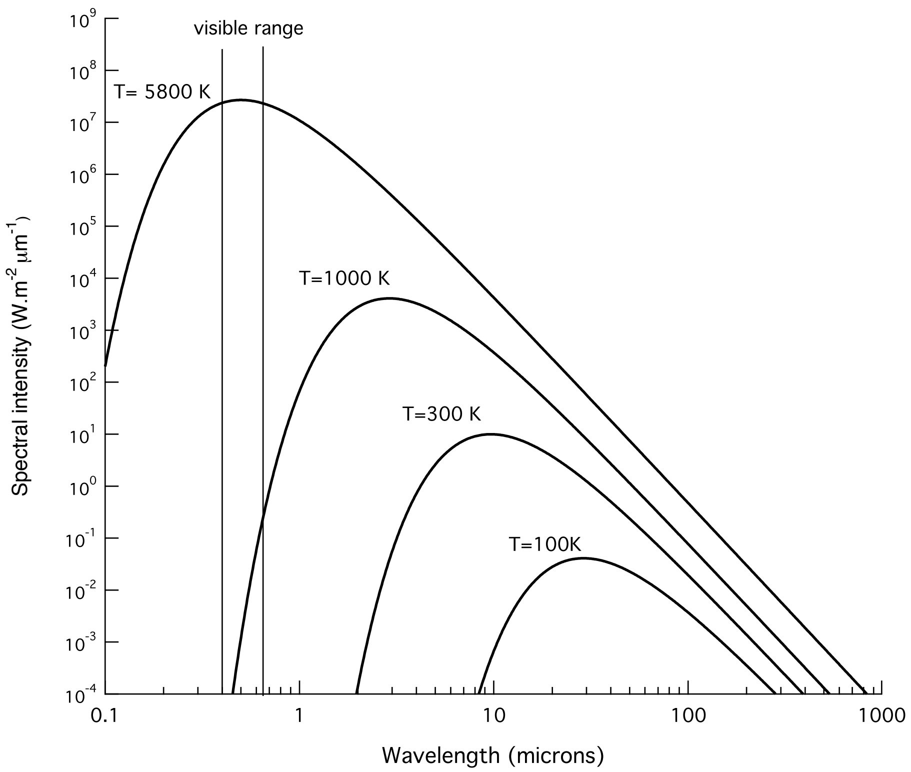
Radiative heat transfer

Radiative heat transfer

Black body radiation

Radiative heat transfer

Black body radiation

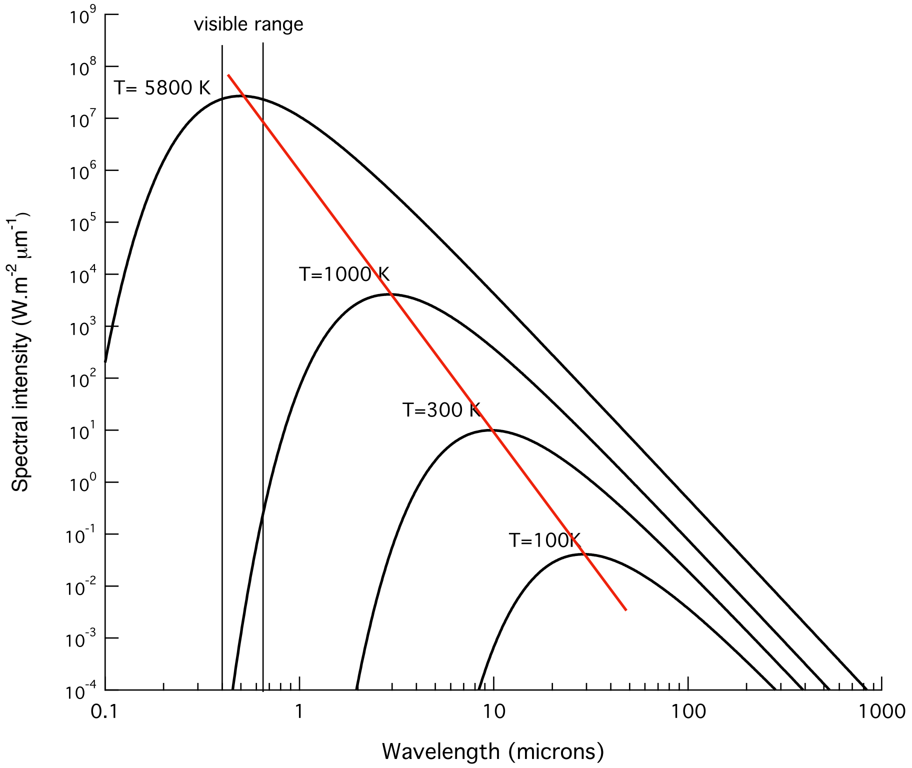


Power emitted per unit surface :

$$P_e(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}$$

Radiative heat transfer

Black body radiation



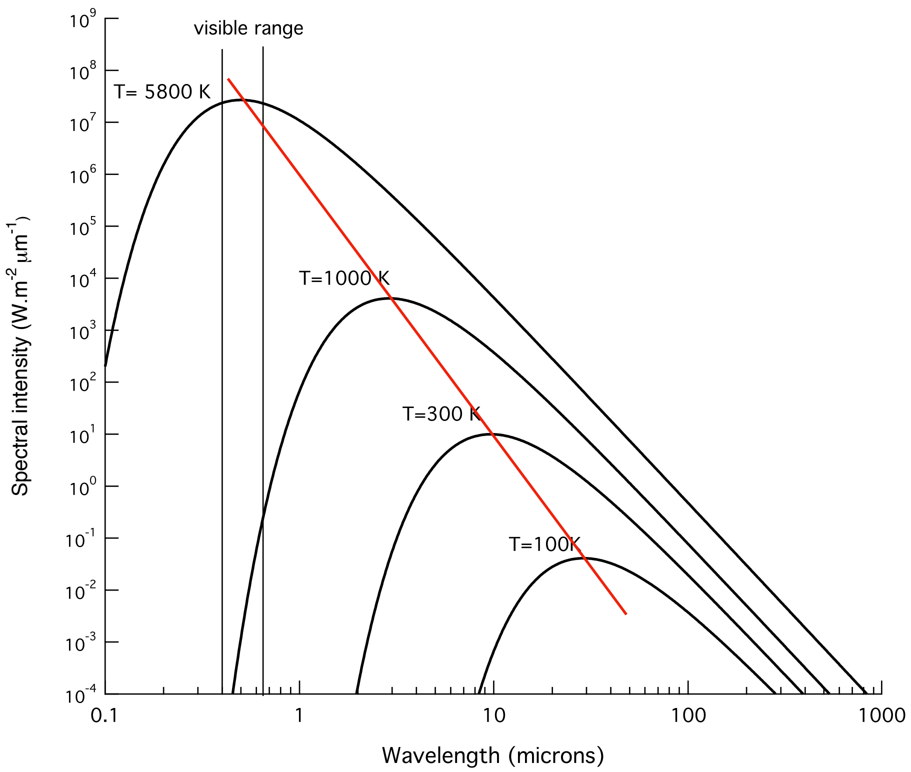
Power emitted per unit surface :

$$P_e(\lambda, T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}$$

$$\lambda_M(\mathrm{m}) = \frac{2.88 \times 10^{-3}}{T}$$ Wien's displacement law

Radiative heat transfer

Black body radiation



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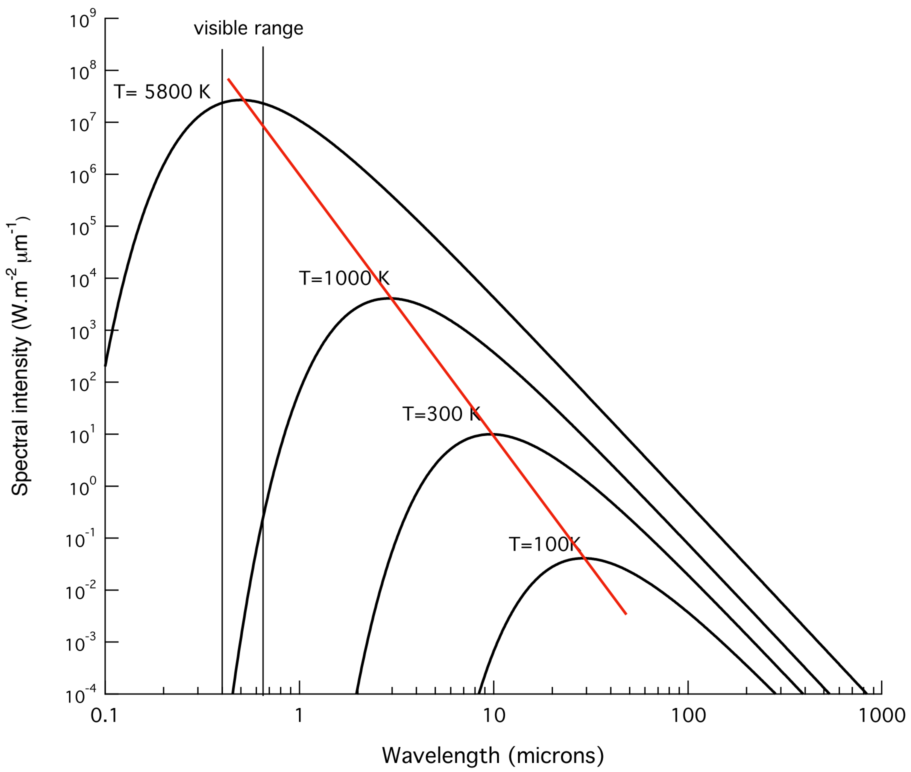
$$P_{BB}(T) = \int_0^\infty P_e(\lambda, T) d\lambda = \frac{2\pi^5 k_B^4}{15\ h^3 c^2} T^4 = \sigma T^4$$

Stefan's law

Radiative heat transfer

Black body radiation

Power emitted per unit surface :



