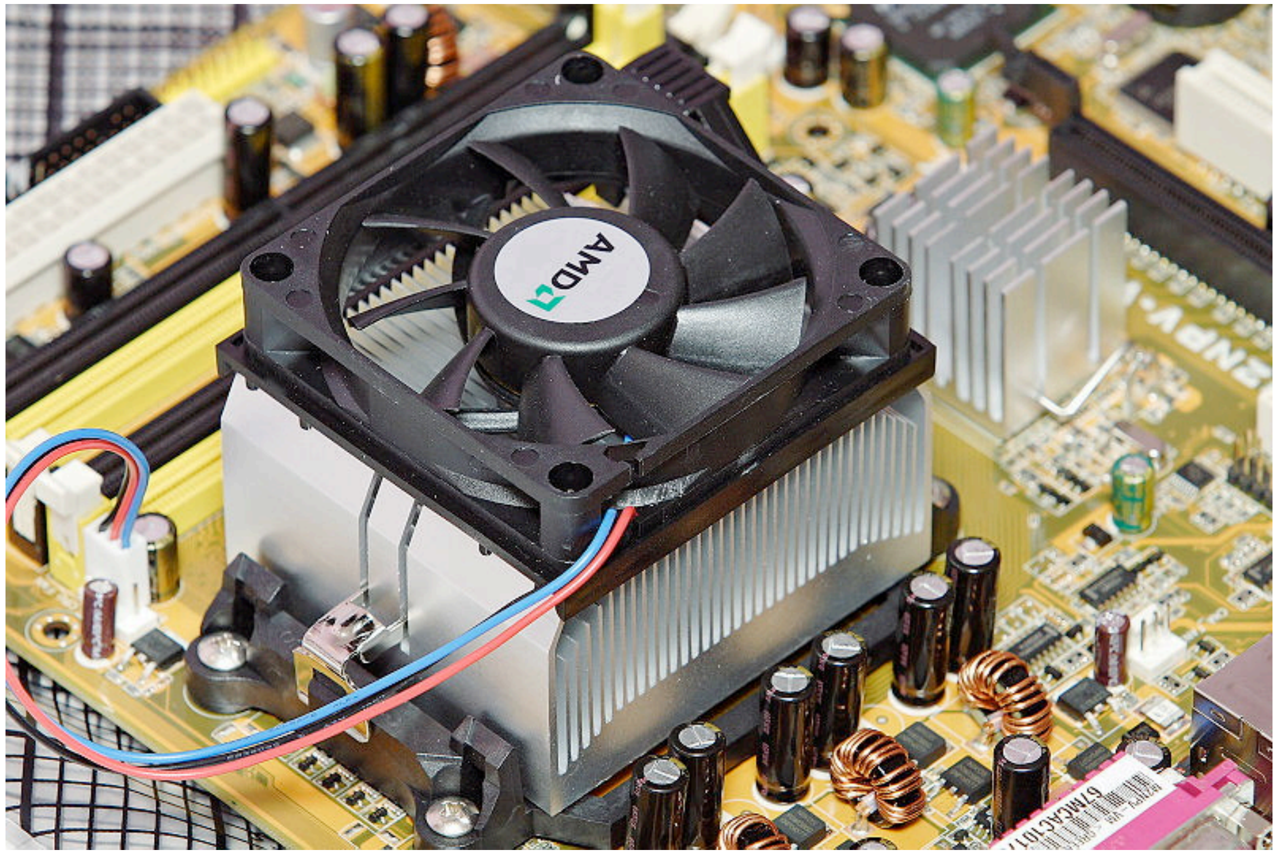
Geometrical effects in diffusion problems, the case of radiator fins



Power to dissipate : 50 W
Typical dimension : L= 3cm
Physical properties of air :
density 1 kg/m^3
specific heat 1000 J/kg.K
thermal conductivity 0.025 W/m.K

