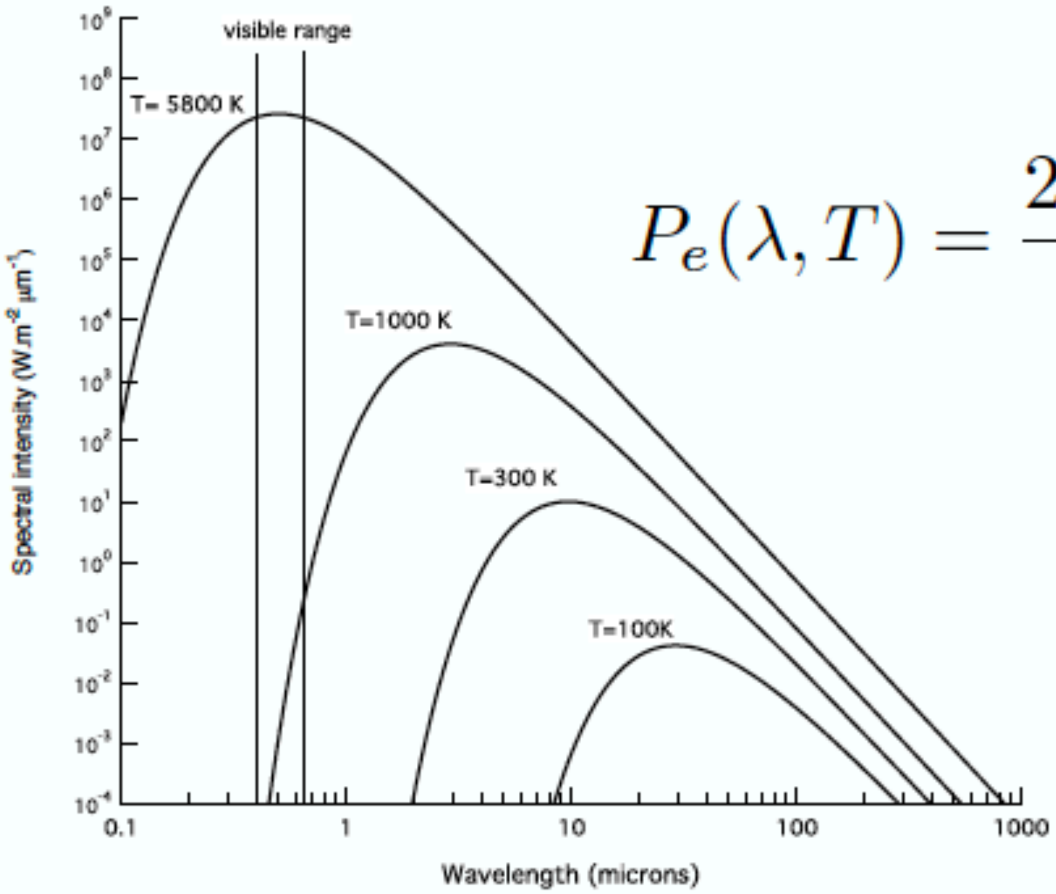
Radiative heat transfer

Black body radiation



$$P_e(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}$$

$$\lambda_M(\mathrm{m}) = \frac{2.88 \times 10^{-3}}{T}$$ Wien's displacement law

$$P_{BB}(T) = \int_0^\infty P_e(\lambda, T) d\lambda = \frac{2\pi^5 k_B^4}{15\ h^3 c^2} T^4 = \sigma T^4$$

Stefan's constant $\sigma = 5.67\ 10^{-8}\ \mathrm{Wm^{-2}K^{-4}}$

Non black bodies

$P(\lambda,T) = \varepsilon(\lambda)\ P_e(\lambda,T)$ emissivity $\varepsilon < 1$

