Dispersion in random velocity fields

Porous media Turbulent flows

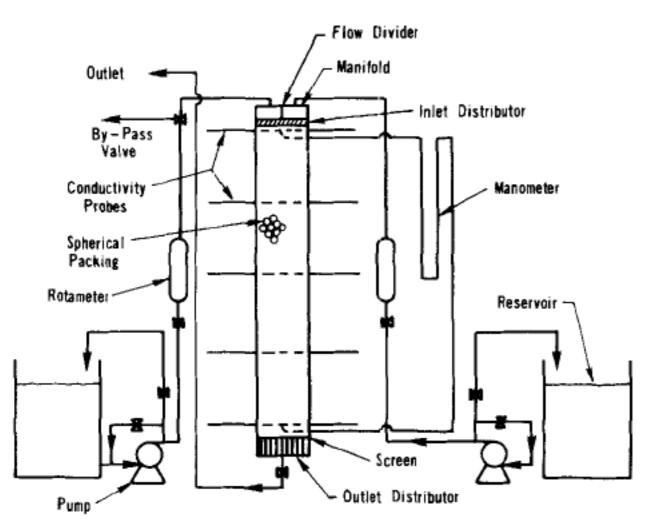


Figure 2. Overall schematic of experimental equipment.

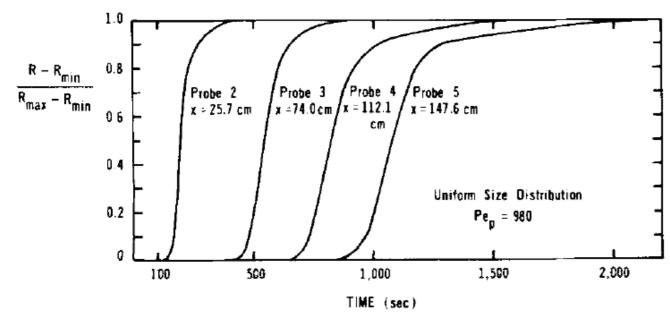


Figure 5. Sample raw data for step response experiment. R is recorded reading.

Dispersion in porous media

Longitudinal and lateral dispersion in packed beds: Effect of column length and particle size distribution, Han et al., AIChE J., 31, 277 (1985)

