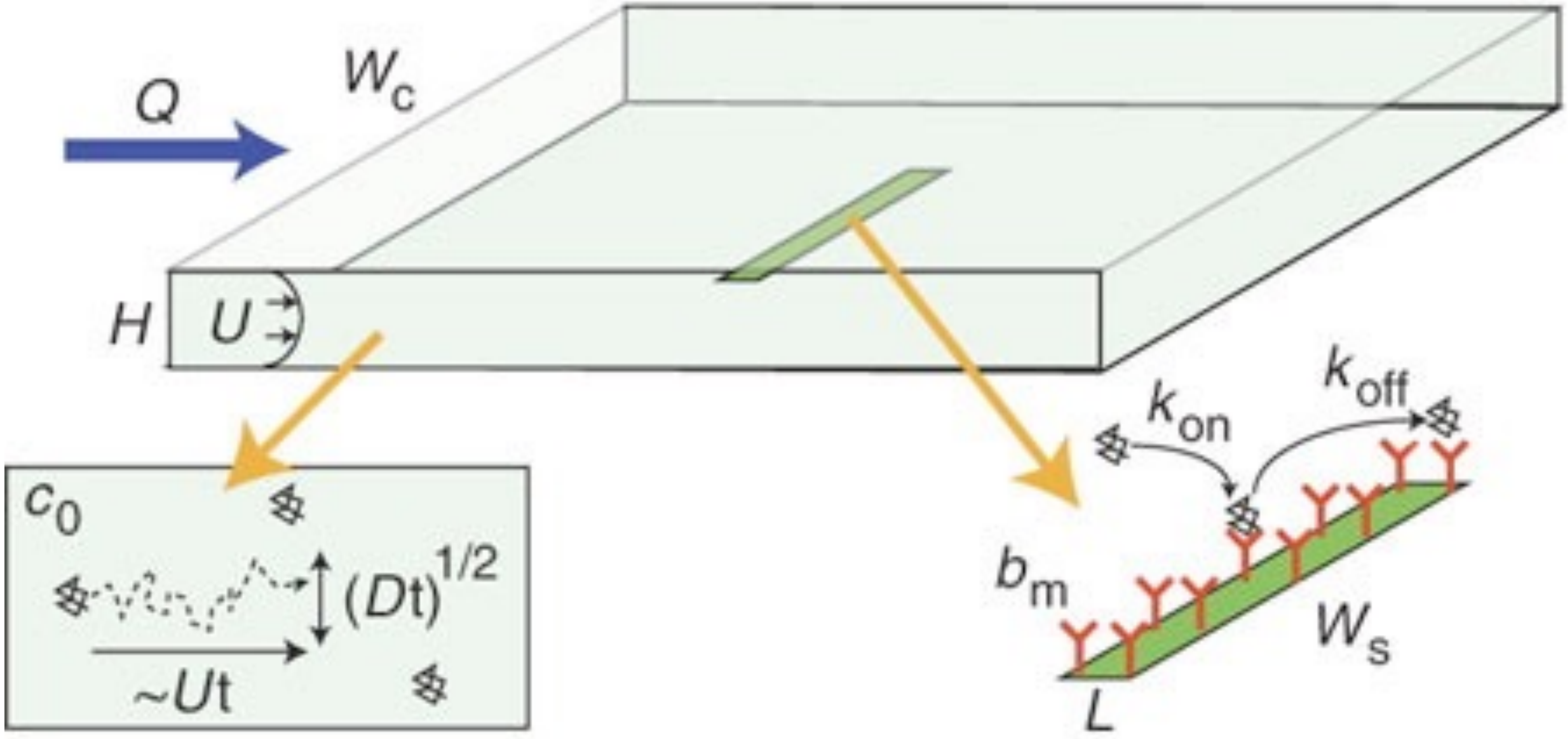
Transport phenomena

Engineering at small scale
Engineering at large scale
Transport in the environment
Transport in living systems