emissivity ε = absorptivity a — Kirchoff's law

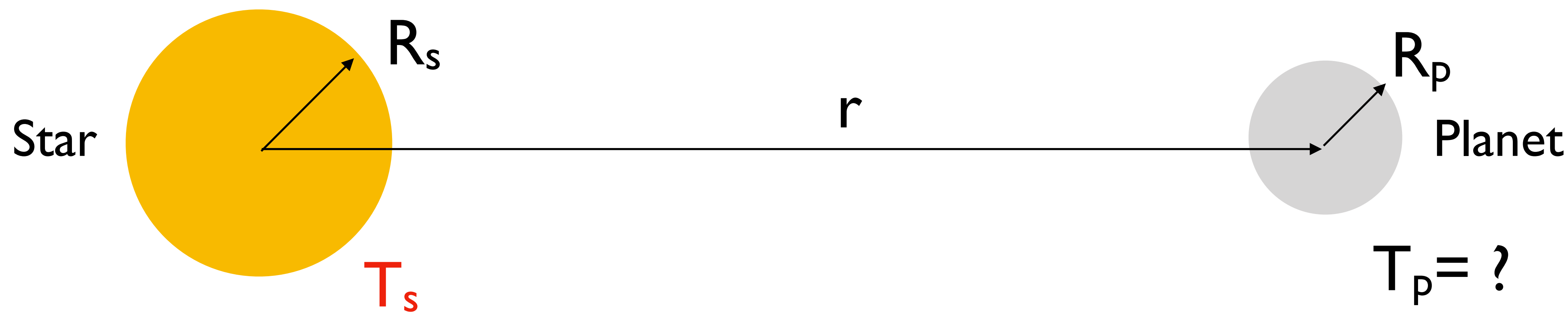
Grey bodies

emissivity ε = C

Radiative equilibrium of planets or The Donald Trump problem at zeroth order :

show that with a completely transparent atmosphere, it would be difficult to grow lawn for a golf course on Earth.

Side result (not for Fox News) : show that, in general, the spectra of incoming and outgoing radiations are well separated.



Radius of the Sun: 7 10^5 km
Surface temperature of the Sun : 5800 K
Distance to the Sun: 1.5 10^8 km
Radius of the Earth: 6350 km
Effective albedo of the Earth: 0.31

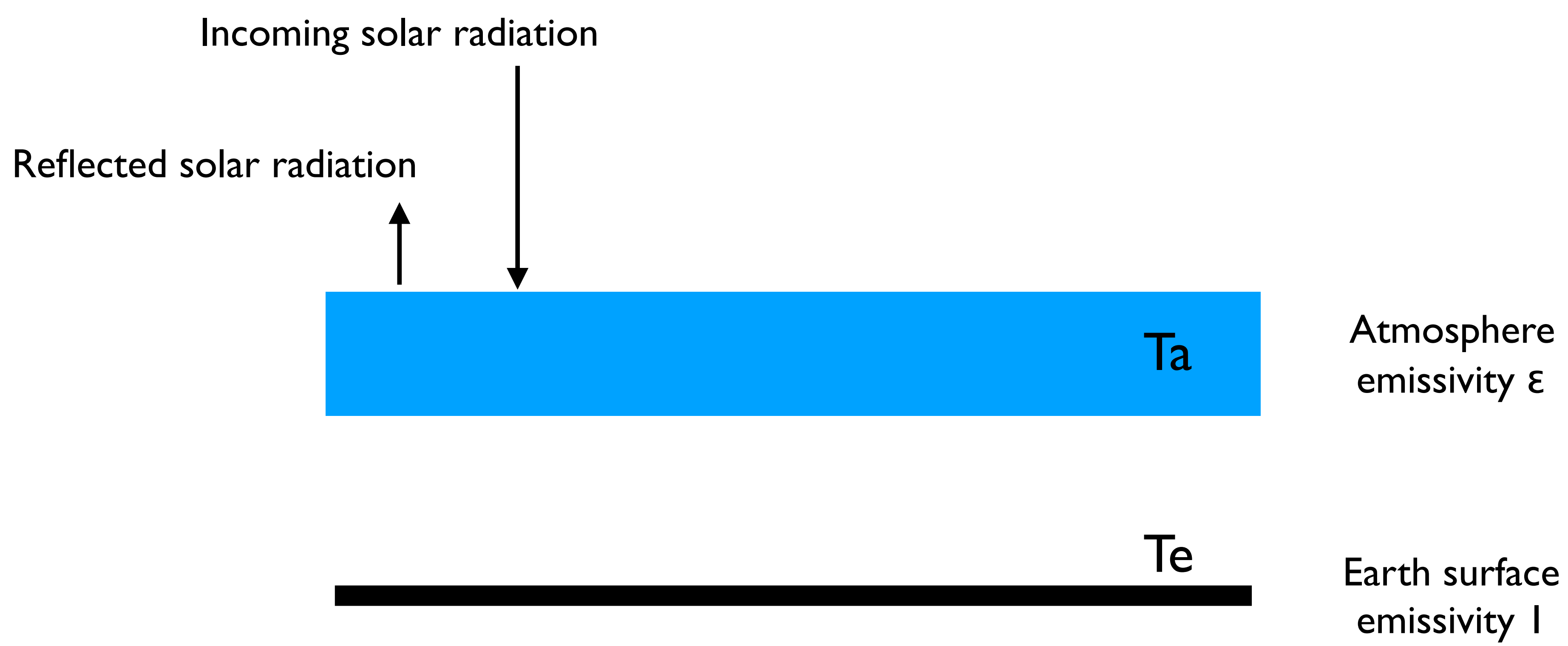
Distance Sun-Venus : 1.1 10^8 km
Effective albedo of Venus : 0.75
Distance Sun-Mars : 2.3 10^8 km
Effective albedo of Mars : 0.25