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Stefan's law

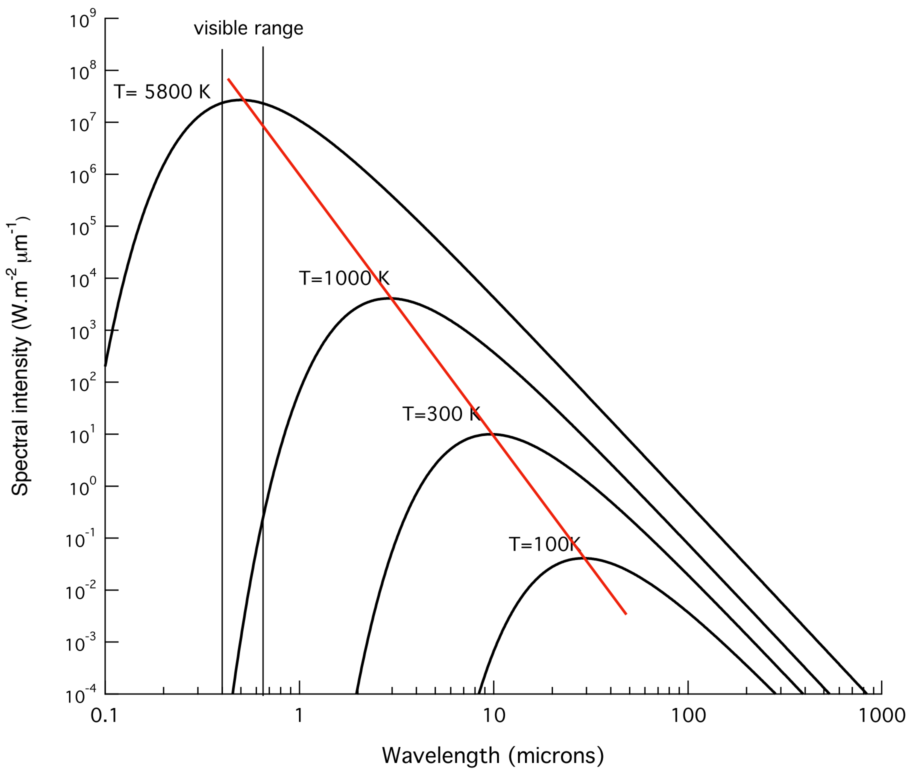
Non black bodies

Emissivity ε = absorptivity a Kirchoff's law

$P(\lambda,T) = P_e(\lambda,T)\ \varepsilon(\lambda)$

Radiative heat transfer

Black body radiation



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Stefan's law

Non black bodies

Emissivity ε = absorptivity a   Kirchoff's law

$P(\lambda,T) = P_e(\lambda,T)\ \varepsilon(\lambda)$

Grey bodies

Emissivity independent of λ

$\varepsilon(\lambda) = a(\lambda) = C$

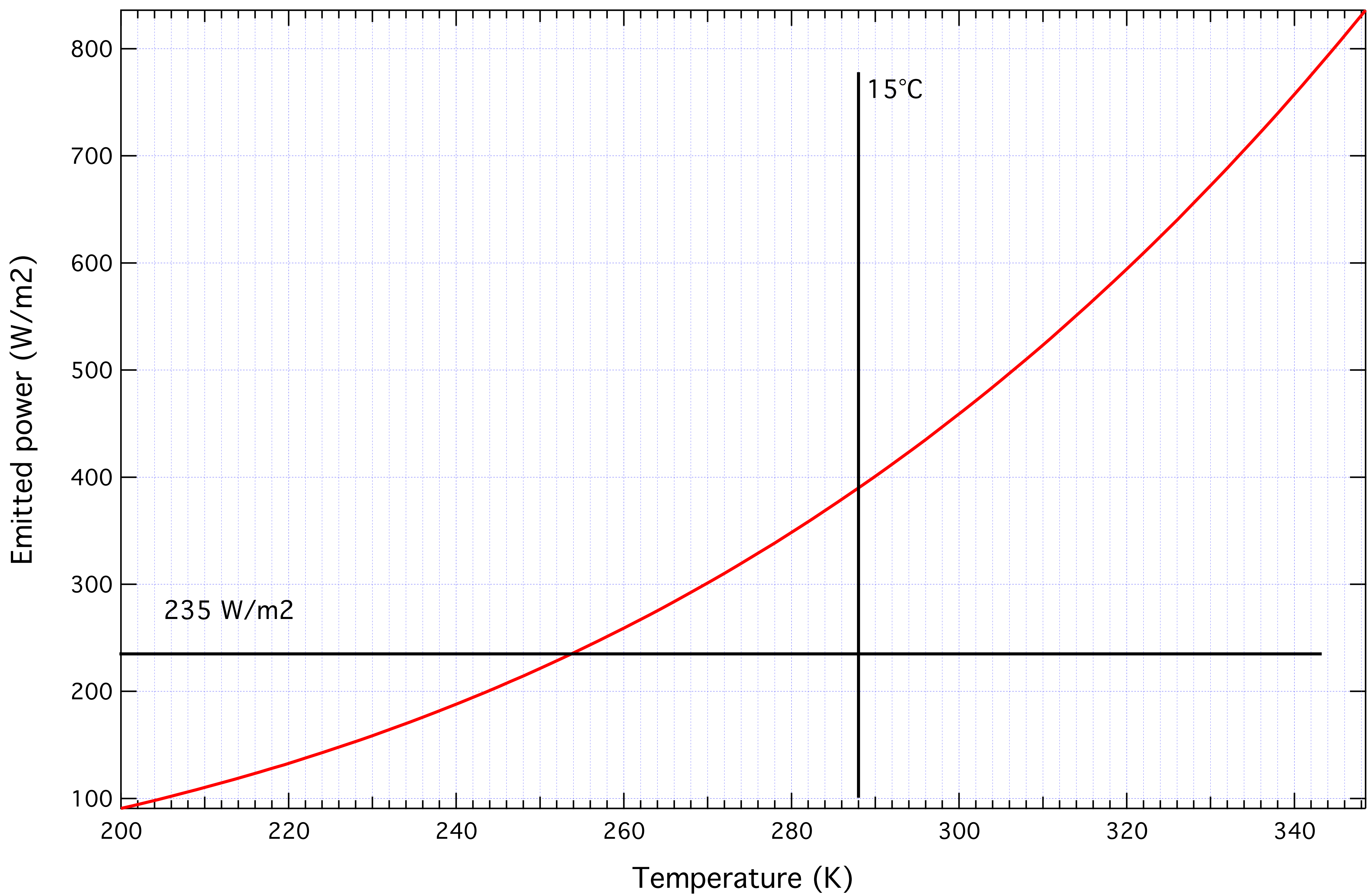
The Donald Trump problem at zeroth order :

show that with a completely transparent atmosphere, it would be difficult to grow lawn for a golf course on Earth.

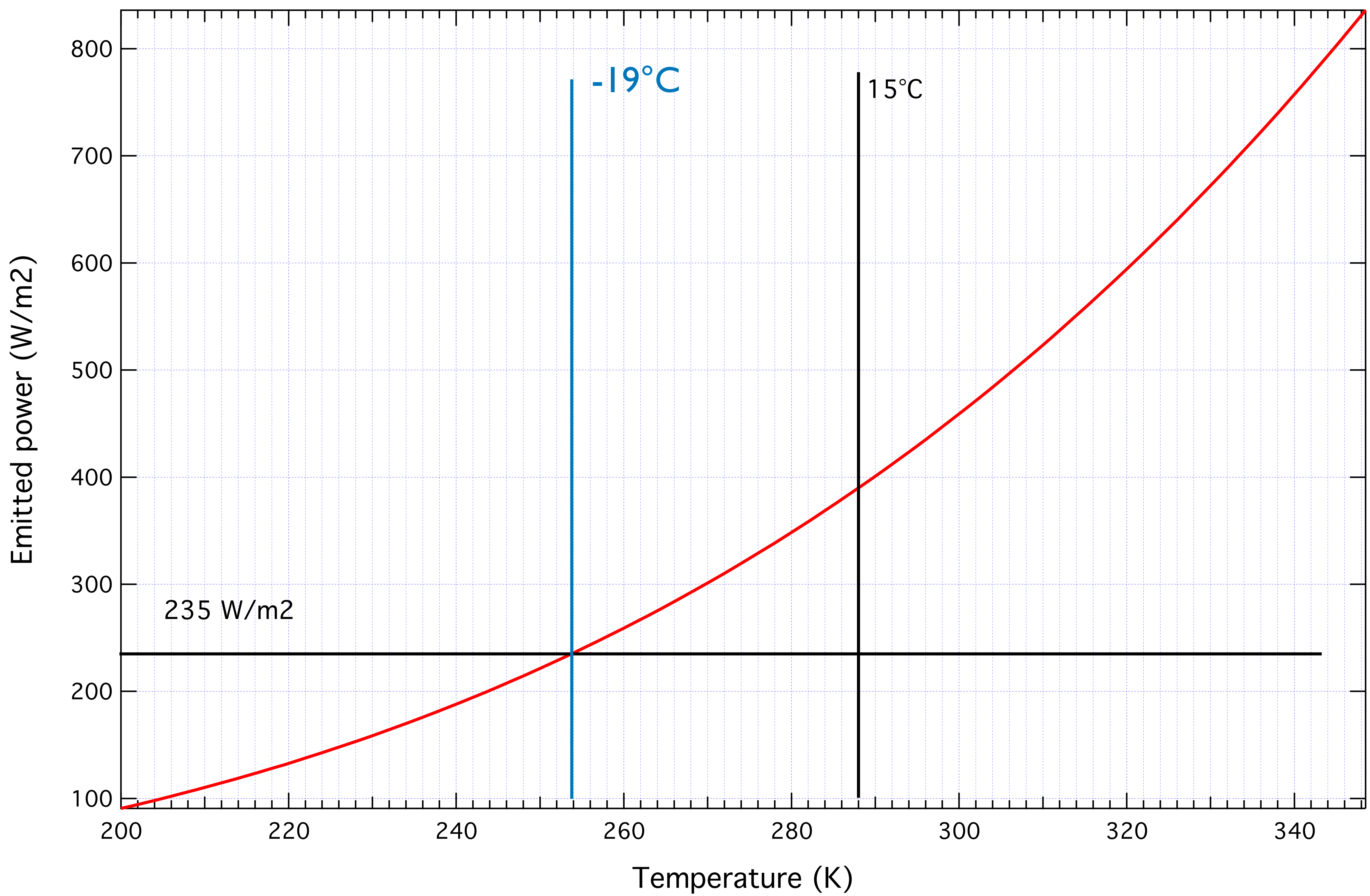
Diameter of the Sun: 1,4 10^6 km
Surface temperature of the Sun : 5800 K
Distance to the Sun: 1,5 10^8 km
Diameter of the Earth: 12700 km

Effective albedo of the Earth: 0.31

Earth and Sun are assumed to be black bodies



15°C
235 W/m2
800
700
600
500
400
300
200
100
200
220
240
260
280
300
320
340
Emitted power (W/m2)
Temperature (K)



-19°C
15°C
235 W/m2
800
700
600
500
400
300
200
100
200
220
240
260
280
300
320
340
Emitted power (W/m2)
Temperature (K)

The Donald Trump problem at zeroth order :
show that with a completely transparent atmosphere, it would be difficult to grow lawn for a golf course on Earth.

Radiative flux from the sun, averaged over entire Earth surface : 342 W/m^2

Flux adsorbed : 235 W/m^2

Corresponding equilibrium temperature : 254°K

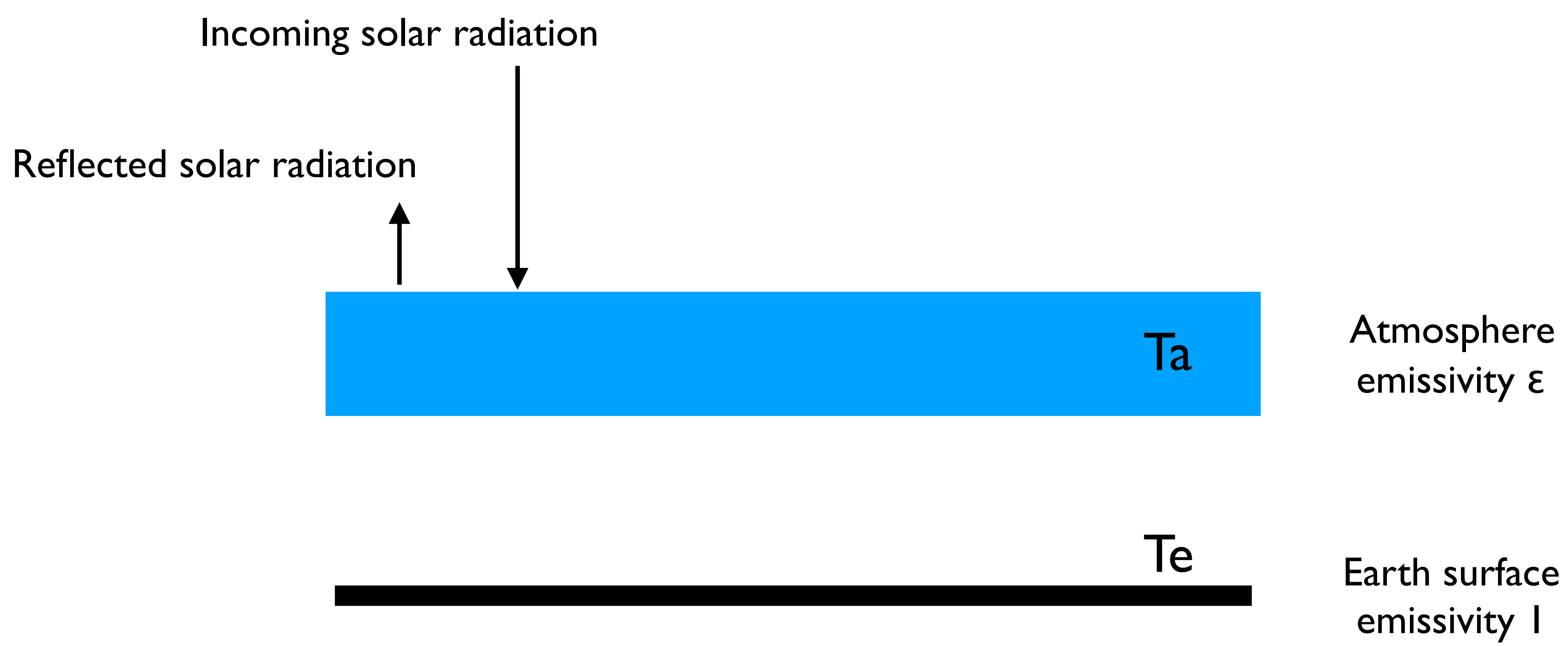
Diameter of the Sun: 1,4 10^6 km
Surface temperature of the Sun : 5800 K
Distance to the Sun: 1,5 10^8 km
Diameter of the Earth: 12700 km

