

Transport due to a macroscopic flow

convection

(advection by a macroscopic flow \mathbf{u})

mass flux $\mathbf{J}_C = C\mathbf{u}$

heat flux $\mathbf{J}_C = \rho C_p T \mathbf{u}$

Convection vs diffusion :

Peclet number

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \Delta T + \frac{R}{\rho C}$$

$$\frac{\partial C}{\partial t} + \mathbf{u} \cdot \nabla C = D \Delta C + R$$

convection-diffusion equations

Heat transfer

$$Pe = \frac{UL}{\kappa}$$

Mass transfer

$$Pe = \frac{UL}{D}$$

$$Pe \rightarrow \infty$$

No diffusion, pure
advection by the flow

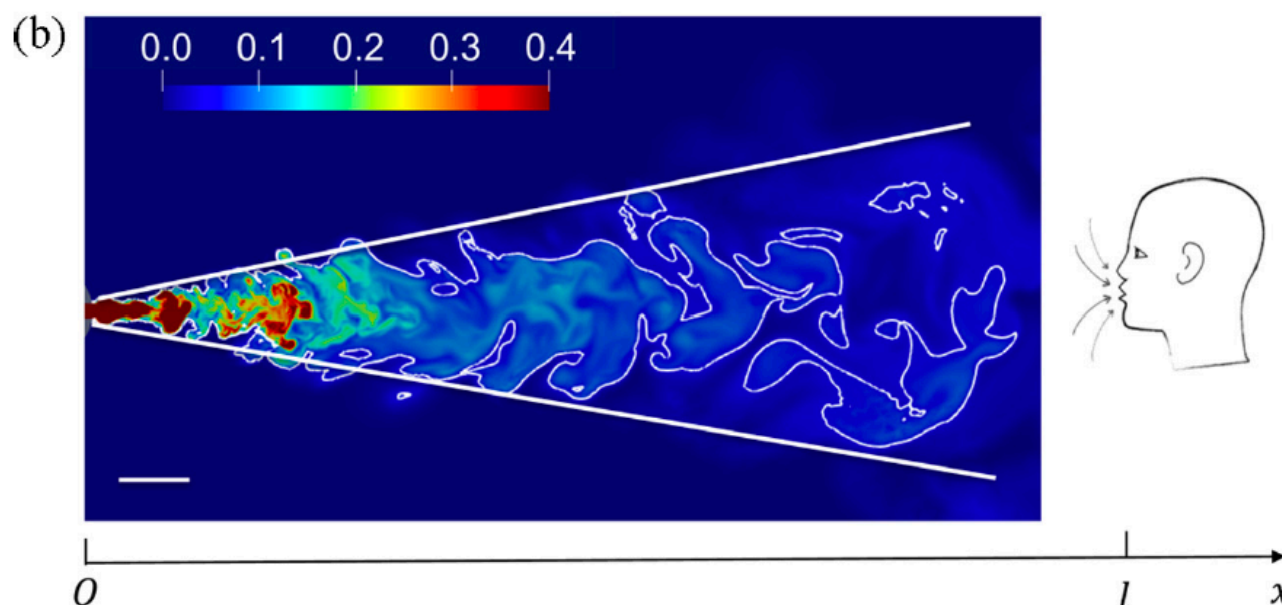
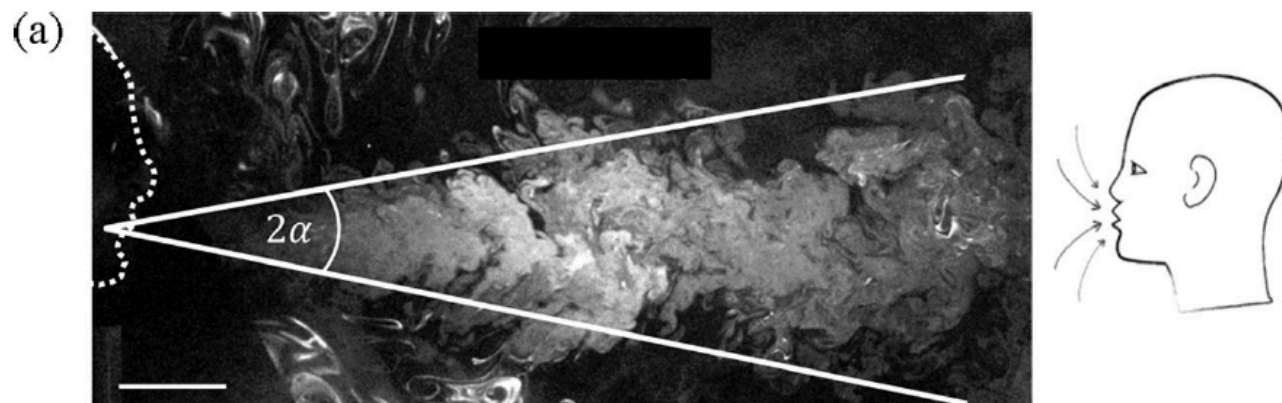
In random velocity fields
(porous media, turbulent
flows) : **dispersion by the
flow**