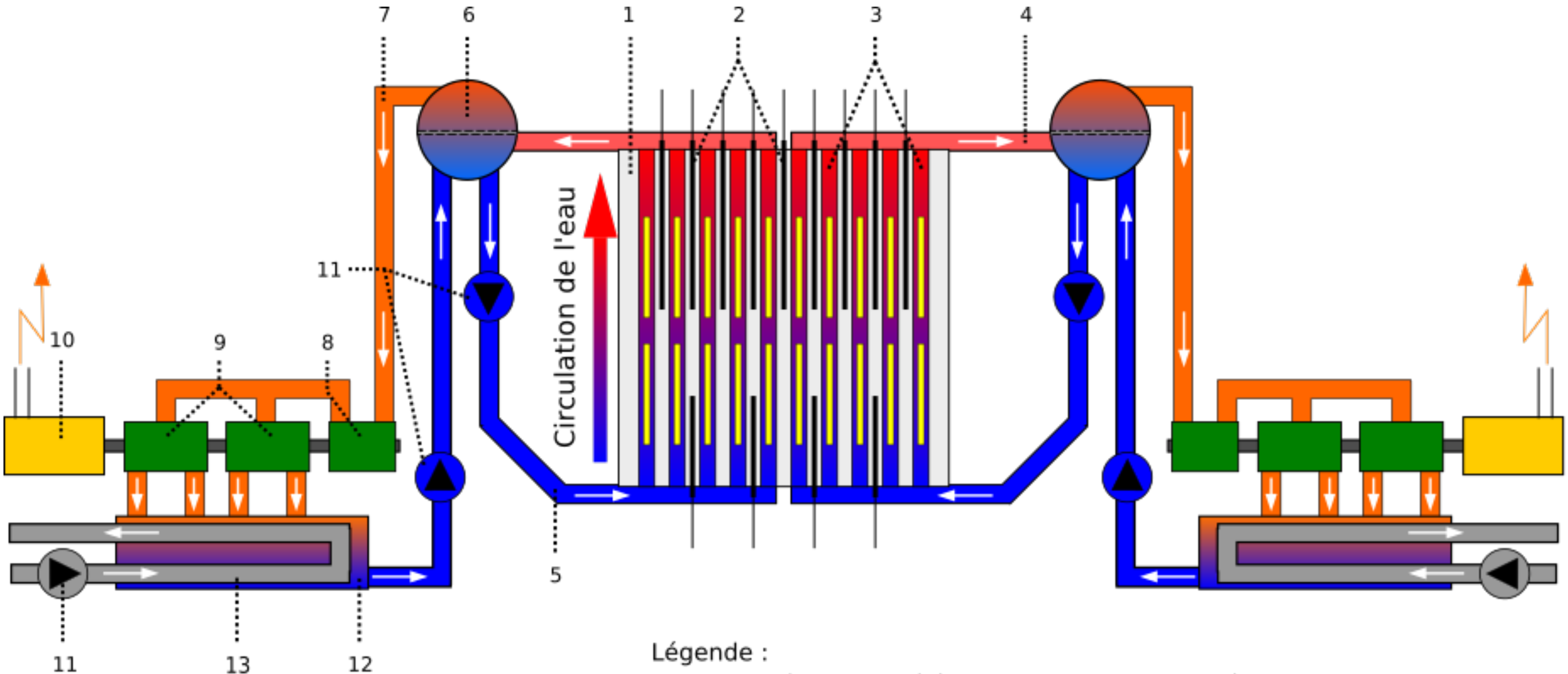
Engineering at small scale : the microchip problem



How do you design precisely a biochemical sensor on a microchip ?