Infrared radiation and planetary temperature, Ray Pierrehumbert, Phys. Today, jan. 2011

	Venus	Earth	Mars
Equilibrium temperature without atmosphere	232 K	255 K	210 K
Equivalent radiation wavelength	12,5 µm	11,3 µm	13,7 µm
Actual temperature	740 K	285 K	210 K

The Donald Trump problem at order one :

show that with a one layer atmosphere adsorbing in the infrared, it would be easier to grow lawn for a golf course on Earth.



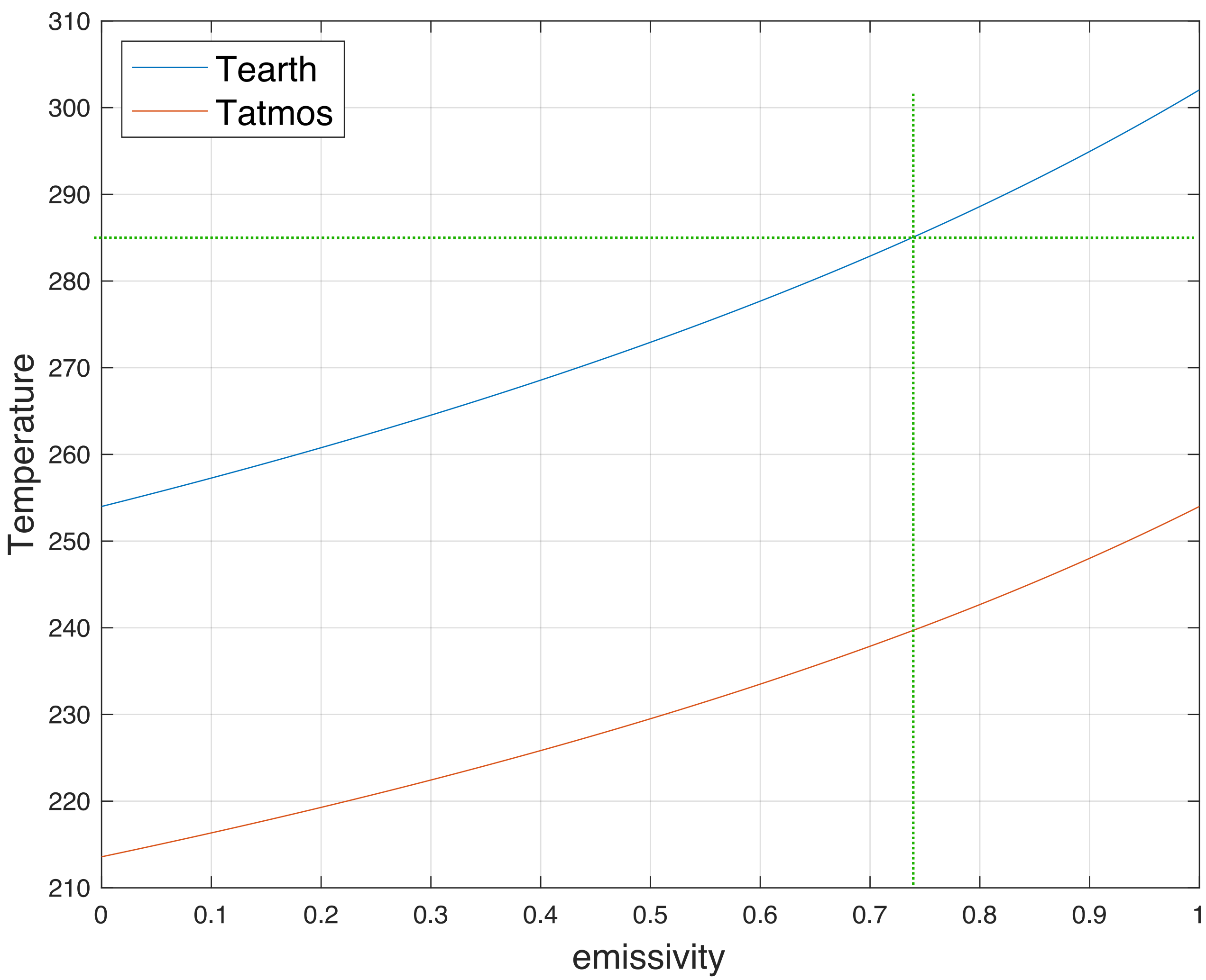
Write the energy flux balance for the earth + atmosphere system