$$Pe \gg 1$$

Convection dominates,
diffusion cannot be ignored
transport fluxes controlled
by **boundary layers**

$$Pe \ll 1$$

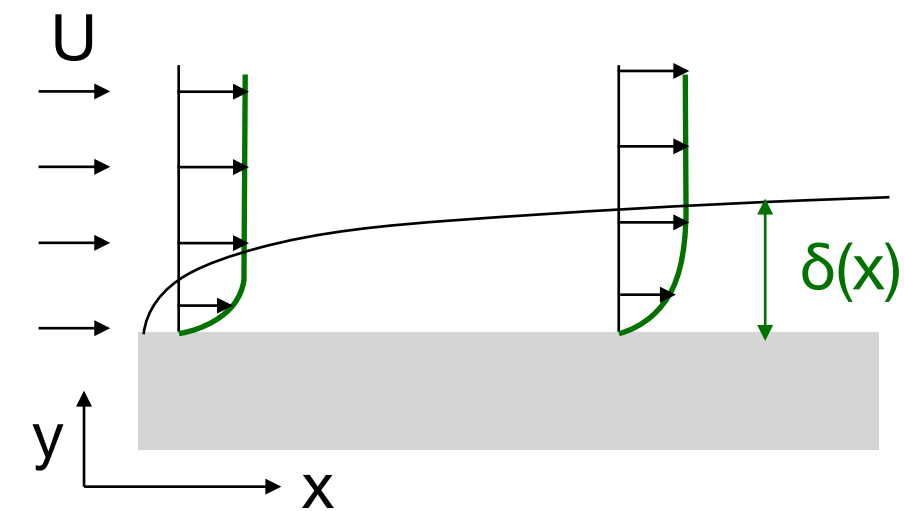
Diffusion dominates,
convection is neglected



Towards improved social distancing guidelines: Space and time dependence of virus transmission from speech-driven aerosol transport between two individuals, F. Yang, et al., Phys. Rev. Fluids 5, 122501(R) (2020)

Boundary layers and the Reynolds analogy

Transport of momentum



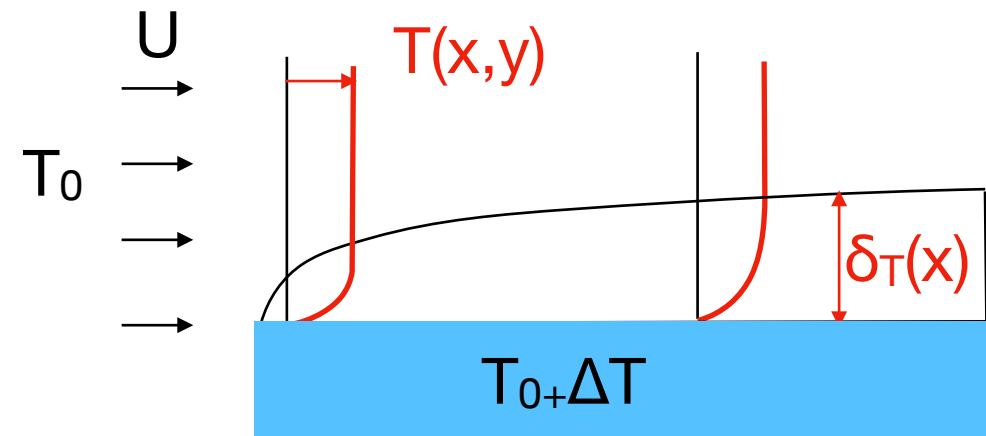
$$\delta(x) \sim \sqrt{\frac{\nu x}{U}}$$

$$\delta(x) \sim x Re_x^{-1/2}$$

Friction drag ~ wall shear stress

$$\sigma_{xy} = \eta \frac{\partial u_x}{\partial y} \sim \eta \frac{U}{\delta(x)}$$