T. Squires et al., Nature Biotech. 26, 417 (2008)

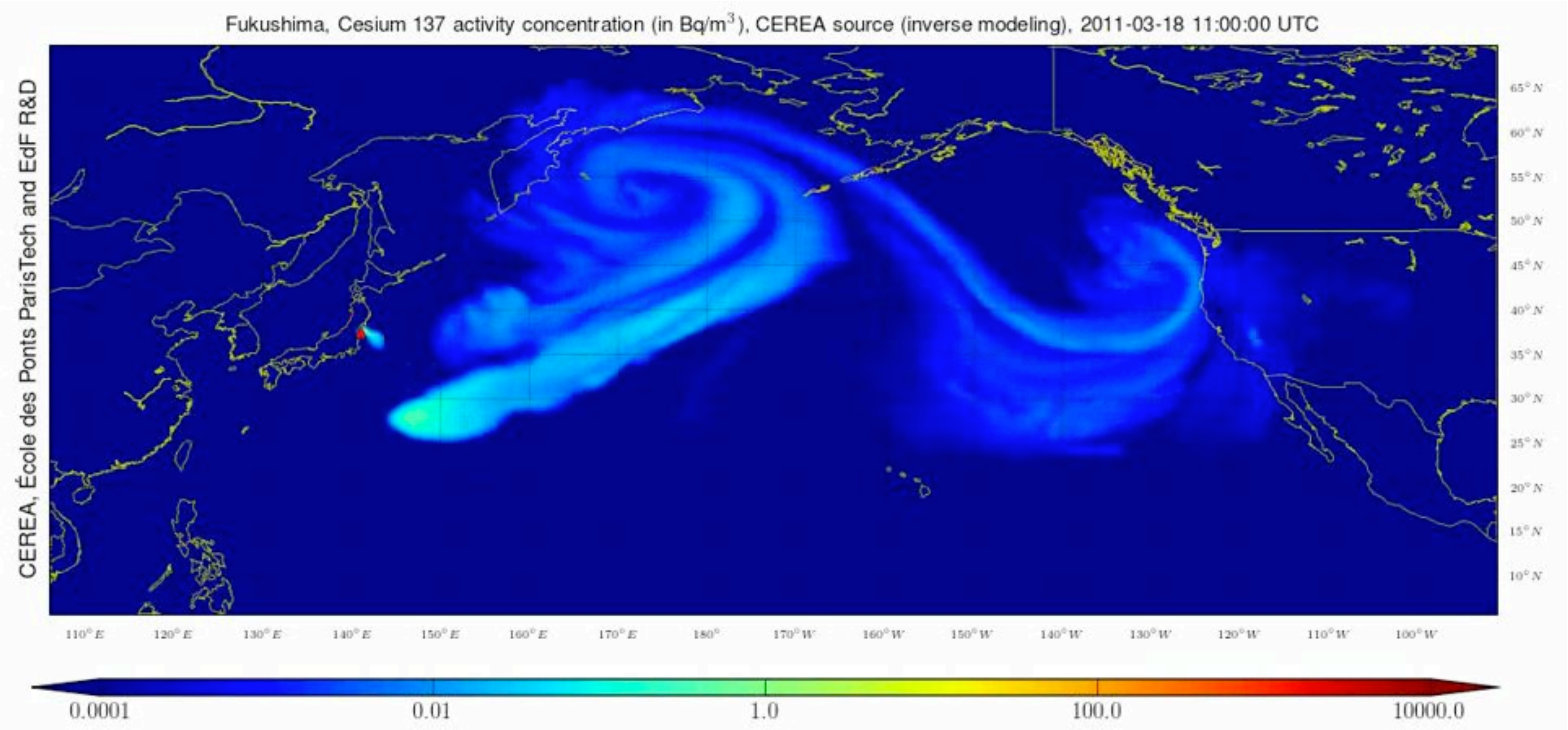
Engineering at large scale : a nuclear reactor



Légende :

1. Cœur du réacteur modéré au graphite
2. Barres de contrôle
3. Tubes de force contenant le combustible
4. Mélange eau/vapeur
5. Eau (légère)
6. Séparateur de vapeur
7. Vapeur entrante
8. Turbine à vapeur haute pression
9. Turbine à vapeur basse pression
10. Génératrice électrique
11. Pompes
12. Condensateurs
13. Eau de refroidissement (fleuve, mer, ...)

