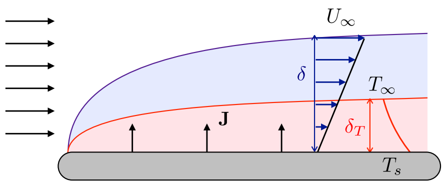
Transport Phenomena:

3. Forced convection

Take home message:

Heat transfer coefficient (Mass transfer coeffcient):

$$\mathbf{J} = -h(T_\infty - T_s) \quad \Rightarrow \quad h \sim \frac{\kappa}{\delta_T}$$



Nusselt number (Sherwood number for mass transport):

$$Nu = \frac{hL}{\kappa} \quad \Rightarrow \quad Nu \sim \frac{L}{\delta_T} = \frac{\text{convective transfer}}{\text{conductive transfer}}$$

Prandtl number (Schmidt number for mass transport):

$$Pr = \frac{\nu}{\alpha} = \frac{\text{momentum diffusivity}}{\text{thermal diffusivity}}$$

Laminar boundary layer over a flat plate:

$$Nu \sim Re^{1/2} Pr^{1/3} \quad (\text{for } Pr > 0.6)$$

3.1 Cooling flow over a plate

Fans are commonly used to evacuate efficiently the heat of CPUs when you are working hard on your computer. We consider here a simplified configuration where a plate of length L and fixed temperature T_s is submitted to a parallel flow of air of initial velocity U_∞ and temperature T_∞ (Fig: 1).

Describe the classical velocity profile in the case of a laminar flow. How does the size of the boundary layer δ_V evolves with x? As a first approximation, what is the expression of the velocity profile in this boundary layer?

Now, the issue is to describe the temperature profile. What equation sets the evolution of the temperature? Propose a solution in terms of scaling laws that put in evidence a boundary layer of local thickness $\delta_T(x)$.

We now need to estimate the heat exchange between the plate and the surrounding air. Propose a scaling law for the conductive heat flux J through this layer.
We define as *heat transfer coefficient* the quantity h such as $J = -h\,(T_\infty - T_s)$. What is the

definition of h in terms of scaling law?

Non-dimensional numbers are widely used in heat transfer problems. One very useful quantity is *Nusselt number* defined as $Nu = hL/\kappa$. Again, what is definition of Nu in terms of scaling law? What is the scaling for Nu in the case of the laminar flow parallel to the plate? Numerous theoretical or empirical expressions for Nu are proposed in the literature for different geometrical configurations. In these relations Nu is generally expressed as a function of *Reynolds number* Re and *Prandtl number* $Pr = \nu/\alpha$. In the present configuration, how does Nu scale with Re and Pr?

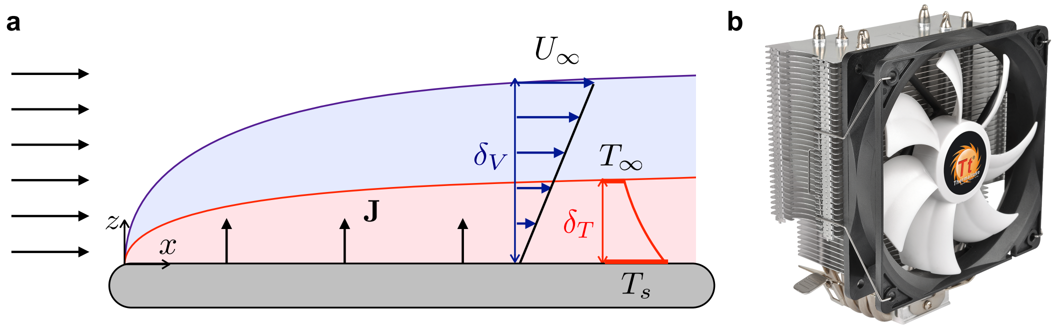


Figure 1: **a.** Parallel laminar flow over a hot plate. The velocity profile is characterized by a boundary layer of extension δ_V, while the temperature profile follows a different boundary layer of extension δ_T. **b.** Practical application to the design of a fan to cool down a CPU.

1 From drying clothes in the wind to sustainable cooling

Figure 2: From clothes drying in the wind to traditional clay pot coolers (images sources: cdn.abcotvs.com, solarcooking.fandom.com, wikipedia).

3.2 Practical exemples

3.2.1 Drying a sheet in the wind

You all have experienced that letting linen outside under a dry wind (even cold) is more efficient than inside a closed room (even warmer). We propose to estimate the time required

to dry a sheet assuming that:
- temperature and material constants remain constant in the system,
- radiative exchanges are neglected,
- the relative pressure of water at the surface of the sheet is equal to the saturating vapor pressure P_{sat}.

The drying sheet is submitted to a lateral wind of velocity U of ambiant temperature T_{amb} and relative humidity r_h ($P_{H_2O} = r_h P_{sat}$).

Estimate the mass flux from the sheet to the air. One interesting quantity is the volume flux which can be interpreted as a front velocity. Why is it better to dry your clothes whet the ambiant temperature is high?