Effective albedo of the Earth: 0.31

Earth and Sun are assumed to be black bodies

The Donald Trump problem at order one :

show that with a one layer atmosphere adsorbing in the infrared, it would be easier to grow lawn for a golf course on Earth.

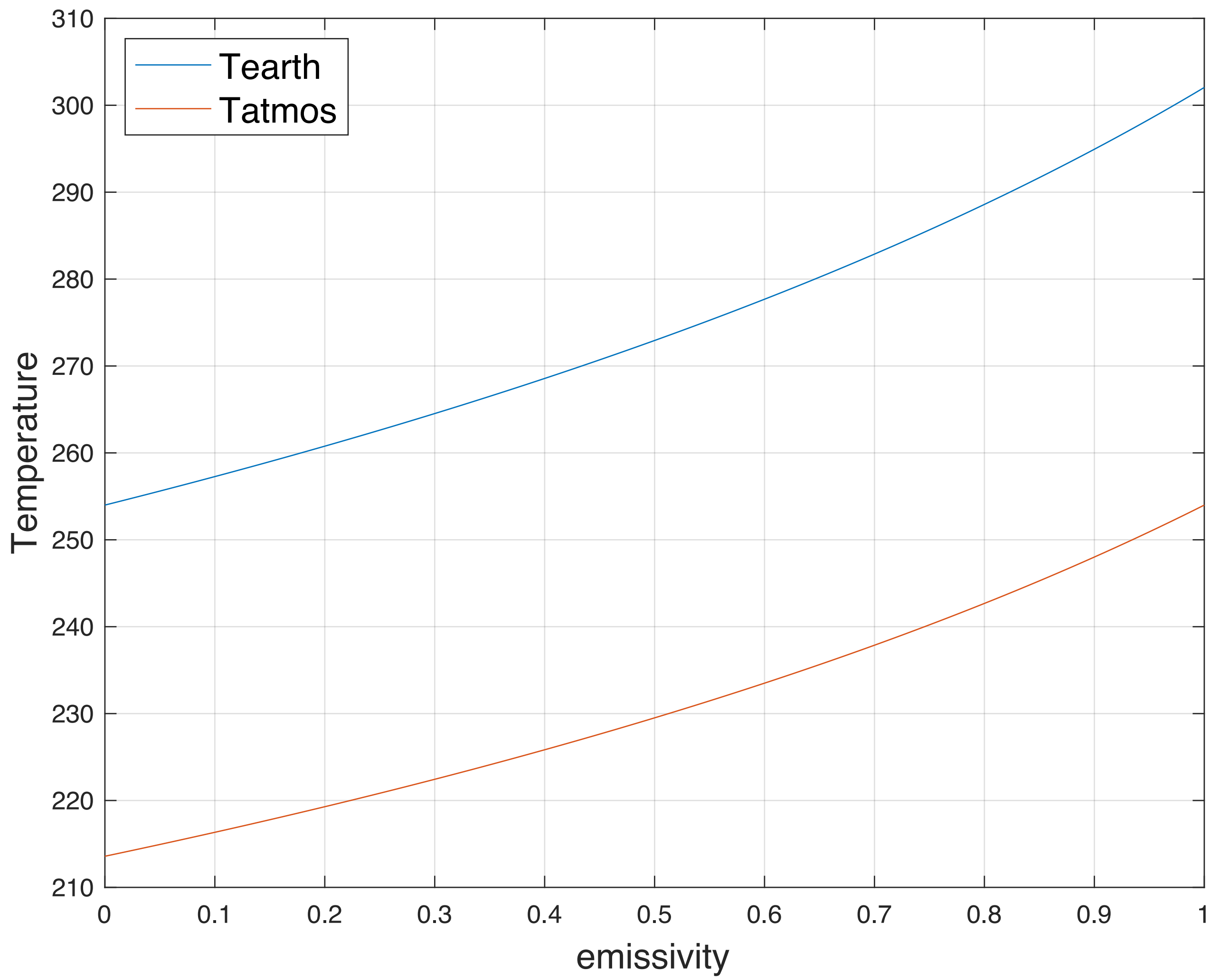


Write the energy flux balance for the earth + atmosphere system

Write the energy flux balance for the atmosphere at temperature Ta

Determine Ta and Te as a function of ε

$$T_e = \left[\frac{(1-a)J_s}{\sigma(1-\epsilon/2)} \right]^{1/4}$$



The Donald Trump problem at order 2 :

Evaluate the temperature distribution in a stable « grey » non scattering atmosphere

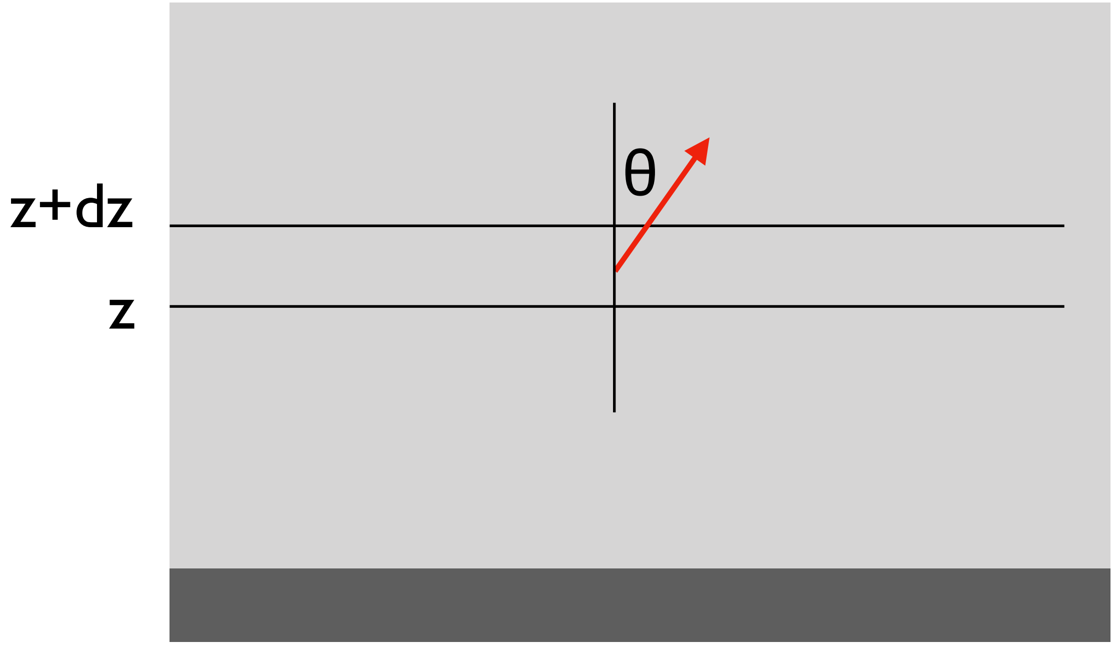
Atmosphere clear to solar radiation ; all absorption takes place at the ground.

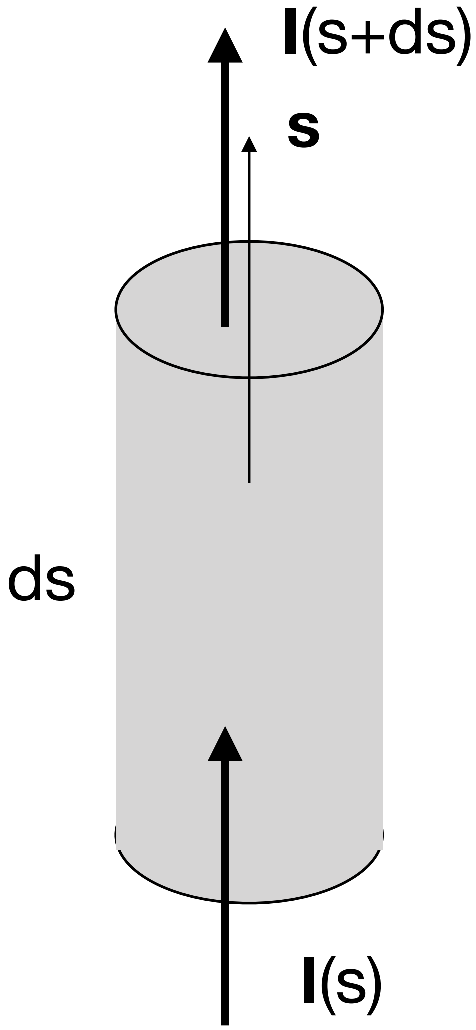
Gray atmosphere with no clouds in the IR (no scattering of light)

Layers of atmosphere are grey thermal radiators

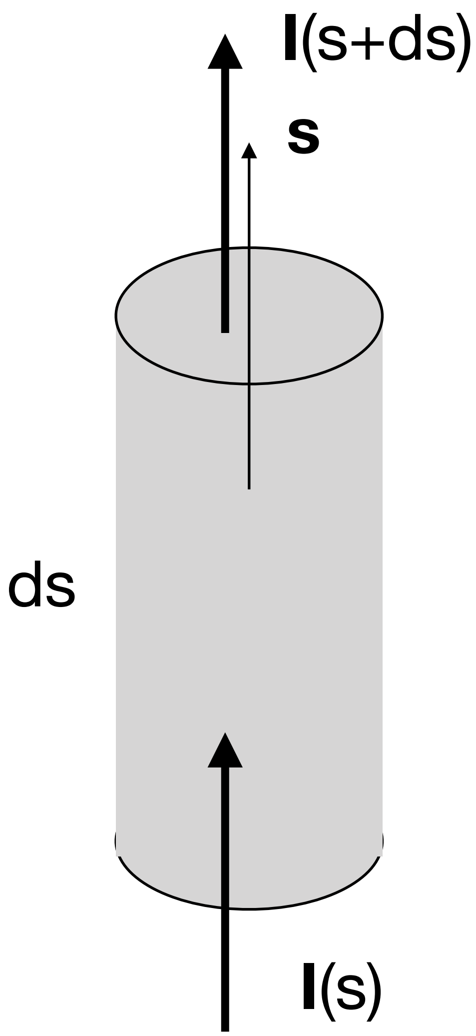
Plane parallel or stratified atmosphere

Eddington approximation for the IR emission : $I(z,\theta) = I_0(z) + I_1(z) \cos\theta$