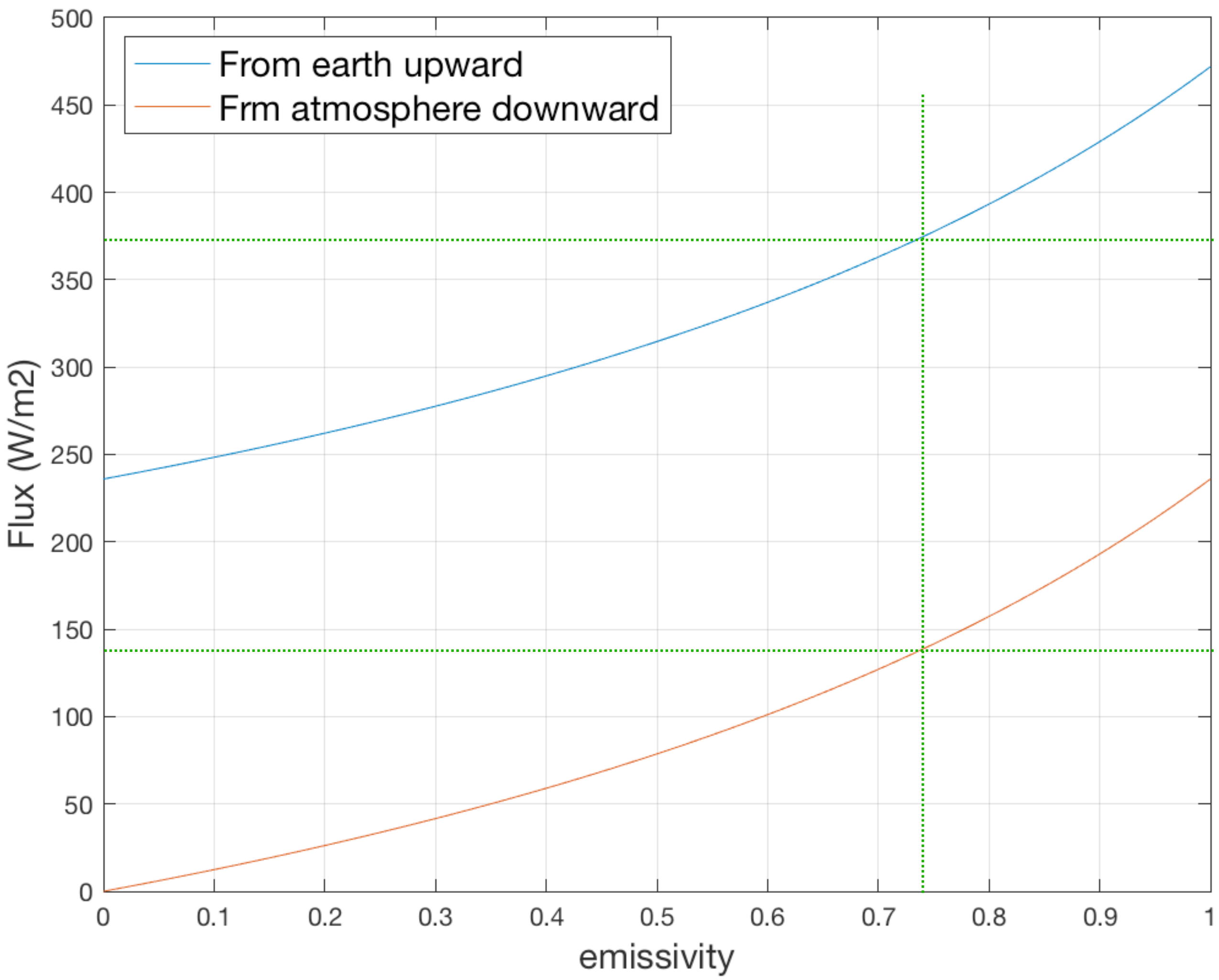
Write the energy flux balance for the atmosphere at temperature Ta