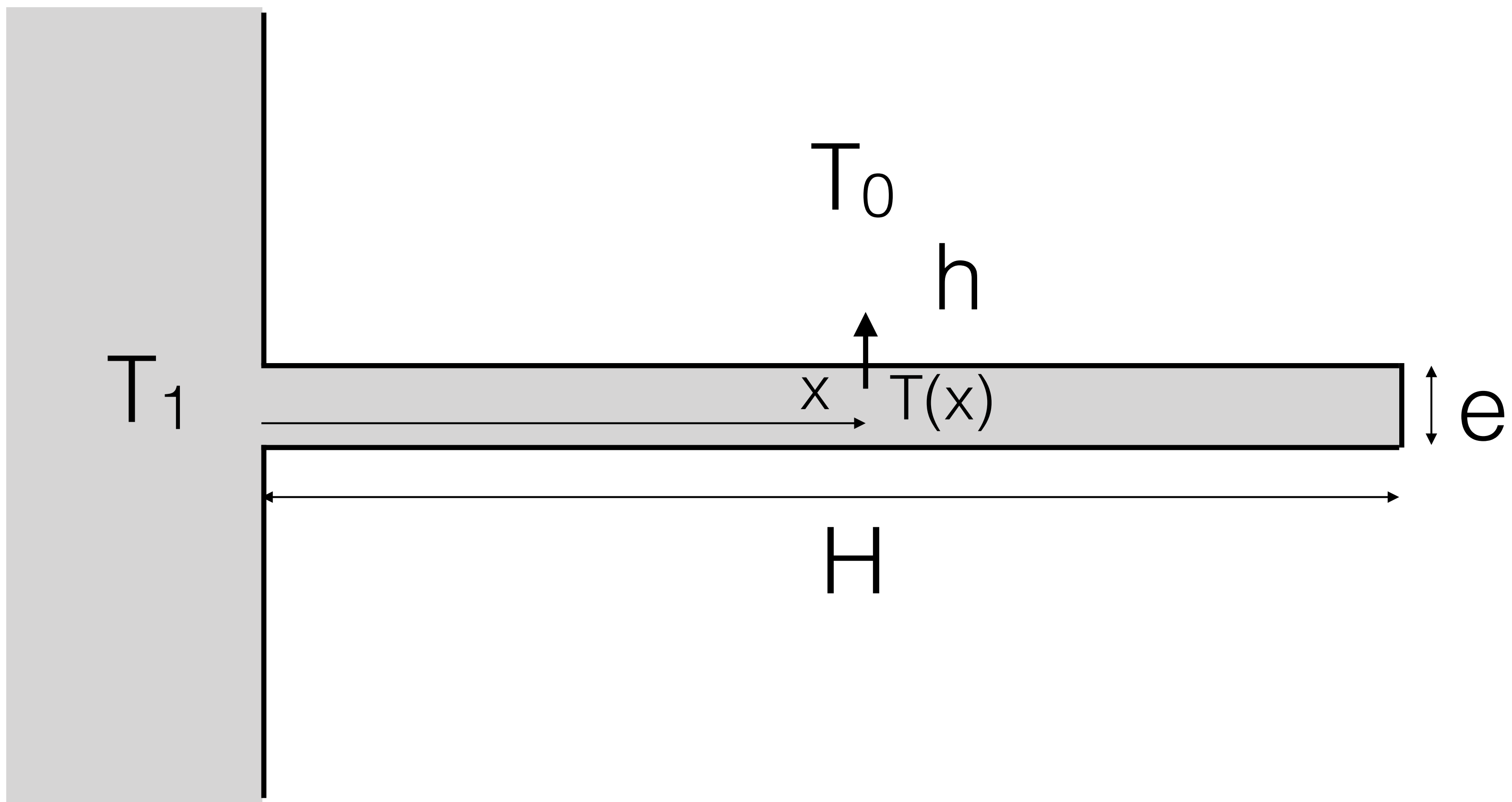
If we rely only on diffusion in air, how much power can we dissipate from the flat chip without fins?

How much power can we dissipate by adding a radiator with fins ?

Is there an optimum length for fins given their thickness e and the heat transfer coefficient *h* between the fins and the surrounding air ?

Physical properties of aluminum (material of the fins) :
thermal conductivity 240 W/m.K

What is the temperature distribution within the fin and the resulting heat flux to the air ?



What are the assumptions on the temperature field within the fin that can be done using the hypothesis e << H ?

Write the heat balance in an element of fin and derive an equation for T(x)

What are the boundary conditions for T(x) ?

Heat flux within the fin $$J_D = -\lambda_M \frac{dT}{dx}$$

Heat flux balances in an element of fin between x and x+dx :

$$J_D(x)\ e = J_D(x + dx)\ e + 2h\ [T(x) - T_0]\ dx$$

$$0 = \frac{dJ_D(x)}{dx}\ e + 2h\ [T(x) - T_0]$$

$$\lambda_M \frac{d^2T(x)}{dx^2}\ e = 2h\ [T(x) - T_0]$$

$$\frac{d^2T(x)}{dx^2} = \frac{2h}{e\lambda_M}\ [T(x) - T_0] = \frac{[T(x) - T_0]}{\ell^2}$$

$$T(x) - T_0 = A\exp(x/\ell) + B\exp(-x/\ell) \qquad \ell = \sqrt{\frac{e\lambda_M}{2h}}$$

Boundary conditions :

x=0 $$T(0) = T_1$$

x=H, assuming $H < \ell$ $$-\lambda_M \frac{dT}{dx}_{\,x=H} = h[T(H) - T_0]$$

x=H, assuming $H \gg \ell$ $$T(x)_{x\to\infty} \to T_0$$

$$T(x) - T_0 = (T_1 - T_0)\exp(-x/\ell)$$

Local heat flux to the air : $$J_L = 2h(T_1 - T_0)\exp(-x/\ell)$$

Total heat flux to the air : $$J_{total} = \int_0^H 2h(T_1 - T_0)\exp(-x/\ell)\ dx$$

$$J_{total} \approx \int_0^\infty 2h(T_1 - T_0)\exp(-x/\ell)\ dx = 2h\ell(T_1 - T_0)$$

$$J_{total} \approx \sqrt{2h\ e\ \lambda_M}(T_1 - T_0)$$