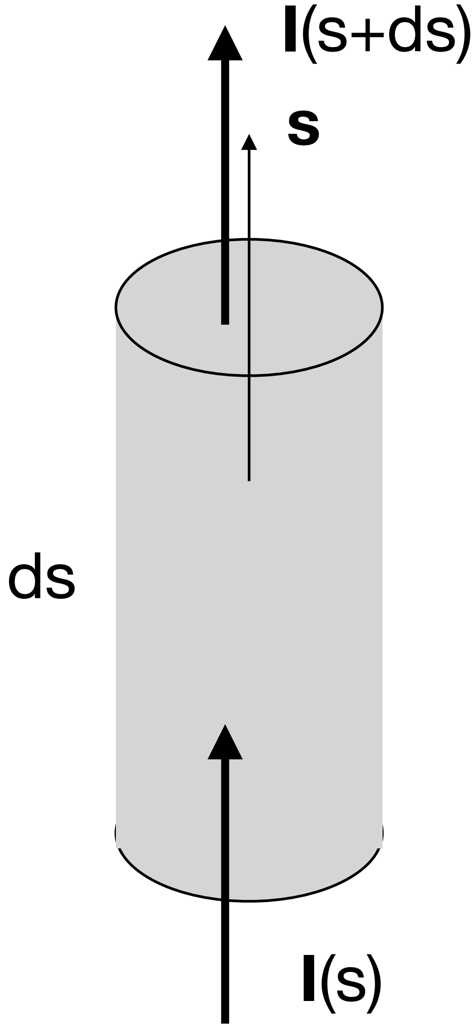


I(s+ds)
s
ds
I(s)



Extinction (absorption, scattering)

$$dI_{ext} = -\kappa_{ext}\ I\ ds$$



Extinction (absorption, scattering)

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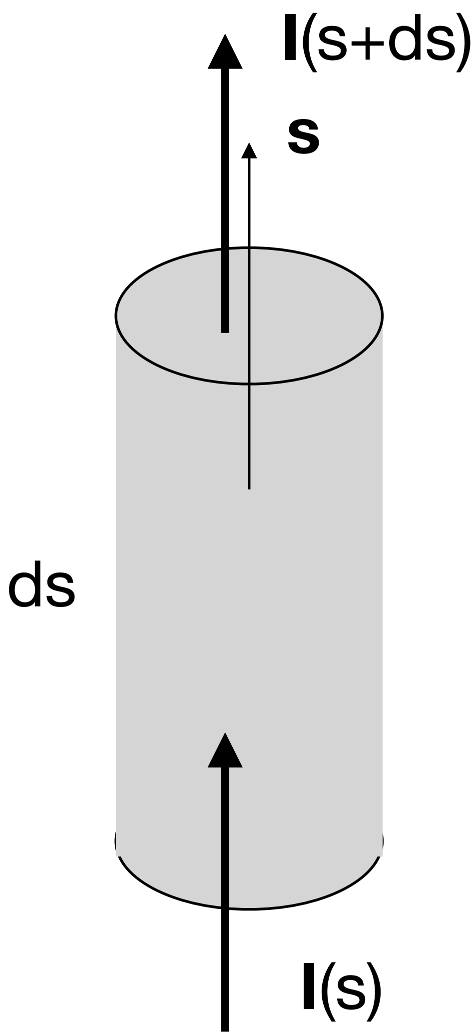
Augmentation (emission, scattering)

$$dI_{aug} = \kappa_{aug}\ J\ ds$$



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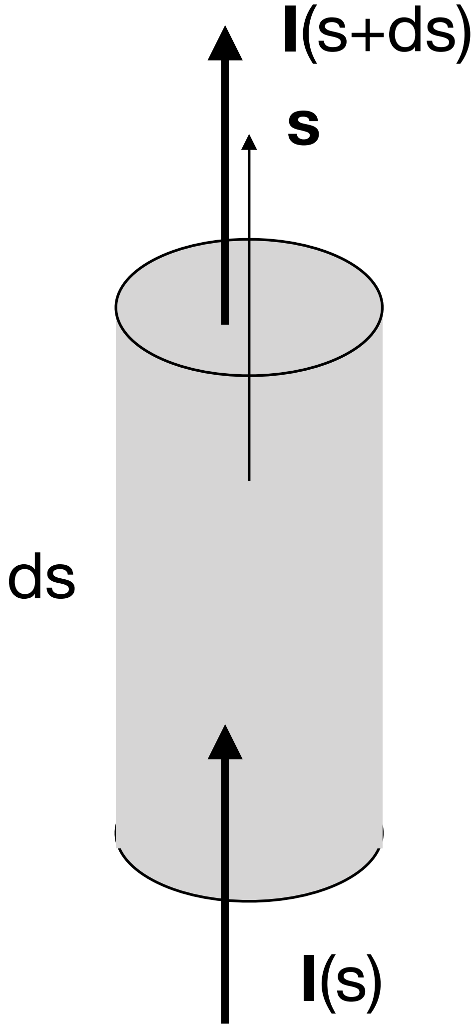
$$dI_{aug} = \kappa_{aug}\ J\ ds$$

If no scattering, emission by thermal radiation

$$dI_{aug} = \kappa_{ext}\frac{\sigma T^4}{\pi} = \kappa_{ext}B(T)$$



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$$\frac{dI}{ds} = \kappa_{ext}(-I + B)$$

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Optical path length (dimensionless) τ

$$d\tau = -\kappa_{ext}\ ds$$

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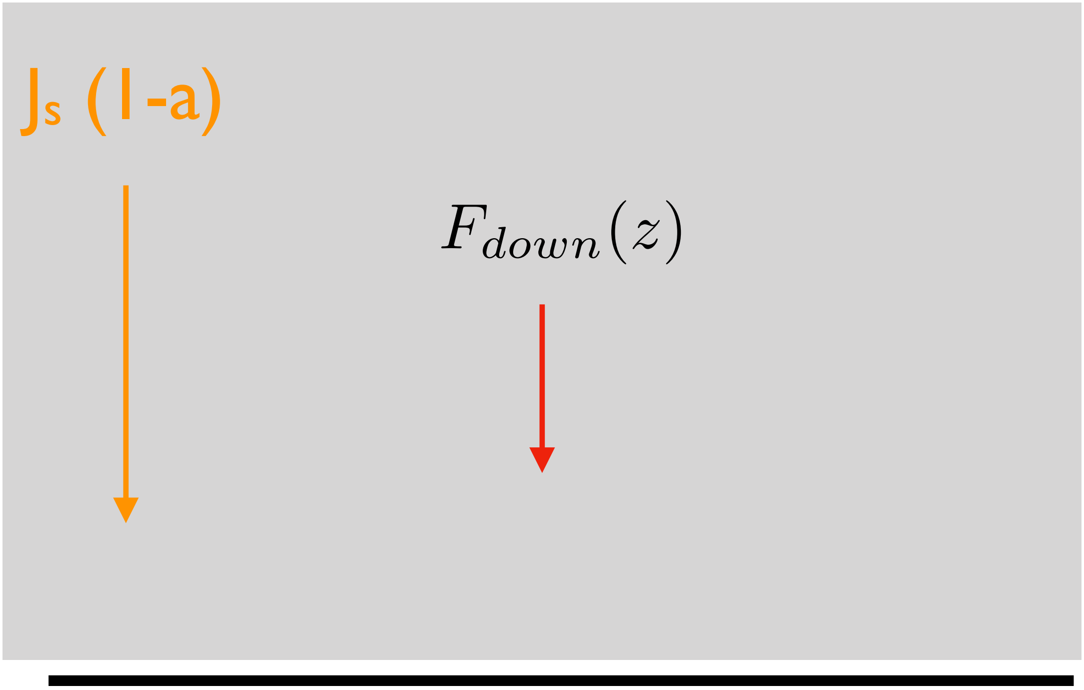
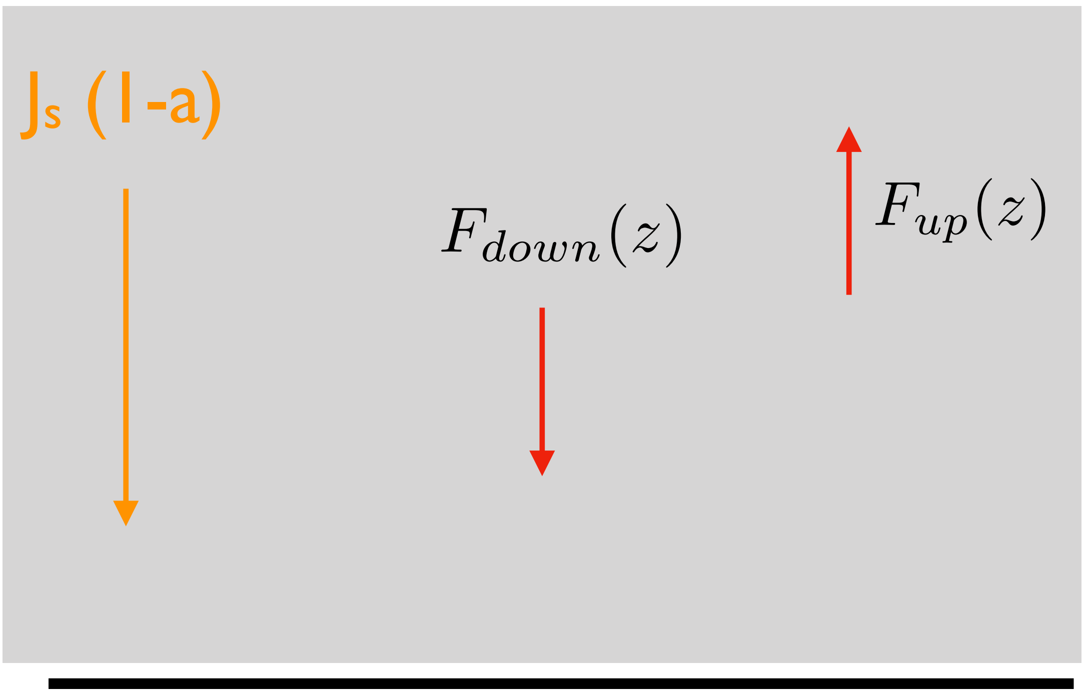
Optical path length (dimensionless) τ

$$d\tau = -\kappa_{ext}\ ds$$

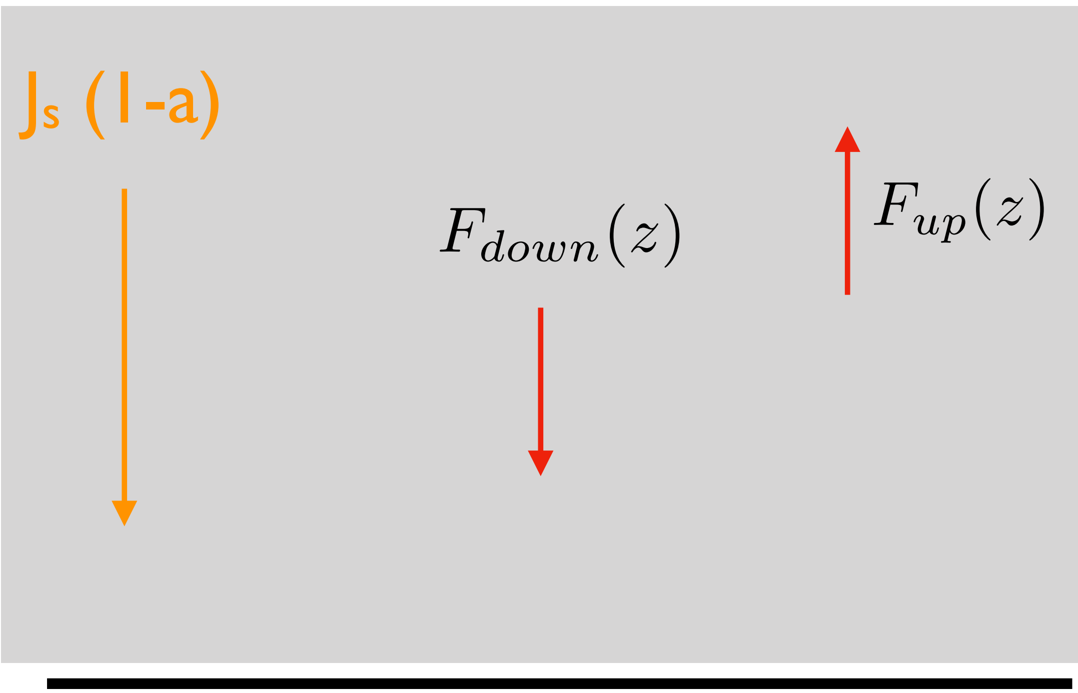
$$\frac{dI}{d\tau} = I - B$$


J_s (1-a)


