Space radiator

January 29, 2021

Fin is at temperature T(x) (assuming temperature is uniform across the section).

Flux received from the solar radiation : $G_s\alpha_s$ on one side. Flux received from the sky : $\epsilon\sigma T_{sky}^4$.

Flux emitted : $\epsilon\sigma T(x)^4$

Net flux radiated towards space : $2\epsilon\sigma T(x)^4 - \epsilon\sigma T_{sky}^4 - G_s\alpha_s$

At equilibrium :

$$\left(-\lambda_M \frac{\partial T}{\partial x}_x + \lambda_M \frac{\partial T}{\partial x}_{x+dx}\right) e - (2\epsilon\sigma T(x)^4 - 2\epsilon\sigma T_{sky}^4 - G_s\alpha_s)dx = 0$$

$$\frac{\partial^2 T}{\partial x^2} - \frac{2\epsilon\sigma T(x)^4 - 2\epsilon\sigma T_{sky}^4 - G_s\alpha_s}{e\lambda_M} = 0$$

Boundary conditions : temperature fixed at fin base $T(x) = T_1 = 353K$ at $x = 0$, flux continuity at tip.

$$-\lambda_M \frac{\partial T}{\partial x}_L = \epsilon\sigma T(L)^4 - \epsilon\sigma T_{sky}^4$$

We have $G_s\alpha_s = 615$ W/m^2, corresponding to a temperature $T_s = 331$ K at $\epsilon = 0.9$ such that : $G_s\alpha_s = \epsilon\sigma T_s^4$.

$$\frac{\partial^2 T}{\partial x^2} - \frac{2\epsilon\sigma}{e\lambda_M}[T(x)^4 - T_{sky}^4 - 0.5T_s^4] = 0$$

$$\frac{\partial^2 \theta}{\partial x^2} - \frac{2\epsilon\sigma T_1^3}{e\lambda_M}[\theta(x)^4 - \theta_{sky}^4 - 0.5\theta_s^4] = 0$$

with $\theta = T/T_1$, $\theta_{sky} = T_{sky}/T_1 = 0.011$, $\theta_s = T_s/T_1 = 0.94$.

Introducing a lengthscale ℓ such that :

$$\ell = \sqrt{\frac{e\lambda_M}{2\epsilon\sigma T_1^3}}$$

and a dimensionless coordinate $\zeta = x/\ell$:

$$\frac{\partial^2\theta}{\partial\zeta^2} - [\theta(\zeta)^4 - \theta_{sky}^4 - 0.5\theta_s^4] = 0.$$

With $\lambda_M = 300$ W/(m.K), $e = 12$ mm, $\ell = 1.6$ m.

In the absence of solar irradiation (satellite in the dark), using the approximation $\theta_s \ll \theta(\zeta)$, the equation reduces to :

$$\frac{\partial^2\theta}{\partial\zeta^2} - \theta(\zeta)^4 = 0.$$

The boundary conditions are in dimensionless coordinates : $\theta(0) = 1$,

$$\frac{d\theta}{d\zeta}_{L/\ell} = \frac{e}{2\ell}\theta(L/\ell)^4 = 6 \times 10^{-3}\theta(L/\ell)^4$$

Since θ is of order one, we can approximate the boundary condition at the tip by :

$$\frac{d\theta}{d\zeta}_{L/\ell} \approx 0$$

The differential equation can be solved numerically with Matlab with the following code. The initial value for $\partial\theta/\partial\zeta$ is adjusted to get a zero temperature gradient at the end of the fin. The temperature profile is shown on fig. 1.

```
[t,y]=ode15s(@radia,[0 0.625],[1 ; -0.44]);
plot(t,y(:,1),'-o');
xlabel('x/l','FontSize',14);
ylabel('T/T1','FontSize',14);
grid on;

function dydt=radia(t,y)
dydt=[y(2);y(1)^4]
end
```

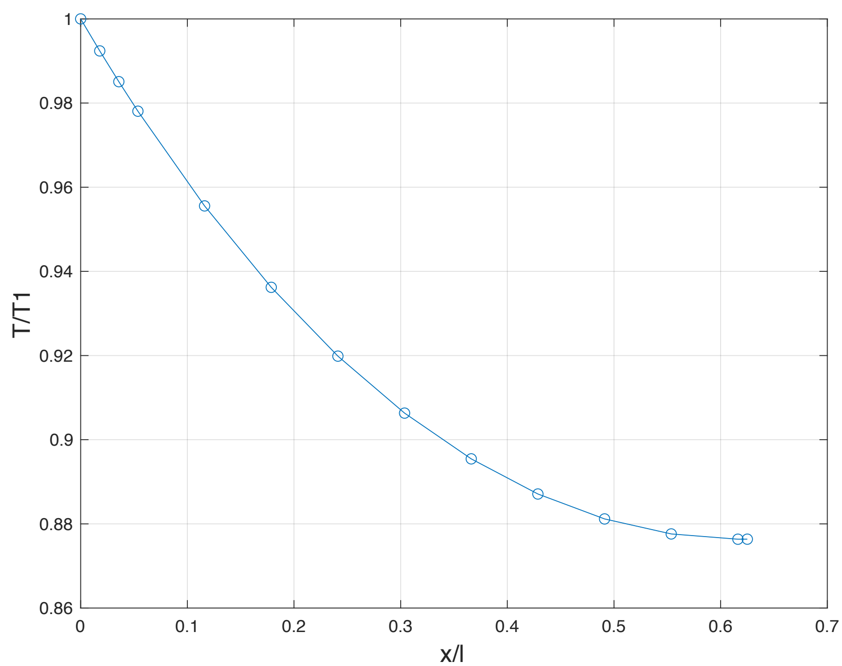

Figure 1: Dimensionless temperature as a function of x/ℓ.