Mass transport in the environment

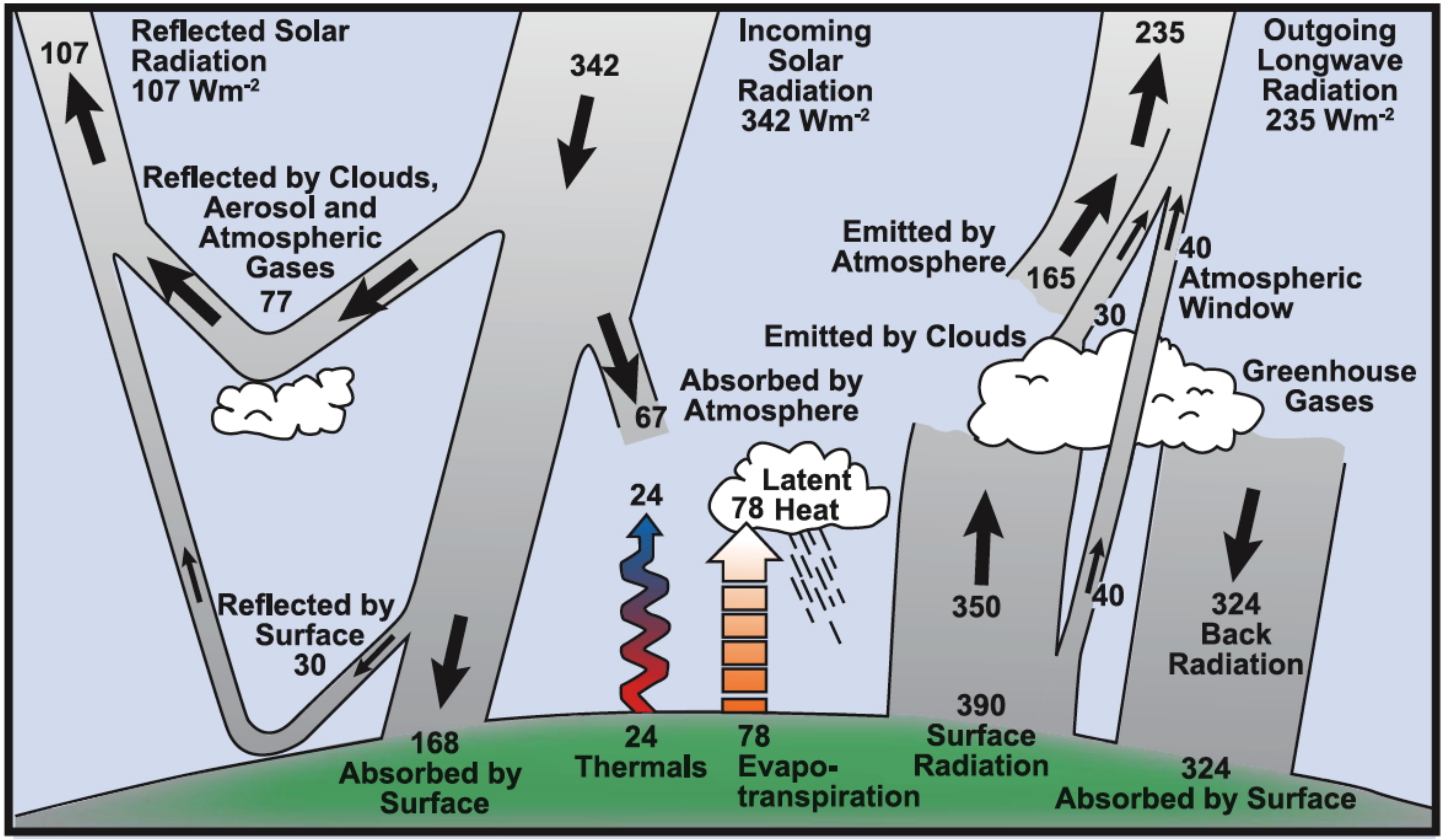


Simulation of Cs^{137} transport in the Pacific Ocean

cerea.enpc.fr/fukushima

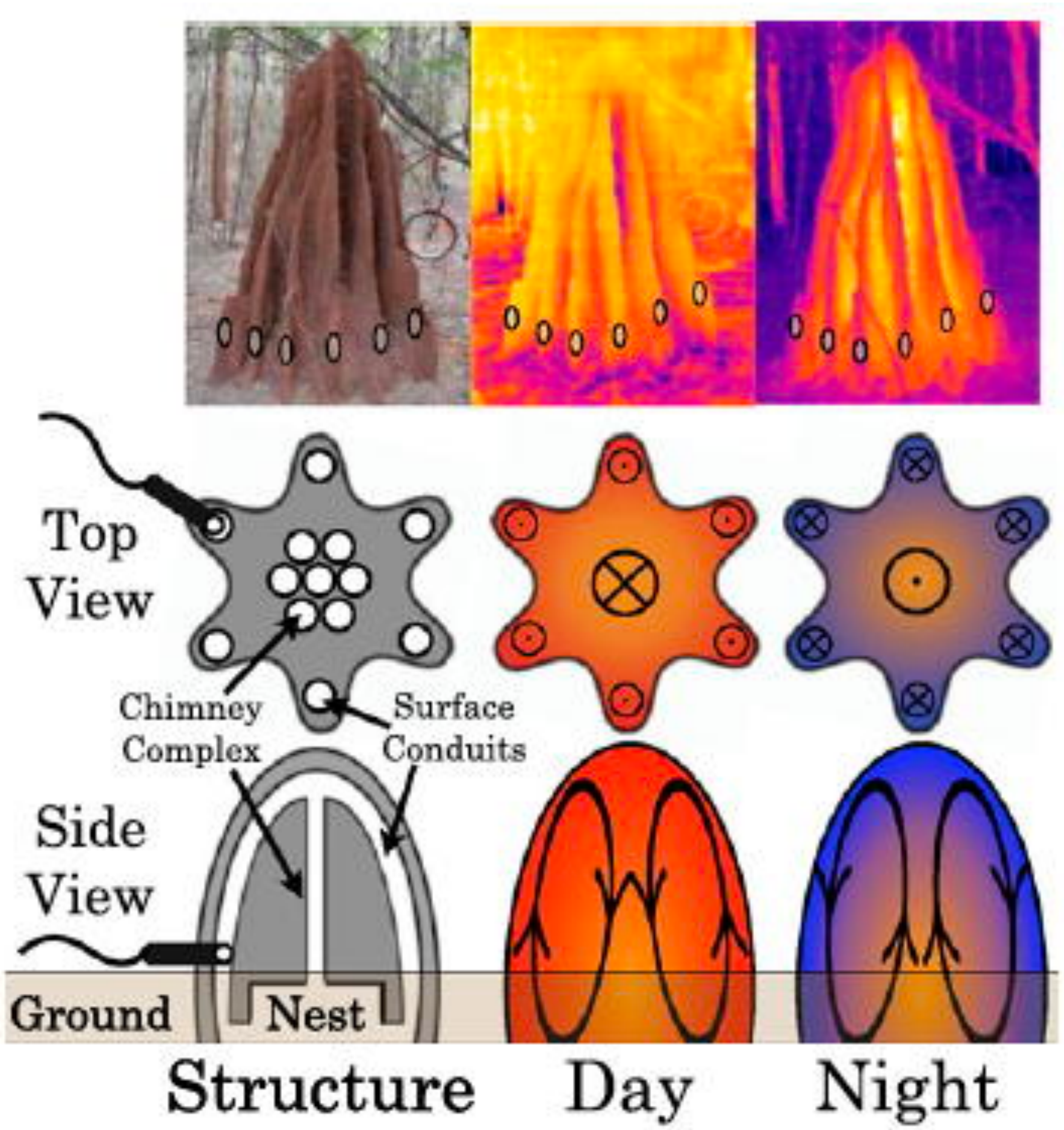
How does turbulence affects heat and mass transport ?