Js (1-a)
$F_{down}(z)$

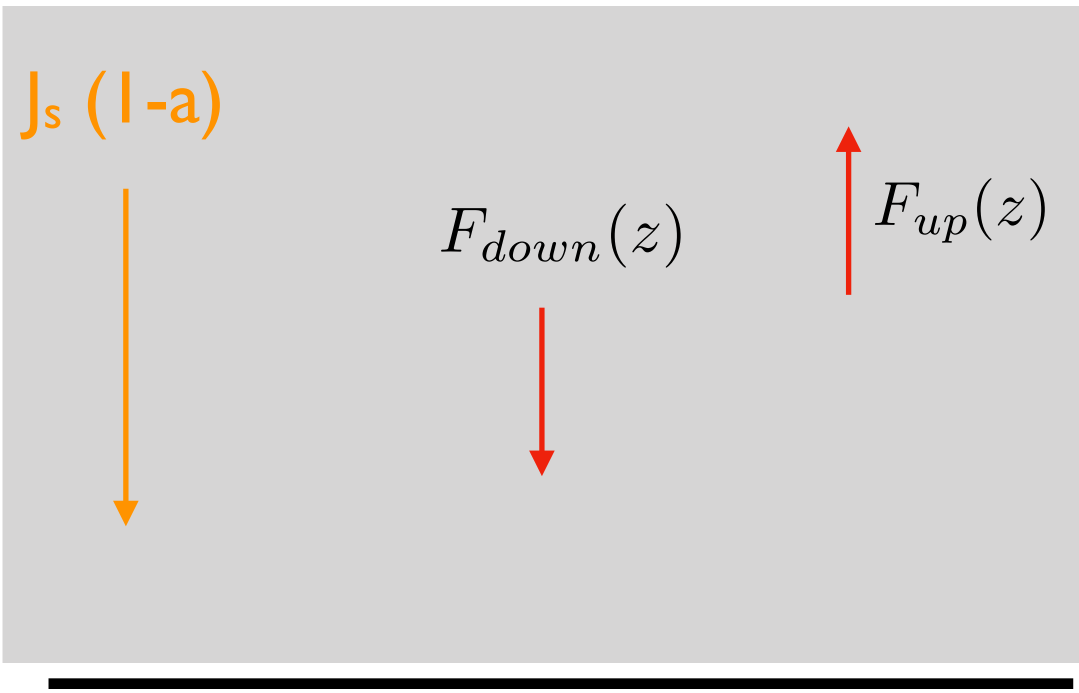


J_s (1-a)
$F_{down}(z)$
$F_{up}(z)$



$$\tau = \int_z^{\infty} \kappa_{ext} \; dz$$

Optical path from atmosphere top



Atmosphere top τ = 0

$$\tau = \int_z^\infty \kappa_{ext} \; dz$$

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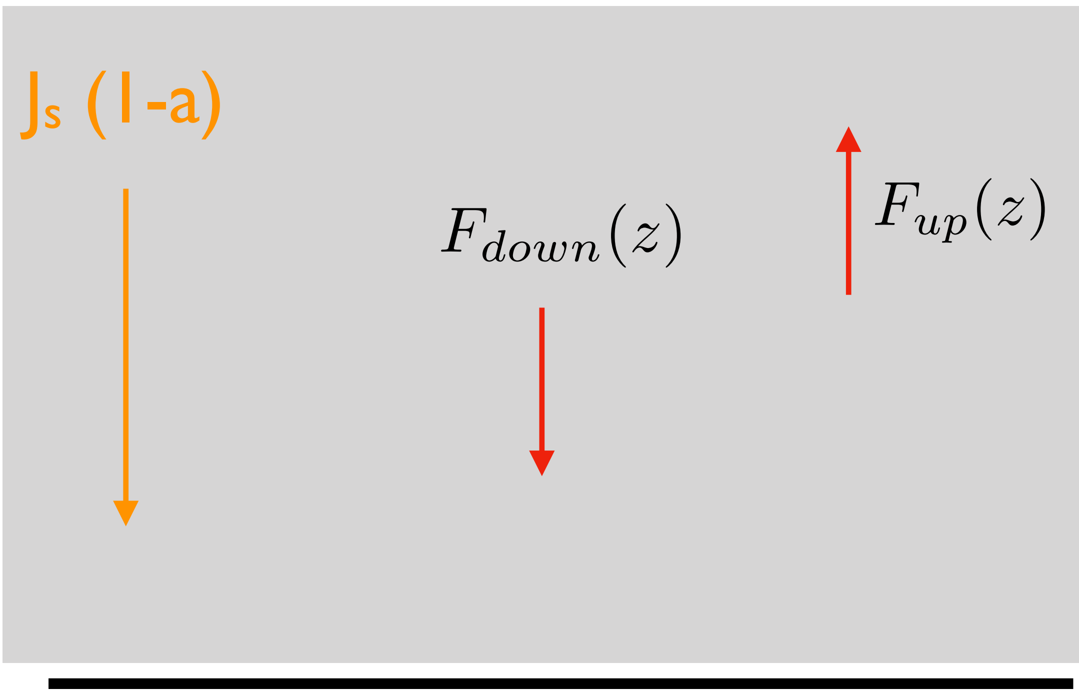

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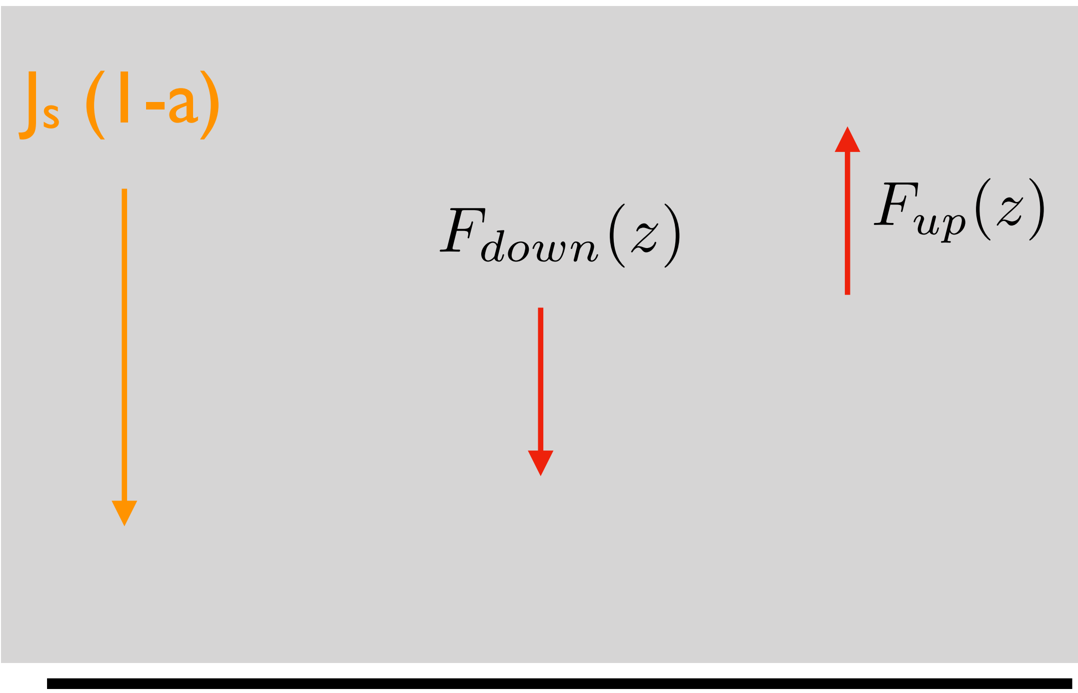

Atmosphere top $\tau = 0$

$$F_{down}(0) = 0$$

Ground $\tau = \tau^*$

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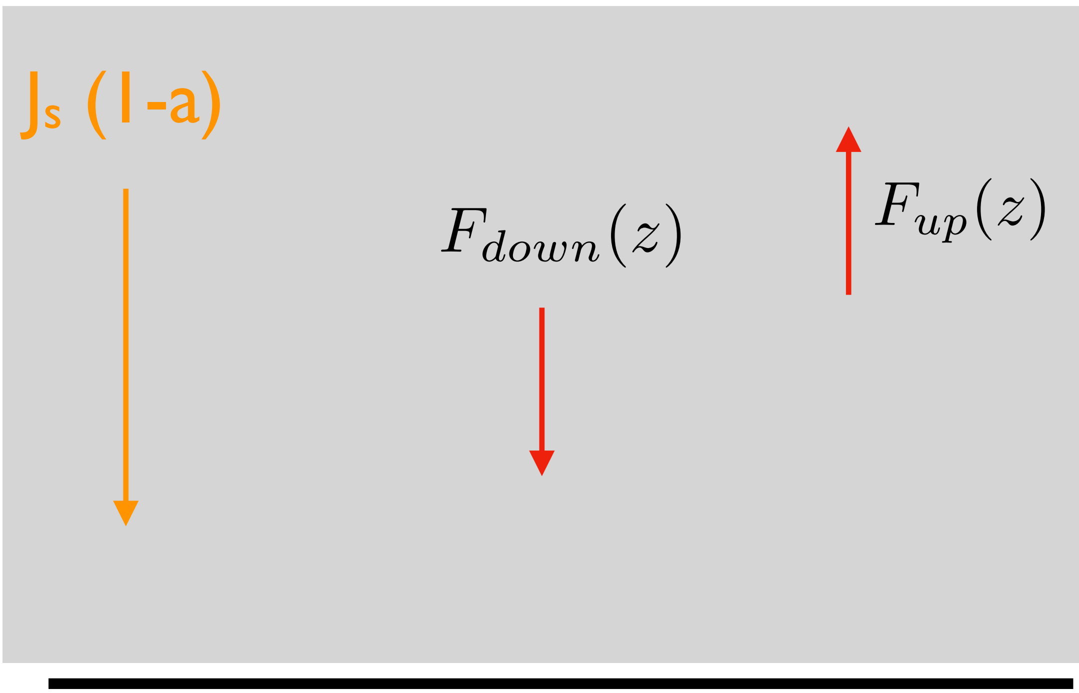
Optical path from atmosphere top