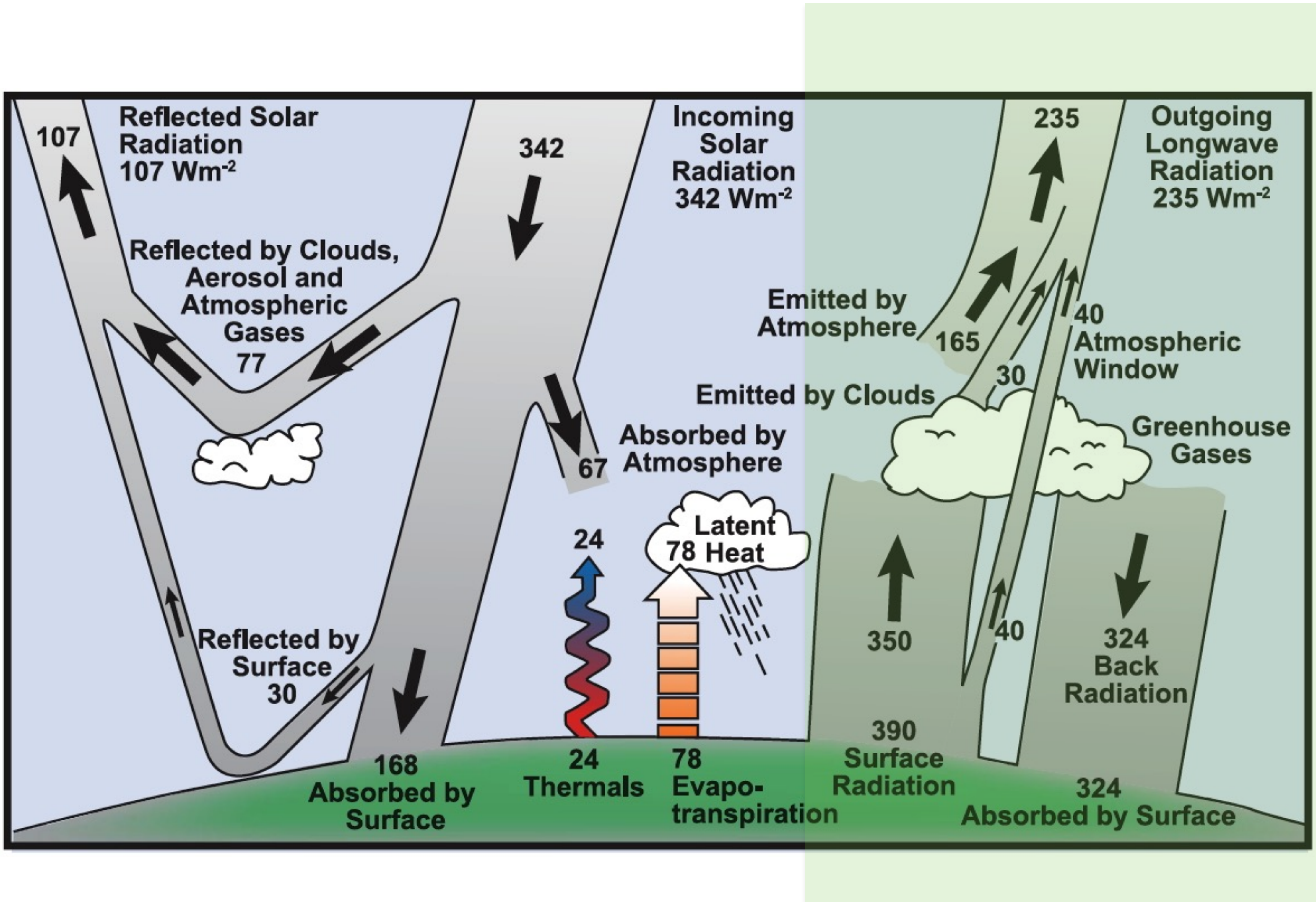
Determine Ta and Te as a function of ε

$$T_e = \left[\frac{(1-a)J_s}{\sigma(1-\epsilon/2)}\right]^{1/4}$$

$$T_a = \frac{T_e}{\sqrt(2)}$$







107
Reflected Solar Radiation 107 Wm-2
342
Incoming Solar Radiation 342 Wm-2
235
Outgoing Longwave Radiation 235 Wm-2
Reflected by Clouds, Aerosol and Atmospheric Gases
77
Emitted by Atmosphere
165
30
40
Atmospheric Window
Emitted by Clouds
Absorbed by Atmosphere
67
Greenhouse Gases
24
Latent 78 Heat