Heat transport in the environment : radiative equilibrium of the Earth



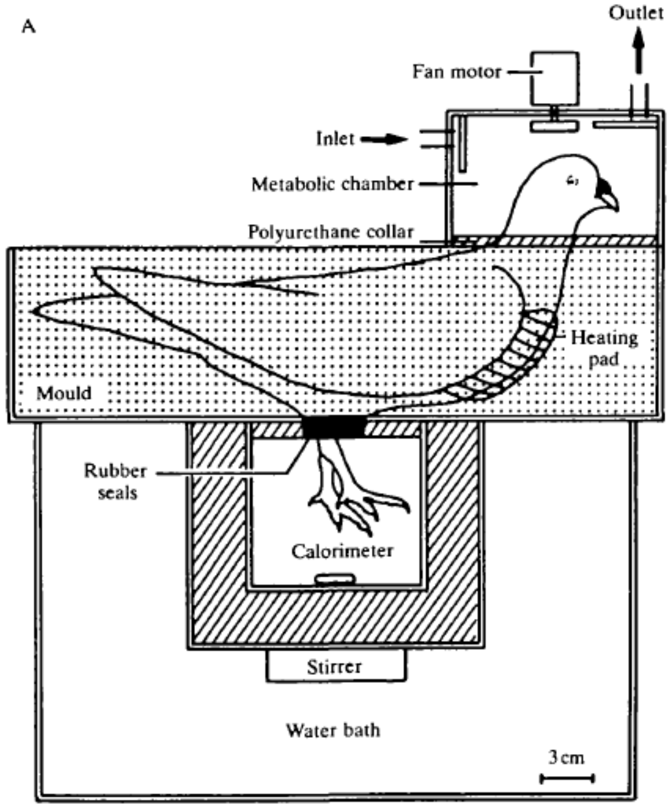
From an IPCC technical report

Heat transport in the environment : thermo-convective flow in a termite mound

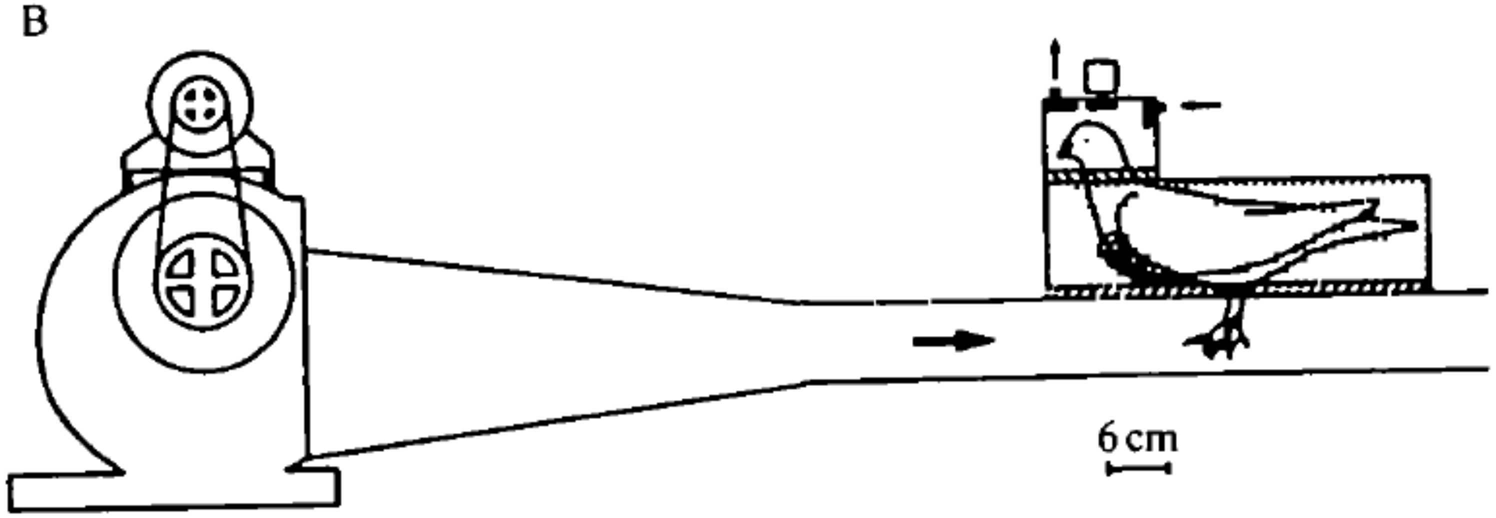