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Ground τ = τ*

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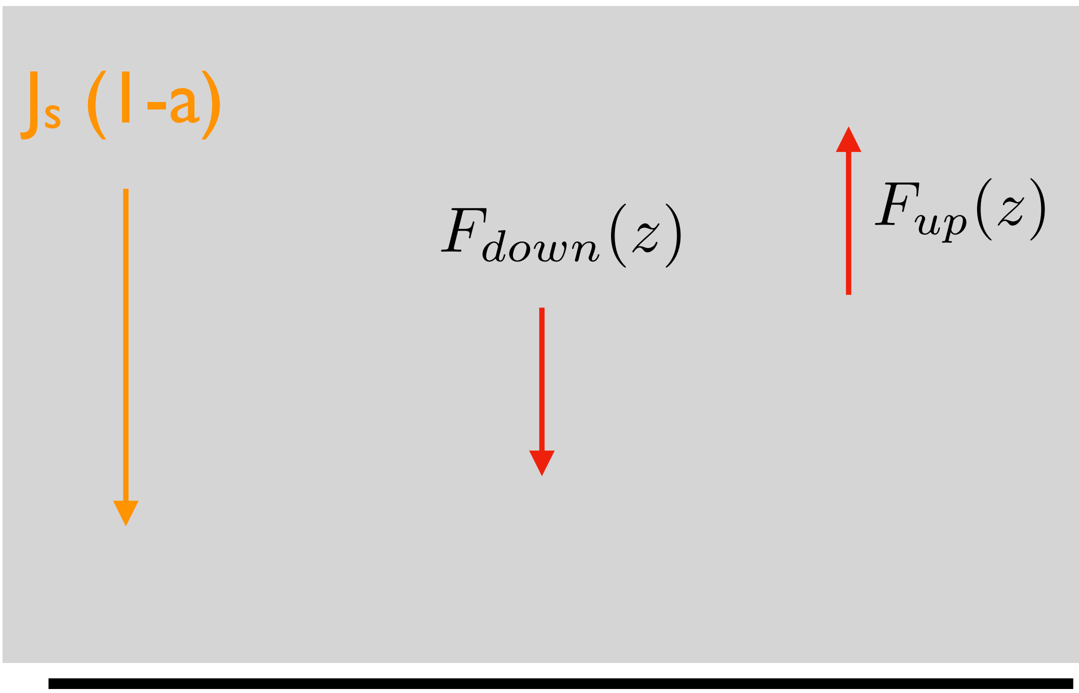
Optical path from atmosphere top

$$\tau^* = \int_0^\infty \kappa_{ext}\ dz$$

Optical depth of atmosphere



Atmosphere top τ = 0

$$F_{down}(0) = 0$$

Ground τ = τ*

Radiation going up :

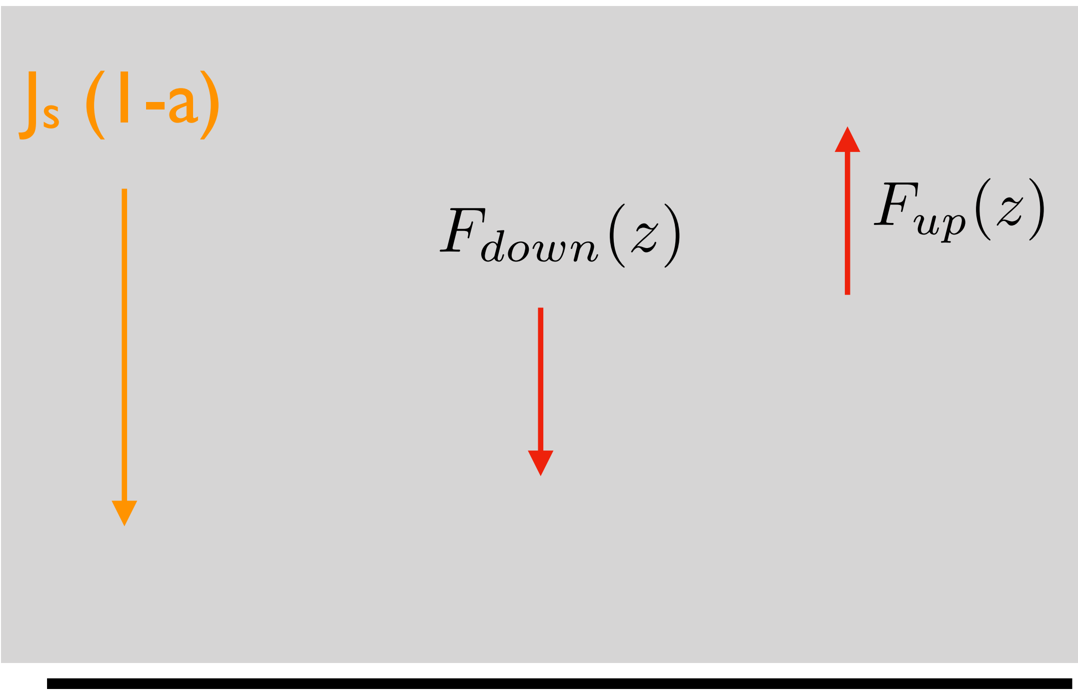
$$F_{up} = F_{up}(\tau^*) = \pi B(T_g)$$

$$\tau = \int_z^\infty \kappa_{ext}\ dz$$

Optical path from atmosphere top

$$\tau^* = \int_0^\infty \kappa_{ext}\ dz$$

Optical depth of atmosphere



Atmosphere top τ = 0

$F_{down}(0) = 0$

Ground τ = τ*

Radiation going up :

$$F_{up} = F_{up}(\tau^*) = \pi B(T_g)$$

Radiation going down :

$$J_s a + F_{down}(\tau^*)$$

$$\tau = \int_z^\infty \kappa_{ext}\ dz$$

Optical path from atmosphere top

$$\tau^* = \int_0^\infty \kappa_{ext}\ dz$$

Optical depth of atmosphere

$$\kappa_{ext} = \kappa_0 \exp(-z/H_w)$$

H_w = 1.6 km (water vapor)

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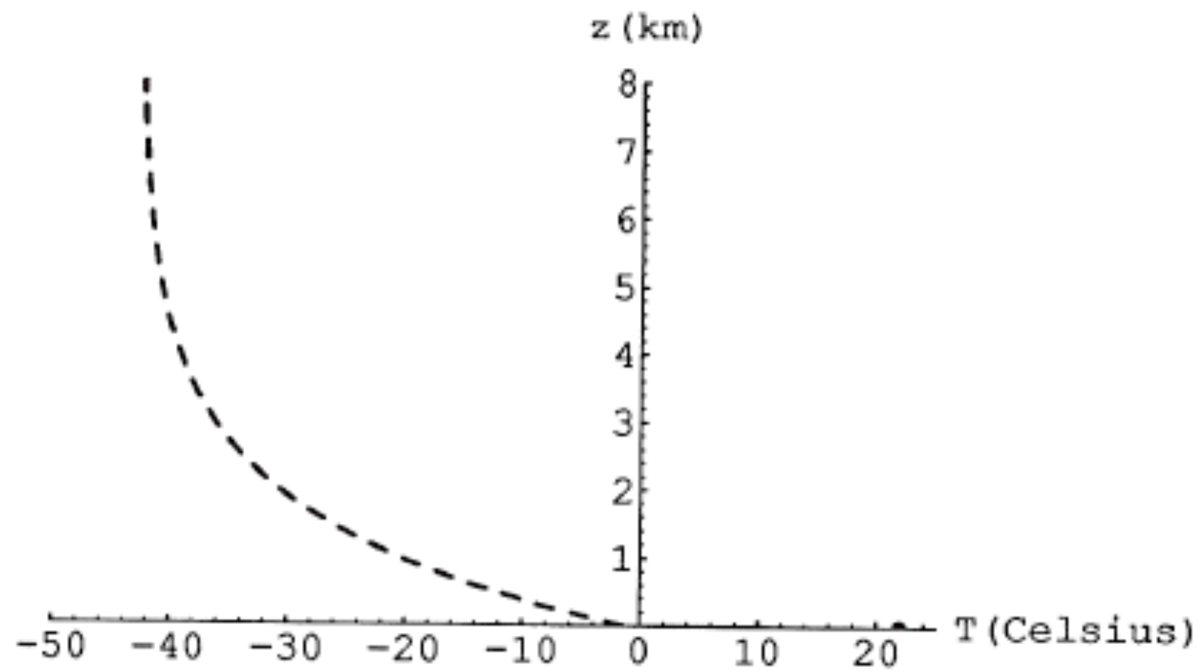
H_w = 1.6 km (water vapor)



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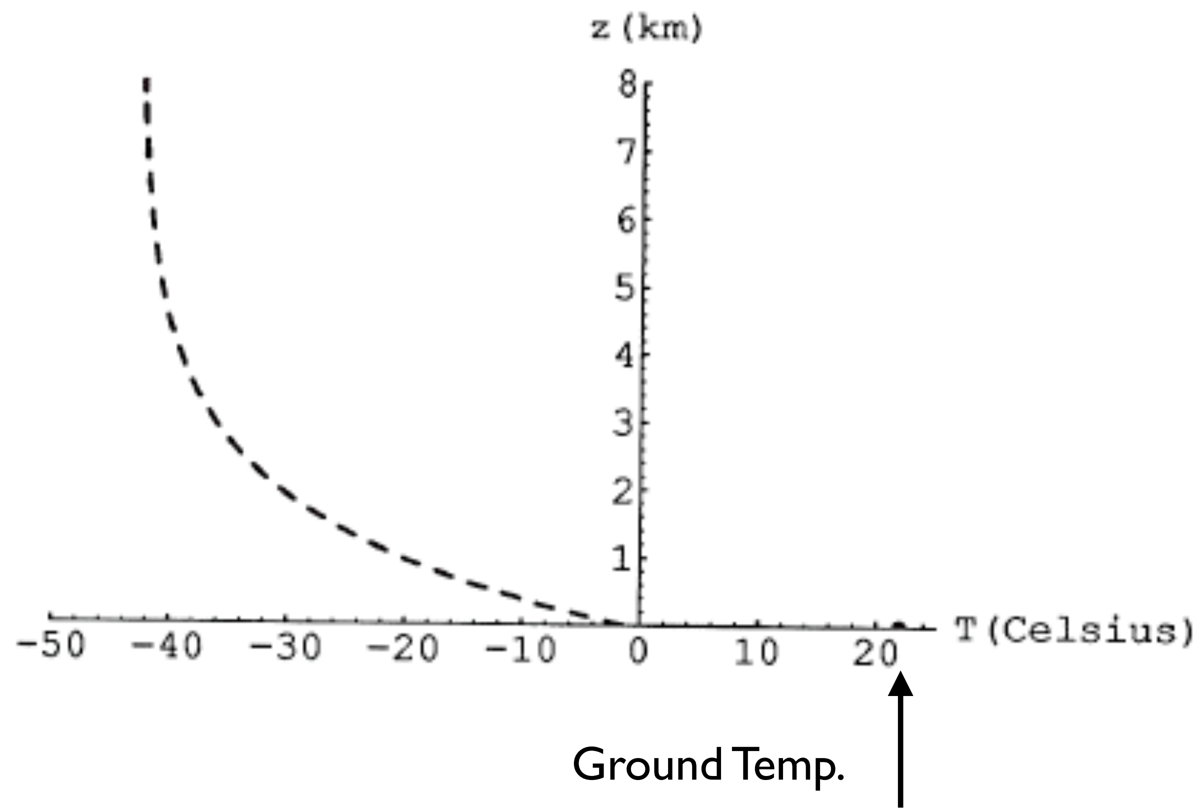
H_w = 1.6 km (water vapor)



