Transport Phenomena:
III. Simple problems, follow-up

1 Classical heat transport problems

Since heat transport follows the same formalism as mass (except for radiation). Solutions from mass transport can readily be adapted to thermal problems.

1.1 Feeling the temperature of a surface

Why at room temperature does a block of metal feels colder than a piece of wood when you touch it? A cool demonstration from EPICS association shows how an ice cubes conversely melts faster on a block of aluminium than on a piece of wood or plastic (Fig. 1). Is it a matter of thermal conductivity? thermal diffusivity? heat specific capacity? a combination of these different quantities? To get an hint, let's estimate the interfacial temperature as two semi-infinite solid blocks with different initial temperatures are put in contact.

Figure 1: **a.** A pair of semi-infinite solids of different initial temperatures are brought in contact. What sets the temperature at the interface? **b.** Ice cube melting faster on a block of Aluminium than on a block of plastic (or wood). Image from Veritasium YouTube channel https://youtu.be/vqDbMEdLiCs . **c.** Simplified model. An ice cube of initial temperature $T = T_f = 0°$C sits on a semi-infinite solid of initial room temperature $T_1 = 20°$C. We assume that the water formed by melting the ice immediately flows away leaving the remaining ice cube in contact with the solid.

If we neglect radiative fluxes, what equation dictates the evolution of the temperature as a function of time along the z axis? Use the previous solutions to determine the time evolution of the temperature along the blocks. What is the temperature at the interface? Show the relevance of *thermal effusivity* $= (\kappa \rho C_p)^{1/2}$ in the problem. Estimate the contact temperature when one touches a piece of wood or of Aluminium both at 20°C.

Numerical application:
wood: $\kappa = 0.15\,\mathrm{W.m^{-1}.K^{-1}}$, $\rho = 600\,\mathrm{kg/m^3}$, $C_p = 1700\,\mathrm{J.kg^{-1}.K^{-1}}$
Aluminium: $\kappa = 237\,\mathrm{W.m^{-1}.K^{-1}}$, $\rho = 2700\,\mathrm{kg/m^3}$, $C_p = 900\,\mathrm{J.kg^{-1}.K^{-1}}$
skin ($\sim$ water): $\kappa = 0.6\,\mathrm{W.m^{-1}.K^{-1}}$, $\rho = 1000\,\mathrm{kg/m^3}$, $C_p = 4180\,\mathrm{J.kg^{-1}.K^{-1}}$

1.2 Melting of an ice cube

How long does it take to melt an ice cube on a block of solid? We consider a very simplified configuration where an ice cube of initial temperature $T = T_f = 0°C$ is deposited on a block of semi-infinite solid initially at room temperature (Fig. 1c). We assume that the water produced by melting the ice flows away and leaves the remaining ice in contact with the solid. Following this assumption, what is the temperature between the melting ice and the solid?
How does the diffusion length δ_T evolves in time? (in scaling laws, but you can do the complete calculation if you wish)
What is the thermal flux at the interface?
Melting ice releases some latent heat L_f (in J/kg). What is the relation between the thermal flux and the variation of the thickness H of the remaining ice?
What is the scaling for the evolution of H(t)? Give an estimate for the time required to melt an ice cube of initial thickness $H_0 = 3\,\text{cm}$ on a block of Aluminium and on a piece of wood. Does this last estimate sounds realistic? What are the other mechanisms involved in the melting process? This will be for the next sessions!

1.3 Depth of penetration

Soil undergoes cyclic temperature variations during the day or even during the year. As a first approximation, we assume that the temperature of the ground follows a simple law of the form $T(z = 0, t) = T_0 + T_1 \cos(\omega t)$ (Fig. 2). How does the temperature evolves as a function of the depth? We can first do the estimate in scaling laws and then derive the exact solution. Give a numerical estimate of the depth of penetration for a daily or annual cycle for a soil of thermal diffusivity $\alpha \simeq 5.10^{-7}\text{m}^2/\text{s}$.
Caves or basements are reputed to have steady temperatures independently of the external conditions. Does it make sense?

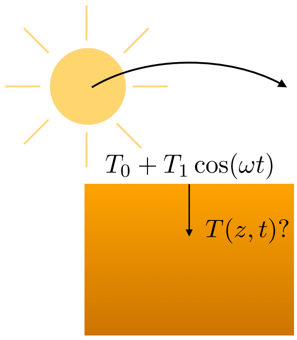

Figure 2: Soil undergoing cyclic variations of the temperature at the surface. What is the temperature profile as a function of the depth?

2 Adding convection

2.1 Condensers

You have probably used a "reflux" while doing to synthesis in organic chemistry. This device is typically composed of an inner tube where the fluid you want to cool down flows in a tube surrounded by a larger tube through which the cooling liquid flows (Fig. 3). This principle is largely used in heat exchangers. You may wonder how you should plug the source of cooling liquid. Is a coflow favorable? A counter flow? Does the answer depend on the configuration?

To get some insight, we will consider a very simplified 2D model for the condenser where two flows of the same fluid are separated by a wall of thickness h and thermal conductivity κ. Both flows have the same absolute velocity U. We will assume that the lateral diffusion is very fast so that the temperature in each side of the condenser only depend on the position x along the condenser (of total length L). Conversely, we will neglect diffusion in the longitudinal direction. Under which condition is this last assumption valid?
We then propose to calculate the evolution of the temperatures in both sides, $T_1(x)$ and $T_2(x)$, respectively.
- *Coflow configuration*: at $x = 0$ we start with $T_1(x = 0) = T_0 + \Delta T_0$ and $T_2(x = 0) = T_0$. Derive a heat transport balance between two sections of the condenser separated by a distance dx. How do temperatures evolve in each side of the condenser if L is large enough?
- *Counter-flow configuration*: at $x = 0$ we start with $T_1(x = 0) = T_0 + \Delta T_0$ and $T_2(x = L) = T_0$. What is now the temperature profiles in each side?
What is the best connnecting configuration?

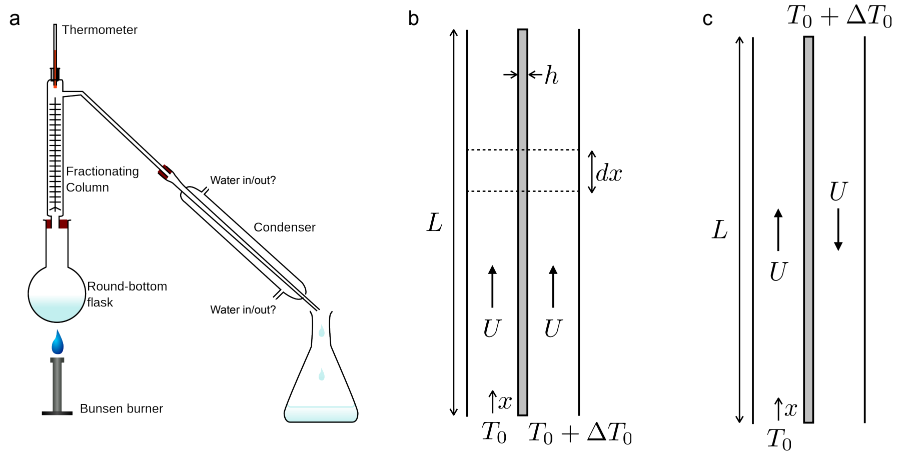

Figure 3: **a.** Condenser used in a distillation setup. At with tip should the water inlet be connected? **b.** Coflow configuration. **c.** Counter flow configuration.