H. King, S. Ocko & L. Mahadevan, Termite mounds harness diurnal temperature oscillations for ventilation, PNAS 112, 11589 (2015)

Heat transport in living systems : thermal regulation of migratory birds

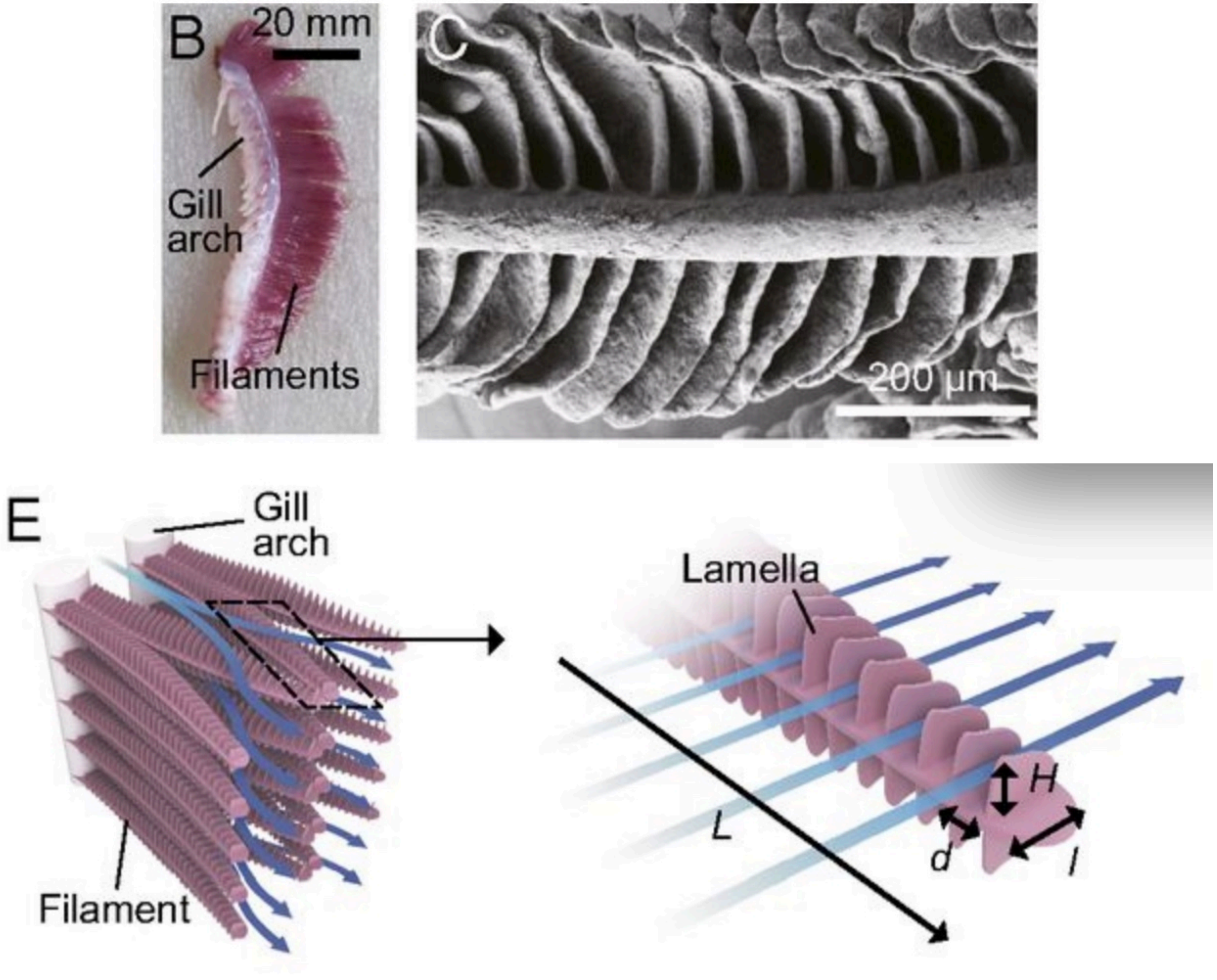


What is the cooling power of pigeon’s legs ?