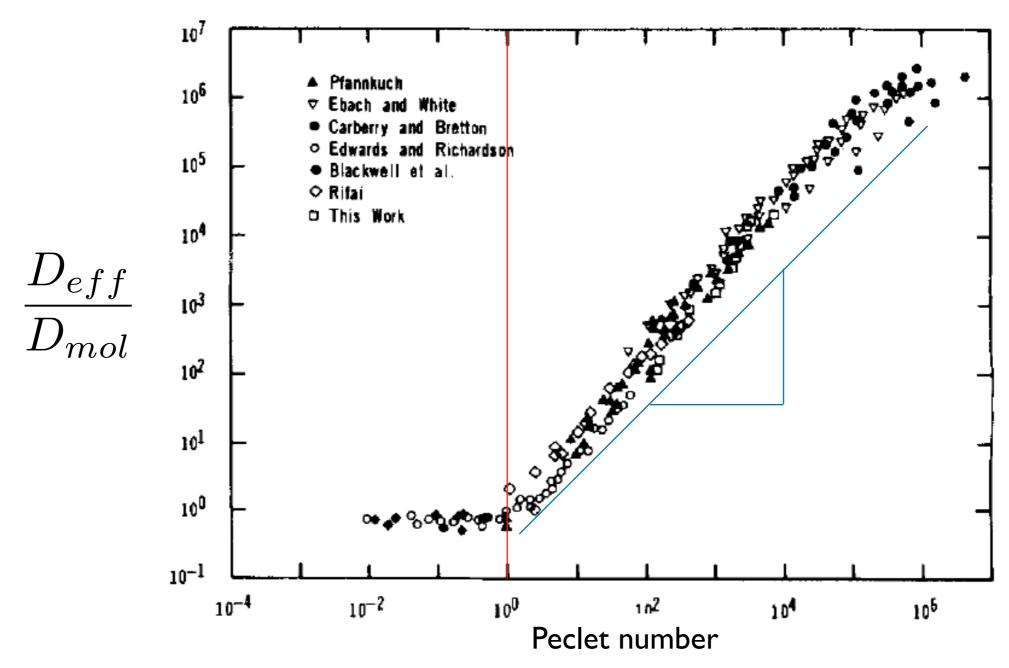
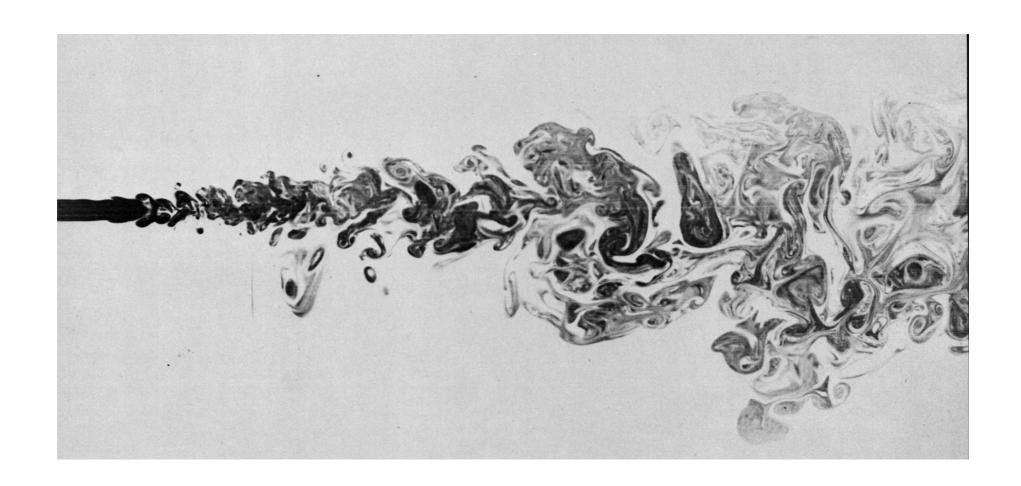


Figure 11. Longitudinal dispersivity values from various experiments.

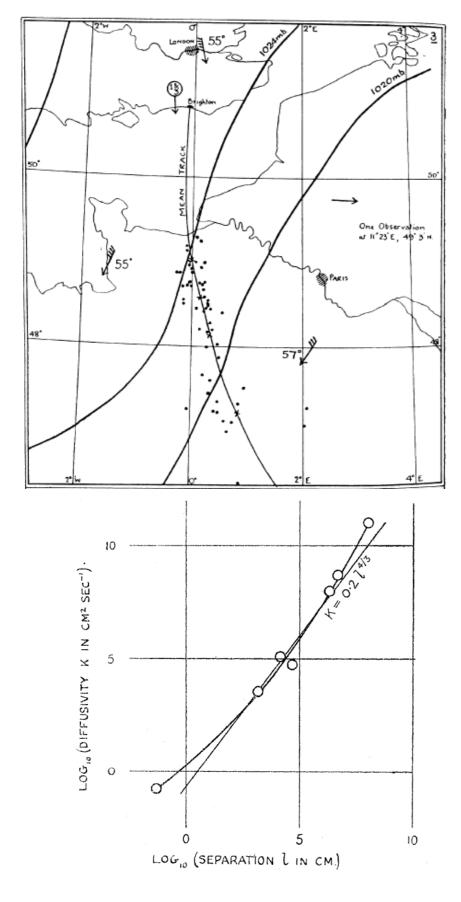
For a random walk of N steps of size d, what is the average distance traveled $\langle r \rangle$? What is the mean square root distance traveled $\sqrt{\langle r^2 \rangle}$?

