,\mathrm{K}^{-1}$, $\nu = 1 \cdot 10^{-6}\,\mathrm{m}^2\mathrm{s}^{-1}$, $\alpha = 0.14 \cdot 10^{-6}\,\mathrm{m}^2\mathrm{s}^{-1}$, $\kappa = 0.6\,\mathrm{Wm}^{-1}\mathrm{K}^{-1}$

2.2 Designing an igloo

Imagine you missed the base camp while hiking in Nepalese mountains and have to spend one night under a very cold weather of $-50°C$. Could you survive inside an igloo made with compacted snow? We will consider an hemispherical igloo of radius $R = 1\,\mathrm{m}$, an inner temperature of $0°C$ (Fig 2c). We may assume that you are able to release $200\,\mathrm{W}$ of heat (this will not be the most confortable night anyway!).

What is the heat flux through the thickness of the igloo? What is the temperature at the outer surface of the igloo? How thick should be your igloo (we will take the worse case scenario where snow behaves as ice of thermal conductivity $\kappa_i \simeq 2\,\mathrm{Wm^{-1}K^{-1}}$).
What is the typical convection velocity in the absence of external wind?
What would be the thickness of the igloo under wind conditions with a wind speed of $10\,\mathrm{m/s}$?

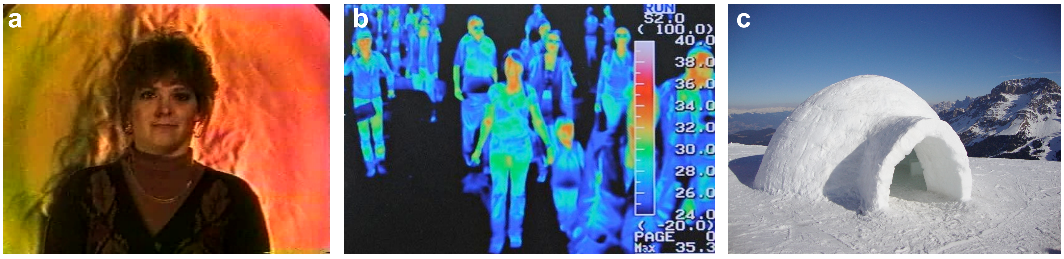


Figure 2: **a** Thermal plume emanating from a human being visualized through the optical "Schlieren" technique (source: https://youtu.be/1MA-zEUepvs). **c** How thick should an igloo be to provide a thermal protection against cold temperatures?