Give a numerical estimate of this evaporation velocity for a sheet of length $L = 1\,\mathrm{m}$, $r_h = 50\%$, $T_{amb} = 25°\mathrm{C}$, $U = 5\,\mathrm{m/s}$, $D_{H_20/air} = 2.6 \cdot 10^{-5}\,\mathrm{m^2/s}$, $\nu_{air} = 1.6 \cdot 10^{-5}\,\mathrm{m^2/s}$, $P_{sat} = 2.3\,\mathrm{kPa}$.

How long does is take to dry a sheet which is initially soaked with $0.5\,\mathrm{kg/m^2}$ of water?

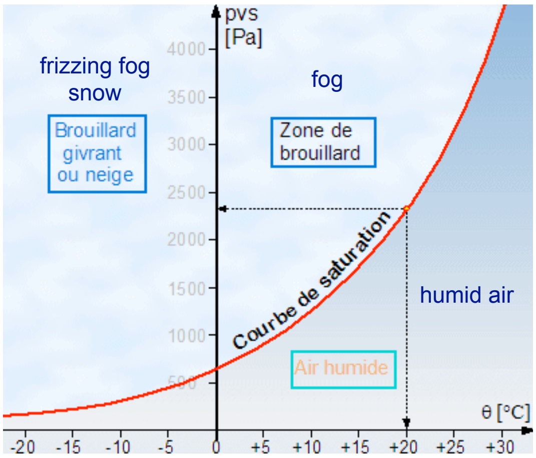


Figure 3: Evolution of water vapor pressure with the temperature at the atmospheric pressure (source: dimclim.fr).

3.2.2 Clay pot cooling

3000 years BC, far before the invention of modern refrigerators, people from middle east regions used clay pots to maintain water and food cool in spite of hot weather conditions. The technique is very simple: two pots are juxtaposed and the gap between the pots is filed with sand and water. The outer pot is porous and is therefore constantly wet. As evaporation occurs, latent heat is lost which cools down the pot (Fig. 3). The latent heat of evaporation of water is of $2265\,\mathrm{kJ/kg}$

What is the thermal power of such a sustainable refrigerator as a function of the ambiant humidity?

3.2.3 Thermal balance: Measuring relative humidity

Never heard about *psychrometry*? This is the science of measuring ambiant humidity. A robust frugal technique is still used today. It consists in measuring the difference in temperature between two thermometers, one with a dry bulb, the other one with a bulb wrapped into a wet fabric (Fig. 4). Both thermometers are submitted to the same wind (that can be created

by spinning the apparatus).
In the previous configuration of the drying sheet, what is the heat flux from the air to the sheet? What is the temperature difference between the sheet and the ambiant air for a given relative humidity?

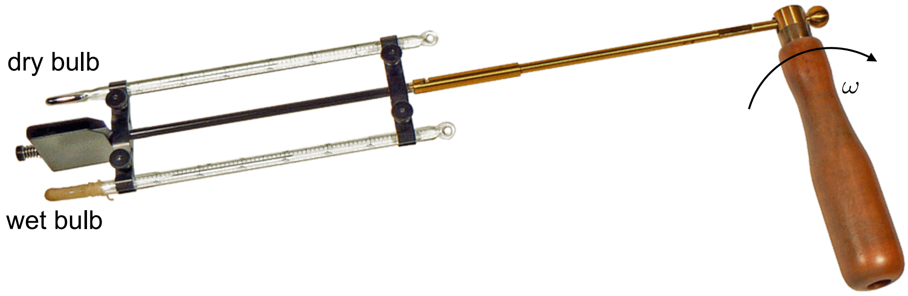


Figure 4: Psychrometer (circa 1850) used to measure the relative humidity of ambiant air (image source: lecompendium.fr).

3.3 Bubbles in a champagne bottle

source: "Kinetics of gas discharging in a glass of champagne: the role of nucleation sites", G. Liger-Belair, M. Vignes-Adler, C. Voisin, B. Robillard & P. Jeandet, *Langmuir*, 18, 1294 (2002)

Fermentation releases CO_2 that partially dissolves in the prepared beverage. When a champagne bottle (or a soda bottle) is opened this excess of CO_2 is released in the form of bubbles. The attached article discuss the nucleation bubbles, the rising dynamics and the inflation of the bubbles as they rise. Beyond being a nice illustration of transport phenomena, the involved mechanisms (especially nucleation) are big issues in a lot of situations less frivolous than champagne bubbles!

A scientific article is always hard to decrypt in details. First look at the figures and then try to get an idea of the message. We will address the different questions:

- *Bubble nucleation:* Is a bubble very likely to nucleate in the bulk of a solution? Show that in order to grow, a bubble has to start with a minimal radius. Hint: think about surface tension.

- *Growth rate of pinned bubble:* Describe the growth mechanism while the bubble remains fixed to the glass (should be easy).

- *Bubble velocity:* What law would you expect for the rising velocity as a function of the radius of the bubble?

- *Growth rate of a rising bubble:* Can we describe the mass exchange transfer with the same relation presented for a plate submitted to a flow? What is the law the authors propose? Much more advanced: try to recover this low by considering the different lengths scales involved in the flow and in the mass exchange.

- *Solving the coupled problem:* rising dynamics / growth rate.