Transport Phenomena:
III. Solving transport equations -2

2.5 Feeling the temperature of a surface

Why does a block of metal feels colder than a piece of wood when you touch it? A cool demonstration from EPICS association shows how an ice cubes counterintuitively melts faster on a block of aluminium than on a piece of wood or plastic (Fig. 1). Is it a matter of thermal conductivity? thermal diffusivity? heat specific capacity? a combination of these different quantities? To get an hint, let's estimate the interfacial temperature as two semi-infinite solid blocks with different initial temperatures are put in contact.

Figure 1: **a.** A pair of semi-infinite solids of different initial temperatures are brought in contact. What sets the temperature at the interface? **b.** Ice cube melting faster on a block of Aluminium than on a block of plastic (or wood). Image from Veritasium YouTube channel https://youtu.be/vqDbMEdLiCs . **c.** Simplified model. An ice cube of initial temperature $T = T_f = 0°$C sits on a semi-infinite solid of initial room temperature $T_1 = 20°$C. We assume that the water formed by melting the ice immediately flows away leaving the remaining ice cube in contact with the solid.

If we neglect radiative fluxes, what equation dictates the evolution of the temperature as a function of time along the z axis? Use the previous solutions to determine the time evolution of the temperature along the blocks. What is the temperature at the interface? Show the relevance of *thermal effusivity* $= (\kappa \rho C_p)^{1/2}$ in the problem. Estimate the contact temperature when one touches a piece of wood or of Aluminium both at 20°C.

Numerical application:
wood: $\kappa = 0.15\,\mathrm{W.m^{-1}.K^{-1}}$, $\rho = 600\,\mathrm{kg/m^3}$, $C_p = 1700\,\mathrm{J.kg^{-1}.K^{-1}}$
Aluminium: $\kappa = 237\,\mathrm{W.m^{-1}.K^{-1}}$, $\rho = 2700\,\mathrm{kg/m^3}$, $C_p = 900\,\mathrm{J.kg^{-1}.K^{-1}}$
skin ($\sim$ water): $\kappa = 0.6\,\mathrm{W.m^{-1}.K^{-1}}$, $\rho = 1000\,\mathrm{kg/m^3}$, $C_p = 4180\,\mathrm{J.kg^{-1}.K^{-1}}$

2.6 Melting of an ice cube

How long does it take to melt an ice cube on a block of solid? We consider a very simplified configuration where an ice cube of initial temperature $T = T_f = 0°$C is deposited on a block of semi-infinite solid initially at room temperature (Fig. 1c). We assume that the water produced by melting the ice flows away and leaves the remaining ice in contact with the solid. Following this assumption, what is the temperature between the melting ice and the solid?

How does the diffusion length δ_T evolves in time? (in scaling laws, but you can do the complete calculation if you wish)
What is the thermal flux at the interface?
Melting ice releases some latent heat L_f (in J/kg). What is the relation between the thermal flux and the variation of the thickness H of the remaining ice?
What is the scaling for the evolution of H(t)? Give an estimate for the time required to melt an ice cube of initial thickness $H_0 = 3\,\text{cm}$ on a block of Aluminium and on a piece of wood. Does this last estimate sounds realistic? What are the other mechanisms involved in the melting process? This will be for the next sessions!

Material properties for ice:
$\kappa = 2\,\text{W.m}^{-1}.\text{K}^{-1}$, $\rho = 920\,\text{kg/m}^3$, $C_p = 2000\,\text{J.kg}^{-1}.\text{K}^{-1}$, $L_f = 334\,\text{kJ.kg}^{-1}$

2.7 Estimating the age of the Earth?

In the 1830's Lamé and Capleyron derived the law of solidification of a sphere of initially molten material. Although they had in mind estimating the age of the Earth, their work reminded mainly theoretical. We propose to follow their steps.
As a main assumption, we consider the the melted part of the sphere of radius R is uniform and equals to its melting temperature T_f. The outer temperature of the crust of thickness h is at fixed temperature T_s. Using the additional assumption $h \ll R$ we simplify the spherical geometry into a 1D problem (Fig. **??**).
What is the heat flux through the crust? How do you expect the thickness of the crust to evolve in time? Try to estimate the age of the Earth considering a crust composed of silica with $\kappa_s \sim 2\,\text{Wm}^{-1}K^{-1}$, $\rho_s \sim 3 \cdot 10^3\,\text{kg/m}^3$, $L_s \sim 400\,\text{kJ/kg}$, $T_f \sim 1100°\text{C}$, $T_s \sim 15°\text{C}$, $h \sim 35\,\text{km}$. What could be wrong in our assumptions?

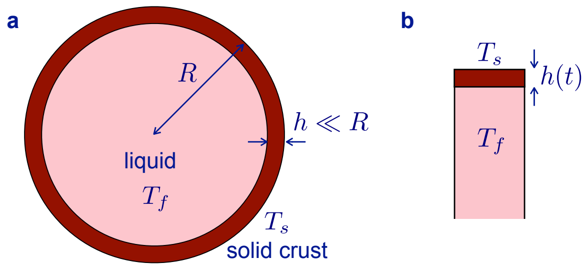

Figure 2: (a) Solidification front on a sphere as proposed by Lamé & Clapeyron. (b) 1D version under the assumption $h \ll R$.