Consider the dispersion in a porous medium as a random walk What is the effective diffusion coefficient as a function of the pore size d and the average velocity U?

Dispersion in turbulent flows

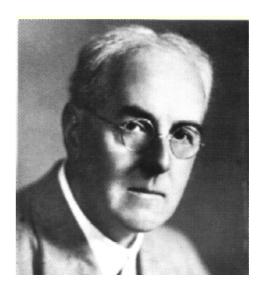


A turbulent jet at Re = 2000

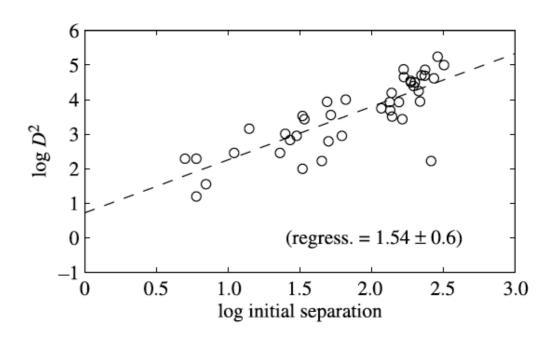


LF Richardson, Atmospheric diffusion shown on a distance-neighbor graph, Proc. Roy. Soc. 1926

Dispersion of pairs of particles in turbulent flows

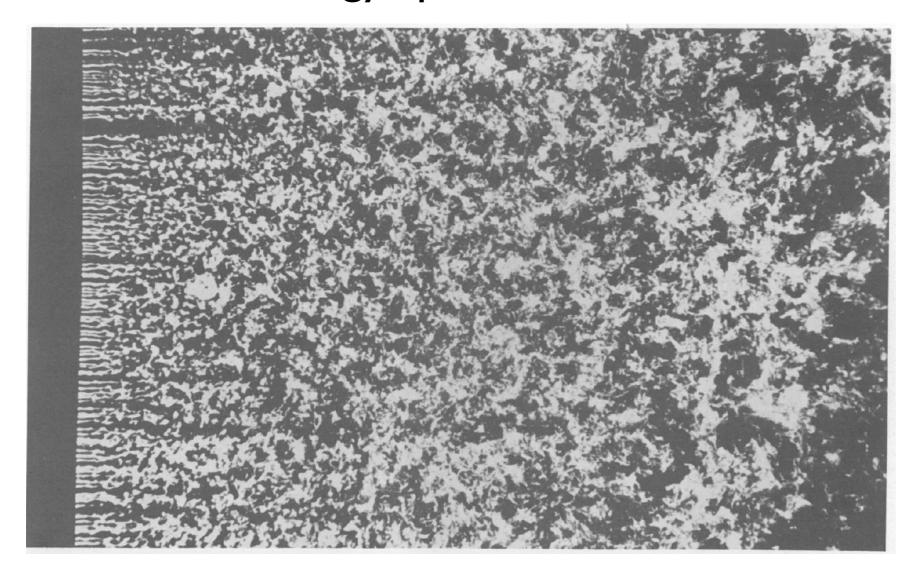


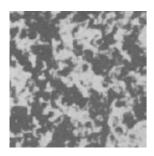
Lewis Fry Richardson



LF Richardson, H Stommel, Note on eddy diffusion in the sea, J. Meteorol., 1948

Fluctuation energy spectra in turbulent flows





At very large Re, fluctuations are isotropic and independent of the large scale structure

