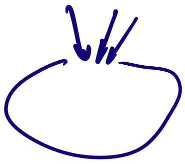
① 8. Radiative exchanges

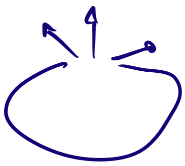
↳ heat transfer possible through electromagnetic waves

1. Basics : black body

Max Planck theory (1900)



absorbs all incident radiation

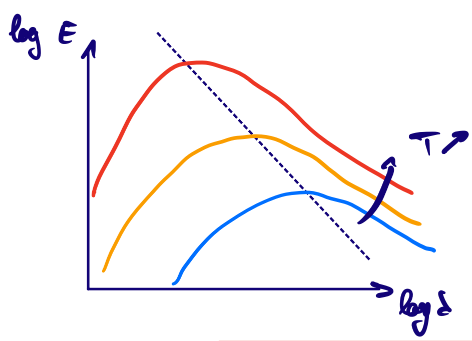


diffuse emitter → same intensity in all directions

↳ emission / area in the range $[\lambda, \lambda + d\lambda]$

$$\boxed{E(\lambda, T)\, d\lambda = \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp\left(hc/\lambda k_B T\right) - 1}\, d\lambda}$$

$h = 6.6 \cdot 10^{-34}$ J.s $c = 3.10^8$ m/s → $E\ [J.m^{-3}.s^{-1}]$



Wien's displacement law: $\boxed{\lambda_{max} = \frac{2898\ \mu m}{T[K]}}$

↳ $T = 300°K \rightarrow \lambda_{max} \simeq 10\,\mu m$

$T = 5800°K$ (sun) $\rightarrow \lambda_{max} = 0{,}5\,\mu m$

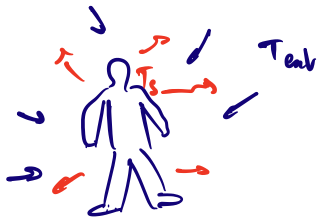
Global flux: $$E_{bb} = \int_0^{\infty} \frac{2\pi h c^2}{\lambda^5} \frac{1}{\exp\left(hc/\lambda k_B T\right) - 1}\, d\lambda$$

$$\boxed{E_{bb} = \frac{2\pi^5 k_B^4}{15 h^3 c^2} T^4 = \sigma T^4}$$

$\sigma = 5{,}67\ 10^{-8}\ W.m^{-2} K^{-4}$ Stefan-Boltzmann

→ ex: Black body at 300°K → 460 W/m²

② Heat loss by radiation



$$\boxed{J_R = \sigma \left(T_s^4 - T_{amb}^4\right)}$$

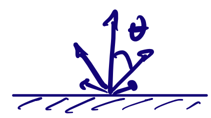
ex : $\begin{cases} T_{ext} = 20°C \rightarrow 293°K \\ T_s = 35°C \rightarrow 308°K \end{cases}$ $\Rightarrow J_R = 70\ W/m^2$

↳ but with clothes $T_s \sim 30°C \rightarrow 303°K$ $\Rightarrow J_R \simeq 37\ W/m^2$

↳ losses by radiation not negligible

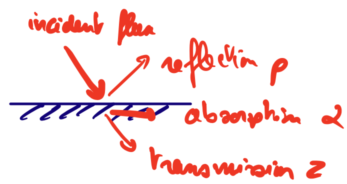
2. Real surfaces :

$E(\lambda, \theta)$



↳ emissivity : $\varepsilon(\lambda,\theta) = \dfrac{E(\lambda,\theta)}{E_{bb}}$

↳ reception :



↳ also depends on (λ,θ).

Kirchhoff's law : $\boxed{\varepsilon(\lambda,\theta) = \alpha(\lambda,\theta)}$

↳ but we can have $\varepsilon(\lambda_1) \neq \alpha(\lambda_2)$

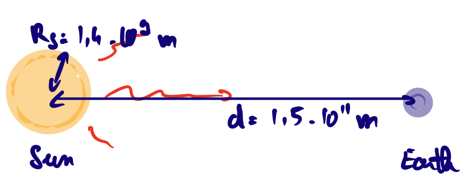
→ ex black paint / white paint

↳ $\lambda = 0{,}5\,\mu m$ (sun) $\rightarrow \alpha_{white} = 0{,}2 \ll \alpha_{black} = 0{,}9$

$\lambda = 10\,\mu m$ (300°K) $\rightarrow \varepsilon_{white} \simeq \varepsilon_{black} \simeq 0{,}9$

3. Radiative balance for the Earth

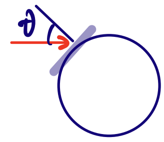
Radiation from the Sun ≃ black body



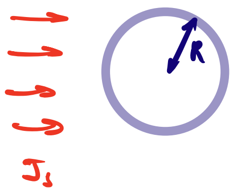
$T_S \sim 5800°K \rightarrow J_e = 6{,}4 \cdot 10^7\ W/m^2$ emitted

③

$$J_s = J_e \frac{R_s^2}{d^2} \simeq 1400 \ W/m^2$$ → maximal absorption



↳ depends on orientation $J_s = J_{max} \cos\vartheta$

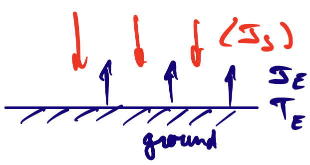


J_s on a section πR^2
but distributed over a surface $4\pi R^2$

$$\Rightarrow \langle J_s \rangle = J_s \frac{\pi R^2}{4\pi R^2} \simeq 350 \ W/m^2$$

Note : in practice ~30% reflected by the atmosphere back to space
~20% absorbed by the atmosphere
~50% reaches the surface