Reflected by Surface 30
350
40
324 Back Radiation
168 Absorbed by Surface
24 Thermals
78 Evapo-transpiration
390 Surface Radiation
324 Absorbed by Surface

The Donald Trump problem at order 2 :

Evaluate the temperature distribution in a stable « grey » non scattering atmosphere

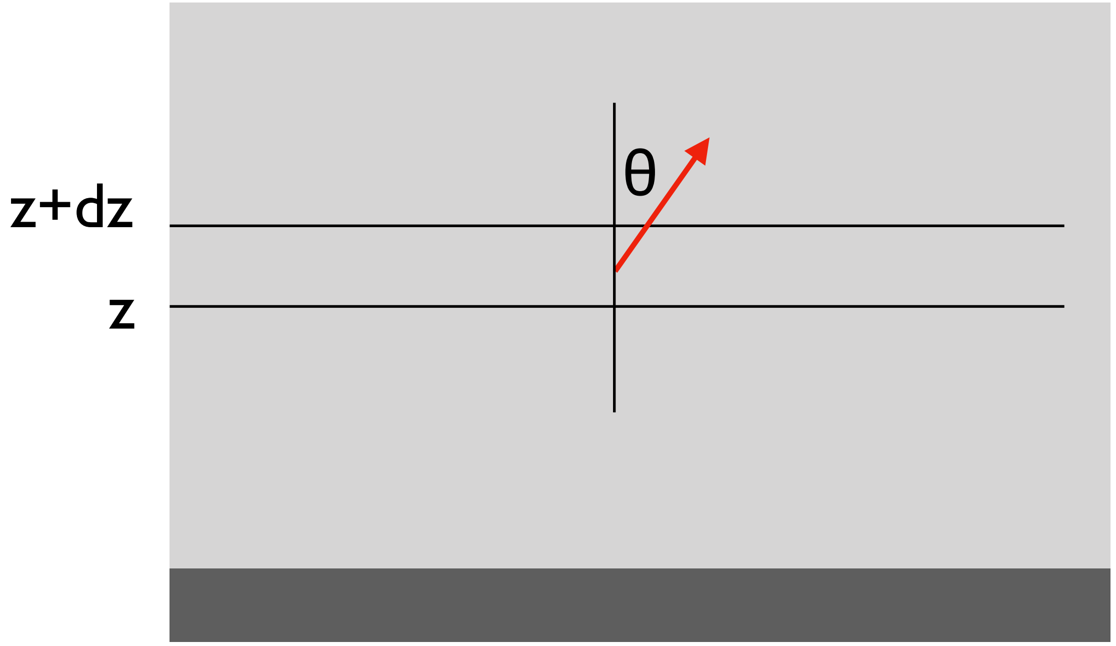
Atmosphere clear to solar radiation ; all absorption takes place at the ground.

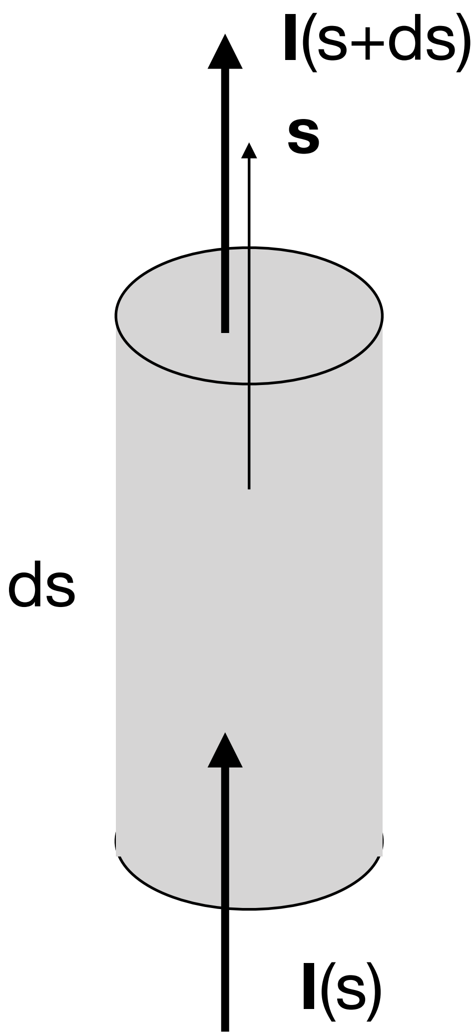
Gray atmosphere with no clouds in the IR (no scattering of light)

