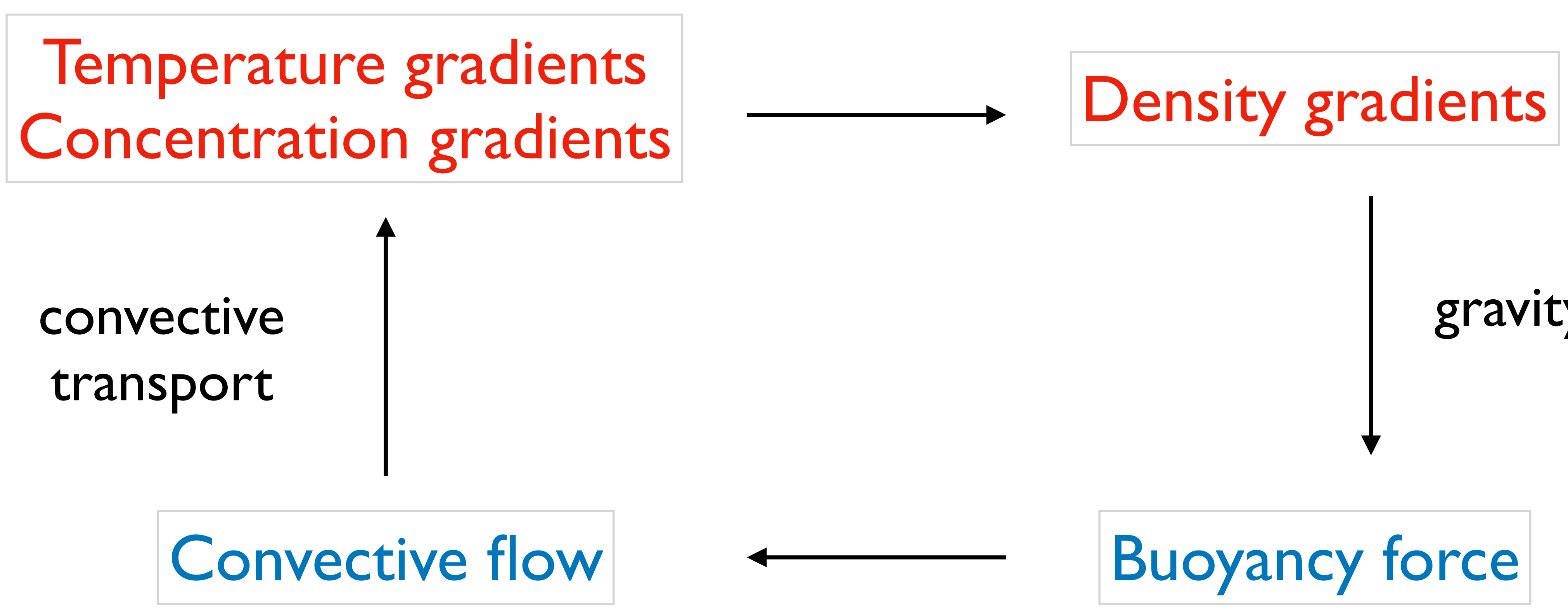
Thermal and solutal convection

Coupling between heat or mass transfer and convection in a gravity field



Visualization with the Schlieren technique

Boussinesq's approximation

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla\mathbf{u} = -\frac{\nabla p^*}{\rho_0} + \nu\Delta\mathbf{u} - \alpha(T - T_0)\mathbf{g}$$

Navier-Stokes

inertia viscosity buoyancy

$$\frac{\partial T}{\partial t} + \mathbf{u}.\nabla T = \kappa\Delta T$$

Transport equation

heat diffusivity

Reynolds number Re = UL/ν = inertia/viscosity

Prandtl number Pr = ν/κ = viscosity/heat diffusivity

Dimensional analysis

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u}.\nabla \mathbf{u} = -\frac{\nabla p^*}{\rho_0} + \nu \Delta \mathbf{u} - \alpha(T - T_0)\mathbf{g}$$