Without atmosphere : assuming $\alpha = \varepsilon = 1$



$$\langle J_S \rangle = J_E = \sigma T_E^4 \Rightarrow T_E = \left(\frac{350}{5{,}7 \cdot 10^{-8}}\right)^{1/4} = 278\,°K = 5\,°C$$

more generally, $T_E = \left(\frac{R_S}{2d}\right)^{1/2} T_S$

If the Earth was painted in white (or covered with snow)

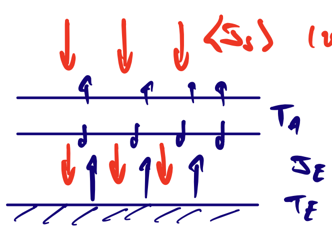
$$\alpha\,(\text{Sun}) = 0{,}2 \qquad \varepsilon_{IR} \simeq 0{,}9$$

$$J_E = \alpha(\text{Sun}) \langle J_S \rangle = \varepsilon_{IR} \sigma T_E^4 \Rightarrow T_E = \left(\frac{0{,}2 \times 350}{0{,}9 \times 5{,}7 \cdot 10^{-8}}\right)^{1/4} = 192\,°K = -81\,°C$$

⇒ large influence of albedo effect

Greenhouse effect:

With atmosphere → simplified version : (transparent to visible light → $\alpha, \varepsilon = 0$
(opaque to IR → $\alpha, \varepsilon = 1$
↳ H_2O, CO_2, CH_4 ...



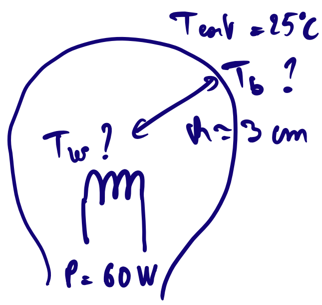
Flux balances :

ground : $\langle J_S \rangle + \sigma T_A^4 = \sigma T_E^4$

atmosphere : $\sigma T_E^4 = 2 \sigma T_A^4$

$$\Rightarrow \frac{1}{2} \sigma T_E^4 = \langle J_S \rangle \Rightarrow T_E = \underbrace{2^{1/4}}_{1{,}19} T_E^0 = 330\,°K = 57\,°C$$

(i) 4. Incandescent ligth bulb



→ Hyp : glass bulb $\left\{ \begin{array}{l} \text{absorbs IR} \\ \text{transparent to visible light} \end{array} \right.$

$L_w = 20$ cm $r_w \simeq 25\ \mu m$

Quick estimate → Hyp: all electrical power radiated

$$\Rightarrow P \simeq 2\pi r_w L_w \sigma T_w^4$$

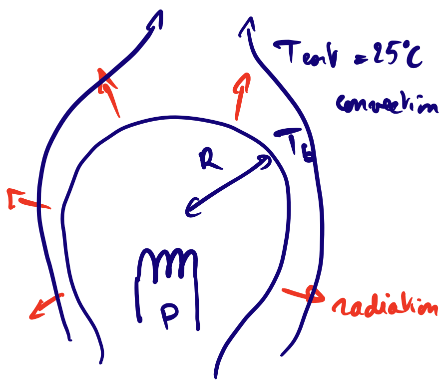
$$\Rightarrow T_w \simeq \left(\frac{60}{2\pi \times 0{,}2 \times 25 \cdot 10^{-6} \times 5{,}7 \cdot 10^{-8}} \right)^{1/4} = 2400\ °K \simeq 2130\ °C$$

$$\Rightarrow \lambda_{max} = \frac{2898}{2400} \simeq 1{,}2\ \mu m \Rightarrow IR$$

$\boxed{\begin{array}{l} \text{incadescent lamp} \Rightarrow \text{more heat than "light"} \\ \text{advantage LED} \rightarrow \text{most power into visible light} \end{array}}$

→ accounting for glass bulb : → absorbs IR

⇒ most of P absorbed and re-diffused



$$\Rightarrow P = 4\pi R^2 \Big[\underbrace{\sigma (T_b^4 - T_{ext}^4)}_{\text{radiated}} + \underbrace{J_c}_{\text{convected}} \Big]$$

$$J_c = \frac{K}{R} (T_b - T_{ext})\ Nu$$

Free convection $Nu \sim Ra^{1/4} = \left(\frac{\beta g (T_b - T_{ext}) R^3}{\nu \alpha} \right)^{1/4}$

$$\Rightarrow J_c \sim K \left(\frac{\beta g}{\nu \alpha R} \right)^{1/4} (T_b - T_{ext})^{5/4}$$

$$\frac{P}{4\pi R^2} = \sigma (T_b^4 - T_{ext}^4) + K \left(\frac{\beta g}{\nu \alpha R} \right)^{1/4} (T_b - T_{ext})^{5/4}$$

$$\frac{60}{4\pi (0{,}03)^2} = 5{,}7 \cdot 10^{-8} (T_b^4 - 302^4) + 0{,}026 \left(\frac{3{,}4 \cdot 10^{-3} \times 9{,}8}{1{,}5 \cdot 10^{-5} \times 2 \cdot 10^{-5} \times 0{,}03} \right)^{1/4} (T_b - 302)^{5/4}$$

$$5300 = 5{,}7 \cdot 10^{-8} (T_b^4 - 302^4) + 6{,}4 (T_b - 302)^{5/4}$$

$$\Rightarrow T_b \simeq 452\ °K \simeq 180°C$$

→ radiation : 36% convection : 64%

→ better estimate of T_w : $P = 2\pi r_w L_w \sigma (T_w^4 - T_b^4)$

$$\Rightarrow T_w = (2400^4 + 452^4)^{1/4} \simeq 2400\ °K$$