L. Martineau & J. Larochelle, J. Exp. Biology 136, 193 (1988)

Mass transport in living systems : is there an optimal geometry for fish gills ?



K. Park et al., PNAS 111, 8067 (2014)

Mass transport and pandemics

Breathing through the mouth

Breathing through the nose

Videos by Gary Settles, Gas Dynamic Lab, Penn State University

Outline

- review of diffusive processes
  - scaling laws in diffusion, self-similar solutions
  - 1D diffusion, with sources and phase changes
- radiative heat transfer
- transfer by convection (advection by a macroscopic flow)
  - combined convection and diffusion
  - transport boundary layer
- thermal convection, coupling velocity and temperature fields
- dispersion in turbulent flows

The « inverted class »

reading material is posted on :
https://blog.espci.fr/marcfermigier/transport-phenomena-2022/

you read and (hopefully) understand it

in class we check that ideas and concepts are understood through problem solving

we explain again ideas and concepts that need to be clarified

Transport processes. Basic relations

molecular diffusion

mass flux : Fick's law $\mathbf{J_D} = -D\nabla C$

heat flux : Fourier's law $\mathbf{J_D} = -\lambda\nabla T$

Flux : quantity exchanged through unit surface per unit time

Total Flux : flux integrated over a whole surface

convection
(advection by a macroscopic flow **u**)

mass flux $\mathbf{J_C} = C\mathbf{u}$

heat flux $\mathbf{J_C} = \rho C_p T\mathbf{u}$

Radiative heat transfer

$$\mathbf{J_R} = \sigma T^4$$

Conservation laws

local equations for concentration and temperature

rate of change = divergence (flux) + source term

Sources of heat and mass

chemical reactions
phase changes
nuclear reactions
dissipative processes

$$\frac{\partial T}{\partial t} + \mathbf{u}.\nabla T = \kappa \Delta T + \frac{R}{\rho C}$$