Grashof number :

buoyancy force/viscous force at Re=1

$$Gr = \frac{\alpha\ \delta T\ \mathbf{g}}{\nu \Delta \mathbf{u}} \sim \frac{\alpha\ \delta T\ \mathbf{g}\ L^2}{\nu U}$$

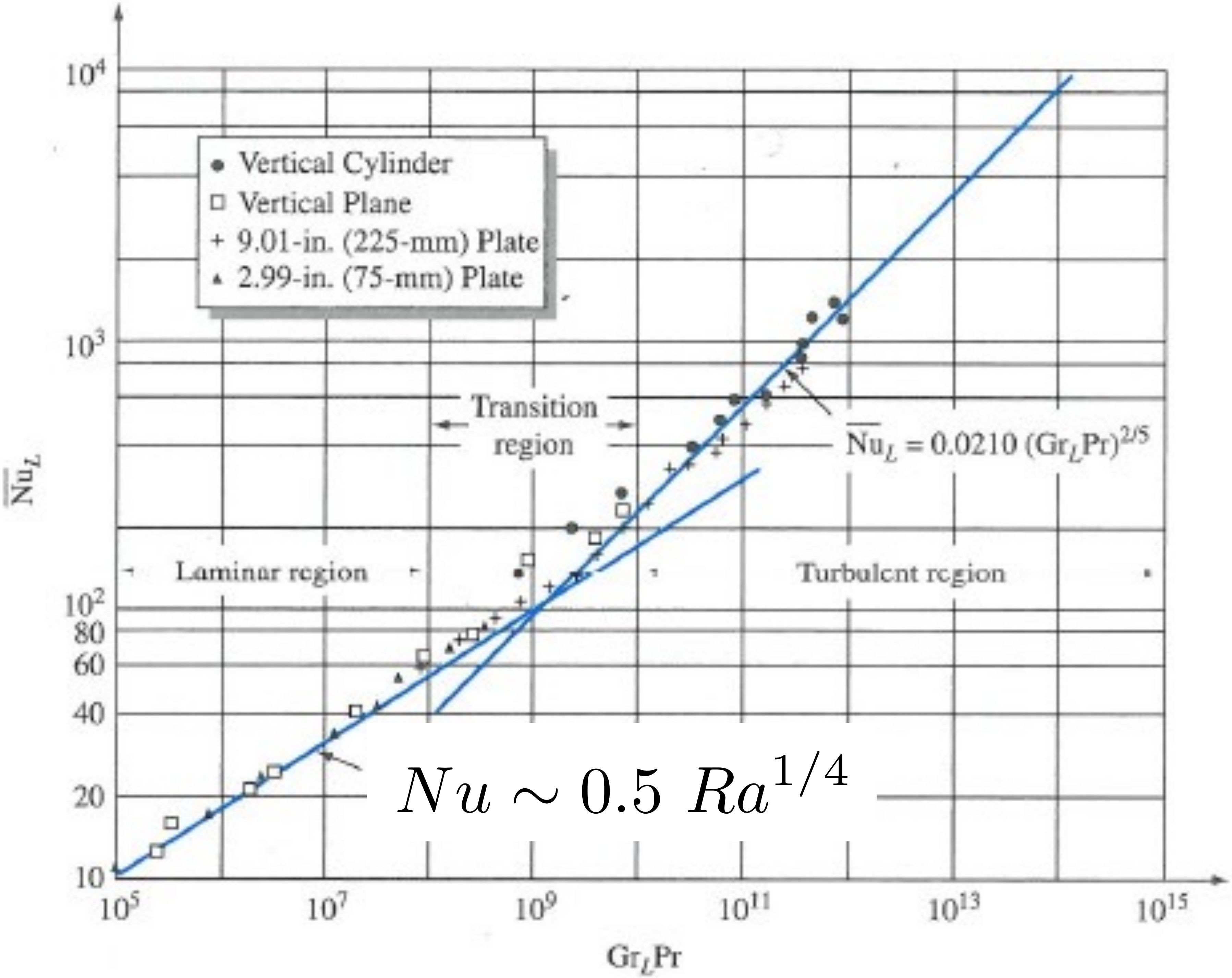
$$Re \sim 1 \rightarrow U \sim \frac{\nu}{L}$$