Velocity fluctuations at different length scales

at lengthscale l, velocity fluctuation u(l)

kinetic energy per unit mass

$$E_c \sim u^2(l)$$

Characteristic time

$$au \sim rac{l}{u(l)}$$

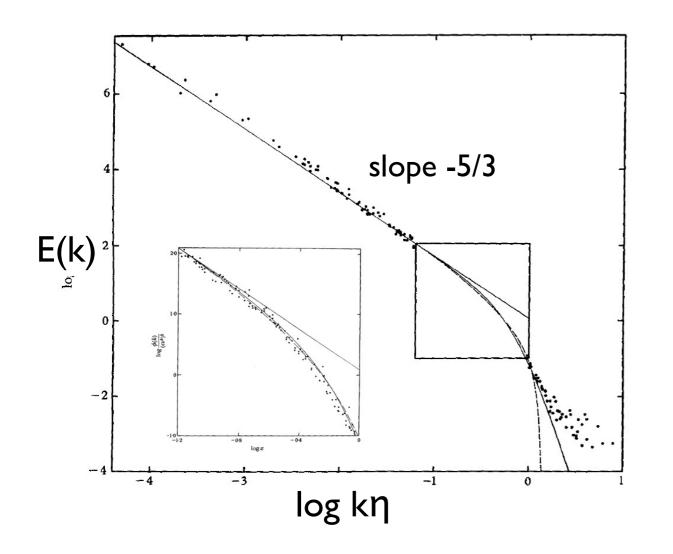
Rate of kinetic energy transfer

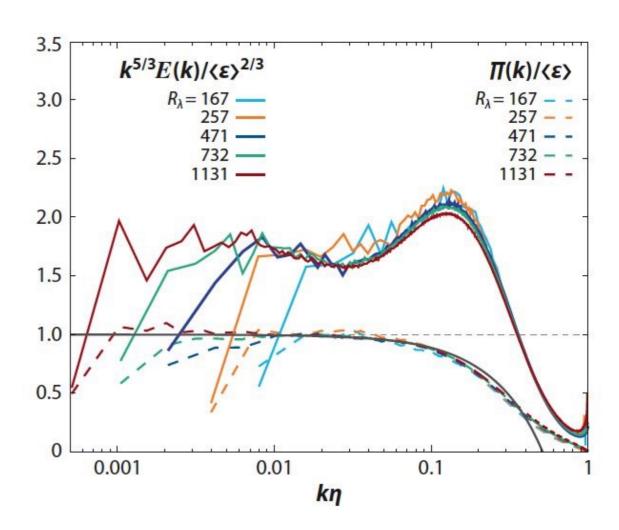
$$\epsilon \sim \frac{u^2(l)}{\tau} \sim \frac{u^3(l)}{l}$$

Kolmogorov's scaling law

$$u(l) \sim l^{1/3} \epsilon^{1/3}$$

Fluctuation energy spectra and Kolmogorov's scaling laws





Measurement in a tidal channel $Re = 3 \cdot 10^8$

η Kolmogorov microscale = $v^{3/4}$ ε-1/4 = L Re -3/4

 $E(k) \sim k^{-5/3}$ is equivalent to $u(l) \sim \epsilon^{1/3} l^{1/3}$

Direct numerical simulation

ε: rate of kinetic energy transfer

Considering the transport in a turbulent velocity field as a random walk, derive a scaling relation for the dispersion coefficient of two fluid elements initially separated by a distance I

I is within the inertial range of the turbulent flow described by the Kolmogorov scaling laws

$$u(l) \sim l^{1/3} \epsilon^{1/3}$$

$$D \sim l \ u(l) \sim l^{4/3} \epsilon^{1/3}$$

A short summary

In fluids, diffusion is not efficient except at small scales (when Pe << 1) Within solids, there is no other transport phenomenon

In fluids, at Pe>> I, convection is dominant, but diffusion cannot be ignored Transport is accelerated by the creation of thin boundary layers (and enhanced gradients) Effective fluxes are given by Nusselt or Sherwood numbers We get scaling laws for Nu or Sh as a function of Pe, Re and Pr The scaling exponents depend on the particular velocity profile

Thermal convection is specific because heat and momentum transport are strongly coupled Thermal convection is essentially governed by the Rayleigh number

Radiative heat transfer is governed by Stefan's law
Emissivity = absorptivity
View factors take into account the geometry of radiative surfaces