$$\frac{\partial C}{\partial t} + \mathbf{u}.\nabla C = D \Delta C + R$$

convection-diffusion equations

Peclet number Pe = U L/D

Pe = convection/diffusion

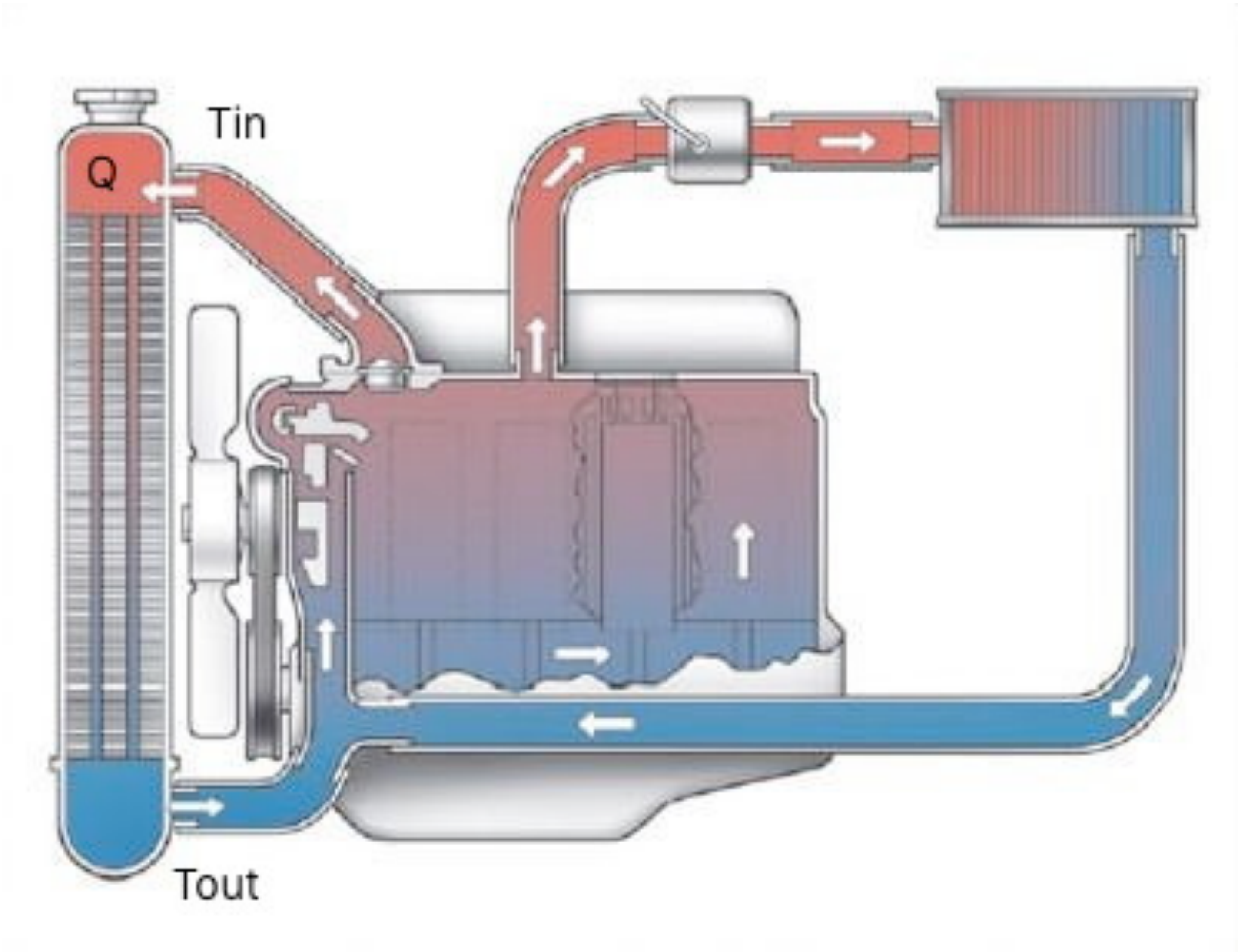
Boundary conditions at interfaces

Continuity of temperature and concentration fields
(infinite gradients would lead to infinite fluxes)

Continuity of heat and mass fluxes
Heat and mass cannot accumulate at an interface (zero volume)
exception : 1st order phase change with latent heat

Boundary conditions for fluid flows
continuity of velocity and stresses

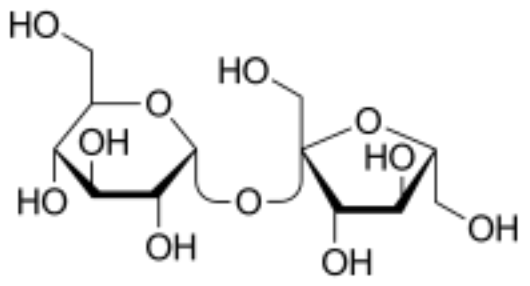
macroscopic balances



The coffee cup problem 1

If you don't stir the sugar in your coffee, why does it get cold way before it is sweetened ?

Coffee (essentially hot water) + sucrose (hydrodynamic radius 0.5 nm)



How long does it take for the sucrose to diffuse to the top ?

The coffee cup problem 1a

What is the molecular diffusion coefficient of sucrose in water ?

$$D = \mu \; k_B T$$

μ : mobility = velocity/applied force

Langevin's equation for the motion of a Brownian object (1908)

$$m\frac{d^2\mathbf{x}}{dt^2} = -\frac{1}{\mu}\frac{d\mathbf{x}}{dt} + \mathbf{F_T}(t) \qquad \langle F_T \rangle = 0$$

$$\langle x\dot{x} \rangle = \frac{1}{2}\frac{d}{dt}\langle x^2 \rangle = \mu k_B T + C \exp(-t/\mu m)$$

$$t \gg \mu\, m \qquad\qquad \langle x^2 \rangle = 2\mu\ k_B T\ t$$

Sphere of radius *a* in a viscous fluid

$$\mu = \frac{1}{6\pi\eta a}$$

$$D = \frac{k_B T}{6\pi\eta a}$$

at 300 K, $k_BT = 4\ 10^{-21}$ J

$a = 5\ 10^{-10}$ m, $\eta = 10^{-3}$ Pa.s

$D = 4\ 10^{-10}\ \mathrm{m}^2/\mathrm{s}$

$t_D = L^2/D \sim 10^6$ s ~10 days

The coffee cup problem 2

How long does it take to cool down to room temperature ?
What if diffusion in air is the only mechanism ?

Physical properties of air :
density $\rho = 1$ kg/m^3
specific heat $C_p = 1000$ J/kg.K
thermal conductivity $\lambda = 0.025$ W/m.K

$\kappa = \lambda/\rho C_p = 2.5\ 10^{-5}$ m^2/s

Physical properties of coffee :
density $\rho = 1000$ kg/m^3
specific heat $C_p = 4180$ J/kg.K
thermal conductivity $\lambda = 0.6$ W/m.K

$t_\kappa = L^2/\kappa \sim 100$ s