Layers of atmosphere are grey thermal radiators

Plane parallel or stratified atmosphere

Eddington approximation for the IR emission : $I(z,\theta) = I_0(z) + I_1(z) \cos \theta$





Extinction (absorption, scattering)

$$dI_{ext} = -\kappa_{ext}\ I\ ds$$

Augmentation (emission, scattering)

$$dI_{aug} = \kappa_{aug}\ J\ ds$$

If no scattering, emission by thermal radiation

$$dI_{aug} = \kappa_{ext}\frac{\sigma T^4}{\pi} = \kappa_{ext}B(T)$$

$$\frac{dI}{ds} = \kappa_{ext}(-I + B)$$

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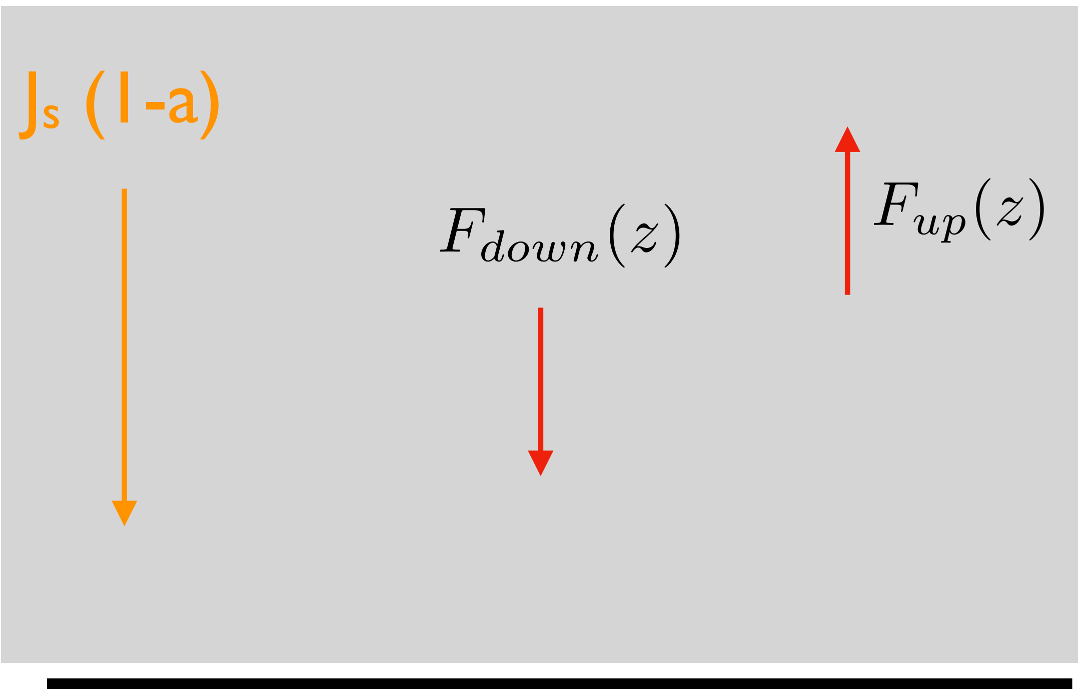
Optical path length (dimensionless) τ

$$d\tau = -\kappa_{ext}\ ds$$

$$\frac{dI}{d\tau} = I - B$$



Atmosphere top τ = 0

$$F_{down}(0) = 0$$

Ground τ = τ*

Radiation going up :

$$F_{up} = F_{up}(\tau^*) = \pi B(T_g)$$

Radiation going down :