$$Gr = \frac{\alpha\ \delta T\ \mathbf{g}\ L^3}{\nu^2}$$

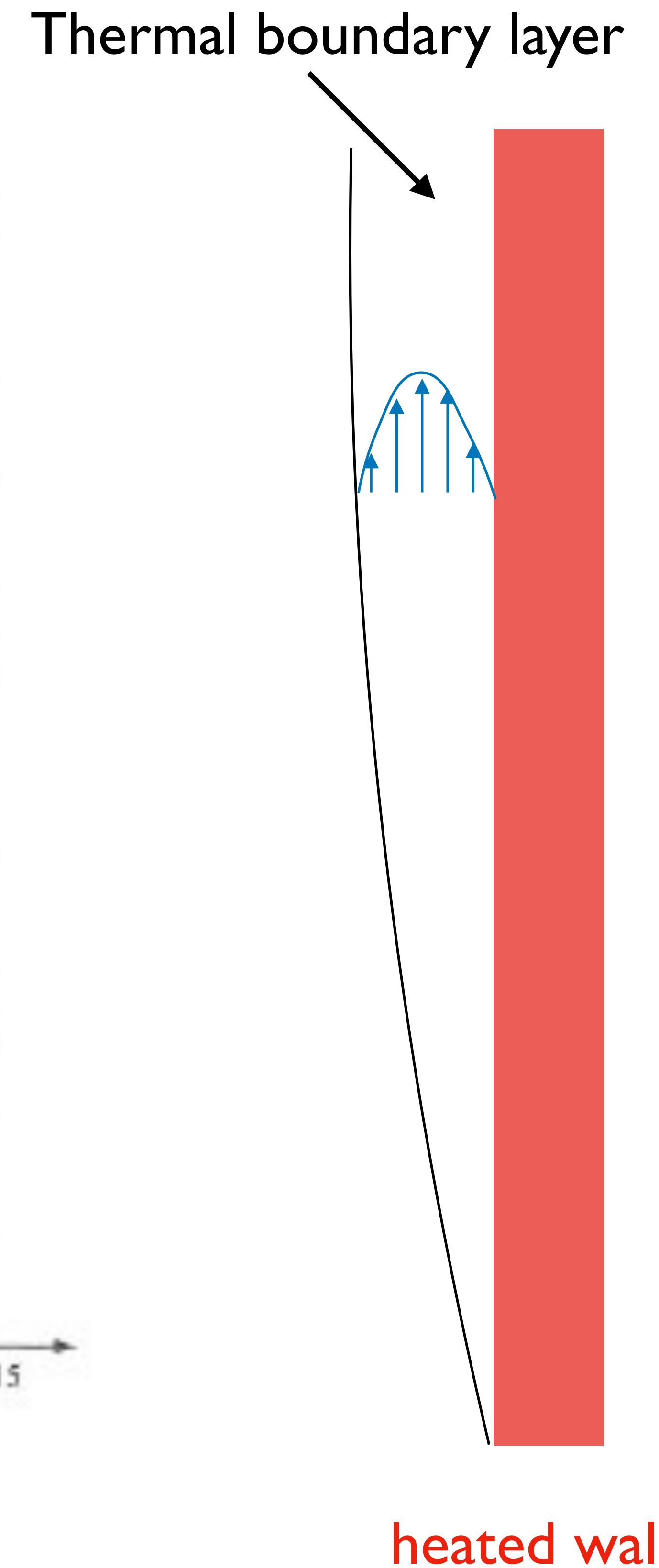
Rayleigh number

$$Ra = Gr\frac{\nu}{\kappa} = Gr\ Pr$$

$$Ra = \frac{\alpha\ \delta T\ \mathbf{g}\ L^3}{\nu \kappa}$$



Dimensionless heat flux (Nusselt number)
vs Rayleigh number



The coffee cup problem

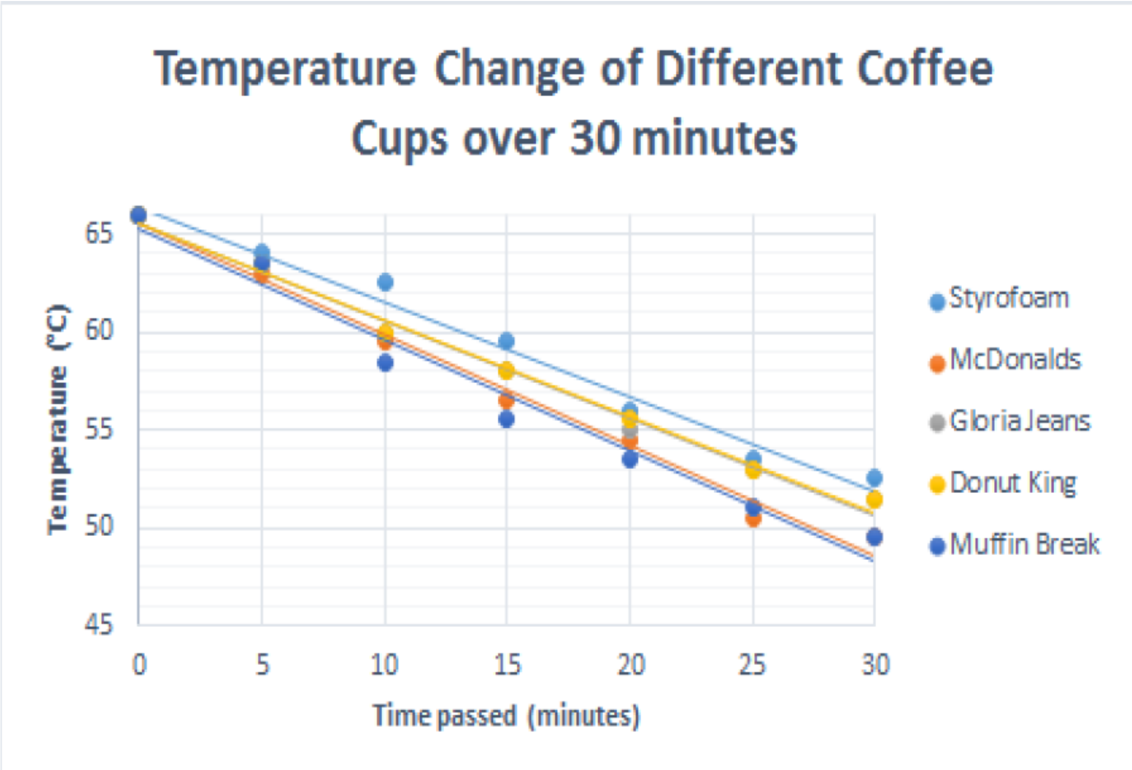
If you don't stir the sugar in your coffee, why does it get cold way before it is sweetened ?

How long does it take for the sucrose to diffuse to the top ?
At least weeks, months for a big cup.

If diffusion in air is the only mechanism, it takes 6 hours to cool down to room temperature …. we need something else.
Something else is thermal convection

Estimate the Rayleigh number for the coffee cup, deduce the relevant Nusselt number and estimate the heat flux due to thermal convection

Physical properties of air :
density 1 kg/m^3
Prandtl number : 0.7
kinematic viscosity $1.5\ 10^{-5}\ m^2/s$
coefficient of thermal expansion : $3.5\ 10^{-3}$



200 ml of water at 65°C initially
diameter of the cup 7 to 8.5 cm