$$\sigma T_g^4 = J_s(1-a)\left(1 + \frac{3\tau^*}{4}\right)$$

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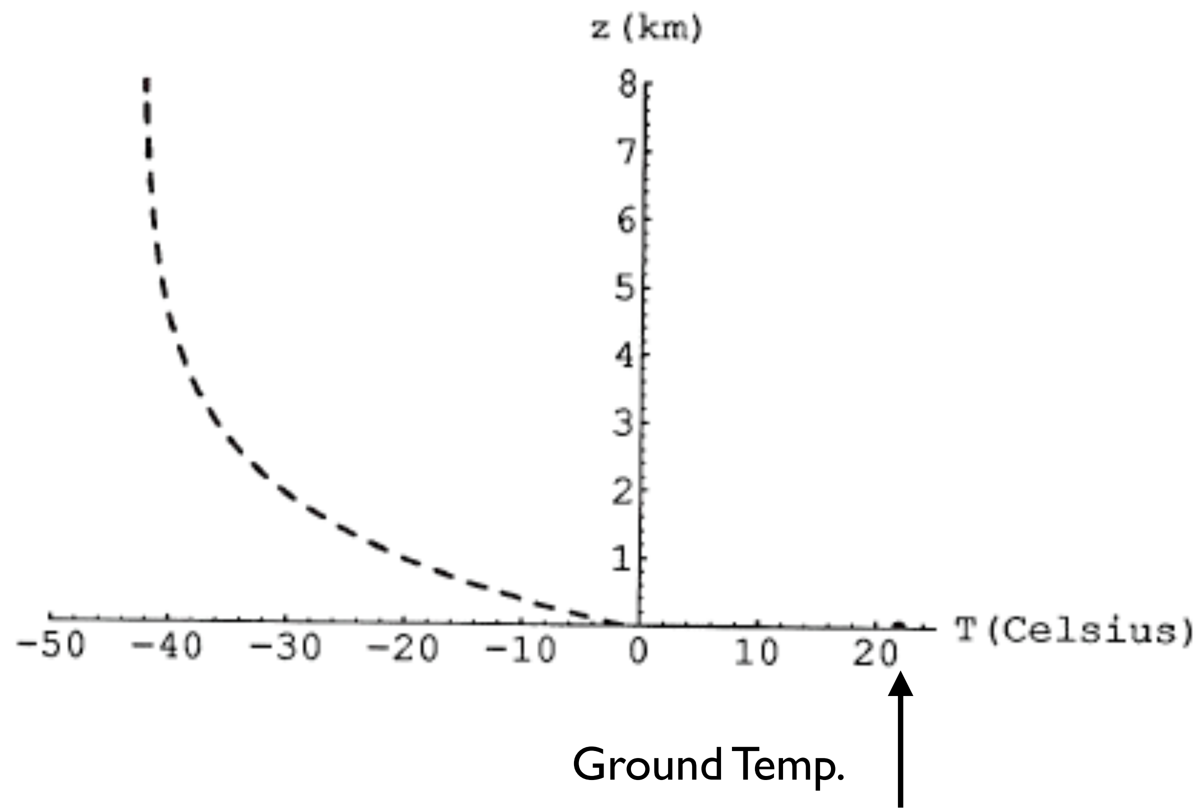
H_w = 1.6 km (water vapor)



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Optical depth of atmosphere

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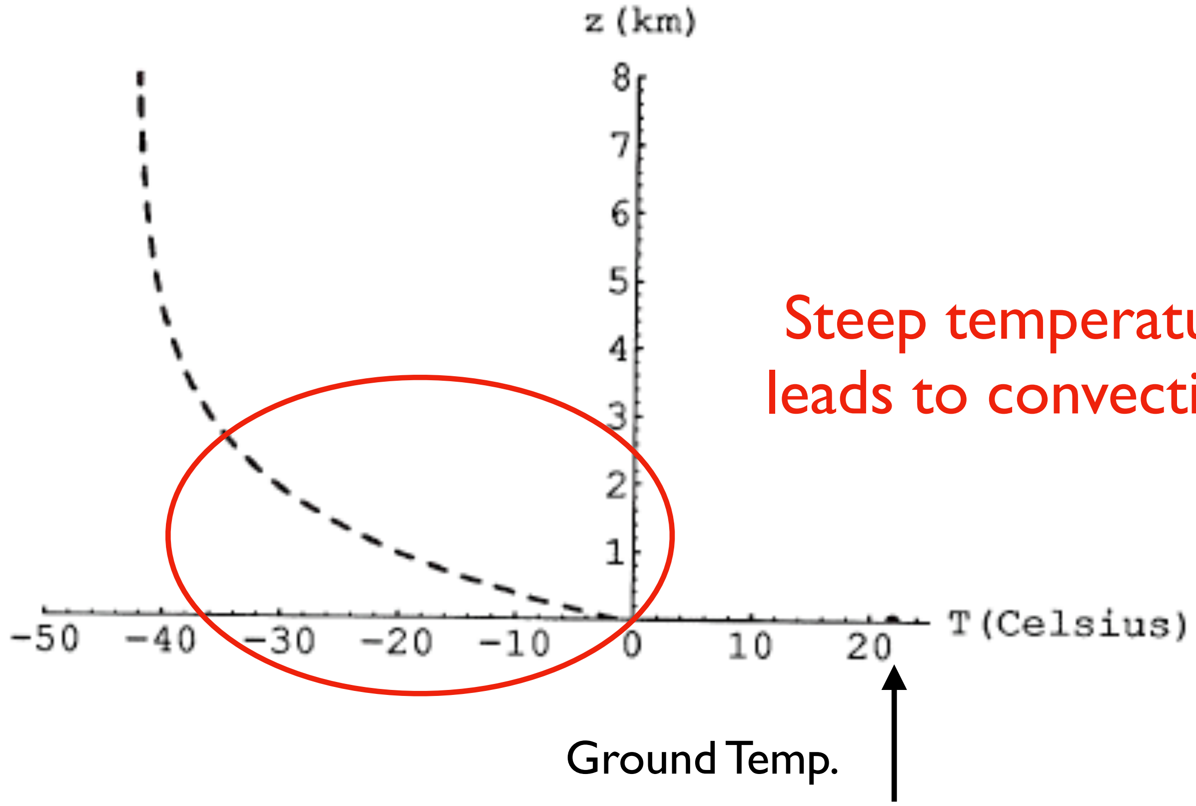
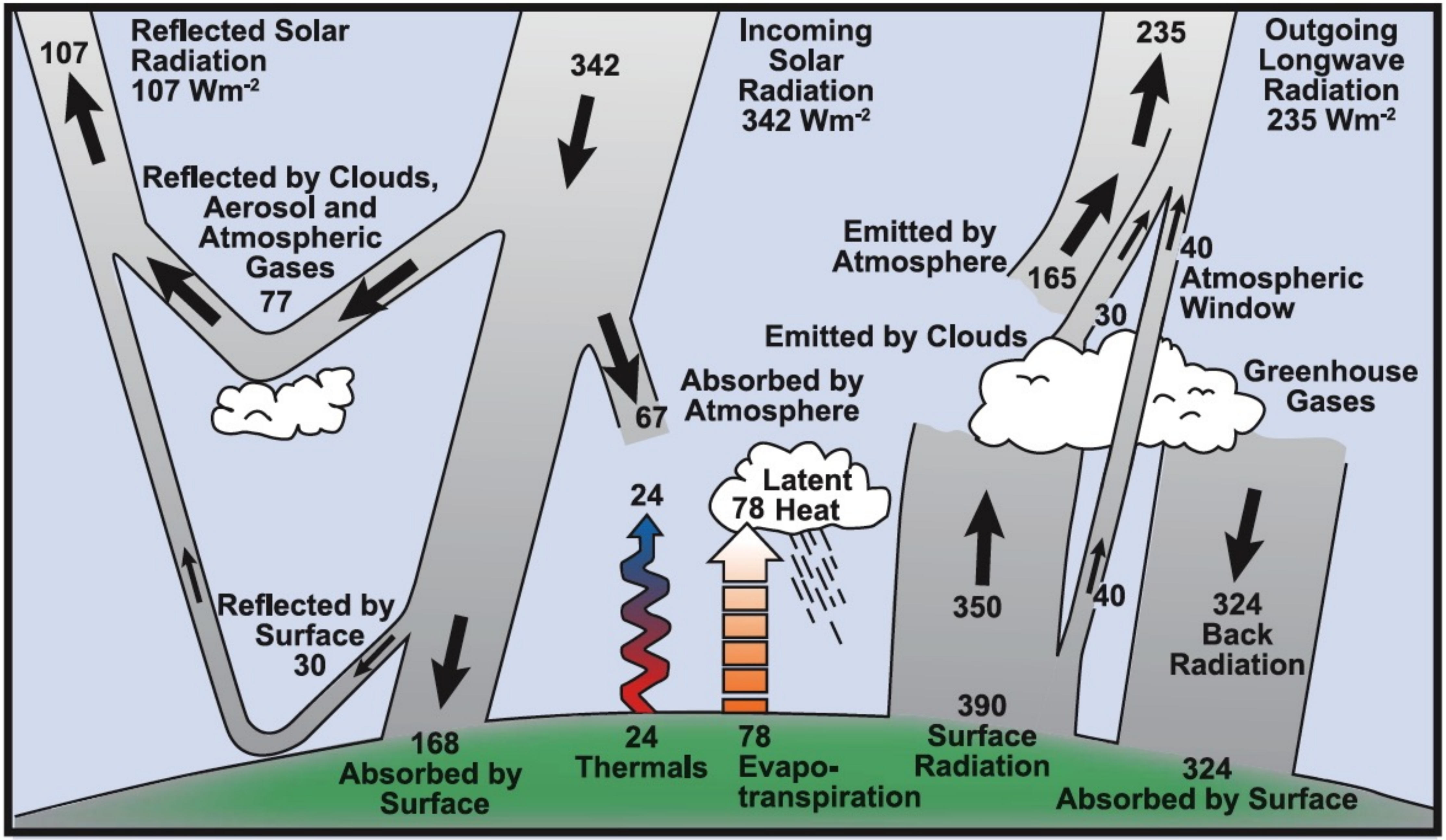