$$J_s(1-a) + F_{down}(\tau^*)$$

$$\tau = \int_z^\infty \kappa_{ext}\ dz$$

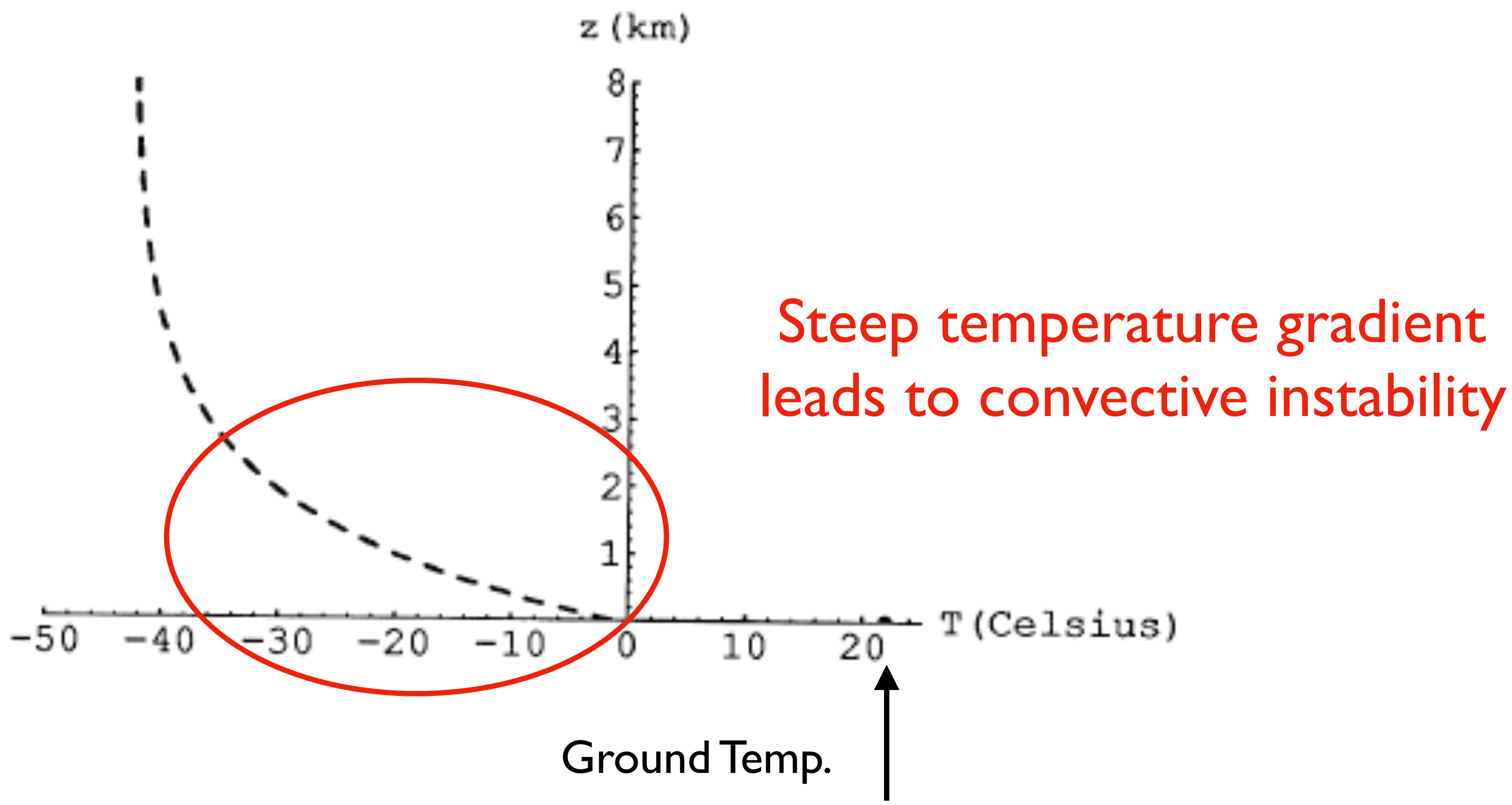
Optical path from atmosphere top

$$\tau^* = \int_0^\infty \kappa_{ext}\ dz$$

Optical depth of atmosphere

$$\kappa_{ext} = \kappa_0 \exp(-z/H_w)$$

H_w = 1.6 km (water vapor)



$$\sigma T_g^4 = J_s(1-a)\left(1 + \frac{3\tau^*}{4}\right)$$

$$\tau^* = \int_0^\infty \kappa_{ext}\, dz$$

Optical depth of atmosphere