Transport of heat or mass



$$\delta_T(x) \sim x \left(\frac{Ux}{\kappa} \right)^{-\alpha}$$

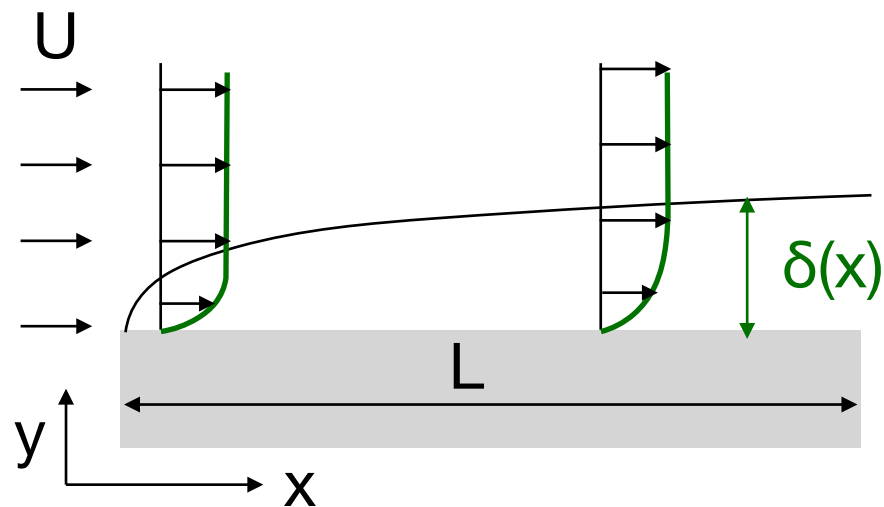
$$\delta_T(x) \sim x Pe_x^{-\alpha}$$

Flux at the wall

$$J_Q(x) = -\lambda \frac{\partial T}{\partial y} \sim \lambda \frac{\Delta T}{\delta_T(x)}$$

Boundary layers and the Reynolds analogy

Transport of momentum



$$Re \gg 1$$



Thin momentum boundary layer

$$\delta \ll L$$

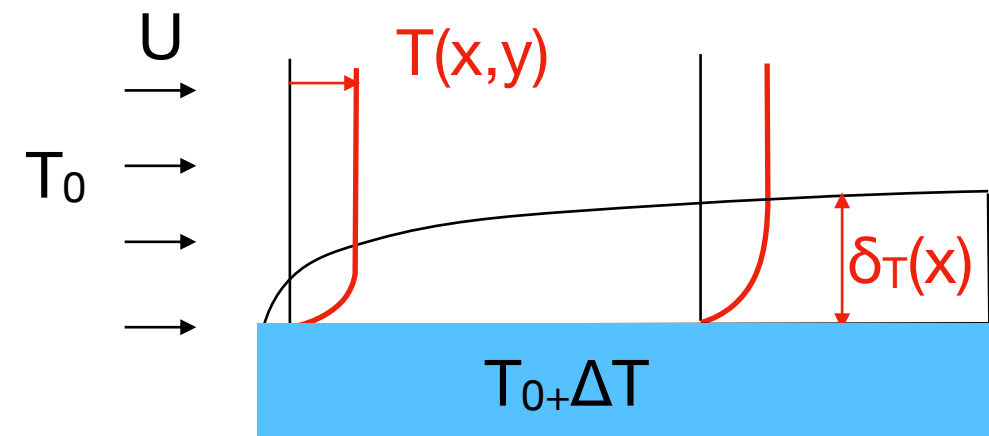


Velocity gradient at the wall



Friction drag

Transport of heat or mass



$$Pe \gg 1$$



Thin temperature boundary layer

$$\delta_T \ll L$$



Temperature gradient at the wall



Heat flux to the wall