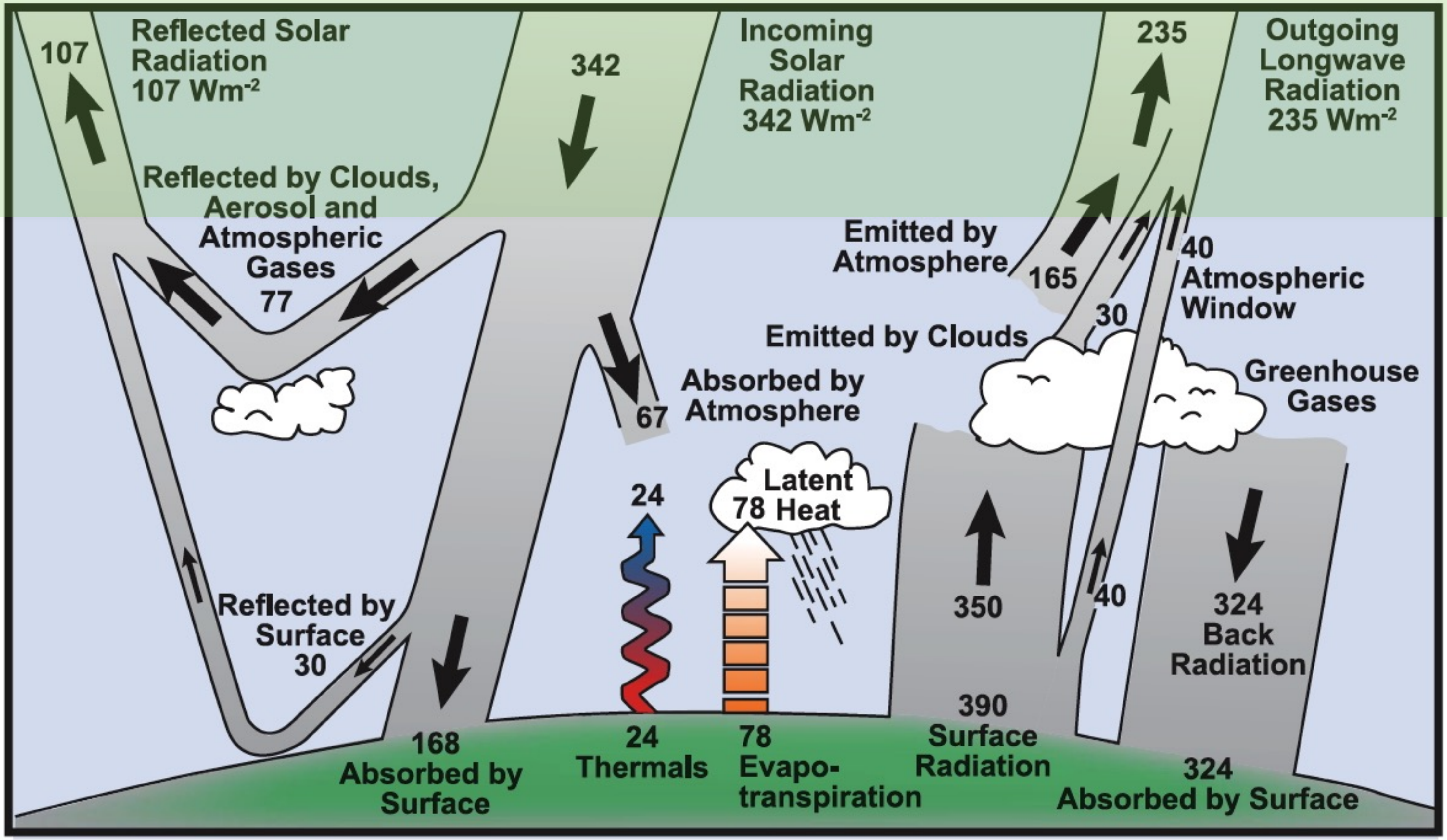
Steep temperature gradient leads to convective instability

$$\sigma T_g^4 = J_s (1-a) \left(1 + \frac{3\tau^*}{4} \right)$$

$$\tau^* = \int_0^\infty \kappa_{ext} \, dz$$

Optical depth of atmosphere


107
Reflected Solar Radiation 107 Wm-2
342
Incoming Solar Radiation 342 Wm-2
235
Outgoing Longwave Radiation 235 Wm-2
Reflected by Clouds, Aerosol and Atmospheric Gases
77
Emitted by Atmosphere
165
40
Atmospheric Window
30
Emitted by Clouds
Absorbed by Atmosphere
67
Greenhouse Gases
24
Latent
78 Heat
Reflected by Surface
30
350
40
324
Back Radiation
168
Absorbed by Surface
24
Thermals
78
Evapo-transpiration
390
Surface Radiation
324
Absorbed by Surface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Absorbed by Atmosphere
Greenhouse Gases
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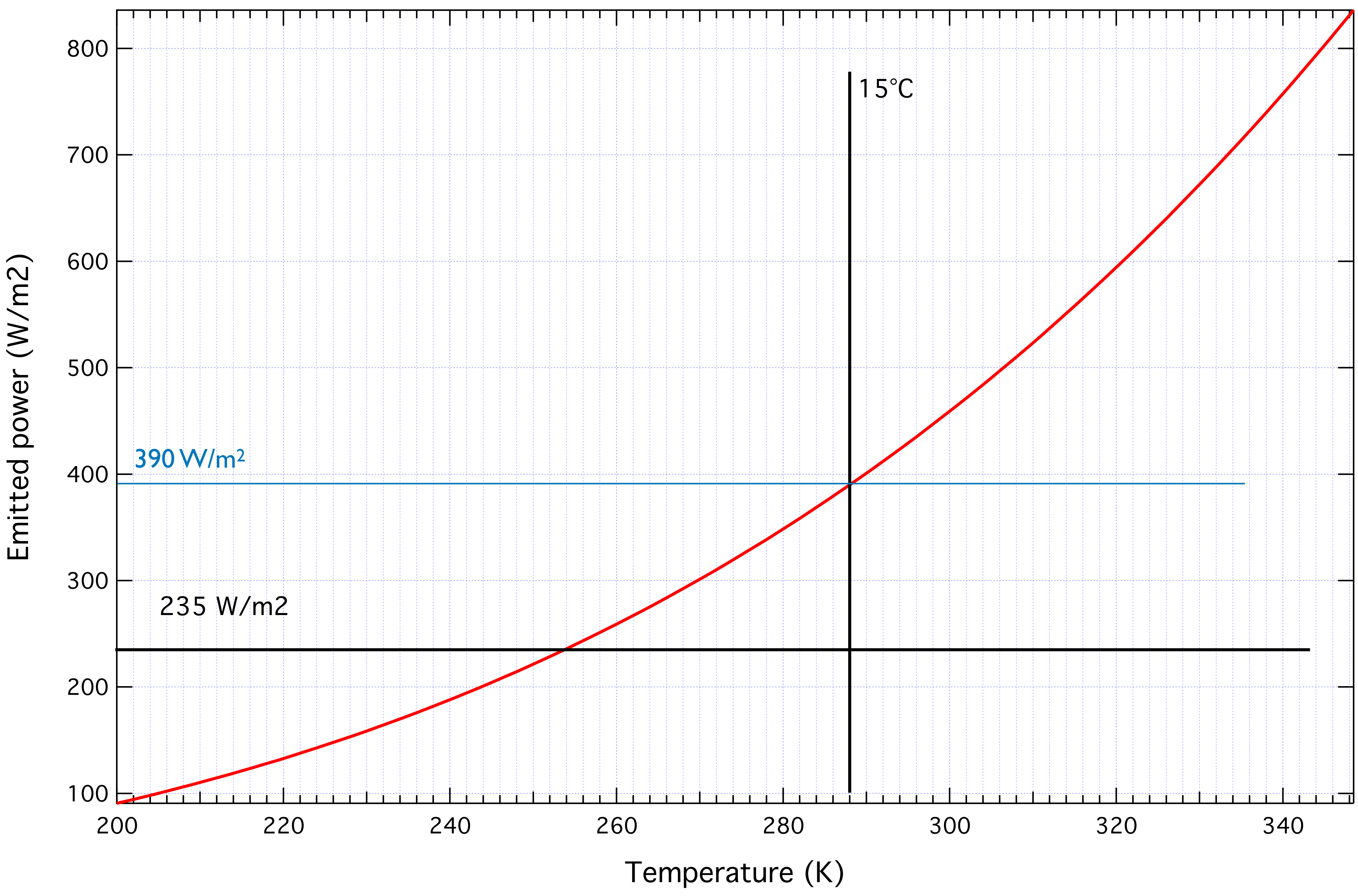
Latent 78 Heat
350
40
324 Back Radiation
Reflected by Surface 30
168 Absorbed by Surface
24 Thermals
78 Evapo-transpiration
390 Surface Radiation
324 Absorbed by Surface



Emitted power (W/m2)
800
700
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320
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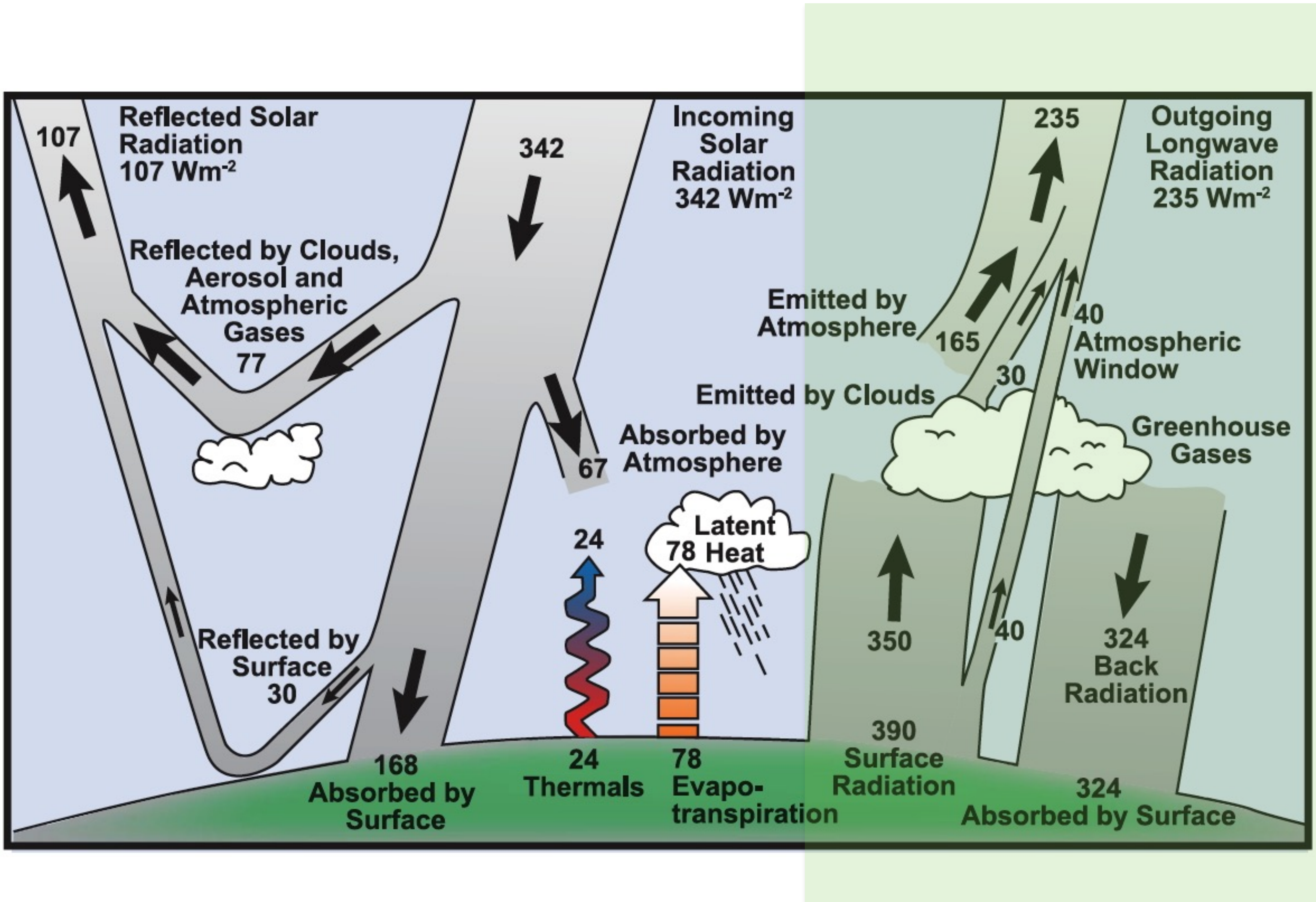
Temperature (K)
15°C
390 W/m2
235 W/m2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